Assignment-1 ME-676A

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1 Problem-1(A)

DDSDDE for abaqus is the tangent stiffness matrix for Jauman rate of kirchoff stress $\underset{\sim}{\tau}^{\nabla}$

DDSDDE in voight notation used by ABAQUS is as follows:

DDSDDE= $\frac{1}{J}\frac{\partial\tau^{\nabla}}{\partial D\epsilon}$

$$\text{DDSDDE=}\begin{bmatrix} \lambda J^{-1}+\frac{2\mu B_{11}}{J} & \frac{\lambda}{J} & \frac{\lambda}{J} & \frac{\mu B_{12}}{J} & \frac{\mu B_{13}}{J} & 0 \\ - & \frac{\lambda+2\mu B_{22}}{J} & \frac{\lambda}{J} & \frac{\mu}{J}B_{21} & 0 & \frac{\mu}{J}B_{23} \\ - & -- & \frac{\lambda+2\mu B_{33}}{J} & 0 & \frac{\mu}{J}B_{31} & \frac{\mu}{J}B_{32} \\ - & -- & -- & \frac{\mu}{2J}(B_{11}+B_{22}) & \frac{\mu}{2J}B_{23} & \frac{\mu}{2J}B_{13} \\ - & -- & -- & -- & \frac{\mu}{2J}(B_{11}+B_{33}) & \frac{\mu}{2J}B_{12} \\ \text{sym} & -- & -- & -- & -- & \frac{\mu}{2J}(B_{22}+B_{33}) \end{bmatrix}$$

2 Problem-1(B)

To update stress at every step is done as follows:

1) At the end of each step udation in deformation tensor is done by abaqus

2) Then we are going to update BB matrix which is F^TF

$\underset{\sim}{B} = \underset{\sim}{F}^T\underset{\sim}{F}$

3) Hence stress will be updated as follows:

$\sigma_{ij} = \frac{\mu}{J}B_{ij} + \frac{(\lambda lnJ-\mu)}{J}\delta_{ij}$

above equation in voight notation:

for i=1,2,3

$\sigma_i = \frac{\mu}{J}B_i + \frac{(\lambda lnJ-\mu)}{J}$

and for i=4,5,6

$\sigma_i = \frac{\mu}{J}B_i$

3 Problem-1(C)

Complete UMAT code for user defined material hyperelastic Neo-Hookean materrial in as follows with two elastic constant μ and λ:

```
      SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
     1 RPL,DDSDDT,DRPLDE,DRPLDT,
     2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
     3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
     4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CMNAME
      DIMENSION STRESS(NTENS),STATEV(NSTATV),
     1 DDSDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
     2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
     3 PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)
C DEFINITIONS
C     ----------------------------------------------
C      ROMIL KADIA(16105045)
C      ANKUR MAURYA(13124)
C      ROHIT KUMAVAT(13587)
C     ----------------------------------------------
C GENERATING RIGHT CAUCHY-GREEN TENSOR:
      DIMENSION BB(6)
      PARAMETER(ZERO=0.0D0, ONE=1.0D0, TWO=2.0D0)
      MU=PROPS(1)
      LAMB=PROPS(2)
C     ----------------------------------------------
C XJ IS DETERMINENT OF (F)
      XJ=DFGRD1(1, 1)*DFGRD1(2, 2)*DFGRD1(3, 3)
     1 -DFGRD1(1, 2)*DFGRD1(2, 1)*DFGRD1(3, 3)
     2 +DFGRD1(1, 2)*DFGRD1(2, 3)*DFGRD1(3, 1)
     3 +DFGRD1(1, 3)*DFGRD1(3, 2)*DFGRD1(2, 1)
     4 -DFGRD1(1, 3)*DFGRD1(3, 1)*DFGRD1(2, 2)
     5 -DFGRD1(2, 3)*DFGRD1(3, 2)*DFGRD1(1, 1)
C     ----------------------------------------------
      BB(1)=DFGRD1(1, 1)**2+DFGRD1(1, 2)**2+DFGRD1(1, 3)**2
      BB(2)=DFGRD1(2, 1)**2+DFGRD1(2, 2)**2+DFGRD1(2, 3)**2
```

```
      BB(3)=DFGRD1(3, 1)**2+DFGRD1(3, 2)**2+DFGRD1(3, 3)**2
      BB(4)=DFGRD1(1, 1)*DFGRD1(2, 1)+DFGRD1(1, 2)*DFGRD1(2, 2)
     1 +DFGRD1(1, 3)*DFGRD1(2, 3)
      BB(5)=DFGRD1(1, 1)*DFGRD1(3, 1)+DFGRD1(1, 2)*DFGRD1(3, 2)
     1 +DFGRD1(1, 3)*DFGRD1(3, 3)
      BB(6)=DFGRD1(2, 1)*DFGRD1(3, 1)+DFGRD1(2, 2)*DFGRD1(3, 2)
     1 +DFGRD1(2, 3)*DFGRD1(3, 3)
C         ---------------------------------------------------
C STRESS UPDATION
      DO I=1, 3
          STRESS(I)=BB(I)*MU/XJ+((LAMB*LOG(XJ)-MU)/XJ)
      END DO
      DO I=4, 6
          STRESS(I)=BB(I)*MU/XJ
      END DO
C         ---------------------------------------------------
      DO I=1, 3
          DDSDDE(I, I)=(LAMB+TWO*MU*BB(I))/XJ
      END DO
      DDSDDE(1, 2)=LAMB/XJ
      DDSDDE(1, 3)=LAMB/XJ
      DDSDDE(2, 3)=LAMB/XJ
      DDSDDE(1, 6)=ZERO
      DDSDDE(2, 5)=ZERO
      DDSDDE(3, 4)=ZERO
      DDSDDE(1, 4)=BB(4)*MU/XJ
      DDSDDE(2, 4)=BB(4)*MU/XJ
      DDSDDE(1, 5)=BB(5)*MU/XJ
      DDSDDE(3, 5)=BB(5)*MU/XJ
      DDSDDE(2, 6)=BB(6)*MU/XJ
      DDSDDE(3, 6)=BB(6)*MU/XJ
      DDSDDE(4, 5)=BB(6)*MU/(TWO*XJ)
      DDSDDE(4, 6)=BB(5)*MU/(TWO*XJ)
      DDSDDE(5, 6)=BB(4)*MU/(TWO*XJ)
      DDSDDE(4, 4)=(BB(1)+BB(2))*MU/(TWO*XJ)
      DDSDDE(5, 5)=(BB(1)+BB(3))*MU/(TWO*XJ)
      DDSDDE(6, 6)=(BB(2)+BB(3))*MU/(TWO*XJ)
      DO I=1, 6
          DO J=1, I-1
              DDSDDE(I, J)=DDSDDE(J, I)
          END DO
      END DO
      RETURN
      END
```

4 Problem-2

Consider Neo-Hookean material as incompressible then,

J=1

$\therefore \lambda_1\lambda_2\lambda_3 = 1$

and for the uniaxial case

$\lambda_2 = \lambda_3$

and, $\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}}$

Calculating $\underset{\sim}{B}$

assuming that deformation gradient is in eigen basis

$$\therefore [F] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

and $\underset{\sim}{B} = \underset{\sim}{F}\underset{\sim}{F}^T$

$$\therefore \text{[B]} = \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix}$$

<u>Modefiying expression of energy density:</u>

for $J = 1$

$\psi_0 = \frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$

because, I_1 for $\underset{\sim}{C}$ is same as I_1 for $\underset{\sim}{B}$ as both are diagonal matrix and $lnJ = 0$

But, $\lambda_2 = \lambda_3$

$\therefore \psi_0 = \frac{\mu}{2}(\lambda_1^2 + 2\lambda_2^2 - 3)$

$= \frac{\mu}{2}(\lambda_1^2 + \frac{2}{\lambda_1} - 3)$

Now, for incompressible neo-hookean material Principal cauchy stress is given by:

$\sigma_i = \frac{\lambda_i}{\lambda_1\lambda_2\lambda_3}\frac{\partial\psi_0}{\partial\lambda_i} + p$

where, p is undetermined pressure

And, $\frac{\partial\psi_0}{\partial\lambda_1} = \frac{\mu}{2}(\lambda_1^2 - \frac{2}{\lambda_1^2})$

$\frac{\partial\psi_0}{\partial\lambda_2} = \frac{\partial\psi_0}{\partial\lambda_3} = 0$

Now, in voight notation

$\sigma_1 = \lambda_1 \frac{\partial\psi_0}{\partial\lambda_1} + p$

$\therefore \sigma_1 = \mu(\lambda_1^2 - \frac{1}{\lambda_1}) + p$

And $\sigma_2 = \sigma_3 = p$

Now, $\sigma_1 - \sigma_2 = \lambda_1 \frac{\partial\psi_0}{\partial\lambda_1}$

But, as per given data $\sigma_2 = \sigma_3 = 0$

Hence, $\sigma_1 = \lambda_1 \frac{\partial\psi_0}{\partial\lambda_1}$

Final expression for Principal Stress in e_1 direction i.e. σ_{11}

$\sigma_1 = \mu(\lambda_1^2 - \frac{1}{\lambda_1})$

5 Problem-3(A)

*.inp file for abaqus to run above UMAT code is as follows:

*inp file for case $\mu = 1.50$ and $\lambda = 1100$, as per fiven data

```
*Heading
** Job name: neo3 Model name: Model-1
** Generated by: Abaqus/CAE 6.13-4
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-1
*Node
      1,          -5.,          -5.,           1.
      2,          -5.,          -4.,           1.
      3,          -5.,          -5.,           0.
      4,          -5.,          -4.,           0.
      5,          -4.,          -5.,           1.
      6,          -4.,          -4.,           1.
      7,          -4.,          -5.,           0.
      8,          -4.,          -4.,           0.
*Element, type=C3D8
1, 5, 6, 8, 7, 1, 2, 4, 3
*Nset, nset=Set-1, generate
 1,  8,  1
*Elset, elset=Set-1
 1,
```

```
** Section: Section-1
*Solid Section, elset=Set-1, material=Material-1
,
*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part-1-1, part=Part-1
*End Instance
**
*Nset, nset=Set-1, instance=Part-1-1
 2,
*Nset, nset=Set-2, instance=Part-1-1
 6,
*Nset, nset=Set-3, instance=Part-1-1
 4,
*Nset, nset=Set-4, instance=Part-1-1
 8,
*Nset, nset=Set-5, instance=Part-1-1
 5,
*Nset, nset=Set-6, instance=Part-1-1
 7,
*Nset, nset=Set-7, instance=Part-1-1
 8,
*Nset, nset=Set-8, instance=Part-1-1
 1,
*Nset, nset=Set-9, instance=Part-1-1
 2,
*Nset, nset=Set-10, instance=Part-1-1
 4,
*Nset, nset=Set-11, instance=Part-1-1
 3,
*End Assembly
**
** MATERIALS
**
*Material, name=Material-1
*User Material, constants=2
 1.5,1100.
** ----------------------------------------------------------------
**
** STEP: Step-1
**
*Step, name=Step-1, nlgeom=YES
*Static
20., 20., 0.0002, 20.
**
** BOUNDARY CONDITIONS
```

```
**
** Name: BC-1 Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
Set-1, 2, 2, 1.
** Name: BC-2 Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
Set-2, 2, 2, 1.
** Name: BC-3 Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
Set-3, 2, 2, 1.
** Name: BC-4 Type: Velocity/Angular velocity
*Boundary, type=VELOCITY
Set-4, 2, 2, 1.
** Name: BC-5 Type: Displacement/Rotation
*Boundary
Set-5, 2, 2
** Name: BC-6 Type: Displacement/Rotation
*Boundary
Set-6, 2, 2
Set-6, 3, 3
** Name: BC-7 Type: Displacement/Rotation
*Boundary
Set-7, 3, 3
** Name: BC-8 Type: Displacement/Rotation
*Boundary
Set-8, 1, 1
Set-8, 2, 2
** Name: BC-9 Type: Displacement/Rotation
*Boundary
Set-9, 1, 1
** Name: BC-10 Type: Displacement/Rotation
*Boundary
Set-10, 1, 1
Set-10, 3, 3
** Name: BC-11 Type: Displacement/Rotation
*Boundary
Set-11, 1, 1
Set-11, 2, 2
Set-11, 3, 3
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
EE, LE, PE, PEEQ, PEMAG, S
```

```
*Contact Output
CDISP, CSTRESS
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
```

To plot graph comparing the abaqus results and analytical results(for incompressible case) are obtained as follows:

1) Take data for principal stress σ_{11} and principal strain E_{11}

2) using the following expression for calculating primary stretch λ_1

$\lambda_1 = \sqrt{1 + 2 * E_{11}}$

3) analytical expression for stress stretch is as follows:

$\sigma_1 = \mu(\lambda_1^2 - \frac{1}{\lambda_1})$

4) from step-1,2 and step-3 we will get abaqus result plot and analytical result(for incompressible case) plot respectively using MatLab:

```
clear all;
clc;
yy=[0 4.61548 11.9563 22.3358 43.7058 88.888 186.263 398.98 440.571];
xx=[1 1.593255 1.846082 2.003516 2.161809 2.31815 2.470736 2.61858 2.637441];
plot(xx,yy,'-*','linewidth',1);
xlabel('Primary Stretch','FontSize',18,'Color','r')
ylabel('Principal Stress','FontSize',18, 'Color','r')
hold on;
lam=[1:0.1:2.7];
sig=zeros(1,length(lam))
for i=1:length(lam)
    sig(i)=1.5*(lam(i)^2*(1/lam(i)))
end
plot(lam,sig,'linewidth',1.5);
legend('Abaqus results','Analytical results');
hold on;
```