RF and Microwave Transmission Lines

ELEC 2230

Section 2

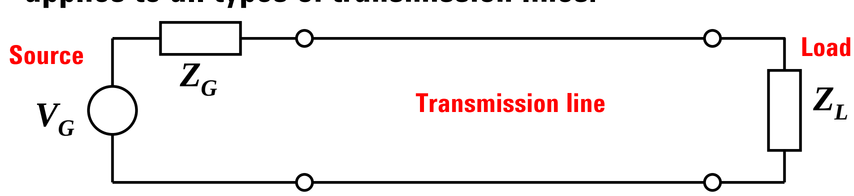
Roger D. Pollard
School of Electronic & Electrical Engineering
The University of Leeds

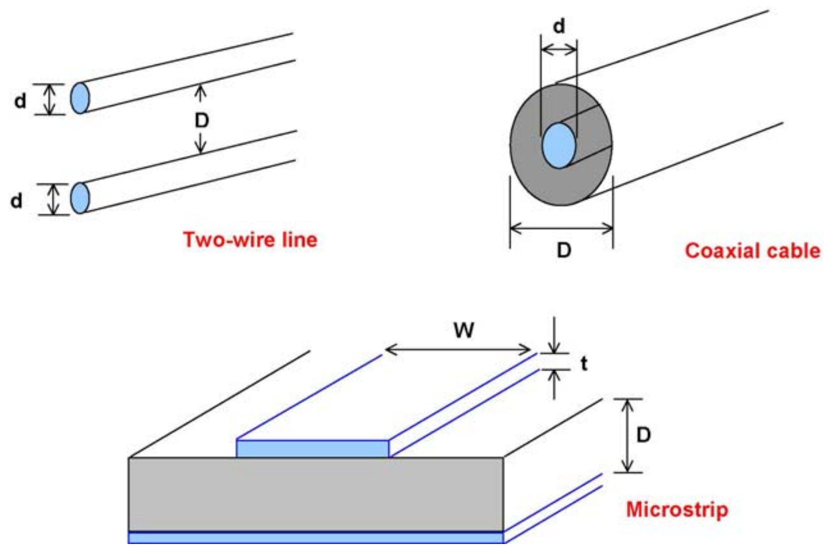
February 2004

Transmission Line Equations

A typical engineering problem involves the transmission of a signal from a generator to a load. A transmission line is the part of the circuit that provides the direct link between generator and load.

Transmission lines can be realized in a number of ways. Common examples are the parallel-wire line and the coaxial cable. For simplicity, we use in most diagrams the parallel-wire line to represent circuit connections, but the theory applies to all types of transmission lines.





If you are only familiar with low frequency circuits, you are used to treat all lines connecting the various circuit elements as perfect wires, with no voltage drop and no impedance associated to them (lumped impedance circuits). This is a reasonable procedure as long as the length of the wires is much smaller than the wavelength of the signal. At any given time, the measured voltage and current are the same for each location on the same wire.

Let's look at some examples. The electricity supplied to households consists of high power sinusoidal signals, with frequency of 50Hz.

Assuming that the insulator between wires is air ($\varepsilon=\varepsilon_0$), the wavelength for 50Hz is:

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}}{50} = 6.0\times10^{6}\ \text{m} = 6,000\ \text{km}$$

Which is about the distance between London and Washington DC

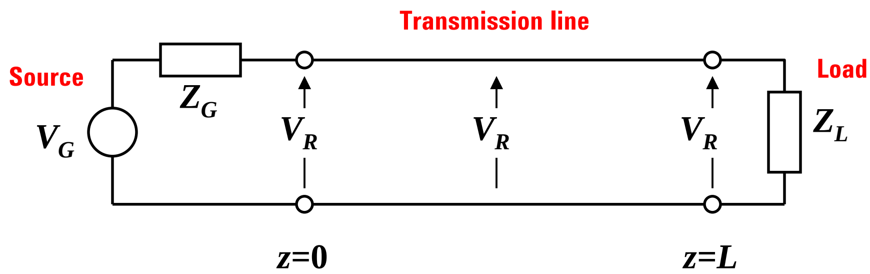
Let's compare to a frequency in the microwave range, for instance 50 GHz. The wavelength is given by

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}}{50\times10^{9}} = 6.0\times10^{-3}\ \text{m} = 6\ \text{mm}$$

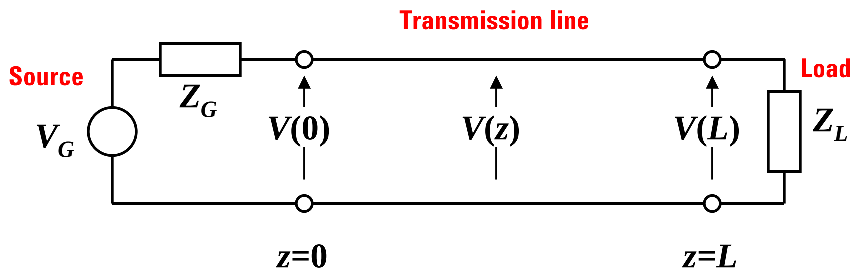
Comparable with the size of a microprocessor chip

For sufficiently high frequencies the wavelength is comparable with the length of conductors in a transmission line. The signal propagates as a wave of voltage and current along the line, because it cannot change instantaneously at all locations. Therefore, we cannot neglect the impedance properties of the wires (a distributed circuit).

$$L << \lambda$$

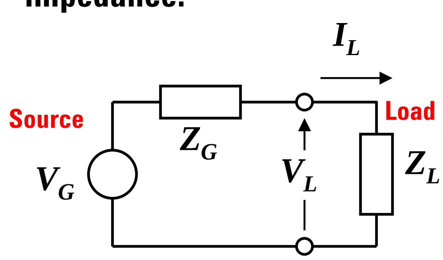


$$V_R = V_G \frac{Z_L}{Z_G + Z_L}$$



$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

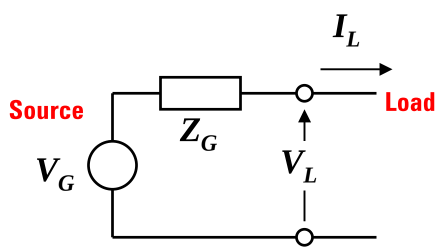
The equivalent circuit of a generator consists of an ideal alternating voltage generator in series with an internal impedance.



$$I_L = \frac{V_G}{Z_G + Z_L}$$

$$V_L = \frac{V_G Z_L}{Z_G + Z_L}$$

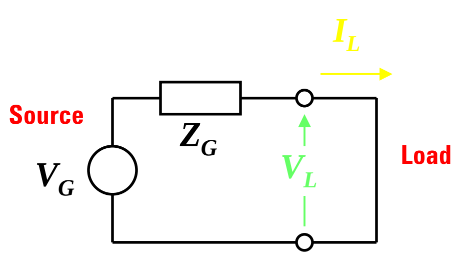
If the load is an open circuit



$$I_L = 0$$

$$V_L = V_G$$

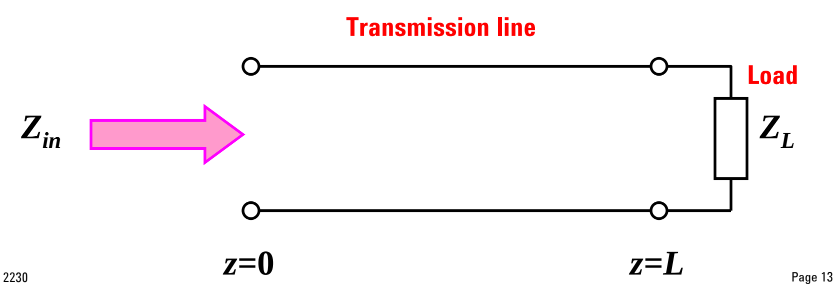
If the load is a short circuit

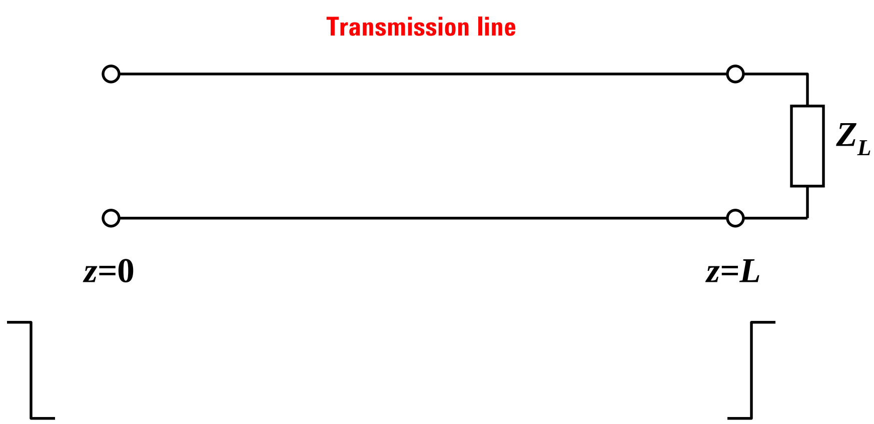


$$V_L = 0$$

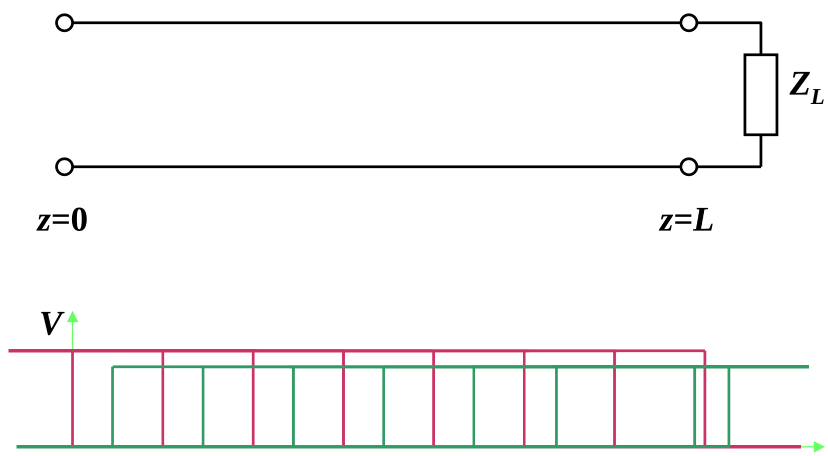
$$I_L = \frac{V_G}{Z_G}$$

Now consider the case of a voltage generator connected to a load through a uniform transmission line. In general, the impedance seen by the generator is not the same as the impedance of the load, because of the presence of the transmission line, except for some very particular cases.





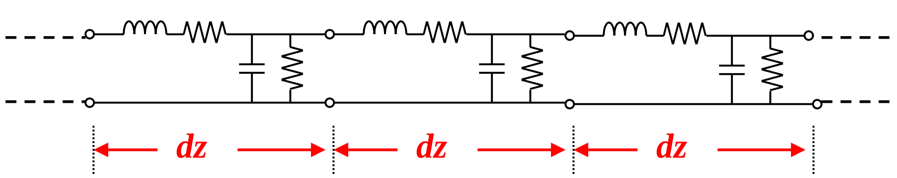
Transmission line

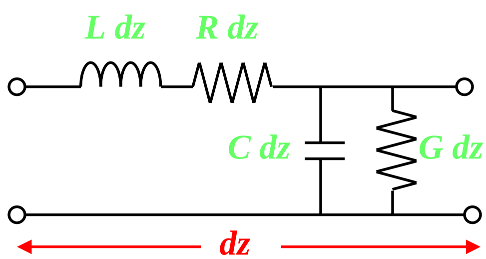


A uniform transmission line is a "distributed circuit"

It can be described as a cascade of identical cells with infinitesimal length.

The wires have series inductance and resistance. There is a shunt capacitance between the conductors, and even a shunt conductance if the medium insulating the wires is not perfect.





L = series inductance per unit length

R = series resistance per unit length

C = shunt capacitance per unit length

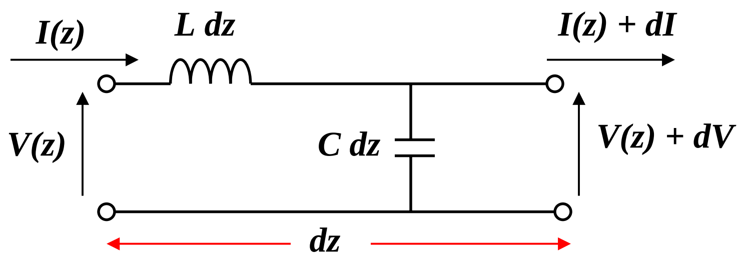
G = shunt conductance per unit length

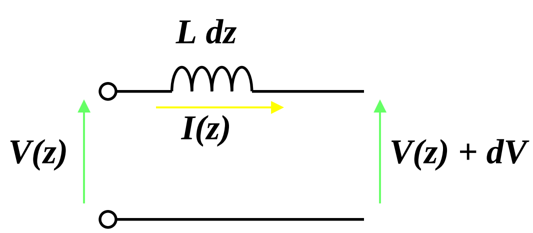
Each cell of the distributed circuit has impedance elements with values: Ldz, Rdz, Cdz and Gdz, where dz is the length of a cell.

From the differential behavior of an elementary cell of the distributed circuit, in terms of voltage and current, a differential equation results that describes the entire transmission line.

IMPORTANT ASSUMPTION - the line is uniform along its length.

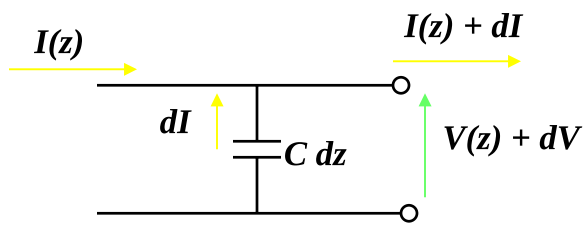
In many practical cases, it is possible to neglect resistive effects in the line - this is the LOSSLESS transmission line.





$$(V + dV) - V = -j\omega L dz I$$

$$\frac{dV}{dz} = -j\omega L I$$



$$dI = -j\omega Cdz(V + dV) = -j\omega CVdz + \overset{= 0}{\cancel{j\omega CdVdz}}$$

$$\frac{dI}{dz} = -j\omega CV$$

Telegraphers' equations for a lossless transmission line

$$\frac{dV}{dz} = -j\omega LI$$

$$\frac{dI}{dz} = -j\omega CV$$

Differentiate w.r.t. z

$$\frac{d^2V}{dz^2} = -j\omega L\frac{dI}{dz}$$

$$\frac{d^2V}{dz^2} = -j\omega L\left(-j\omega CV\right)$$

$$\frac{d^2V}{dz^2} = -\omega^2 LCV$$

Differentiate w.r.t. z

$$\frac{d^2I}{dz^2} = -j\omega C\frac{dV}{dz}$$

$$\frac{d^2I}{dz^2} = -j\omega C\left(-j\omega LI\right)$$

$$\frac{d^2I}{dz^2} = -\omega^2 LCI$$

$$\frac{d^2V}{dz^2} = -\omega^2 LCV$$

$$\frac{d^2I}{dz^2} = -\omega^2 LCI$$

Solutions for V and I will have the same form:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

Wave propagation constant

$$\beta = \omega\sqrt{LC}$$

The complex exponential terms have unitary magnitude and purely "imaginary" argument, therefore they only affect the "phase" of the wave in space. The terms represent waves:

(a) Amplitude V^+ travelling in the $+z$ direction

(b) Amplitude V^- travelling in the $-z$ direction

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{\omega}{v_p}$$

wavelength $$\lambda = \frac{v_p}{f}$$

phase velocity $$v_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_r\varepsilon_0\mu_r\mu_0}}$$

$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$\frac{dV}{dz} = -j\beta V^+ e^{-j\beta z} + j\beta V^- e^{+j\beta z} \quad = -j\omega LI$$

$$I(z) = \sqrt{\frac{C}{L}}\left(V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right)$$

$$I(z) = Z_0\left(V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right)$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

Real

Units of Ohms

Depends only on physical properties of the line, independent of length