

EFFECTS OF COMPLIANCE ON THE STABILITY OF JETS



AND WAKES

Ryan Poole, rp00378@surrey.ac.uk

University of Surrey



Introduction

We carry out a spatio-temporal analysis¹ on a piecewise linear inviscid planar jet or wake with uniform density and infinitely thin shear layers at $z = \pm 1$, see Fig.1, when bound above and below by Kramer compliant walls [4]. We take the parallel flow approximation, and assuming small normal mode perturbations for velocity and pressure, we linearise the Euler equations about some base flow $U(z)\hat{x}$. From these linear equations, and with a suitable boundary condition, we derive a dispersion relation² (1), governing the absolute instability³ characteristics of the flow. Scan the QR-code to read further on any points with a superscript.

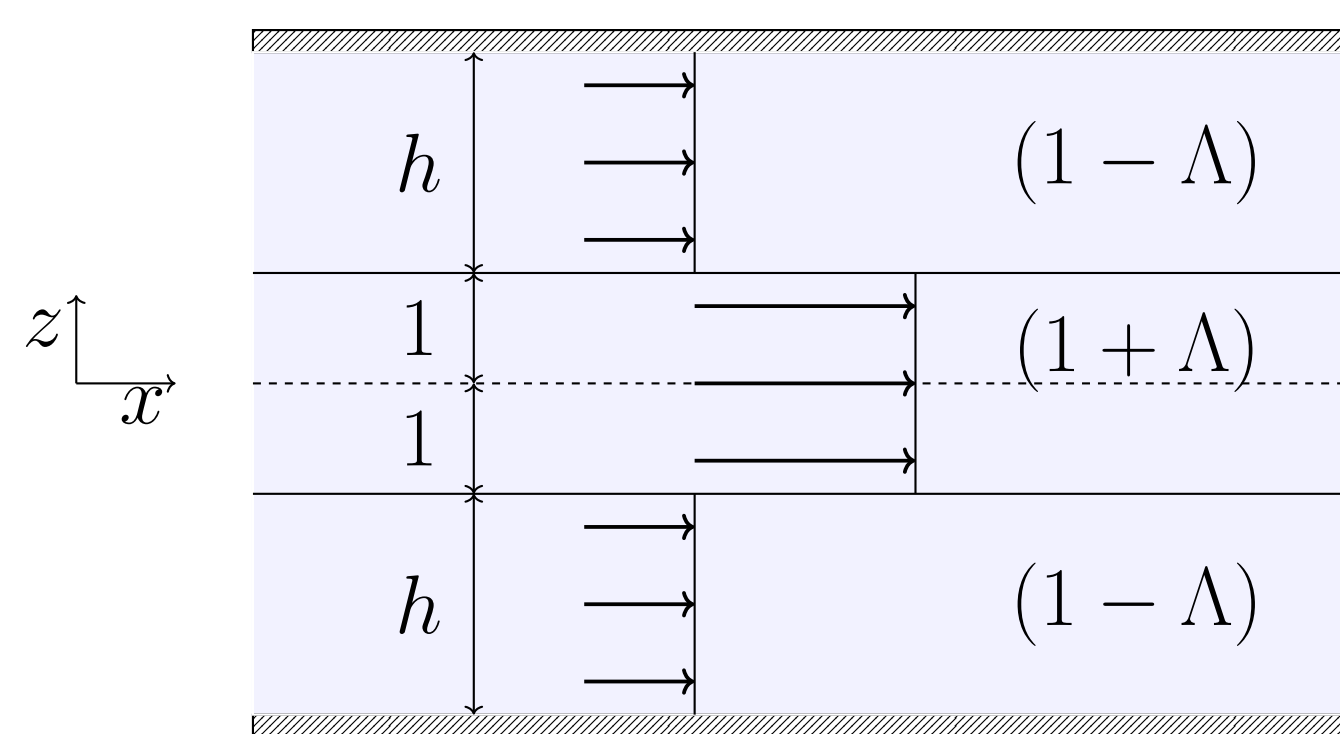
Flow setup and Dispersion relation

Our Kramer compliant wall consists of an elastic plate attached to a rigid back plate with identical springs. Between these springs there is a viscous damping fluid [1]. These confine a piecewise linear flow⁴ as shown in Fig.1, whose simplicity allows us to write down an explicit dispersion relation of the form

$$\mathbb{D}^{(v,s)}(\alpha, \omega) = \alpha Q X^{(v,s)} - (\alpha(1 - \Lambda) - \omega)^2 Y^{(v,s)} = 0, \quad \text{where } Q = -m\omega^2 - id\omega + B\alpha^4 + T\alpha^2 + K. \quad (1)$$

Here, superscript v represents varicose (symmetric) modes, while s represents sinuous (anti-symmetric) modes⁵.

Here, Λ and h represent the flow shear and confinement ratios respectively⁶. Confinement effects on jets and wakes is explored further in [2, 3, 5]



Dimensionless wall parameters m, d, B, T, K describe wall mass, damping, rigidity, tension and spring stiffness respectively. When these parameters are large, we tend to the rigid wall limit. In our AI analysis, we let $(B, K, m) = (2, 10, 0.1)$ and $d = T = 0$.

Fig. 1: Flow configuration, in dimensionless form, subject to confinement by compliant walls at $z = \pm(1 + h)$.

Absolute Instability Analysis⁷: results

Absolutely unstable is a term that is given to wavepackets, groups of waves, where the dominant behaviour shows growth both up and down stream of the initial perturbation, while also growing in time⁸. These modes of instability exist in the rigid wall case, and are able to be modified by confinement by compliant walls. We find that compliant walls are able to stabilize or destabilize AI modes⁹, i.e. increase how fast or slowly they grow in time, depending on the stability of the flow. They also introduce new modes of instability which lead to larger regions of (Λ^{-1}, h) -space where we see AI, see Fig. 2, which means we see instability for a wider range of shear values than we could in the rigid wall case.

Regions of AI

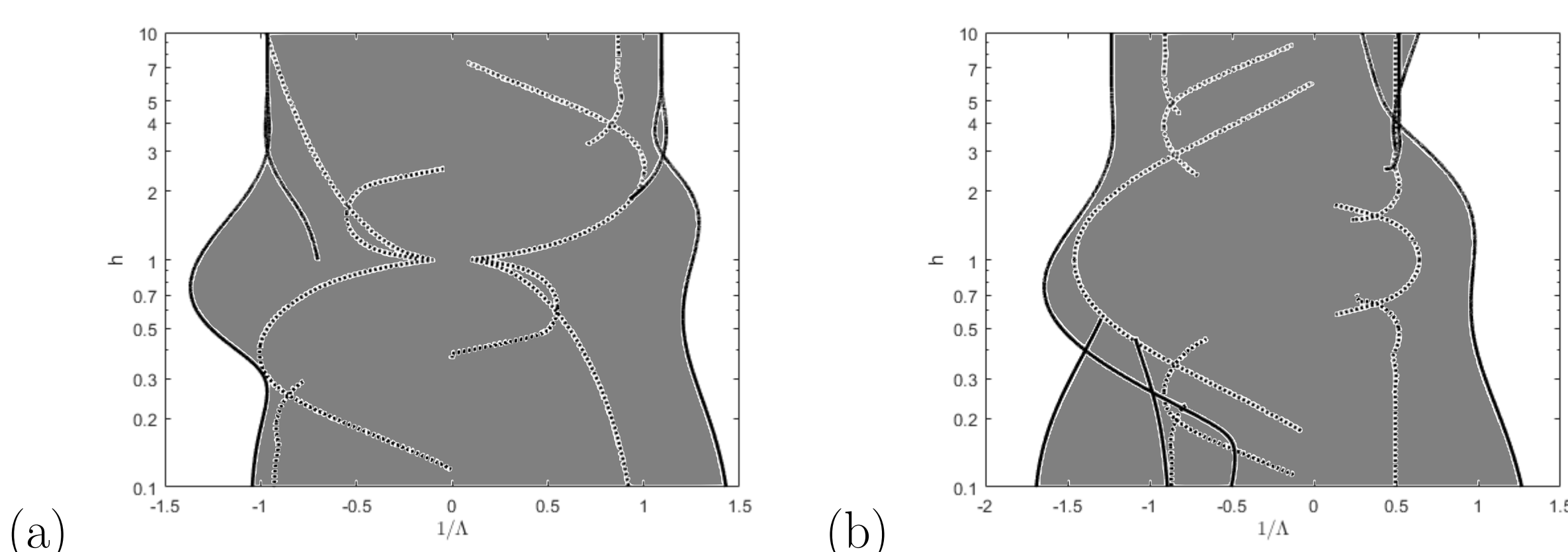


Fig. 2: Modified regions of AI (grey) in $(\frac{1}{\Lambda}, h)$ -space. Dotted and solid lines represent rigid wall and compliant wall regions respectively.

References

- [1] P. Carpenter and A. Garrad. The hydrodynamic stability of flow over kramer-type compliant surfaces. part 1. tollmien-schlichting instabilities. *J. Fluid Mech.*, 155:465–510, 1985.
- [2] M. P. Juniper. The effect of confinement on the stability of two-dimensional shear flows. *J. Fluid Mech.*, 565:171, 2006.
- [3] M. P. Juniper. The full impulse response of two-dimensional jet/wake flows and implications for confinement. *J. Fluid Mech.*, 590:163, 2007.
- [4] M. O. Kramer. Boundary-layer stabilization by distributed damping. *Journal of the Aerospace Sciences*, 27(1):69–69, 1960.
- [5] S. Rees and M. Juniper. The effect of confinement on the stability of viscous planar jets and wakes. *J. Fluid Mech.*, 656:309, 2010.

