

Statistical_Inference_Notes

Coursera Course by John Hopkins University

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Intro

Instructor's Note:

"Statistical inference is the process of drawing conclusions about populations or scientific truths from data. There are many modes of performing inference including statistical modeling, data oriented strategies and explicit use of designs and randomization in analyses. Furthermore, there are broad theories (frequentists, Bayesian, likelihood, design based, ...) & numerous complexities (missing data, observed and unobserved confounding, biases) for performing inference. A practitioner can often be left in a debilitating maze of techniques, philosophies and nuance. This course presents the fundamentals of inference in a practical approach for getting things done. After taking this course, students will understand the broad directions of statistical inference and use this information for making informed choices in analyzing data.

All the best,

Brian Caffo"

Statistical inference help us extend beyond a small subset of data to give answers about a population.

Course Description:

“In this class students will learn the fundamentals of statistical inference. Students will receive a broad overview of the goals, assumptions and modes of performing statistical inference. Students will be able to perform inferential tasks in highly targeted settings and will be able to use the skills developed as a roadmap for more complex inferential challenges.”

GitHub Link for Lectures

Statistical Inference Lectures on GitHub

Course Book

The book for this course is located on **LeanPub**

Data Science Specialization Community Site

The site is created using **GitHub Pages**

Homework Problems

The homework problems are optional, they are a good opportunity to practice the skills covered in the course. There are also worked out solutions on youtube (linked to from the book)

Here's all four homeworks as interactive web pages (it's probably better to just keep up with them from the book):

- * **HW 1**
- * **HW 2**
- * **HW 3**
- * **HW 4**

Probability & Expected Values

Introduction to Probability

Intro

Probability assigns a number between 0 and 1 to events to give a sense of the “chance” of the event. These sections will look at the basics of probability calculus.

An additional resource is the class Mathematical Biostatistics Boot Camp 1

Probability

Given a random experiment (i.e. rolling a die) a probability measure is a population quantity that summarizes the randomness

Specifically, probability takes a possible outcome from the experiment and:

* assigns it a number between 0 and 1

* so that the probability that something occurs is 1 (the die must be rolled)

* so that the probability of the union of any two sets of outcomes that are mutually exclusive is the sum of their respective probabilities ($P(E \cup F) = P(E) + P(F)$)

Rules probability must follow

- The probability that nothing occurs is 0
- The probability that something occurs is 1
- The probability of something is 1 minus the probability that the opposite occurs, the conjugate
- If an event **A** implies the occurrence of event **B**, then the probability of **A** occurring is less than or equal to the probability that **B** occurs
- For any two events the probability that at least one occurs is the sum of their probabilities minus their intersection ($P(E \cup F) = P(E) + P(F) - P(E \cap F)$)

Probability mass functions

- Probability calculus is useful for understanding the rules that probabilities must follow.
- We need ways to model and think about probabilities for numeric outcomes of experiments
+ Densities and mass functions for random variables are the best starting point for this
- The goal is to use the data to estimate properties of the population
- A **random variable** is a numeric outcome of an experiment and can be **discrete** or **continuous**
 - + For **discrete** a probability can be assigned for every value it can take
 - + for **continuous** a probability can be assigned for the ranges of values it can take

Some examples of variables that can be seen as random variables

* The outcome of the flip of a coin (discrete)

* The outcome from the roll of a die (discrete)

* The web site traffic on a given day

+ Since this discrete variable has no upper bound we'd view it as a poisson distribution

* The BMI of a subject four years after a baseline measurement (continuous)

* The hypertension status of a subject randomly drawn from a population (binomial discrete variable)

* The number of people who click on an ad (discrete; poisson)

* Intelligence quotients for a sample of children (continuous)

PMF

- A **Probability mass function** evaluated at a value corresponds to the probability that a random variable takes that value. To be a valid pmf a function, p , must satisfy:
 - 1) It must always be larger than or equal to 0
 - 2) The sum of the possible values that the random variable can take has to add up to one

Example: Coin Flip

$X = 0$ represents tails and $X = 1$ represents heads

$$p(x) = (1/2)^x * (1/2)^{1-x} \text{ for } x = 0, 1$$

And for a loaded coin this could be generalized as

$$p(x) = \theta^x * (1 - \theta)^{1-x} \text{ for } x = 0, 1 \text{ where } \theta \text{ represents probability of heads}$$

When evaluating this we get..

$$\text{Probability of heads is } p(1) = \theta^1 * (1 - \theta)^{1-1} = \theta \text{ and}$$

$$\text{Probability of tails is } p(0) = \theta^0 * (1 - \theta)^{1-0} = 1 - \theta$$

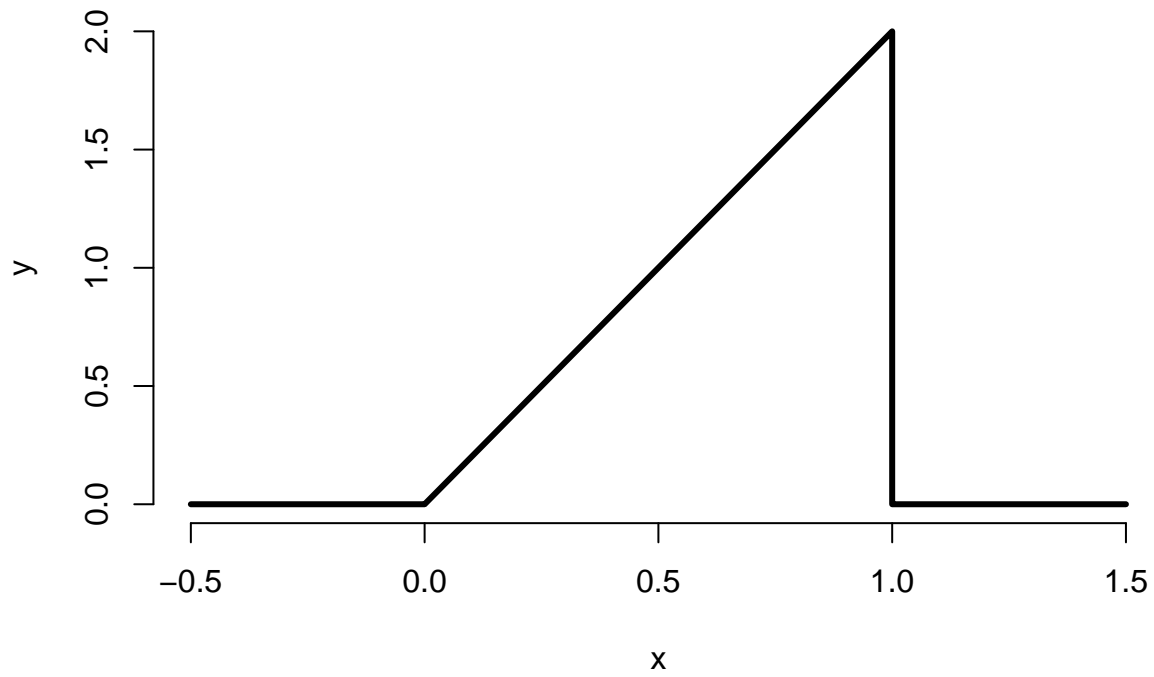
Probability density functions

- A **probability density function** (pdf), is a function associated with a continuous random variable
- To be a valid pdf, a function must:
 - 1) Be larger than or equal to zero everywhere
 - 2) The total area under it must be one
- Areas under pdfs correspond to probabilities for that random variable

Example with beta density (triangle & piecewise fn)

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

```
x <- c(-0.5, 0, 1, 1, 1.5)
y <- c( 0, 0, 2, 0, 0)
plot(x, y, lwd = 3, frame = FALSE, type = "l")
```



To check if this is a valid PDF we can calculate the total area, using geometry we have a triangle with a *base of 1* and a *height of 2* $A = 1/2bh = h/2 * b = 2/2 * 1 = 1$, therefore it satisfies the rules for a PDF.

Me using integrals for primary school math problem



$$[x^2]_0^1 = 1^2 - 0^2 = 1$$

$$\int_0^1 2x dx =$$

Assume this pdf is for the porportion of health calls that get addressed in a given day. What's the probability that 75% or fewer of calls get addressed?

$$\int_0^{0.75} 2x dx = [x^2]_0^{0.75} = 0.75^2 - 0^2 = 0.5625$$

The beta function is also a function in R:

```
#first param is for the quantile  
#second param is the height of the distrbution  
#third param is the width of the dist.  
pbeta(0.75, 2, 1)
```

```
## [1] 0.5625
```

Cumulative Distribution Function (CDF) and Survival Function

The **cumulative distribution function** (CDF) of a random variable, X , returns the probability that the random variable is less than or equal to the value of x

$$F(x) = P(X \leq x)$$

The **survival function** of a random variable X is defined as the probability that the random variable is greater than the value x

$$S(x) = P(X > x)$$

Notice that $S(x) = 1 - F(x)$

We can evaluate multiple quantiles at once with `pbeta`

```
pbeta(c(0.4, 0.5, 0.6), 2, 1)
```

```
## [1] 0.16 0.25 0.36
```

Quantile

The α^{th} **quantile** of a distribution with distribution function F is the point x_α such that $F(x_\alpha) = \alpha$

* A **percentile** is simply a quantile with α expressed as a percent

* The **median** is the 50th percentile

* The `qbeta` function will take a quantile and return the value of x_α

```
sqrt(.5) #Solving  $x^2 = 0.5$  manually
```

```
## [1] 0.7071068
```

```
qbeta(0.5, 2, 1)
```

```
## [1] 0.7071068
```

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Conditional Probability

Bayes' rule

Independence

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Expected values

Expected values, simple examples

Expected values for PDFs

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Lessons with `swirl()`

Quiz 1

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Variability, Distribution, & Asymptotics

Introduction to Variability

Variance Simulation Examples

Standard Error of the Mean

Variance Data Example

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Binomial Distrubtion

Normal Distribution

Poisson

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Asymptotics and the Law of Large Numbers (LLN)

Asymptotics and the Central Limit Theorem (CLT)

Asymptotics and Confidence Intervals

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Lessons with `swirl()`

Quiz 2

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Intervals, Testing, & P-values

T Confidence Intervals

T Confidence Intervals Example

Independent Group T Intervals

A Note on Unequal Variance

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Hypothesis Testing

Example of Choosing a Rejection Region

T Tests

Two Group Testing

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P-Values

P-Value Further Examples

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Lessons with `swirl()`

Quiz 3

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Power, Bootstrapping, & Permutation Tests

Power

Calculating Power

Notes on Power

T Test Power

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Multiple Comparisons

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Bootstrapping

Bootstrapping Example

Notes on the Bootstrap

Permutation Tests

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Lessons with `swirl()`

Quiz 4

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