A Critical Assessment of Satellite Drag and Atmospheric Density Modeling

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This paper examines diverse approaches to representing gasdynamic drag effects on Low Earth Orbit (LEO) satellites. Although the total drag force on a satellite can be measured, the physics of gasdynamic resistance, dynamics of the orbiting body, and characteristics of the atmosphere are inextricably combined. Investigators must hypothesize physical relationships among the drag force, body shape, size, and orientation, the distribution of density, and the predictive assessment of density. Drag coefficients determined under one set of hypotheses are often employed improperly in orbital assessments that use a different set of hypotheses. Our goal is to consolidate the existing information, establish a framework for future research, and expose practical issues.

I. INTRODUCTION

Significant research has taken place to determine the proper modeling for atmospheric density. The literature contains information in several different disciplines, ranging from basic physics, aerodynamic gas / surface dynamic interactions, to complex models of density, and corrections to existing atmospheric models. Much of the basic research took place almost a half century ago. Gaposchkin and Coster (1988) developed a comprehensive assessment of satellite drag and thermospheric density distribution. Graziano (2007) added significant value in his recent doctoral thesis. Nonetheless, the orbit estimation community often overlooks the assumptions upon which commonly used drag coefficients and atmospheric models were developed.

Atmospheric models generally use aggregated parameters based on observables that are considered indicators of atmospheric density, which are often difficult to measure directly (a_p or $K_{p,}$ and $F_{10.7}$). Researchers usually fix one parameter, such as the drag coefficient, and add others to the state vector to be estimated. In some cases, such as spherical objects, this is appropriate. But for more complex shapes this simply moves the uncertainty to another parameter that might not have any real physical connection to the observables. Occasionally, researchers neglect to recognize that the drag coefficients they estimate depend on the model and assumptions they invoke to make the estimates. This leads to physically unrealistic outcomes.

Recent papers (Bowman 2005) have implied that the coefficient of drag is significantly mis-modeled by current orbit determination approaches and that by simply using a “corrected atmosphere”, one can remove all un-modeled density variations and achieve improved results. While this has merit, the assumption is valid only within the context of the specific computer code implementation. In addition, there are other factors that affect the overall behavior of atmospheric drag that are not standardized, nor well discussed in the literature. Without the proper understanding of the interrelationships of each contributing parameter, one could erroneously couple pieces of information and obtain incorrect conclusions that would only partially solve the problem.

This paper seeks to clarify the sources of uncertainty in the drag problem and quantify the process by which fixing one parameter can artificially influence the values of another. Much of the work is inherently related to orbit determination and we use the Kalman filter in Analytical Graphic Inc’s Orbit Determination Toolkit (ODTK). This program is used in many operational and high precision applications because it is trusted as a robust and highly accurate processing platform.

There are five major areas that we examine (See Fig. 1). This is our approach to organizing and examining the coupled issues of atmospheric density and it’s affect on satellite motion. In some cases we are able to develop metrics to indicate the affect of one parameter on the overall accuracy. We examine the predominant approaches to

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predicting drag forces based on their assumptions and hypotheses. We illuminate physical inconsistencies in determining and employing approximations to thermospheric density, drag coefficients, and other important parameters. Finally, we propose a framework to guide research to an internally consistent and focused set of density and drag representations for precision orbit determination.

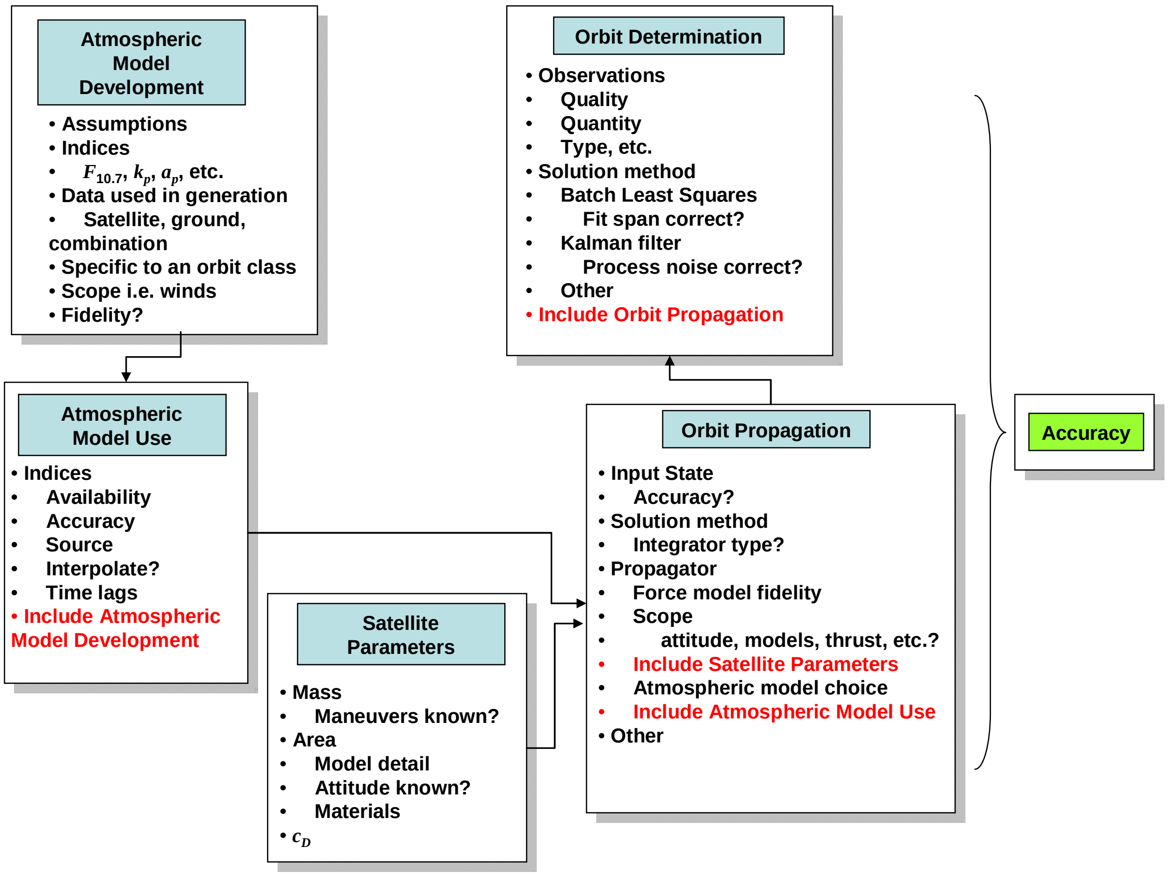


Figure 1. Variables in the Determination of Atmospheric Drag. Several disciplines are required to properly model atmospheric drag. The relationships show the broad areas we will explore. Note that each of these areas has error bars, some of which are significant. None of them are without error, and many are inter-related.

A. Basic Introduction

There are two fundamental operations that are involved in atmospheric drag: the determination of atmospheric density, and the interaction of the satellite surface with the atmosphere. Much of the recent literature deals with the former, but the latter is an equally important component to the overall effect of atmospheric density.

Vallado (2007:Sec 8.6.2) provides the basic fundamentals related to atmospheric drag so we will not repeat all that information here. The underlying cause of atmospheric drag is momentum transfer from molecules in the atmosphere. Gravitation also results in hydrostatic pressure and density variations within the atmosphere. The development of both the static and time-varying atmospheric models relies on a few basic hydrostatic principles which are used to model atmospheric effects. The ideal-gas law relates the absolute pressure, p_o, the mean molecular mass of all atmospheric constituents, M, the acceleration due to gravity, g_o, the universal gas constant, R (per mole), and the absolute temperature, T (Kelvin):

$$\rho = \frac{p_o M}{g_o R T} \tag{1}$$

The linkage with temperature is important because it causes much of the difficulty in developing an exact model for the density. The Earth's rotation exposes the atmosphere to the Sun, and the resulting solar heating affects density.

The second relation is the hydrostatic equation, which relates the change in pressure, Δp, to the density, gravity, and change in altitude, Δh:

$$\Delta p = -\rho g \Delta h \tag{2}$$

Baker and Makemson (1967:210–213) and Escobal ([1968] 1979:18–25) develop the density equations and associated relations. The static models are the simplest to use because we assume all the atmospheric parameters remain constant. Yet, some factors affect even static models.

The widely accepted equation for atmospheric drag is

$$\vec{a}_{drag} = -\frac{1}{2}\rho\frac{c_D A}{m}v_{rel}^2\frac{\vec{v}_{rel}}{\left|\vec{v}_{rel}\right|} \tag{3}$$

Despite the innocuous form, atmospheric drag is probably the most elusive of the force models used for satellite operations.

Aerodynamic forces and moments are the consequence of an object doing work on the medium it operates in. Momentum and energy are exchanged between the object and the medium. The nature of the momentum exchange depends on the distribution of mass in the medium and the geometry and characteristics of the surface of the object. If the medium is dense and the object smooth and impermeable, the interaction is much like determining the forces experienced when redirecting an energetic, collimated stream of liquid, a fire hose. The duration of the encounter between an element of the medium and the object is important. If the encounter is short compared to the time during which elements of the medium can transfer momentum or energy to each other, the fire hose analogy applies. Otherwise, interactions among fluid particles redistribute energy and momentum. Time and length scales are very important. The common approximation:

$$c_D = \frac{2m\vec{a}_{drag}}{\rho v_{rel}^2 A}\frac{\left|\vec{v}_{rel}\right|}{\vec{v}_{rel}} \tag{4}$$

is valid only when the length scale of the problem is much greater than the mean free path between collisions among gas particles or the duration of the interaction between the medium and the object is much greater than the time between particle collisions. There has to be enough time for collisions among gas particles to equilibrate the consequences of the interaction. The dependence of drag force upon area, velocity, and density is not necessarily as straightforward in the sparse atmosphere of orbital regimes.

This approximation is valid for hypervelocity, continuum flow. If the oncoming flow is reflected from a vertical surface, the drag is twice the momentum of the free stream intercepted by the surface, c_D = 4. If the surface were inclined at 45 degrees and the oncoming flow deflected upward, c_D = 2, and there would be a corresponding downward force on the surface. ***This roughly bounds drag coefficient***, but it is not sufficient for accurate prediction of the drag force. Using Eq. 4, we estimate the significance of drag relative to other forces that a satellite experiences.

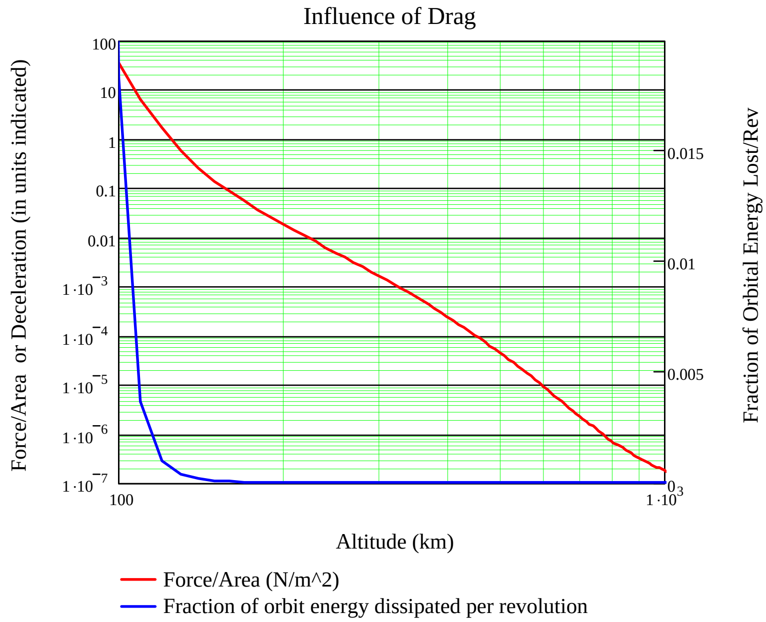


Figure 2: Influence of Drag for a 1000 kg satellite, 100 square meter drag area, and c_D = 2.2. Harris-Preister model atmosphere.

Let's also look at the relative velocity. $\vec{v}_{rel}$, or the velocity relative to the rotating atmosphere depends on the accuracy of the *a-priori* estimate, and the results of any orbit determination processes. Because it's generally large, and squared, it becomes a *very* important factor in the calculation of the acceleration, yet it has received surprisingly little analysis in the literature. A common assumption is that the lower atmosphere rotates with the Earth, allowing a vector summation for the velocity values. The upper atmosphere winds can be several hundred m/s (Gaposchkin and Coster, 1988) which can have a large effect of the drag acceleration. However, they are particularly unknown, un-modeled, and unpredicted.

$$\vec{v}_{rel} = \vec{v}_{sat} + \vec{v}_{atmos} \tag{5}$$

The other parameter we do not mention specifically in the following sections is the actual density. Density is a product of all the other parameters we discuss, but ultimately, it's the atmospheric density that affects the trajectory of the satellite. The density usually depends on the atmospheric model, EUV, $F_{10.7}$, K_p, a_p, prediction capability, atmospheric composition, etc. There are a lot of parameters here, each can cause significant changes, and each has uncertainty both in measured values and in prediction. The popular parameters to examine today are the density and the exospheric temperatures. This single parameter represents the largest contribution to error in any orbit determination application.

The gas properties that are consistent with the simple drag formula are statistical measures of the fact that gas particles move randomly with respect to the bulk medium. Temperature is a statistical measure of the randomly distributed kinetic energy of the molecular and atomic constituents of a gas. This motion is manifested by momentum fluxes whose statistical measure is called pressure. There is also a continuous exchange of gas particles within a control volume. The statistical average number of gas particles within a unit volume is density. Transport properties such as viscosity and conductivity can also be determined as statistical characteristics of very large quantities of molecules.

The random velocities of gas constituents depend on molar mass (and hence the attention given in modern atmospheric models that rely on the molecular composition of the atmosphere). For a given temperature, velocities of light constituents are skewed toward higher velocities than heavy constituents. Even at ambient temperatures, a noticeable fraction of atmospheric helium moves more rapidly than orbital speed, and helium departs the atmosphere.

This leads to the inference that the upper atmosphere is composed more of lighter elements and that the random velocities of gas particles are greater the higher the altitude. However, orbital velocity decreases with increasing altitude. Therefore, the interaction of the atmosphere with satellites is not that of a continuum fire hose. It is much better described by the interaction of individual molecules with a surface. The higher the altitude, the more these individual particle events dominate.

These fundamental physical principles imply that the simple drag formula is not valid in much of the low Earth orbit regime.

II. ATMOSPHERIC MODEL DEVELOPMENT

Developing atmospheric models is extremely difficult and time consuming, but there are a plethora of available models. Relatively few new models have appeared in recent years with the largest advances coming in the form of corrections to existing models. Because of data availability problems and other technological limitations, many of the early models were focused more on the theoretical implications that could be modeled. For instance, the early Jacchia model (J60 for instance) was more a “mean” approach as compared to the modern NRLMSISE-00 which is an atmospheric constituent based model that tries to accurately model molecular compositions at different altitudes, and hence the overall density.

Figure 3 shows some of the models (Vallado, 2007:563).

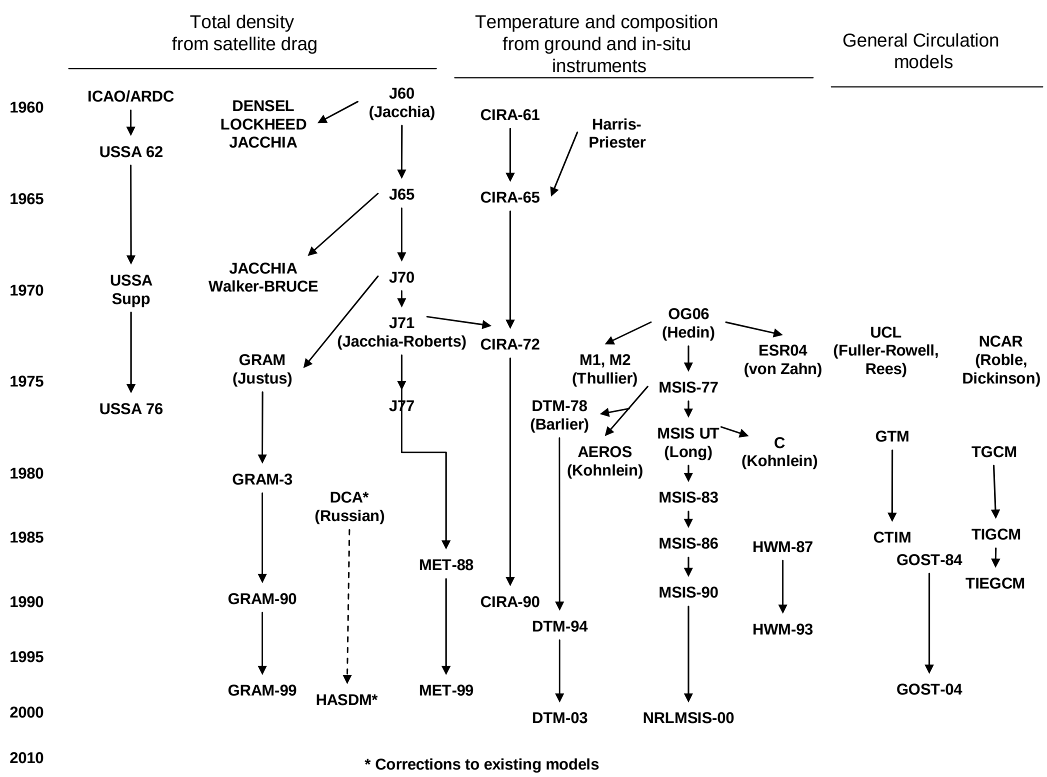


Figure 3. Development of Atmospheric Models. Notice the variety of models. Flow of information among the three overall categories is limited (Marcos, et al., 1993, 20). The main models in use today are the Standard Atmosphere, USSA76; variations of the Jacchia-Roberts, J71, J77, and GRAM99; COSPAR International Reference Atmosphere, CIRA90; Mass Spectrometer Incoherent Scatter, NRLMSIS-00; Drag Temperature Model (DTM), Marshall Engineering Thermosphere (MET), the Russian GOST and general circulation models. Dynamic Calibration of the atmosphere (DCA) and High Accuracy Satellite Drag Model (HASDM) are corrections to other models, usually J70. The dashed line indicates that the concept for correcting the atmosphere followed the Russian work, but the models themselves are different.

A. Assumptions

There are numerous assumptions that are required when creating an atmospheric model. In fact, there are probably more assumptions with atmospheric drag than with any other perturbing force used with satellites. We list some of the more obvious assumptions in this section.

- Assume $F_{10.7}$ is a suitable proxy for EUV which is actually believed to be the major factor in upper atmospheric heating. (Jacchia, 1970). Thuillier and Bruinsma (2001) indicate the existence of a 13-day periodicity in EUV that is not represented in the $F_{10.7}$ data.
- The lower atmosphere rotates with the Earth.
- The sea-level atmosphere is composed of nitrogen (78.11%), oxygen (20.955%), argon (0.934%), helium (0.0006%), and hydrogen in "mixing to 105 km, and in diffusion above this height" (Jacchia, 1971)
- The change in mean molecular mass in the mixing region below 105 km is caused by Oxygen disassociation (Jacchia, 1970)
- The shape of the temperature profiles remains constant with respect to basic models. Jacchia (1971) said this is *a-priori* unlikely.
- The coefficient of drag equation is correct and complete as shown in Eq 4.
- The atmospheric model precisely accounts for the variations in atmospheric drag including effects of diurnal variations, including the tilt of the Earth, the 27-day Solar cycle rotation, the 11 year solar cycle, semi-annual and seasonal variations, cyclical variations in the 11 solar cycle, upper atmosphere winds, magnetic storm variations, irregular short-periodic variations, tides, the proper altitude and spatial varying molecular components of the atmosphere, the appropriate lags for all factors affecting density, and other factors still under development.
- The precise method of employing the existing atmospheric indices is known and used
- All the input parameters (solar flux, geomagnetic indices (and any newer indices), atmospheric winds, distribution of molecular concentrations, etc) are available with [known] precision into the future
- Any interpolation of the indices introduces no weak time dependence errors into the solution
- The proper times are used with each index, and any interpolation is matched to the actual time-varying behavior of the index
- The atmospheric model is appropriate for the orbital class of satellite under consideration

B. Indices

The complex interaction of the solar wind and the geomagnetic effects on the atmosphere might indicate a plethora of indices and expressions to properly model the total atmospheric drag effect. However, there are surprisingly few parameters in this category. Common parameters that have been used in model development to date include the following:

$F_{10.7}$, many decades of values from http://www.swpc.noaa.gov/Data/index.html
$E_{10.7}$, only two days of data from http://www.spacewx.com/data/e107_nowcast.txt
S_{10} (from the SOHO satellite in a halo orbit at the L_1 Lagrange point since about December 1995),
Mg_{10} (from the NOAA and Nimbus satellites from 1978, Thuillier and Bruinsma 2001), "*The Mg II core-to-wing ratio is derived from the ratio of the h and k lines of the solar Mg II feature at 280 nm to the background or wings at approximately 278 nm and 282 nm. The h and k lines are variable chromospheric emissions while the background emissions are more stable. The result is a robust measure of chromospheric activity. The ratio is a good measure of solar UV and EUV emissions*." Thuillier and Bruinsma (2001) shows an approximate relation using the solar flux index. Mg II index = $0.000128F_{10.7} + 0.25068$. ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_UV/NOAAMgII.dat.
a_p or K_p for geomagnetic effects, data for many decades from http://www.swpc.noaa.gov/Data/index.html

Most atmospheric models use the $F_{10.7}$, a_p, and K_p indices, primarily because they have been available for about 70 years. Some models are designed using a preferred geomagnetic index – MSIS uses a_p, Jacchia uses a_p or K_p but preferentially uses K_p. Others atmospheric models use K_p.

Ultimately, the indices are a crucial step in determining the accuracy of any atmospheric model. If the variability of the parameter is greater than the effect it tries to model, the solution can become corrupted. We'll discuss the prediction aspect for these indices later, but this is another important limitation in the use of indices.

C. Available Observational Data

Development of atmospheric models is particularly challenging due to the complex interactions of atmospheric molecules, vehicle characteristics, solar environment, etc. It is further impeded by the essential non-availability of unclassified observational data for unclassified satellites. Bowman has repeatedly cited his unique advantage to access the AFSPC observational database, but has refused to release any observations for independent verification of his claims.

Historically, data was available, but very limited. The following sample atmospheric models list some of the data used in their development.

The Jacchia models used sounding rockets (6 firings on Jan 24, 1967 from Cape Kennedy), and satellites

Name	Norad #	Period	Incl	Ap alt	Per alt	RCS	Launch / Decay
EXPLORER 17 (S-6)	564	87.9	57.6	186	151	N/A	04/03/1963 to 11/24/1966
EXPLORER 32 (AE-B)	2183	87.5	64.5	162	140	0.4954	05/25/1966 to 02/22/1985
and compared to 5 satellites							
ECHO 2 (A-12)	740	94.5	81.4	522	465	N/A	01/25/1964 to 06/07/1969
EXPLORER 19 (ADI-1)	714	90.1	78.7	301	259	13.7332	12/19/1963 to 05/10/1981
EXPLORER 8	60	94.8	49.9	664	353	0.5910	11/03/1960
EXPLORER 1	4	88.5	33.1	215	183	N/A	02/01/1958 to 03/31/1970
INJUN 3	504	88.0	70.2	185	161	N/A	12/13/1962 to 08/25/1968

The model development used Least Squares solutions and the satellites used c_D = 2.2 below 300 km, which is a representative value across many approximations to aerodynamic drag. Jacchia mentions that c_D = 2.4 might be better, in which case the densities would be about 10% too high. The sample size is rather small, and the altitude dispersion is not too great. Also note that most of the Jacchia satellite data consisted of Baker-Nunn optical observations. The accuracy of this data is considerably less than what is currently available today – making it even more important to re-evaluate using today's' modern radar measurements!

The MSIS models generally used all the data available from the previous (Jacchia) models, plus "*Barlier data, accelerometer data, and the atmospheric decay model data from the Jacchia models. (1) satellite drag [Jacchia, 1970; Barlier et al., 1978; Hedin, 1988], orbit determination (1961–1973); (2) accelerometer [Hedin, 1988; F. Marcos, private communication, 1987], Atmosphere Explorer (AE-C, D, E) MESA [Champion and Marcos, 1973], Air Force SETA [Rhoden et al., 2000], CACTUS [Boudon et al., 1979], San Marco 5 [Arduini et al., 1997]; (3) incoherent scatter radar, exospheric temperature (T_{ex}), Millstone Hill (1981–1997 [Buonsanto and Pohlman, 1998]), Arecibo (1985–1995 [Melendez-Alvira et al., 1998]); (4) ISR, Lower thermospheric temperature SIA 15 - 2 PICONE ET AL.: TECHNIQUES (T_{low}), Millstone Hill (1988–1997 Goncharenko and Salah [1998]); (5) Solar Maximum Mission (SMM) O_2 density data derived from occultation of solar UV emissions [Aikin et al., 1993].*" (Picone et al. 2002)

Today, there are additional sources of data with which to develop atmospheric drag models. Satellite Laser Ranging Data (SLR) exists for many satellites, and GPS and accelerometer data are available from some sources. However, obtaining these data at the same time a Precise Orbit Ephemeris (POE) is developed and available becomes difficult. In the absence of raw observational data, one can also use the POE itself as a data source – a case we use later. This requires that the POE be long enough that you could get a "converged" solution and still have enough data to perform a comparison with. The absence of maneuvers is also desirable.

D. Orbital Classes

The early researchers had tough problems to overcome as mentioned previously. While they were able to obtain some observations, there weren't very many satellites in orbit (compared to today), or the proper orbital classes to ensure adequate and global coverage. As a result, many of the early models were very limited in their ability to generically represent atmospheric drag over a variety of orbital inclinations and altitudes. Nevertheless, the researchers were able to assemble models that are still relevant today.

We cite an example from J70. The Jacchia models (J70) state a range of altitudes from 90-2500 km, but the comment is made that above 1100 km there was no reliable observational data (as is seen from the available satellite data), and all the results are simple extrapolations (Jacchia, 1970). The overall technique is analogous to the gravity field tuning that is regularly done for precise orbit determination applications.

E. Fidelity

The previous discussion of assumptions, data etc. produce a wide range of accuracy for the atmospheric models. Numerous studies (Gaposchkin 1987 is one example) find that no single atmospheric model is conclusively better in all applications for all satellites, and this makes sense given the limitations of the indices and satellite data.

Often, a program simply uses the available models that were programmed long ago. Implementing new models is difficult due to many older computer programming styles and practices, and so it's often easier to use the old models that already exist. For example, AFSPC still primarily uses a version of Jacchia 70 despite several advances noted in the development of newer and more modern methods (J71, NRLMSISE-00, DTM-04, etc).

III. ATMOSPHERIC MODEL USE

A. Indices

Most models use the $F_{10.7}$, K_p/a_p parameters because they are available for the longest period of time. In fact, $F_{10.7}$ measurements have been made since the 1930's. Some of the newer indices are available only after the satellite was launched (1995, etc). This limits the period of time for analysis to older data and for independent verification, but it is still relevant.

Not often discussed is the accuracy of the measurement indices. Each measurement has an uncertainty, but this is virtually never addressed. Coupled with this fact is the missing data points which occur occasionally. The Sun hasn't gone out for a day, but there are cases, usually in the past, where zero solar flux values are reported. We have corrected those points in the EOP and Space Weather files on CelesTrak by doing a simple linear interpolation between the adjacent points. As a minimum, this should mitigate the impact of having a zero input.

To further complicate the accuracy issue, there are ***observed*** and ***adjusted*** values for some of the parameters – a process that adjusts the value to one that would be recorded at an average Earth-Sun distance, or to simply use the observed value that varies during the course of the year due to the Earth's distance and orientation from the Sun.

$$F_{10.7(obs)} = \frac{F_{10.7(adj)} AU^2}{r_{\oplus-\odot}^2} \tag{6}$$

The published data does not always match Eq 6 (see Fig. 4), and the differences can be quite large.

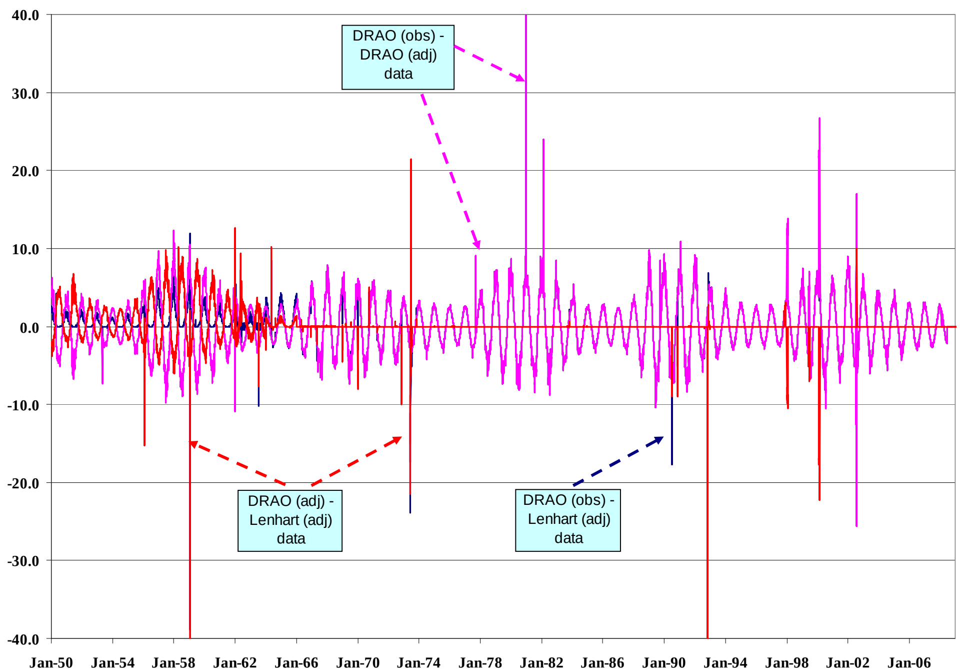


Figure 4. Difference of Observed and Adjusted Solar Flux Values. The solar cycles are evident in the observed minus adjusted solar flux differences, along with the seasonal variations that drive the original variations over time. The data spikes occur in both observed and adjusted values, and in the DRAO and Lenhart data for a variety of reasons including [erroneous] multiple Sun-Earth distance corrections.

Jacchia and most other atmospheric models use the adjusted values, published by NGDC:

ftp://ftp.ngdc.noaa.gov/STP/GEOMAGNETIC_DATA/INDICES/KP_AP/

MSIS is intended to use the actual observed data, and not the adjusted value at 1.0 AU.

ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SOLAR_RADIO/FLUX/DAILYPLT.OBS

Gathering the data for a program can be challenging. Vallado and Kelso (2005) developed and automated a process to automatically combine the several sources of data into a single file that is constantly updated to produce the current historical and predicted values. CelesTrak provides this resource. The files are updated continuously, and completely free for use. Documentation is included on the assembly. This single format is very useful for operational programs needing constant updates as they are published, but not wishing to expend the resources to assemble the data.

http://www.celestrak.com/SpaceData/

The primary quantity, $F10.7$, has been used as a proxy for the EUV radiation for many years. Several new indices have been under consideration for the past few years—Mg II (from NOAA and other satellites at about 280 nm), EUV (from the NASA/ESA SOHO satellite at the Lagrange point at 26-34 nm), etc.—but none has been unequivocally proven better than another and established as a new leader that will dramatically improve the accuracy of satellite operations. New parameters have begun to appear with the advent of satellite based sensors. Some of these data are available (some must be purchased) from the Space Environment website.

http://www.spacewx.com/index.html

Many of the newer indices are available only back to 1997, they lag the current time by about a month, and they contain no predictions. The data has the observed $F_{10.7}$, centered 81-day average, S_{10}, S_{10} 81-day centered average, Mg_{10}, Mg_{10} 81-day centered average, Ssrc. There are several costs, registrations, and update intervals are limited to those outside the AF.

B. Predicting Solar Flux

Solar flux receives a lot of attention because it is an important parameter in determining atmospheric density. Moreover, predicting the $F_{10.7}$ values into the future poses significant challenges, and introduces significant uncertainty into operations requiring accurate forecasts of atmospheric behavior. Schatten and Sofia (1987) and Schatten (1988) have developed a monthly estimate of $F_{10.7}$ and a_p. Notice the dramatic changes during the course of a single solar cycle.

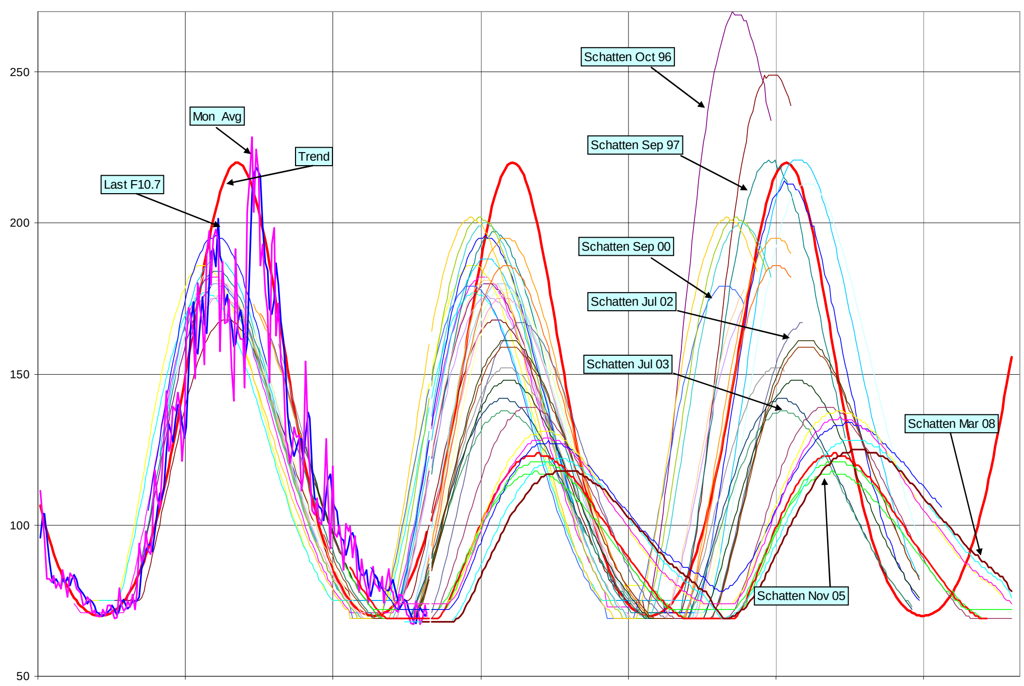


Figure 5. Recent Solar Flux Predictions. Several Schatten predictions of solar flux are shown along with the polynomial trend. Notice that each Schatten prediction covers about two solar cycles and the latest predictions suggest a lower solar max for cycles 24 and 25.

Figure 6 examines a shorter span of time and current data. The variability of the predictions is readily apparent. Most predictions rely on the solar sunspot activity. This has been routinely monitored since the 1700's. A relation exists between the sunspot number, R, averaged over a month or longer, and $F_{10.7}$.

$$F_{10.7} = 63.7 + 0.728\, R + 0.000\,89\, R^2 \tag{7}$$

The polynomial is intended only to match the last few solar cycles, assuming the next will not change dramatically from the previous. In the next equation, t is the number of days from January 1, 1981.

$$F_{10.7} = 145 + 75\, \mathrm{COS}(\;0.001\,696\, t + 0.35\, \mathrm{SIN}(0.000\,016\,95)\,) \tag{8}$$

While no claim is made to its' accuracy, it offers an alternative for programs requiring some estimate of solar flux into the future. The best course of action is to design for a more rigorous solar flux period, and if the actual is much less, the lifetime of the satellite will likely be longer. Fine tuning the fuel budget based on an overly conservative solar flux estimate could shorten satellite lifetime unnecessarily.

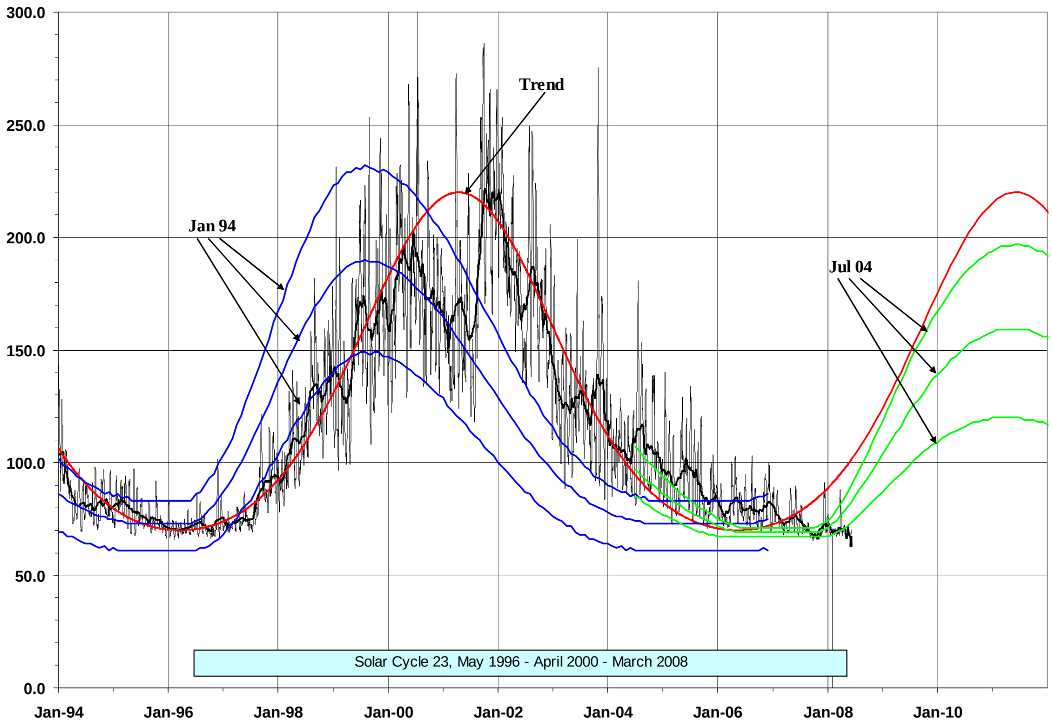


Figure 6. Recent Solar Flux Predictions. Two Schatten predictions of solar flux are shown along with the polynomial trend. Notice that each Schatten prediction has a minimum, medium, and maximum prediction that vary quite a bit. They also suggest a lower solar max for cycles 24 and 25.

Recognize that these predictions are at the macro-level. To properly determine the 81-day averages for a practical orbit propagation scenario, one needs the detailed predictions for 50-60 days in advance (depending of course on the length of the propagation). Thus, a monthly average is merely a guide.

As part of their work to maintain and conduct mission operations for the European Space Agency (ESA), the European Space Operations Center (ESOC) has developed a prediction service for solar flux. ESOC uses *P*re*d*iction of *Fl*ux and *Ap* (PDFLAP) (for short-term forecasts) and (SOLMAG) (for long-term forecasts). The programs were developed in the 1990's by the British Geomagnetic Service BGS/Edinburg under ESA management PDFLAP looks at the traditional $F_{10.7}$ daily values and the daily a_p indices, for about one solar rotation period in advance. The resulting indices are then used with MSIS-like atmospheric models, and the results indicate they perform quite well with observational data. The ESA Space Debris Office have embedded PDFLAP and SOLMAG in automated programs to upload recent NOAA data, perform daily short-term forecasts, and to calculate long-term forecasts each month.

Another interesting approach is given by Oltrogge and Chao (2007). Their approach for extrapolating and interpolating atmospheric density is based on adjusting or modifying the density values, not the proxies. The rationale is that the dependence of density on the proxies is nonlinear; hence averaging or interpolating proxies does not yield correct average or interpolated density. They were examining the effect of solar flux prediction estimates on satellite lifetimes and noted that most orbit lifetime prediction errors were caused by (1) the unreliability of current 'mean' solar activity predictions as compared to actual activity variations; (2) non-availability of predictions more than one solar cycle away; and (3) nonlinear density as a function of solar and geomagnetic activity. To circumvent these obstacles, they used the complete set of solar and geomagnetic data available (from Feb 1947). Thus, five solar cycles were used and combined into a single cycle of 10.82546 years (3954 days). Rather than attempting to produce a single mean solar cycle, they chose to directly use the solar flux and geomagnetic values from the previous cycles. Essentially, the technique creates a day of solar and geomagnetic data (a combined triad of values including the $F_{10.7}$ 81-day average) from one of the five cycles at random, and does so for each day of the simulation. Thus, the existing data is used to construct a future solar cycle, while retaining all the variability and interrelations of the existing data. Their resulting solar and geomagnetic activity computer code 'tdt2f10ap_60yr' is available upon request from the author (Oltrogge@1EarthResearch.com) for users wishing to explore the model in greater detail. This technique is certainly worthy of additional analysis and use as it avoids the vagaries of other prediction techniques that appear to vary widely over just a single solar cycle.

The bottom line for prediction is that it possibly represents one of the largest contributing factors to differences in atmospheric modeling. Solar flux can easily vary by 30-50 SFU which, as we saw in Fig. 8 (the constant cases), represented a difference of about 1 to 10 km. Not including the proper geomagnetic indices (the adjusted daily case), also from Fig. 8, indicated a potential for differences of about the same amount. Furthermore, if a satellite was designed in 1994 with then available predictions, its lifetime would be significantly different one solar cycle later as the new estimates for the next solar max are nearly 120-150 SFU less than the previous. This represents a difference as large as the mean solar max to solar min difference. Of course, here the estimate was downward, and satellite lifetime would likely be lengthened. However, a prediction that was too low would have dramatic differences on satellite lifetime.

C. To interpolate or not?

The question evokes considerable discussion and is usually centered on the geomagnetic indices, but also applies to the solar flux and other parameters. Some models (NRLMSISE-00 for instance) even suggest that they were developed to use the step-function values of the 3-hourly geomagnetic indices (Picone et al. 2002), although they did use the daily a_p during "quiet" periods ($a_p < 10$) and then switched to the 3-hourly a_p during "active" periods ($a_p > 50$). Because the space weather data is provided at discreet time intervals, it's reasonable to imagine that one should interpolate the indices to provide a smooth data input. This causes two problems. First, some researchers do not feel that interpolation should be done due to the original definitions of the parameters. In addition, some models state that they were developed for use without any interpolation. Notwithstanding these statements, nature seldom operates in a digital fashion, rather operating in a continuous analog manner. The rates of change can be quite rapid, but common sense indicates a continuous representation should better replicate the actual environmental conditions under consideration.

To further support the idea that interpolation is all but required for atmospheric models, consider the remaining processing of a model such as the Jacchia models. Every parameter, temperature, and density is formulated with a polynomial to represent the tabular observed or calculated parameters. If continuous functions are necessary (sufficient) for these parameters, they should be for the input indices as well. Jacchia (1971:38) states *"all these equations give only a smoothed version of the real variations, for which no model exists … Even in this case, however, the formulas should not be used with the original 3-hourly K_p indexes but rather with a smoothed K_p index in which the smoothing is commensurate with the resolution of the observed data".* Tanygin and Wright (2004) showed how not interpolating the input parameters caused a failure of the McReynolds Filter-Smoother test.

The conclusion implies the need for interpolation, but which method of interpolation? Vallado (2007:555-557) shows a cubic spline interpolation that recovered the data at each measured point. This should be adequate for most applications. However, be aware of the differences that alternate interpolation techniques can insert into the propagation or orbit determination results.

We summarize our current understanding of the possibilities:

- K_p is a set of discrete categories defined by Chapman and Bartels (1940). As such, they should not be interpolated. In addition, atmospheric models should ideally use a_p as the input because it affords additional sensitivity not available in the K_p scale.

- a_p values should be interpolated. The cubic spline routines discussed in Vallado and Kelso (2005) are recommended.

- For models using the K_p index, the ideal method of operation is to interpolate the a_p values, and then select the discreet K_p value. However, this is not usually feasible, thus interpolation of K_p is an acceptable variation.

- Programs should strive to use the newer Northern and Southern geomagnetic proxies (a_n and a_s respectively) as they represent a more rigorous approach. The older a_p values are still valid for historical continuity.

- The options for using a_p or K_p (and $F_{10.7}$) should be:

  - Daily: Just the daily values are used without interpolation. All 3-hourly values are ignored.

  - 3-hourly: Just the 3-hourly values are used. The daily values are ignored and there is no interpolation. This will produce step function discontinuities for both solar flux and geomagnetic values.

  - 3-hourly interp: This could use the cubic splines, but may use other interpolation techniques. It should produce the smooth transitions from one time to the next while preserving the discrete values. The measurements should reproduce exactly at the measurement times (e.g., 0000, 0300, 0600 UTC), and be smooth in between. Both solar flux and geomagnetic values are interpolated.

D. Timing
Most atmospheric models use a 3-solar rotation (81-day) average for the solar flux. The technical descriptions usually specify a centered 81-day average. However, this is often practically difficult as the predicted values of space weather parameters are not particularly well known. Thus, programs should have an option to use either the last $F_{10.7}$ 81-day average, or the centered 81-day average. In either case, the differences can be quite large depending on which data is used. Atmospheric density model descriptions generally cite a centered average, but this is impractical for many operational systems, and a trailing 81-day average is often used.

The time of the space weather measurements are also relevant, particularly the solar flux. Program codes should treat all $F_{10.7}$ measurements at the time the measurement is actually taken. The offset (2000 UTC after 1991 May 31, 1700 UTC before) should be used with all $F_{10.7}$ and average $F_{10.7}$ values. Any model specific "day before," "6.7 hours before," etc., should account for this offset. This can be a km-level effect. Note that some organizations may not use these time constraints, and therefore require a different time offset. It's not a good idea to introduce unnecessary confusion by ignoring the reality of a timed observation.

There is also a lag time for the density to increase after a particular index changes. The a_p (K_p) lag values are often fixed to 6.7 hours, but other times have been proposed. Software should accommodate future changes to lag times without the need to re-compile.

E. Tests
A series of tests were run to determine the variability of different atmospheric models for a given satellite using a single flight dynamics program, and the differences resulting from the diverse treatment of the input solar weather data. The state vectors, epoch, *BC*, and solar radiation pressure coefficient ($m/c_r A_{sun}$) were held constant for all runs. The baseline used the Jacchia-Roberts atmospheric model. The simulations were run during a time of "average" solar flux (January 4, 2003, $F_{10.7}$ ~ 140). Minimum solar flux periods ($F_{10.7}$ ~ 70) will show little difference. Maximum periods ($F_{10.7}$ ~ 220) will show much larger excursions. Figure 7 shows the results for JERS (NORAD satellite #21867 in a Sun synchronous about 600 km altitude orbit). Additional runs were performed with different satellites and as expected, the results were larger for lower and more eccentric orbits.

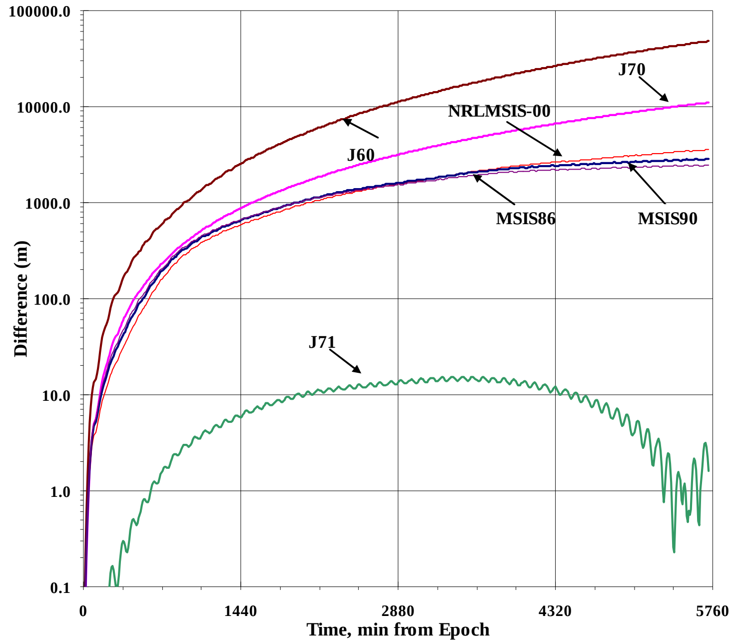


Figure 7: Sample Atmospheric Drag Sensitivity: Positional differences are shown for satellite 21867. Jacchia-Roberts is the baseline for all runs with 3-hourly values. The graph shows the variations by simply selecting different atmospheric models.

If we select a single atmospheric model, Jacchia-Roberts-71 for instance, we can also evaluate how the propagation is affected by treating the data differently. We chose some generic parameters since this is only a simulated case. The interpolations addressed by using either the daily values, the 3-hourly values with no interpolation, or the 3-hourly values with spline and polynomial interpolations. The observed and adjusted parameters are also tested.

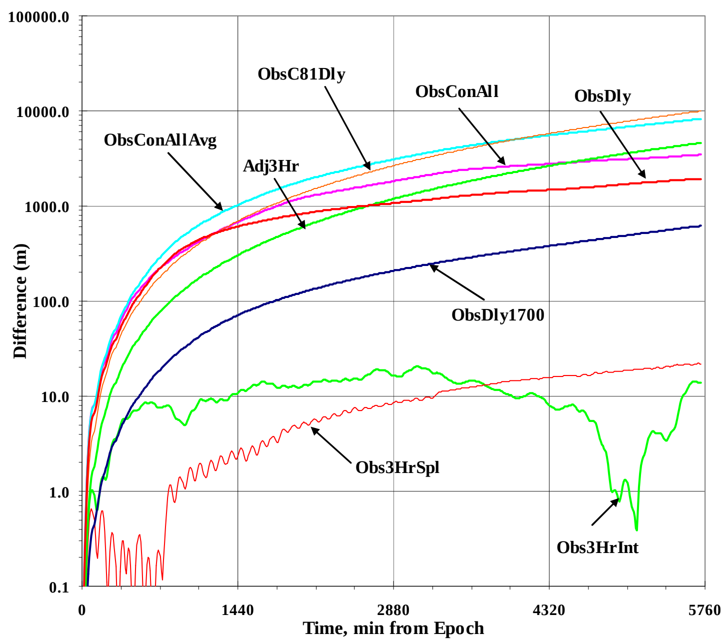


Figure 8: Sample Atmospheric Drag Sensitivity: Positional differences are shown for satellite 21867. Jacchia-Roberts is the baseline for all runs with 3-hourly values. The graph shows the effect of various options for treating solar weather data. Specific options are discussed in the text. Note that the scale is the same as Fig. 7, the relative effect of different models and solar data options are about the same, and any transient effects quickly disappear as the effect of drag overwhelms the contributions.

From Fig. 8, note that simply changing the solar flux to be a constant – a difference of only a few SFU's – causes nearly a 10 km difference at 4 days. This indicates that the prediction of solar flux is very important, and if it differs by more than just a few SFU's, can have dramatic effects on the results of a propagation. ***The important point to understand from Fig. 7 and Fig. 8 is that the choice of atmospheric model has as much effect on the overall results as the various ways to use the input data.*** This is remarkable as many organizations "go to the mat" for a particular model. However, based on how they use the input data parameters, they may actually obtain worse results than had they simply used the input parameters differently!

To summarize, the major error sources in using atmospheric models are listed below (note that density, *BC*, etc., are not listed as they are derivative effects from the items listed below). This list is generally ordered in decreasing magnitude of effect, although the exact effect will differ over different orbital regimes and solar conditions.

- Using predicted values of $F_{10.7}$, K_p, a_p for real-time operations
- Not using the actual measurement time for the values ($F_{10.7}$ in particular at 2000 UTC)
- Using step functions for the atmospheric parameters vs interpolation
- Using the last 81-day average $F_{10.7}$ vs. the central 81-day average
- Using observed or adjusted space weather parameter values
- Using undocumented differences from the original atmospheric model definition
- Not accounting for [possibly] known dynamic effects – changing attitude, molecular interaction with the satellite materials, etc.
- Inherent limitations of the atmospheric models
- Use of differing interpolation techniques for the atmospheric parameters
- Using approximations for the satellite altitude, solar position, etc.
- Using a_p or K_p and converting between these values
- Use of $E_{10.7}$ vs $F_{10.7}$ in the atmospheric models (this is not well characterized yet)

IV. SATELLITE PARAMETERS

A. Mass

Satellite mass is probably the easiest parameter to chose/set in calculations because it is either known by the owner operator, is constant because there is no thrusting capability on the satellite, or is unknown because the data is not released. The latter category presents additional difficulties and will not be considered directly here, noting that the general case for which satellite information is not known is handled via the general solution case in which the entire ballistic coefficient (*BC*) is solved as a free parameter.

B. COEFFICIENT OF DRAG

The coefficient of drag was investigated in the early 1960's (Kork, 1962, Sentman, 1961) and tested mainly for use in the developing ICBM programs. Some work continued but not much work applied to satellites. Gaposchkin (1994) assembled a wealth of information about the early research, and coupled it with his own knowledge and research into atmospheric drag. He summarizes many things:

- c_D The coefficient of drag is related to the shape, but ultimately a difficult parameter to define. Gaposchkin (1994) discusses that the c_D is affected by a complex interaction of reflection, molecular content, attitude, etc. It will vary, but typically not very much as the satellite materials usually remain constant.
- Plots of c_D vs altitude are important. From the historical literature, note the limited altitudes in the plots. One could assume the graphs increase without changing, but the very nature of c_D and it's interaction of materials plus atmospheric constituents suggest that may be an incorrect assumption.

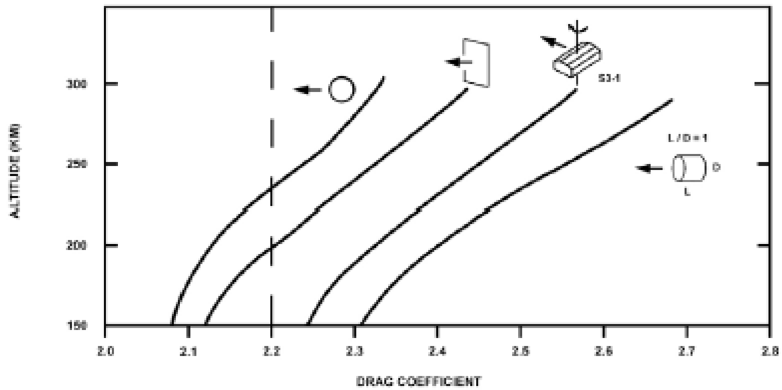


Figure 9: Coefficient of Drag Values. Sample cD values from Sentman (1961).

Ben Graziano (2007) recently examined the principal approaches to modeling aerodynamic drag across the low Earth orbit regime. Figure 10 (Graziano, 2007) delineates the major approaches to determining aerodynamic drag across low Earth orbit altitudes (or correspondingly large ratios of mean free path to satellite characteristic dimensions).

The key discriminator is the magnitude of random, thermal velocities relative to the bulk velocity of the medium, or, equivalently, the orbital velocity of the satellite. In the hyperthermal regime, random motion is small compared to bulk directed velocity. This prevails over much of the low Earth orbit regime. Continuum flow in which thermal velocities are large compared with directed motion applies only at very low altitudes, calling further in question using the simple drag formula. However, the ratio of thermal speed to bulk velocity does not vary monotonically with altitude.

At high altitudes, the residual atmospheric gases separate into strata according to molecular mass. Gaskinetic temperatures increase with altitude due to absorption of highly energetic solar radiation by the small amount of residual oxygen. Temperatures are highly dependent on solar activity. The few particles of gas in this area can reach 2,500°C (4532°F) during the day. Better said, the random velocities of gas particles can be greater than the orbital velocity, because the statistical definition of temperature fails. This region is called the thermosphere. It begins about 90 km above the earth. The upper reaches of the near Earth orbit regime may be sub-hyperthermal. Marcos et al. (2007) and others have recently reported Thermospheric cooling, which impacts orbit lifetime for many satellites.

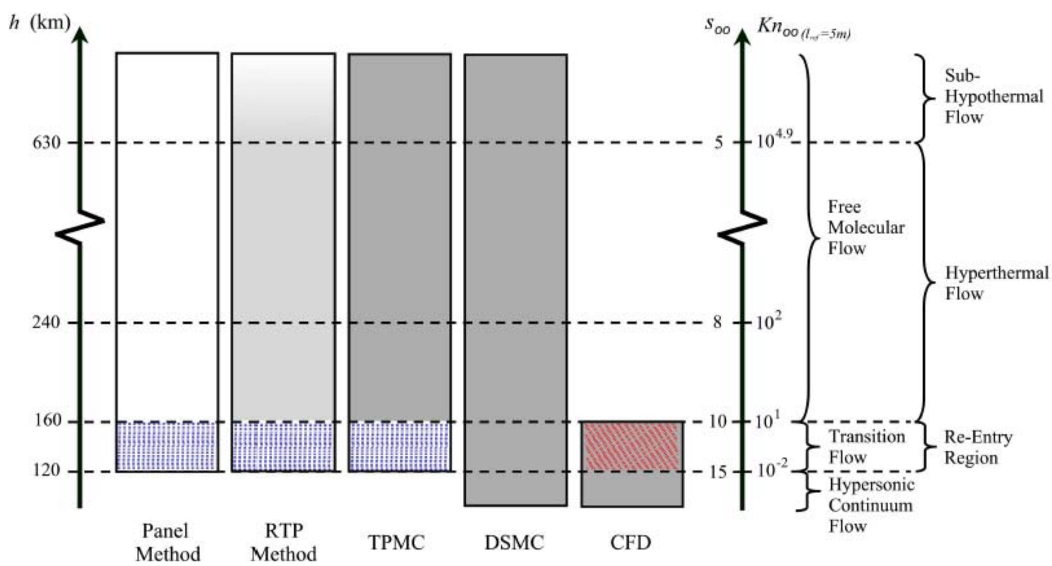
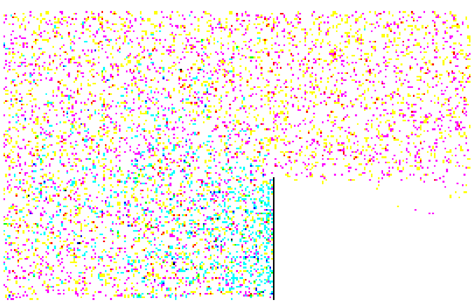


Figure 10: Approaches to spacecraft aerodynamics. (Graziano, Ref 3)

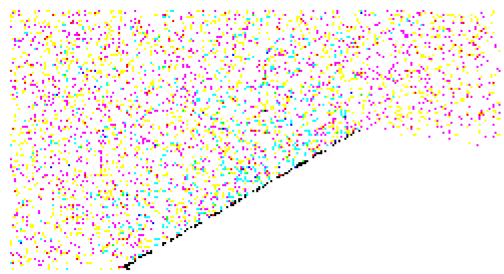
Panel methods decompose the spacecraft into discrete flat facets and model the interaction of gas particles with the surface. The Ray Tracing Panel (RTP) method follows the trajectories of scattered particles so that collisions with other satellite surfaces may be included. Test Particle Monte Carlo (TMPC) fires at the surface aggregates of many molecules with like characteristics upon which a drifting Maxwellian distribution is imposed. This also accounts for multiple encounters with satellite surfaces but not of particles with each other. The most widely applicable approach is Direct Simulation Monte Carlo (DSMC) in which the laws of physics are applied to a statistically significant number of individual particles. The particles can interact with each other according to a prescribed interparticle potential function, which includes elastic, billiard ball collisions. The particles can exchange momentum and energy (including that in internal degrees of freedom) with each other and with the surface. Computer Fluid Dynamics (CFD) includes a spectrum of mathematical and numerical techniques to solve the continuum Navier-Stokes equations, the existence of individual particles need not be considered.

There isn't much data to compare these theories with, although some missions require very precise knowledge of non-conservative forces. For example, GRACE uses two identical satellites in the same orbit and a precisely characterized distance apart to recover gravity variations. If the satellites have identical drag characteristics, differences in the orbits at the same sub-satellite point must be due to changes in gravitation.

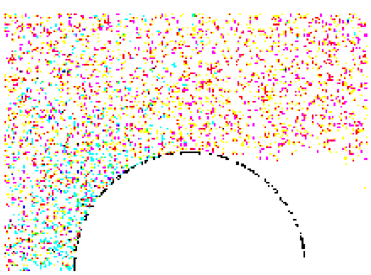
All of these techniques require hypotheses about the interaction of the gas with solid surfaces. For example, in profound continuum at low altitudes, the gas is assumed to "stick" to each surface. There can be no relative motion between the surface and the gas next to it. When the medium is less dense, there can be slip between the gas and the surface. The surface interaction depends on the degree to which the gas "accommodates" to surface conditions. The accommodation coefficients depend on the surface material and the condition of the surface, for example, the scale of surface roughness. These properties vary widely among materials and with the events that a surface experiences during its lifetime. Gas particles might adhere to a surface during flight, or the surface might grow hot, and its surface characteristics change during its mission lifetime. Gaposchkin (1994), among others, explain most used approaches to determining aerodynamic forces in rarefied flows including surface accommodation characteristics. Ultimately there is great uncertainty.



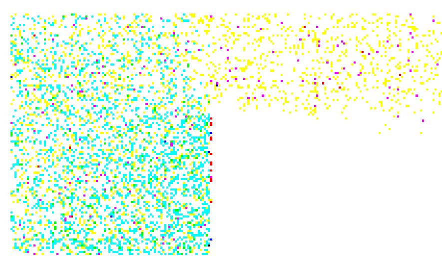
Circular Disk: $c_D = 3.05$



30 degree Cone: $c_D = 2.54$



Sphere: $c_D = 2.67$



Vertical Flat Plate: $c_D = 3.04$

Figure 11: DSMC simulations, M=10, Kn=1000, 10 collisions required for relaxation of internal degrees of freedom, surface temperature = free stream stagnation temperature. Red particles are unaffected by the presence of the sphere. Blue particles have collided only with the surface. Yellow particles have experienced multiple interparticle collisions. All objects have the same cross-sectional dimension. Over millions of test particle there are only hundreds of collisions.

The simulations in Figure 11 illustrate the variation of predicted drag with velocity, altitude, and surface characteristics. We chose a warm surface as though it were heated by the Sun. Note that the drag coefficient is not 2.2, as commonly assumed, but it is in the range 2-4 predicted by the simple continuum formula.

It is very important that density, temperature, and Mach number cannot strictly be defined under such rarefied conditions. Drag does not necessarily scale as velocity squared or linearly with area. For example, there is little difference between a circular disc with finite area and a vertical flat plate with infinite or undefined area because there are few collisions between particles that have encountered the surface either with each other or with the oncoming flow. It is better that one calculate the relevant forces and moments and employ them directly.

This is what the most comprehensive and significant operational analyses do. The ESTEC model suite developed by Fritsche and others, exemplifies complex, careful analysis of aerodynamic (and radiation induced) forces on satellites (Koppenwallner and Fritsche, 2006). The ESTEC Rarefied Aerodynamic Modeling System for Earth Satellites (RAMSES) model uses the TPMC approach and accommodates surface shadowing. Klinkrad and Fritsche (1998) has used RAMSES incorporated in the more comprehensive Analysis of Non Gravitational Accelerations due to Radiation and Aerodynamics (ANGARA) formalism for radiation and kinetic momentum transfer to estimate forces and moments on ERS-1 and ENVISAT . The magnitudes of the forces are consistent with previous estimates in this paper, on the order of 10^{-5} Nt.

There is a sound body of literature and research for estimating satellite drag and other non-conservative forces. The most important analyses predict forces and torques that affect operational satellites once the orbit is known. This is necessary for spacecraft design and stabilization. For example, satellite resources must be able to accommodate the range of anticipated torques, and propulsion systems must be sized for anticipated disturbing forces. In normal operations, satellite mass and cross sectional area can be inferred over time using simple formulas (perhaps beyond their region of validity) and the energy required to maintain or change orbits can be estimated well. We opine that few investigators have addressed the impacts of physically consistent drag on orbit estimation.

C. Area

The satellite area can be easy (spherical shape or constant area to the velocity vector), or complex (all others). The easy case is not very common within the context of the entire satellite catalog, but it nevertheless provides opportunities to investigate the variability of the other parameters. The cross sectional area changes constantly (unless there is precise attitude control, or the satellite is a sphere). This variable can change by a factor of 10 or more depending on the specific satellite configuration. Macro models are often used for modeling solar pressure accelerations, but seldom if ever, for atmospheric drag.

The most scientifically accurate approach is to input the attitude (quaternions, direction cosines, etc) into the orbit determination solution and simply account for the actual or predicted attitude, and thus the actual frontal area exposed in the relative velocity direction. Very few programs are able to accomplish this.

Clearly, the shapes of various satellites are not spherical – with the exception of a few calibration spheres. Notice the shape of the satellites in Fig. 12.

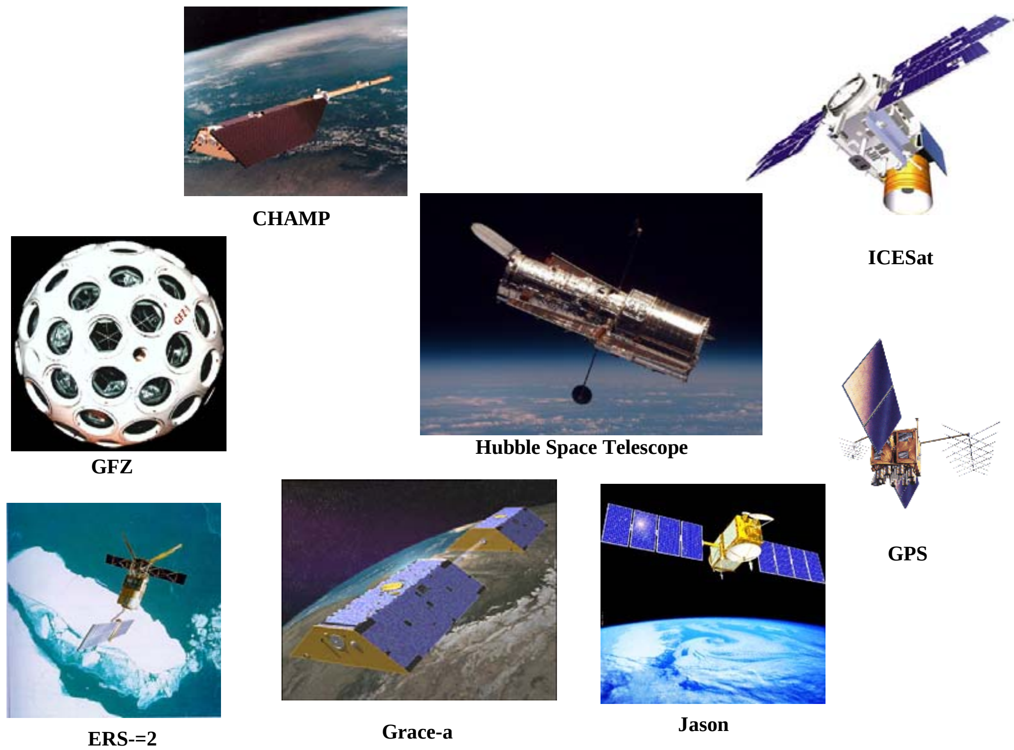


Figure 12. Example Satellite Shapes. While a few satellites have spherical shapes (GFZ), the majority do not. Some satellites exhibit aspect ratios that are near unity, but again, this is not very common.

D. Ballistic Coefficient

The previous individual parameters are not know for a majority of the space catalog. Thus, it is common to use a combined parameter which incorporates mass, area, and coefficient of drag. The ballistic coefficient (BC) is generally recognized as $m/c_D A$, although the reciprocal is used in some places. Be sure of the proper definition before using values! The ballistic coefficient *will* vary, sometimes by a large factor. Several initiatives are examining the time-rate of change for this parameter, but not looking at the variable area, and its effect in this combined factor. It's probably best not to model this parameter because it includes several other time-varying parameters that are perhaps better modeled separately.

V. ORBIT PROPAGATION

At first glance, this may appear to be similar to the next section on orbit determination. However, there is an important difference. In the orbit determination section, the filter was able to adjust the final state and parameters to match the input observations. We saw that changing the atmospheric models made a difference, but it wasn't too great. In this section, we are given a single state and parameters. We then vary the various inputs and show the results. In the absence of the OD processing, we find that the differences are magnified any times. We introduce the various options for change.

A. Initial State

The uncertainty of the initial state is critical in determining the results of an atmospheric drag perturbed orbit. However, as Vallado (2007) showed, even a perfect initial state (< 10 cm radial position) quickly degrades to many kilometers due to other factors discussed in the paper. Generic results from this study are shown in the final Table.

B. Solution Method

The Integrator type is generally a predictor corrector, or other high fidelity technique. This factor used to be more demanding than today with modern compilers and techniques.

C. Propagator

The force model fidelity is important, but most highly accurate programs have the usual full gravity fields, selection of atmospheric models, third body effects, and solar radiation pressure models. The key for atmospheric drag though is the ability to include attitude, albedo, tides (solid, ocean), and thrusting models into the propagation process. This permits a better physically representative model that leaves fewer effects unknown that would be absorbed by drag parameters. Vallado (2005) presented several plots of different satellites showing the effect of various forces on each orbit. The drag orbits dominated the effects in low Earth orbit.

D. Implementation

The selection of one atmospheric model over another evokes lots of discussion. However, it's likely that the selection is not as important as some of the other choices that can be made. From the previous section on Orbit Determination, we saw that the particular atmospheric model made a difference in the prediction accuracy. In this section, we'll examine the prediction portion in greater detail, varying many of the input parameters (See Table 1 also).

The process of taking the OD from a portion of the POE, and then predicting from that point is the most general case, and the one selected for analysis in the paper. The goal was then to examine the propagation performance using the final state from the OD filter run, and then look at different options of atmospheric models and treatment of the space weather data input. Notice how similar the results are when comparing to the POE (Fig. 13), to the previous simulated analysis (Fig 7).

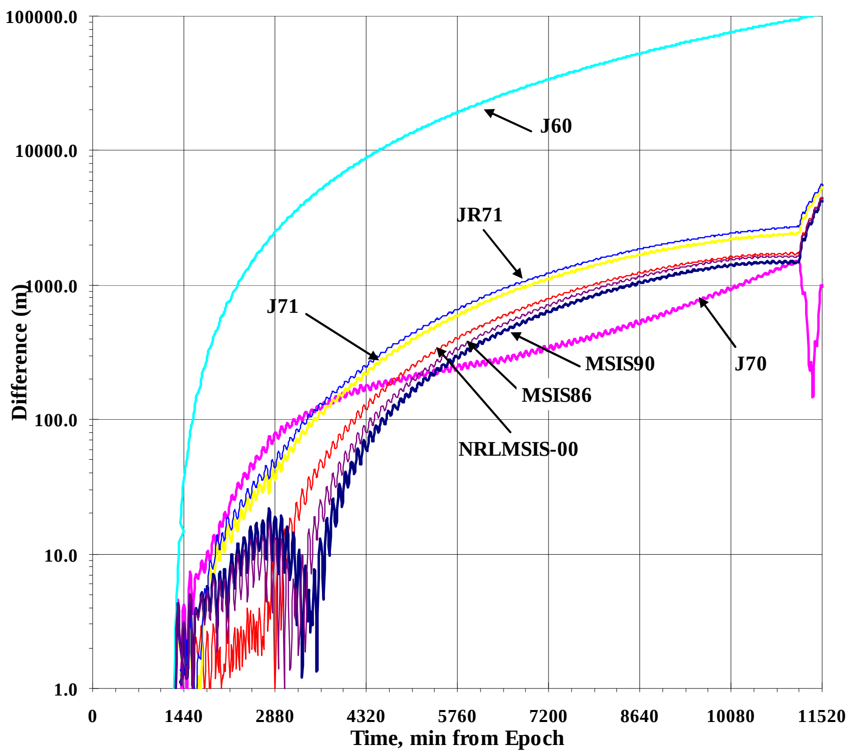


Figure 13: Sample Atmospheric Drag Sensitivity: Positional differences are shown for ICESat (satellite 27642). The POE comparison was used to evaluate each atmospheric model. Note that each of the atmospheric models used a 3-hourly geomagnetic interpolation, observed solar flux, and a centered 81-day average. The same initial conditions were used for each run.

We can also examine the effect of treating the input data differently. Table 1 shows the various test combinations, and the results are shown in Fig. 14.

Table 1: Test setup: Several combinations of treating the input data are analyzed. The labels on the left are simply short-hand to identify each case. Each letter represents a test – last or centered $F_{10.7}$ average, observed or adjusted values to the Earth's radius, and Daily, Three-hourly, Three-hourly Interpolated, or Spline interpolation for the indices. The final numbers represent the time that the $F_{10.7}$ measurement is assumed to take place – 1700 or 2000 UTC.

name	fluxfile	fluxupdate	Drag model	F10OBSTIMEOFDAY	F10MEASUREMENTTYPE	F10AVERAGEMETHOD
Obs3Hr	sw20030101L81Obs.txt	3Hour	Jacchia-Roberts	72000	OBSERVED	TRAILING81
ConAll	sw2003ObsConAll.txt	3Hour	Jacchia-Roberts	72000	OBSERVED	TRAILING81
ConAllAvg	sw2003ObsConAllAvg.txt	3Hour	Jacchia-Roberts	72000	OBSERVED	TRAILING81
LOD20	sw2003Obs.txt	Daily	Jacchia-Roberts	72000	OBSERVED	TRAILING81
LAD20	sw2003Adj.txt	Daily	Jacchia-Roberts	72000	ADJUSTED	TRAILING81
COD20	sw2003ObsC81.txt	Daily	Jacchia-Roberts	72000	OBSERVED	CENTERED81
CAD20	sw2003AdjC81.txt	Daily	Jacchia-Roberts	72000	ADJUSTED	CENTERED81
LOT20	sw2003Obs.txt	3Hour	Jacchia-Roberts	72000	OBSERVED	TRAILING81
LAT20	sw2003Adj.txt	3Hour	Jacchia-Roberts	72000	ADJUSTED	TRAILING81
COT20	sw2003ObsC81.txt	3Hour	Jacchia-Roberts	72000	OBSERVED	CENTERED81
CAT20	sw2003AdjC81.txt	3Hour	Jacchia-Roberts	72000	ADJUSTED	CENTERED81
LOI20	sw2003Obs.txt	3HourInterp	Jacchia-Roberts	72000	OBSERVED	TRAILING81
LAI20	sw2003Adj.txt	3HourInterp	Jacchia-Roberts	72000	ADJUSTED	TRAILING81
COI20	sw2003ObsC81.txt	3HourInterp	Jacchia-Roberts	72000	OBSERVED	CENTERED81
CAI20	sw2003AdjC81.txt	3HourInterp	Jacchia-Roberts	72000	ADJUSTED	CENTERED81
LOS20	sw2003Obs.txt	3HourSpline	Jacchia-Roberts	72000	OBSERVED	TRAILING81
LAS20	sw2003Adj.txt	3HourSpline	Jacchia-Roberts	72000	ADJUSTED	TRAILING81
COS20	sw2003ObsC81.txt	3HourSpline	Jacchia-Roberts	72000	OBSERVED	CENTERED81
CAS20	sw2003AdjC81.txt	3HourSpline	Jacchia-Roberts	72000	ADJUSTED	CENTERED81
LOT17	sw2003Obs1700.txt	3Hour	Jacchia-Roberts	61200	OBSERVED	TRAILING81
LAT17	sw2003Adj1700.txt	3Hour	Jacchia-Roberts	61200	ADJUSTED	TRAILING81
COT17	sw2003ObsC811700.txt	3Hour	Jacchia-Roberts	61200	OBSERVED	CENTERED81
CAT17	sw2003AdjC811700.txt	3Hour	Jacchia-Roberts	61200	ADJUSTED	CENTERED81

Using the same configuration as in Fig. 4 we obtain (Filter run uses MSIS-00, COT20) the plots in Fig. 14.

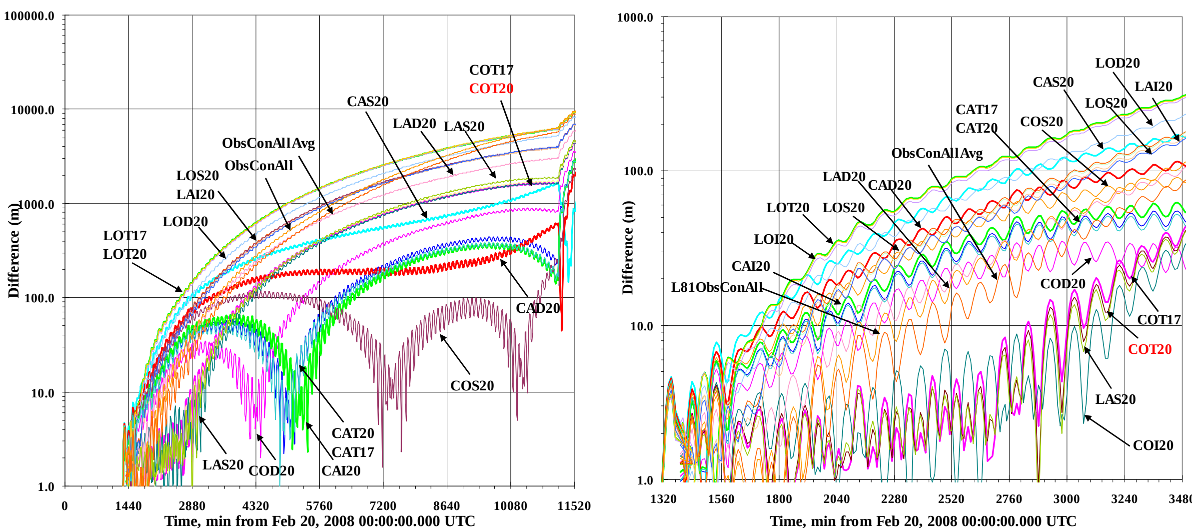


Figure 14: Sample Atmospheric Drag Sensitivity: Positional differences are shown for ICESat (satellite 27642) using various data approaches with the NRLMSISE-00 model. The POE comparison was used to evaluate each atmospheric model. Note that each of the atmospheric models used a daily interpolation on the geomagnetic indices. The same initial conditions were used for each run, but the axes scales are different.

Notice that the short term results are reasonably similar, while the longer term results show a gradual separation of the approaches. After careful study, we could conclude that the centered solar flux generally fared better, and the observed solar flux was also better in most cases. The LAS20 showed good results in the short term, but quickly degraded in the longer term.

If we use Jacchia-Roberts instead of MSIS inside the filter, we get very different results.

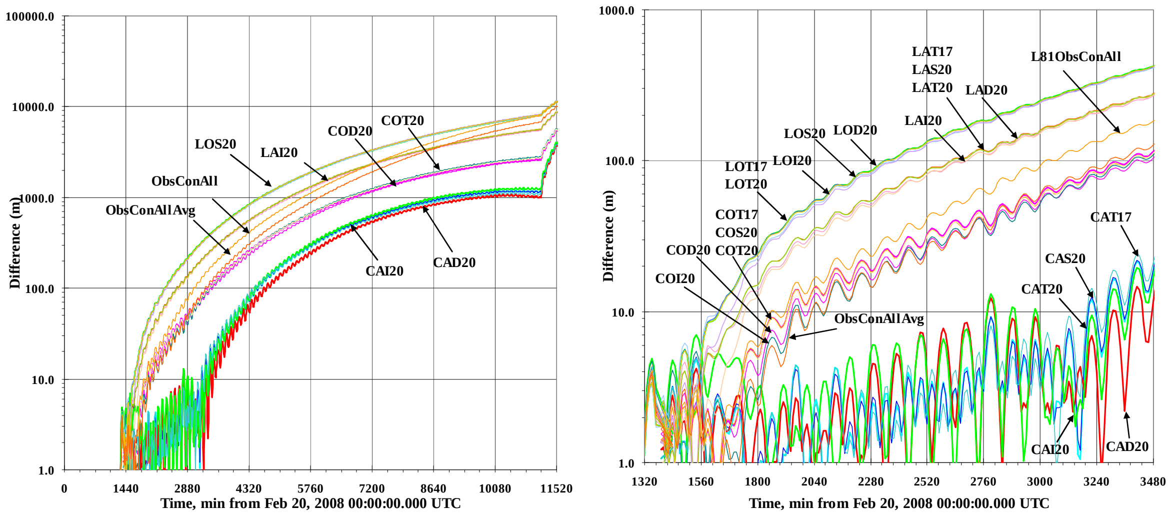


Figure 15: Sample Atmospheric Drag Sensitivity: Positional differences are shown for ICESat (satellite 27642). The POE comparison was used to evaluate each atmospheric model. Note that each of the atmospheric models used a daily interpolation on the geomagnetic indices. The same initial conditions were used for each run.

The Jacchia-Roberts results show much more definitively that the adjusted solar flux, and centered solar flux averages performed the best, in fact, they differed only by about 1 - 10 km at the end of a week. The method of interpolation made some minor differences in the short term, but were not a real factor.

The primary difference between the two models was the performance of the adjusted and the observed solar flux in the predictions.

VI. ORBIT DETERMINATION

The orbit determination and propagation areas are closely tied, but there are some distinctions. Orbit determination encompasses the primary processing of observational data to determine the orbit. As such, the particular form of processing must be very carefully selected to balance the model type, development, and satellite parameters that are available. The assumptions from the atmospheric model development are important, but often overlooked at this step. Some researchers discount the effect of these assumptions in their studies, or assume that one parameter or model will resolve many of the assumptions. Unfortunately, such considerations must be independently verified and tested to validate the appropriateness of the assertions.

In general, the orbit determination area imparts some error into the process, but it also can mask many of the parameters discussed earlier in this paper simply by its specific implementation. Often, techniques are applied to mitigate omissions in the force model, model use, model itself, type of processing, etc. These are not wrong, but if they are not coupled with as much physical reality as is known, they are at best incomplete. Thus, while results may achieve a certain level of accuracy in one program, transferring the final state to another program can give dramatically different results. This further stresses the importance of striving to use proper techniques, coding, indices, etc to achieve the best possible results. Documenting the actual practice is perhaps the most important step.

A. Observations

The accuracy we obtain from observational data is a function of many variables including the number, type, and quality of the observations in a pass over a sensor, the location of the data within a pass, the total number of tracks, number of sensors, location of the sensors, the processing technique, etc. To obtain the best accuracy, you must trade-off the merits of each of these aspects versus their costs. The Central Limit Theorem implies that more data is better, and that you will get better results with more data. In other words, the optimal estimate of a state as determined from multiple, independent observations of the same attribute will improve (in a least squares sense) the more observations there are, independent of the quality (variances) of those observations. There are of course limits to how much data can be acquired, but much of the operational space surveillance mission relies on a very few observations per pass. For limited analytical theories this is sufficient but this is simply not enough when the goal is achieving highly accurate estimates for the orbit. Experiments have confirmed that additional observations can lead to dramatic improvements (Phillips 1995). In this experiment, AFRL received dense observations directly form a sensor site and processed them with numerical techniques. The resulting vectors were accurate to about 10-15 m as confirmed by laser tests. The comparable AFSPC vector produced with a handful of observations per pass and somewhat limited force models produced a vector only accurate to about 400 m, again confirmed by laser tests.

The inherent noise in the observations makes it all that more difficult to properly use the time changing space weather parameters. Thus – if one treatment of the space weather parameters works better in an OD, is it because the process was correct, or it accounted for an inconsistency in the observational data?

B. Solution Method

Orbits are estimated by comparing observations of diverse types to physical hypotheses. The hypotheses include drag models representations. They also include gravitational hypotheses both for the Earth and other massive bodies, hypotheses about the atmosphere, solar radiation and light pressure, and other important phenomena. These hypotheses are generically called "processes" and their uncertainty is "process noise." The state of an object is propagated forward from a previous state (*a priori* state) with an error state transition matrix that represents the influence of each state variable and control on all of the state variables. The error state transition often includes a control matrix that operates on a vector of exogenous control inputs to influence deterministically the future state. The propagation depends on observations of existing states through a measurement model which includes uncertainties in measurements (measurement noise). Minimizing the difference between predicted and observed states in a least squares sense with respect to the independent variables in the hypotheses creates a gain matrix or weighting which, when applied to the difference between predictions and observations updates or corrects the estimates of future states. The error covariance matrix appears naturally in the mathematics, and it must also be propagated into the future to quantify uncertainty in the predicted state. The covariance matrix connects uncertainty in each estimated state variable with uncertainties in the quantities that are observed and measured. This generic approach is well described in the technical literature.

The "state" that is estimated is more than just the state of physical motion. In simple conservative Newtonian mechanics the state of physical motion would be fully determined by knowing position and velocity at one point in time. This is not sufficient when there are non-conservative forces (such as drag) and parametric hypotheses. We also treat as dependent variables some of the parameters in the hypotheses, such as ballistic coefficient, and augment them to the state vector for simple physical motion. These additional state variables help us determine drag, density, and other quantities that are not directly observable but which are connected to observable quantities. The credibility of these additional state variables is only as good as the process models we created, and they represent only the dependent variables in those models.

What we described above is called "filtering." There are three classes of filters. Estimators focus on the current time using all past data. Estimators are static. Predictors estimate values ahead of the time span of current data. Smoothers estimate a past state using current data. Predictors and smoothers are dynamic. They can be employed iteratively to predict a future state and work back from that estimate to a less uncertain assumption of the current state, and so on. Filters will work with any set of hypotheses. They only reveal how well the observations match the hypotheses. The residual differences between data and hypotheses measure the quality of the hypotheses.

Consistently large residuals or covariances indicate that the hypotheses are poor, that the data is of poor quality, or both. Covariances that are consistently large may indicate that the state vector omits potentially important processes. Practitioners who are sufficiently confident that this is the case occasionally diminish the covariances by values determined through intuition rather than physics or mathematics. These values are called "consider parameters," and no corresponding states are propagated.

Similarly, very small covariances may not be reflective of the true parameters under consideration. Fundamental assumptions, such as the statistical independence of each observation, might have been violated. We believe there are some organizational centers that are experiencing this effect today.

This phenomenon is strikingly analogous to quantum mechanics. Striving to strike the connection between the probabilistic nature of quantum mechanics and the determinism of classical physics, philosophers of quantum mechanics conjectured that there are "hidden variables" that, if revealed, would restore deterministic stability. Heisenberg's Uncertainty Principle might be the covariance or residual that accommodates probabilistically the influence of hidden variables – consider parameters.

If we use the simple, continuum drag hypothesis, the quantity we determine is generally not drag. It is simply a parameter that is physically related to density by hypothesis, velocity squared, area, and mass (Eq 4). As stated earlier, the real drag can only be determined once the orbit and the space environment are known. Once we recognize this, it follows that the orbit estimates and propagation that employ this parameter are only strictly valid for the specific satellite, orbit, and environment they were determined for. Lacking a valid physical connection between these parameters and outcomes, it may not be advisable to apply those values to other satellites in other orbits under different environmental conditions.

We unequivocally recommend sound physical processes rather than correlations without physical reasoning. We also note that not recognizing the potential interdependencies of state variables and hypothetical processes leads to circular reasoning. For example, inferring density from degradation of an otherwise persistent orbit and then using that density field to infer drag is circular. Using that density field to infer drag on other satellites in other orbits, potentially with different descriptions of other orbit perturbation processes, is absolutely inconsistent.

Batch Least Squares (BLS) methods have an inherent difficulty when processing LEO satellite data – the fit span is generally a few days. Over this time, the satellite may make 40-60 or more revolutions. Averaging the observations reduces the amount of information which may be extracted for solution. Some effort has been accomplished with "track weighting" and segmented parameters in which individual tracks or parameters are processed or accounted for in the orbit determination. Although this has shown some success, it is a simplistic short cut to accurately processing each measurement.

Kalman filters avoid the ambiguities and additional required logic to account for the long fit span needed with BLS techniques. Some mention has been made about filters being unstable and requiring tuning, however, the ODTK filter-smoother combination avoids this by determining the process noise via mathematical processes based on the uncertainty of the particular force model (gravity, drag, third body, etc). This has proved extremely reliable and dependable in operational applications for almost two decades in all orbital regimes.

Maneuvers present yet another difficulty in orbit determination. BLS methods may be configured to process through a maneuver, but the usual process requires a restart. If the maneuver is long, or even a continuous thrusting application, BLS techniques will fail. Kalman filters can actually process through maneuvers and because the maneuver can be input to the process, the recovery time after a maneuver is significantly reduced, usually within a revolution of the maneuver.

C. Implementation

We can expand the earlier results and perform OD to demonstrate the previous statements. Consider a POE from 2003 with ICESat (same study interval as used in Vallado (2007). The reference ephemeris (POE) is usually accurate to about 10-20 cm radial position throughout, so we will consider that to be truth for our purposes. We can then try several options (as with the previous JERS case) and observe the effects. Using a setup of

70×70 WGS-84-EGM-96 gravity
NRLMSISE-00 atmosphere reading a_p values, 3-hourly a_p values
Mass = 950 kg and area = 5.21 m^2 (and therefore area/mass ratio = 0.0054842)
Sun and Moon third body
Solar radiation pressure (c_{srp} = 1.04) with dual cone boundary mitigation
Solid and time dependant solid tides and full ocean tides
RK 7/8 Integrator, relative error of 1×10^{-15}, 1-600 sec step sizes

We run the filter for 1 day. The Earth Fixed (ITRF) initial and final position and velocity vectors (km and km/s) from the POE are (osculating orbital elements are included for each time)

19 Feb 2003 20:59:47.000 UTCG	**814.732027000**	**-6735.434964000**	**1620.790872000**	**-0.8173538675**	**-1.8748560790**	**-7.3333578195**
20 Feb 2003 20:59:47.000 UTCG	**791.990709000**	**-4288.658540000**	**5435.575745000**	**-0.0759493638**	**-5.9813522276**	**-4.6964130956**

	a	**e**	**incl**	**raan**	**argp**	**nu**	**m**			
19 Feb 2003 20:59:47.000 UTCG	**6979.966219**	**0.001583**	**93.992**	**200.242**	**100.611**	**65.935**	**65.770**	**0.013820**	**1.04**	
20 Feb 2003 20:59:47.000 UTCG	**6969.395729**	**0.001219**	**93.995**	**200.751**	**41.976**	**86.606**	**86.467**	**0.013820**	**0.01403 (sm)**	**1.073359**

the 0.01403 bc results in .01403/.013820 * 2.52 = 2.55829 – didn't work better…

From the OD, the residual ratio and smoother position uncertainty are shown in Fig. 16.

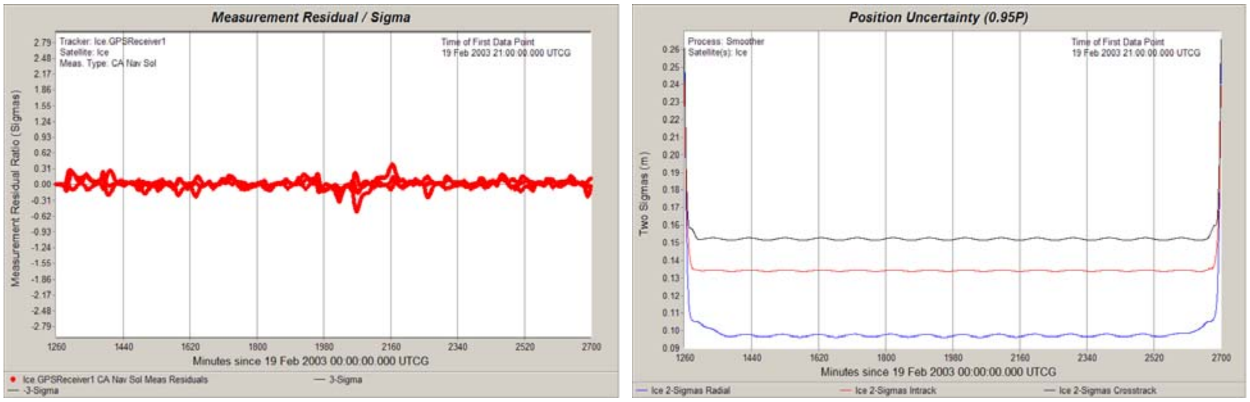


Figure 16: Filter Results for ICESat Processing: The residual ratios (residuals divided by the sigma values) and the position uncertainty are shown. Note that the filter is determining the orbit to about 15 cm.

A unique report in ODTK is the filter-smoother consistency test. The report is found using the filter and smoother runs. Essentially, the report is an instantiation of the McReynolds filter-smoother consistency test. It states that the difference of the filter and smoother runs is a zero mean Gaussian vector with a covariance given by the difference of the two covariances. The differences let you examine the transponder, station biases, etc. to determine if the filter modeling is proper. The position consistency check is shown in Fig. 17. Note that the results should all fall within the ±3 limit. If the modeling is off in any parameter, the test will exceed the 3 limit.

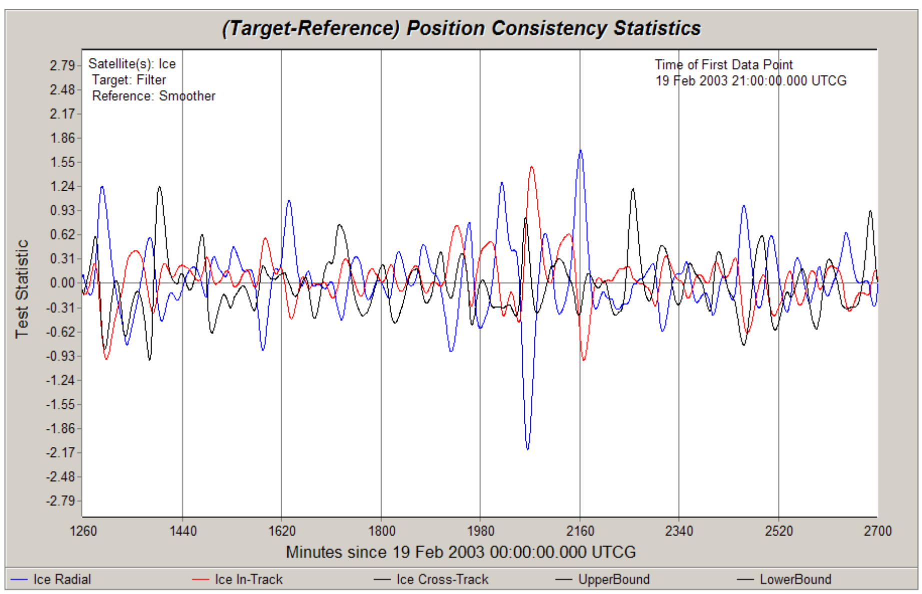


Figure 17: Filter Smoother Results for ICESat Processing: The position consistency test is shown for the day that the filter was run. Note that all the points are well within the limit of 3. This gives us added confidence that the modeling is proper.

The resulting state is from the filter file is shown in Fig. 11. The original values from the POE are given in red. Notice the proximity of the values. Although we could take the state directly from the POE and use the published force models in the development of the POE, it is more accurate to insert an additional OD into the process to ensure the force models line up exactly. Vallado (2007) showed this and we repeat it here because it's important. The inclusion in the development of the POE of 1 cpr parameters, process noise, solve for parameters, polynomial approximations, etc, in the formation of the POE make it virtually impossible to precisely line-up the force models. The results are significantly better if an additional OD is conducted. However, we compare to the original POE for maximum accuracy. It would be disingenuous to compare to an ephemeris created by our OD run, and would negate any benefit of the comparison to an independent reference source.

OD 19 Feb 2003 20:59:47.000 UTCG 814.732041294 -6735.434967609 1620.790849817 -0.8173541620 -1.8748559808 -7.3333573606
OD 20 Feb 2003 20:59:47.000 UTCG 791.990757127 -4288.658528446 5435.575681403 -0.0759502171 -5.9813519021 -4.6964129808

POE 20 Feb 2003 20:59:47.000 UTCG 791.990709000 -4288.658540000 5435.575745000 -0.0759493638 -5.9813522276 -4.6964130956

	a	e	incl	raan	argp	nu	m
OD 19 Feb 2003 20:59:47.000 UTCG	**6979.965375**	**0.001583**	**93.992**	**200.242**	**100.607**	**65.939**	**65.774**
OD 20 Feb 2003 20:59:47.000 UTCG	**6969.394971**	**0.001219**	**93.995**	**200.751**	**41.971**	**86.611**	**86.472**
POE 20 Feb 2003 20:59:47.000 UTCG	**6969.395729**	**0.001219**	**93.995**	**200.751**	**41.976**	**86.606**	**86.467**

The figure shows both the performance during the filter (the first day), and during the prediction until the time of the first maneuver (about Feb 27, 2008).

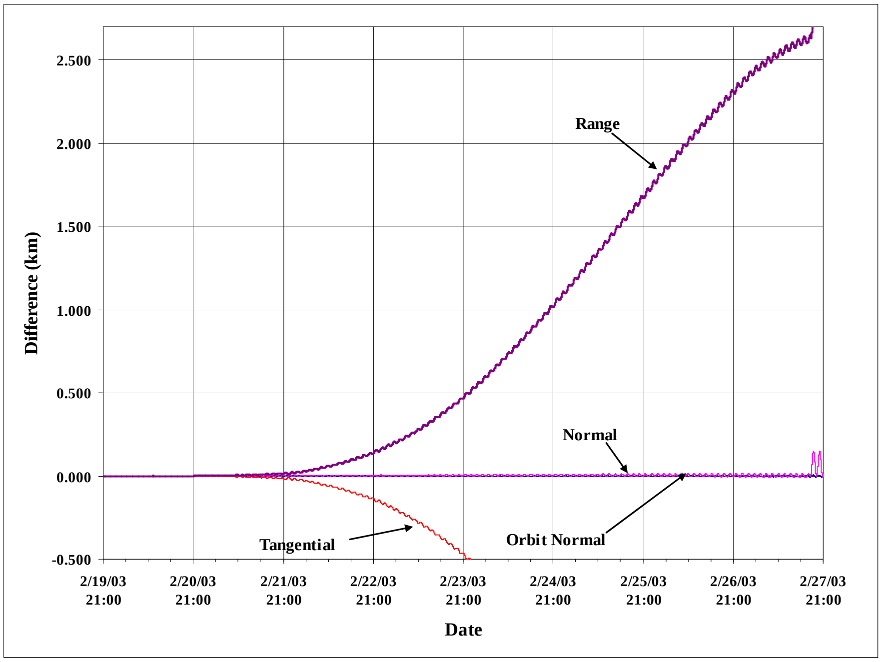


Figure 18: Prediction Results for ICESat Processing: The position uncertainty is shown for a filter-smoother run using the NRLMSISE-00 atmospheric model. The noise at the end of the span is a small maneuver in the ephemeris.

If we change the filter to use the Jacchia-Roberts atmospheric model, we get similar results, but there are differences. Figure 19 shows the results.

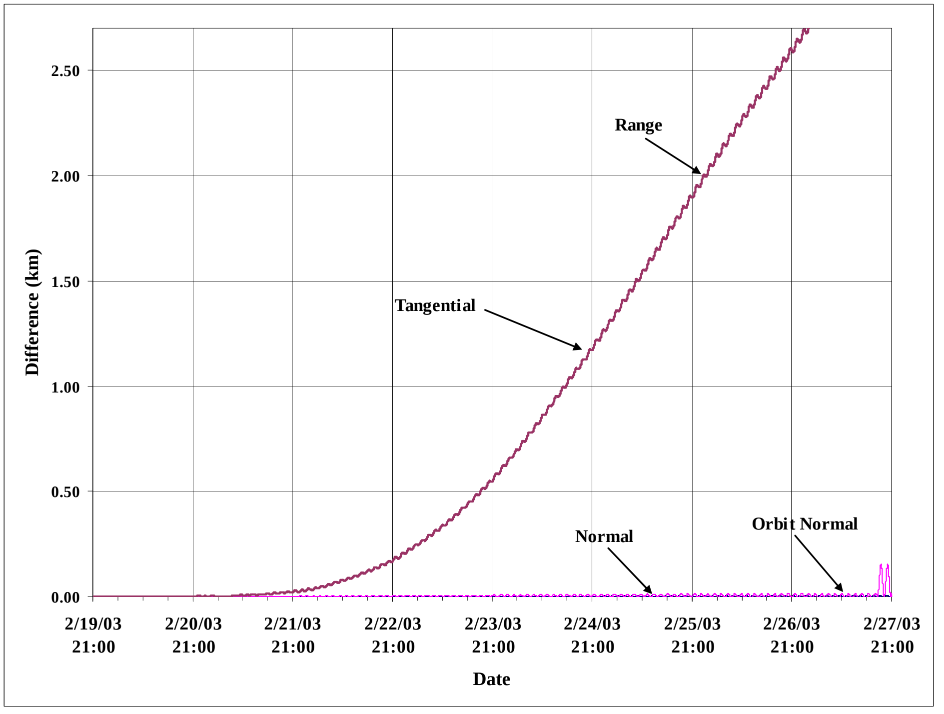


Figure 19: Prediction Results for ICESat Processing: The position uncertainty is shown for a filter-smoother run using the Jacchia-Roberts atmospheric model.

After many trials, we could find that simply by changing the atmospheric model used during the filter run, we can expect about 500 m of difference (to the POE) after about a week of propagation. While this isn't as great as some of the differences we'll see later in the propagation section, it represents yet another error source that should be accounted for in a flight dynamics program. It also follows that the effect should not be as great (switching atmospheric models in an orbit determination example as in a propagating scenario. This is because the orbit determination program is simply using the atmospheric model to estimate the density, and the OD will arrive at a slightly different solution using the different atmospheric model. That one filter state produces more accurate results than another is simply a consequence of the overall fidelity of the model.

D. Implementation using Observations

We expect that processing observations will produce the same results as the previous section, but time did not permit an adequate examination, so this will be deferred to a later presentation this year at the AGI User Conference. Interested researchers may check the www.CenterforSpace.com website for an update after that event.

VII. Summary and Conclusions

The proceeding discussion has provided numerous options for consideration in determining atmospheric drag. Perhaps the simplest statement is to state that attempting to solve one or two parameters may result in improved results, but unless that underlying assumptions and conventions are precisely known (which they are not), you simply shift the problem from one variable to another. Depending on your solution technique, this may make certain problems unobservable and give you a false outcome. We summarize the major choices and their approximate effects in Table 2 below.

Table 2: Atmospheric Drag Summary and Effects: Each of the various topics discussed is identified with a brief summary of the topic, and numerical results if available. Unless each of these characteristics are well known and appropriate, results are suspect. Note that the effects are cumulative.

	Topic	Summary of Topic	Effect
Atmospheric Model Development			
	Indices	Which indices available when developed? How were they used?	

		Span of time indices available	
	Data used in generation	Satellites were limited in atmospheric model development	
	Orbital Class	Atmospheric model is only good for certain orbital classes	
	Fidelity	Many studies, no conclusive “winner”	10-15% or more inaccuracy
Atmospheric Model Use			
	Which Model	Jacchia, MSIS, DTM, etc.	100 to 50,000 m in 4 days
	Implementation	Code implements technical approach exactly?	??
	Accuracy	Inherent inaccuracy of the model	10-15%
	Data Indices Availability	Publically available? Time span available	??
	Data Correction	Observed vs adjusted	6,000 m in 4 days
	Daily vs Hourly values	Frequency of updates to indices	3,000 m in 4 days
	Interpolation	Interpolate or not Which parameters to interpolate, or all? Method of interpolation	10-5,000 m in 4 days 3,000 in 4 days 10-20 m
	Which Index to use	Use a_p or K_p preferentially?	??
	Time lags	6.7 hours, other	??
	Time of observation	1700 vs 2000 UTC for $F_{10.7}$	1,000 m in 4 days
	Averages	Centered or trailing?	10,000 m in 4
	Prediction	Source, Schatten, ESA, Dan, Other? Variability over time Short term accuracy	120 – 150 SFU (long) Varies 1-5 SFU (short)
	Winds		??
	All previous categories		
Satellite Parameters			
	Mass	Accuracy	5 – 10%?
	Area	Time varying?	??
	c_D	Aerodynamics, gas dynamics, contium flow	??
	Model detail	Sphere, plate models, detailed CAD	??
	Attitude known	Consider a 10 m long 3 m diameter cylinder end vs side	424% difference
	Materials	Interaction to atmosphere – DSMC method?	??
	Maneuvers	Known or unknown Magnitude, direction	Can be very large 100’s of km
Orbit Propagation			
	Input state accuracy	Example 1m initial error in the position	5,000 – 10,000 m in 7 days (extrapolated)
	Integrator type		Small
	Force model fidelity	Drag force is 10 – 100 km or more effect in 4 days	Varies
	All previous categories		
Orbit Determination			
	Obs Quality	Not tested	
	Obs Quantity	Phillips Lab (1995) report	400 m at epoch
	Solution method BLS	Not tested	??
	Solution EKF		
	Force Models	Using a different atmospheric model	300-500 m in 7 days
	All previous categories		

There are three general observations that are important—the difference between atmospheric density models, treating the input data differently, and differing implementations of an approach. Each can contribute substantial differences to propagation results, and each must be considered collectively when evaluating results from a flight dynamics program.

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