We want to consider a set G of simple graphs with N nodes

We can define P(G) the probability of observing a specific G network

$$\sum_{G \in \mathcal{G}} P(G) = 1$$

We can say: we want a network with some specific metrics (costraints), and ask ourselves what is the probability of observing a certiain graph with those costraints.

We use an approach usually used in statistical physics:

$S = \ldots$

we maximize the entropy with 1 constrain (sum of P(G) =1) + one constrain for each metrics we want respected

$P(G) = \ldots \quad e^{H(G)} \ldots$

$E[y] = \ldots$ for any $y$ quality

Let's assume that:

$$H(G) = \beta\, m$$

this is the average of a quantity we will see in a bit

if we have only one constraint so i=1

lagrange multiplier

$$\Rightarrow \quad P(G) = \frac{e^{\beta m}}{Z}$$

$$Z = \sum_{G \in \mathcal{G}} e^{\beta m}$$

how do we compute such a sum over all the graphs in the set G?
We sum over every possible values of the elements of the adjacency matrix

$$m = \sum_{i<j} A_{ij} \equiv$$ the number of links in our graphs

The only constrain here is a fixed number of links! It's erdos-renyi

(The number of nodes is fixed from the very start of this model, it's always fixed)

We can also derive $\beta$ the lagrange multiplier

$$\underbrace{p_{vw}} = \langle A_{vw} \rangle = \frac{1}{Z} \sum A_{vw} \exp\left(\beta \sum_{i<j} A_{ij}\right) = \dots = \frac{\langle m \rangle}{\binom{n}{2}}$$

This is the probability of having a link between node v and node w

It's just like the prob that generate Erdos-Renyi with $p = \frac{\langle m \rangle}{\binom{n}{2}}$

**We can get also the configuration model:**

Let's assume $H(G) = \sum_{i=1}^{N} \beta_i k_i$ k is the degree, so it's fixated for every node!

We compute Z as well

$$\Rightarrow \quad p_{vw} = \langle A_{vw} \rangle = \ldots = \frac{e^{(\beta_v+\beta_w)}}{1+\underbrace{e^{(\beta_v+\beta_w)}}_{\text{if } \ll 1}}$$

$$\Rightarrow \quad p_{vw} = e^{(\beta_v+\beta_w)} = \ldots = \underbrace{C}_{\frac{1}{2m}}{}^{2} \langle k_v \rangle \langle k_w \rangle$$

and we find the values of $\beta$