

On the Various Translations between Classical, Intuitionistic and Linear Logic

Gilda Ferreira^{a,b,1,*}, Paulo Oliva^c, Clarence Lewis Protin^{d,3}

^a*DCeT, Universidade Aberta, Lisboa, 1269-001, Lisboa, Portugal*

^b*CMAFcIO, Faculdade de Ciências, Universidade de Lisboa, 1749-016, Lisboa, Portugal*

^c*School of Electronic Engineering and Computer Science, Queen Mary University of London, London E1 4NS, United Kingdom*

^d*Centro de Filosofia, Faculdade de Letras da Universidade de Lisboa, Portugal*

Abstract

Several different proof translations exist between classical and intuitionistic logic (negative translations), and intuitionistic and linear logic (Girard translations). Our aims in this paper are (1) to show that all these systems can be expressed as extensions of a basic logical system (essentially intuitionistic linear logic), and that (2) with this common logical basis, a common approach to devising and simplifying such proof translations can be formalised. Via this process of “simplification” we get the most well-known translations in the literature.

Keywords: Intuitionistic linear logic, classical linear logic, negative translations, Gödel–Gentzen translation, Kuroda translation, Girard translations, embeddings into linear logic.

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*Corresponding author

Email addresses: gmferreira@fc.ul.pt (Gilda Ferreira), p.oliva@qmul.ac.uk (Paulo Oliva), cprotin@sapo.pt (Clarence Lewis Protin)

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