ME 685A (2016-2017 Summer)

Assignment Policy

- **Three kinds of problems**
  (A) Practice (B) Modelling (B) Methods
  Modelling related problems are open ended seeds for potential projects. A reasonable amount of work from formulation to post-processing is expected.
  Methods related problems are to be implemented in any programming language (e.g, Fortran, C, C++, Python etc) or perhaps MATLAB (for 90% credit), and validated for at least 3 sets of data:
  (1) Data suggested in the problem,
  (2) Student's own data, and
  (3) Data given at the time of evaluation.

- **Submission**
  (1) A hard copy of a brief report, in the *beginning* of the class.
  (2) A soft copy (zip file) of the code (**in any language**) done **individually** and the pdf file of the report, to mankur@ and sunjac@ roughly two hours before the class.
  (3) Typical time for completing an assignment: 6 days; due dates strict, with mild penalties for late submission.

- The code for a problem on a particular method must not use the **corresponding** library function.

- To the extent possible the methods are to be implemented as **functions or subroutines** rather than cluttering the main program.

ME685 (2017) Assignment 1

(Full marks = 100)

Deadline:
Soft complete version (zip and pdf) with codes— 06 Jun 12:00;
Hard copy without codes— 06 Jun 14:00.

1. **Problem for variable data sets.**
   Consider the function $f : R^2 \to R$. Measuring the *size* of the input by $|x| + |y|$, and taking the size of the output as $|f(x, y)|$, determine the condition number of the problem of evaluating the function f. In which regions of the x-y plane would you expect less reliability?
   Suggested case: $f(x, y) = x - y$. (10)

2. **Problem for variable data sets.**
   Write a programme to solve the quadratic equation $ax^2 + bx + c = 0$ using the *standard formula*
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   and also by the alternative formula that you can develop by multiplying the numerator and denominator in the above with $-b \mp \sqrt{b^2 - 4ac}$ (even though the idea looks funny).
   Make your programme robust against unusual input data (values of a, b, c) that may cause numerical inaccuracies in one solution or the other. Test your programme using lots of *unusual* data sets and check whether these measures give better results than the standard formula.
   Suggested data sets: (i) $a = 6 \times 10^{30}$, $b = 5 \times 10^{30}$, $c = -4 \times 10^{30}$; (ii) $a = 1$, $b = -4$, $c = 3.999999$; (iii) $a = 10^{-30}$, $b = 10^{-30}$, $c = 10^{30}$. (10)

3. **Problem for variable data sets.**
   The first derivative of a function $f(x)$ was computed using the usual central difference formula
   $$f'(x) = \frac{f(x + h) - f(x - h)}{2h},$$
   with its usual error of the second order. After that, the need for the third derivative was felt, for which the function was evaluated at two more points, namely $x - 2h$ and $x + 2h$. Develop the formula for the third derivative in terms of these four function values (even if you have done it once in Quiz 1) and work out its order of error. Further, now that you have as many as four function values, use them to develop an alternative formula for the first derivative which will be (hopefully) more accurate.
   Still later, there arose the need for the second derivative as well, for which the usual (second order) formula needs the value of $f(x)$, which we *do not* have yet. We decide that we will not evaluate it, rather we will use the four function values that we already have to work out the second derivative. Develop a formula for this purpose and determine its error order.
   Test all your formulae against diverse functions.
   Suggested case: $f(x) = x^3 e^{-2x} \sin x$. (15)

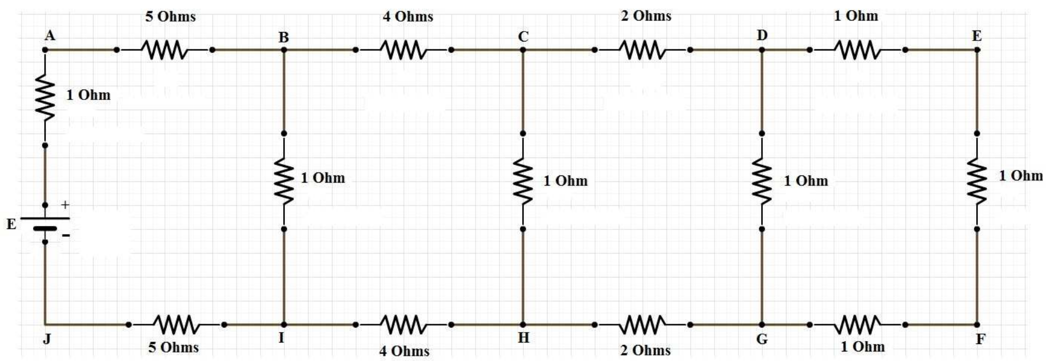
4. A surveyor reaches a remote valley to prepare records of land holdings. The valley is a narrow strip of plain land between a mountain ridge and sea, and local people use a local and antiquated system of measures. They have two distant landmarks: the lighthouse and the high peak. To mention the location of any place, they typically instruct: *so many* bans *towards the lighthouse and so many* kos *towards the high peak.* Upon careful measurement, the surveyor and his assistants found that (a) one *bans* is roughly 200 m, (b) one *kos* is around 15 km, (c) the lighthouse is 10 degrees south of east, and (d) the high peak is 5 degress west of north; and both the lighthouse and the high peak are actually far away — they are just distant direction indicators.

   The surveyor's team, obviously, uses the standard system, with unit distances of 1 km along east and along north. Now, to convert the local documents into standard system and to make sense to the locals about their intended locations, work out

   (a) a conversion formula from valley system to standard system, and (5)

   (b) another conversion formula from standard system to valley system. (5)

5. Find out the current in each branch of the circuit in terms of the emf E (in volts). (10)



   Next, instead of just four loops, suppose there are more such loops on the right side, in the same pattern. Work out the current through the battery in the case of 7, 8 and 10 such loops. (10)
   Taking advantage of the pattern of coefficients, try to make the computation *efficient.* (BONUS)

6. **Problem for variable data sets.**
   Implement a complete and robust linear system solver that will work in all kinds of cases.
   Suggested data:

   (i) $\begin{bmatrix} 3 & 2 & -1 \\ 4 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix} X = \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 3 & 4 \\ -1 & 1 & 2 \end{bmatrix} X = \begin{bmatrix} 4 \\ 11 \\ 1 \end{bmatrix}$

   (iii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 3 & 1 \\ 4 & 2 & -3 \\ 1 & 0 & 2 \end{bmatrix} X = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 2 & 3 & 5 & 4 \\ 1 & 9 & 10 & 3 \\ 3 & -3 & 0 & 5 \end{bmatrix} X = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ (35)