ME685 (2017) Assignment 2

(Full marks = 100)

Deadline:
Soft complete version (zip and pdf) with codes— 19 Jun 12:00;
Hard copy without codes— 19 Jun 14:00.

1. **Problem for variable data sets.**
   Given four data points, determine the interpolating cubic polynomial using

   (a) the monomial basis,

   (b) the Lagrange basis,

   (c) the Newton basis.

   Show that the three representations give the same polynomial.
   Suggested data: $(-1, 4), (0, 3), (2, 7), (8, 11)$. (20)

2. **Problem for variable data sets.**
   Use Richardson extrapolation to improve the estimate of the first derivative of a function $f(x)$ at a given point. Compare the predicted value with the actual analytical derivative.
   Suggested case:

$$f(x) = \frac{\sin(2x + \pi/3)\sqrt{3x^2 + 2x - 4}}{\ln(2x + 4)} \quad \text{at} \quad x = 2\,.$$

(15)

3. **Problem for variable data sets.**
   Write a programme to approximate any given function using an 8th degree interpolating polynomial. With the same data, develop a cubic spline approximation and compare the two. Which method gives a better representation in the given domain?
   Suggested case:

$$p(t) = \cos(10 \cos^{-1} t + \pi/6) + \ln(2t + 5) \quad \text{over} \quad [-0.3, 0.8]\,.$$

(15)

4. Consider the problem of finding a root of the function $h(x) = 30 \sin x + x^3 - 5$ with two points 0 and 1.44 identified with opposite signs of the function. (Up to two places of decimal is enough.)

   (a) Use the method of false position with these two points as starting values to find the root.

   (b) Use the Newton-Raphson method, starting with 0, to find the root and note the comparative performance.

   (c) Attempt the Newton-Raphson method, starting with 1.42, 1.44 and 1.46, and explain your experience of the first four iterations in each case.

(20)

5. Starting from $\mathbf{x} = [1 \;\; 1 \;\; 1]^T$, solve the system of equations

$$16x_1^4 + 16x_2^4 + x_3^4 = 15, \quad x_1^2 + x_2^2 + x_3^2 = 3, \quad x_1^3 - x_2 = 0$$

by Newton's method. (10)

6. Consider the function $f : R^2 \to R$ defined by

$$f(\mathbf{x}) = 2x_1^2 - x_1^4 + x_1^6/6 + x_1 x_2 + x_2^2/2 \ .$$

Find out the stationary points and classify them as minimum, maximum and saddle points. Using a penalty for constraint violation, minimize $f(x)$ in the domain defined by

$$2x_1^2 - 12x_1 - x_2 + 23 \le 0 \, .$$

Verify and display your results with a contour plot. (20)