RF and Microwave Transmission Lines

ELEC 2230

Section 5

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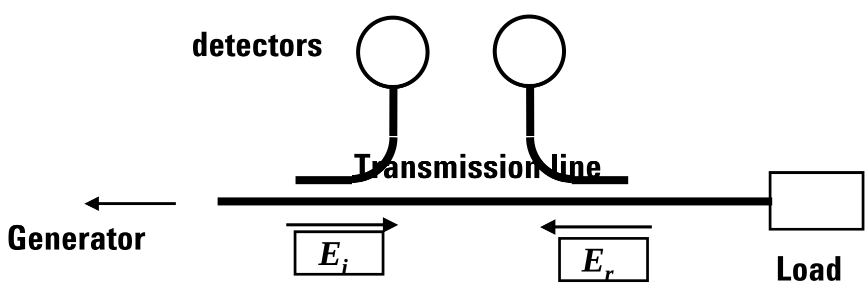
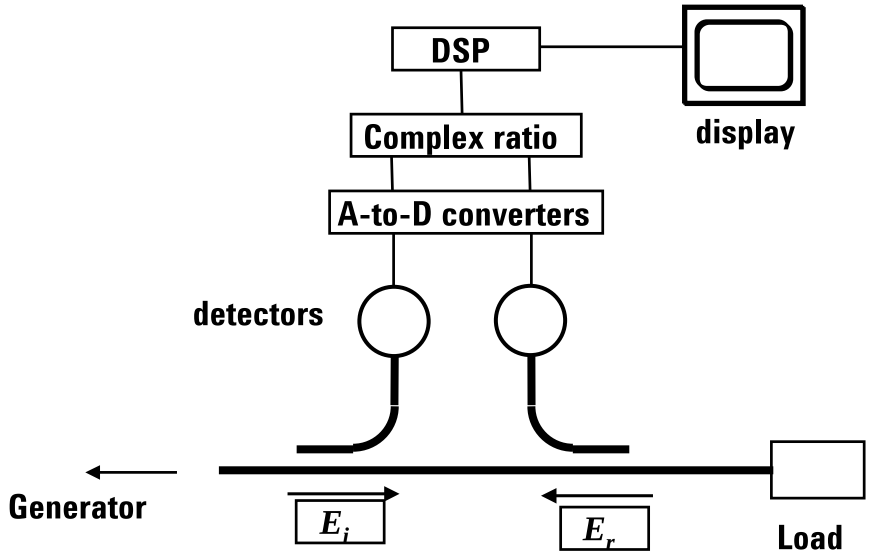
February 2004

The network analyzer



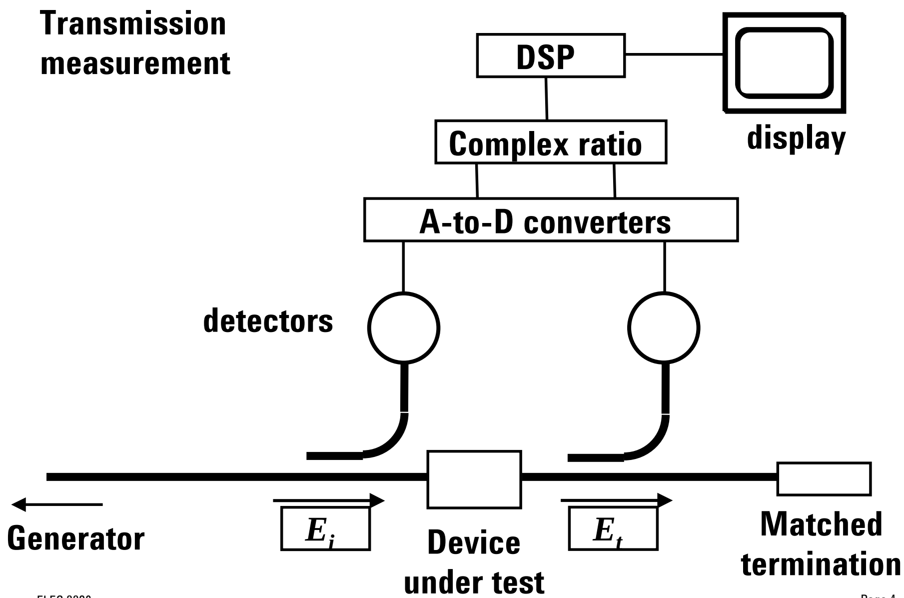
$$\Gamma = \frac{E_r}{E_i}$$

The network analyzer



The network analyzer

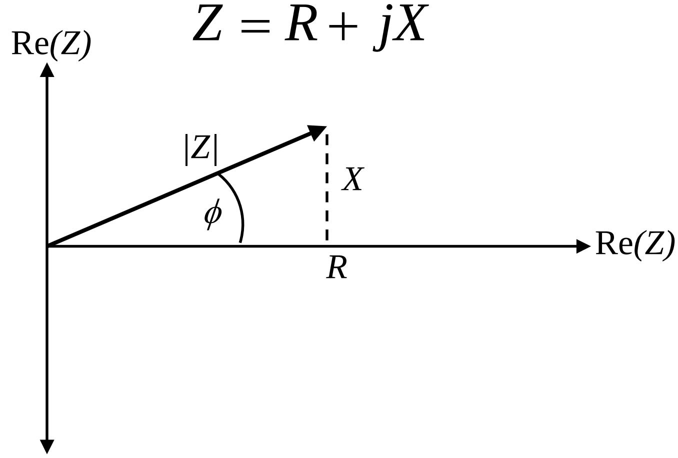
Transmission measurement



Impedance and Admittance relationships

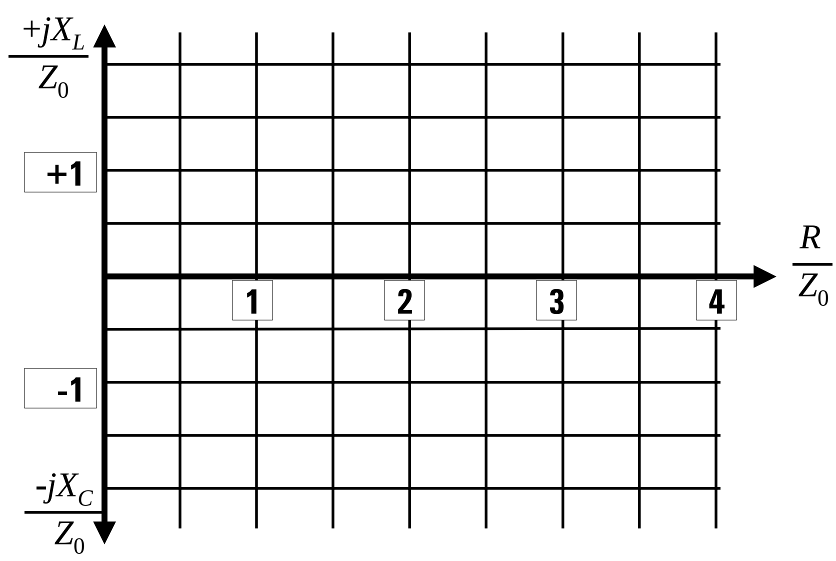
Termination impedance is, in general, complex

$$Z = R + jX$$

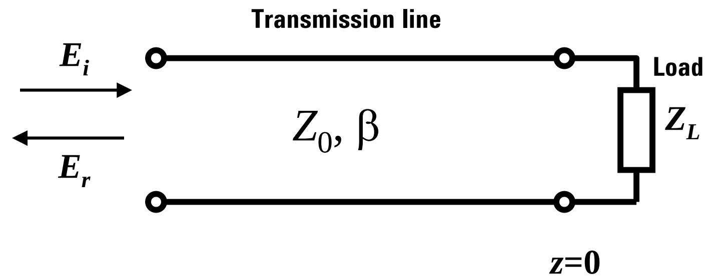


Reactance chart

Normalised to Z_0



Terminated lossless transmission line



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Terminated lossless transmission line

Incident wave $$E_i = V^+ e^{-j\beta z}$$

Reflected wave $$E_r = V^- e^{+j\beta z}$$

Ratio of voltage to current for travelling wave on the transmission line must be Z_0

But since the termination $Z_L \neq Z_0$ the ratio of voltage to current at the load must be Z_L and a reflected wave must be excited to satisfy that condition.

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Terminated lossless transmission line

Total voltage

$$V(z) = E_i + E_r = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

Total current

$$I(z) = \frac{V^+}{Z_0} e^{-j\beta z} - \frac{V^-}{Z_0} e^{+j\beta z}$$

So, at the load, where z = 0

Terminated lossless transmission line

So, at the load, where z = 0

$$Z_L = \frac{V(0)}{I(0)} = \frac{V^+ + V^-}{V^+ - V^-} Z_0$$

Now, solve for

$$V^- = \frac{Z_L - Z_0}{Z_L + Z_0} V^+$$

Terminated lossless transmission line

Recall that the reflection coefficient is

$$\Gamma = \frac{V^-}{V^+} \qquad = \frac{Z_L - Z_0}{Z_L + Z_0}$$

And can write the total voltage and current as

$$V(z) = V^+\left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right]$$

$$I(z) = \frac{V^+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right]$$

Terminated lossless transmission line

At any point on the line, ($z = -l$)

$$\Gamma(l) = \frac{V^- e^{-j\beta l}}{V^+ e^{+j\beta l}} \qquad = \Gamma(0) e^{-2j\beta l}$$

So note that, as you move along the line, the magnitude of the reflection coefficient is unchanged. The phase of the reflection coefficient increases by $2\beta l$ - the electrical length.

Terminated lossless transmission line

Or the impedance seen looking in to the line looking towards the load is

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{V^{+}\left[e^{+j\beta l} + \Gamma e^{-j\beta l}\right]}{V^{+}\left[e^{+j\beta l} - \Gamma e^{-j\beta l}\right]} Z_0$$

$$= \frac{\left[1 + \Gamma e^{-2j\beta l}\right]}{\left[1 - \Gamma e^{-2j\beta l}\right]} Z_0$$

Transmission line impedance equation

Can re-arrange as

$$Z_{in} = Z_0 \frac{\left(Z_L + Z_0\right)e^{+j\beta l} + \left(Z_L - Z_0\right)e^{-j\beta l}}{\left(Z_L + Z_0\right)e^{+j\beta l} - \left(Z_L - Z_0\right)e^{-j\beta l}}$$

$$Z_0 \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l}$$

$$= Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

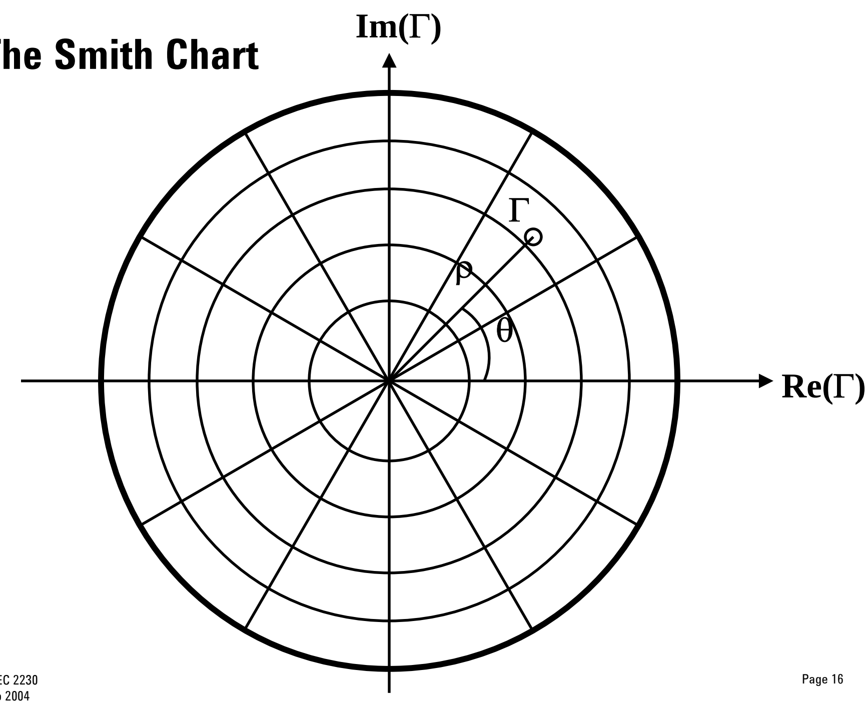
The Smith Chart

Tool to help understand and solve transmission line problems

It is a polar polar plot of reflection coefficient G overlaid with contours of constant resistance and reactance

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{z - 1}{z + 1}$$

The Smith Chart



The Smith Chart

Now consider some specific values of Z_L

(a) Short circuit: $Z_L = 0$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - Z_0}{0 + Z_0} = -1$$

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The Smith Chart

Now consider some specific values of Z_L

(b) Open circuit: $Z_L = \infty$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \dfrac{Z_0}{Z_L}}{1 + \dfrac{Z_0}{Z_L}} = \frac{1 - \dfrac{Z_0}{\infty}}{1 + \dfrac{Z_0}{\infty}} = +1$$

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The Smith Chart

Now consider some specific values of Z_L

(c) Match: $Z_L = Z_0$ ($z = 1$)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z-1}{z+1} \quad = 0$$

The Smith Chart

Now consider some specific values of Z_L

(d) Inductor: $z = 0 + j1$

$$\Gamma = \frac{z-1}{z+1} = \frac{j-1}{j+1} = \frac{(j-1)(-j+1)}{(j+1)(-j+1)} = j$$

(e) Capacitor: $z = 0 - j1$

$$\Gamma = \frac{z-1}{z+1} = \frac{-j-1}{-j+1} = \frac{(-j-1)(j+1)}{(-j+1)(j+1)} = -j$$

The Smith Chart

Now consider some specific values of Z_L

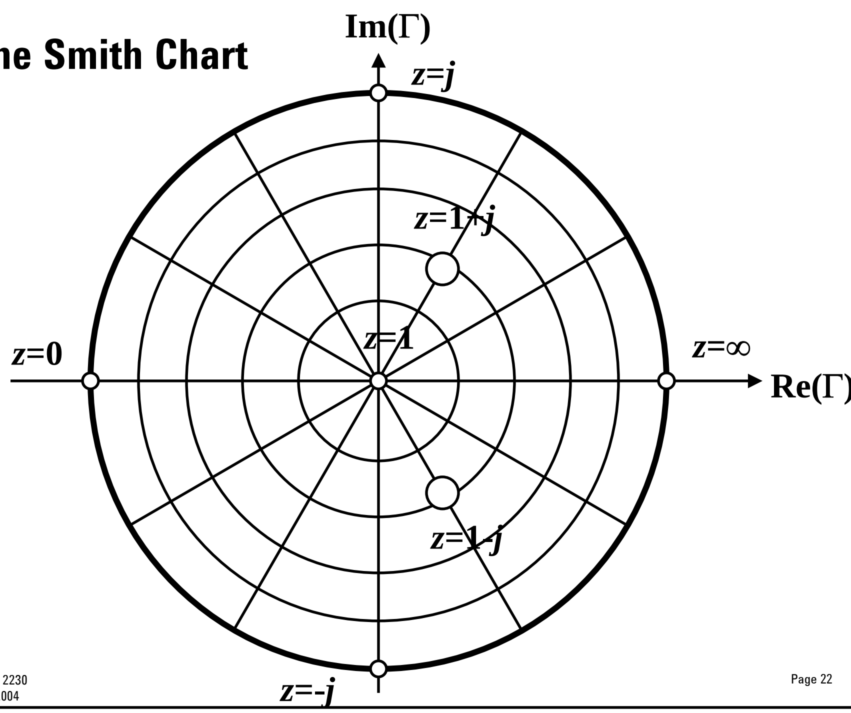
(d) Inductive impedance: $z = 1 + j1$

$$\Gamma = \frac{z-1}{z+1} = \frac{1+j-1}{1+j+1} = \frac{j}{2+j} = \frac{1+2j}{5}$$

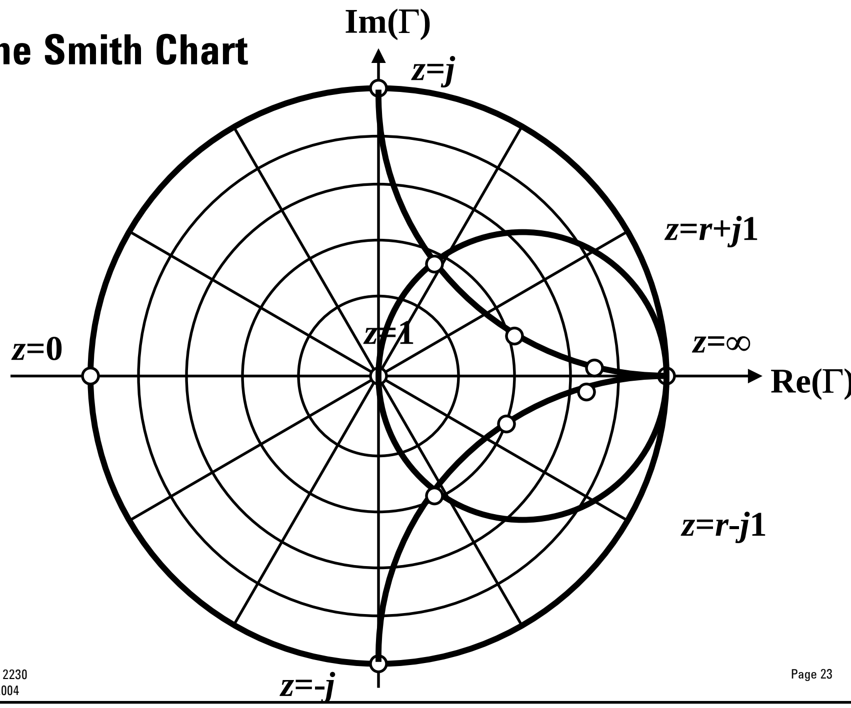
(e) Capacitive impedance: $z = 1 - j1$

$$\Gamma = \frac{z-1}{z+1} = \frac{1-j-1}{1-j+1} = \frac{-j}{2-j} = \frac{1-2j}{5}$$

The Smith Chart



The Smith Chart



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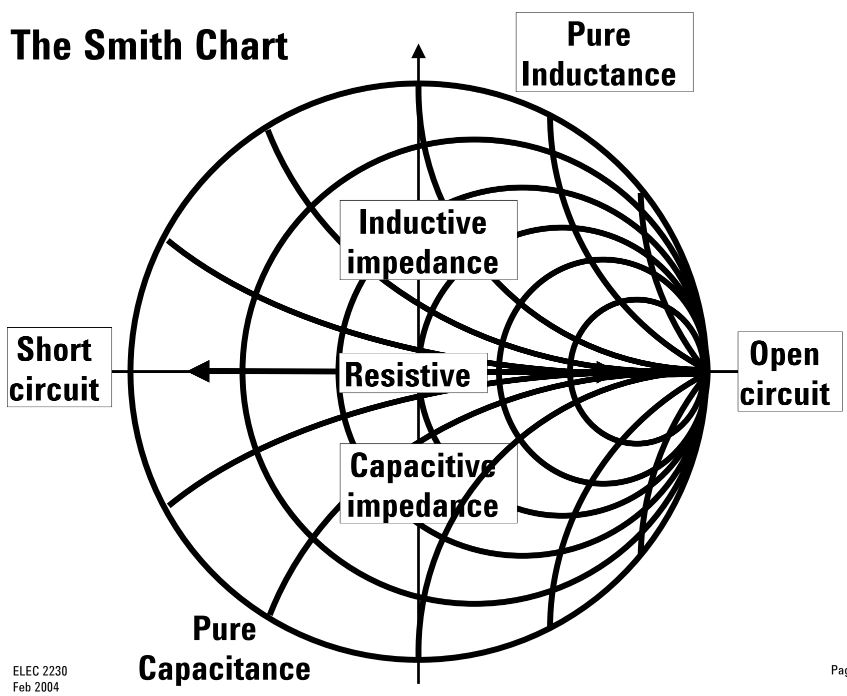
The Smith Chart



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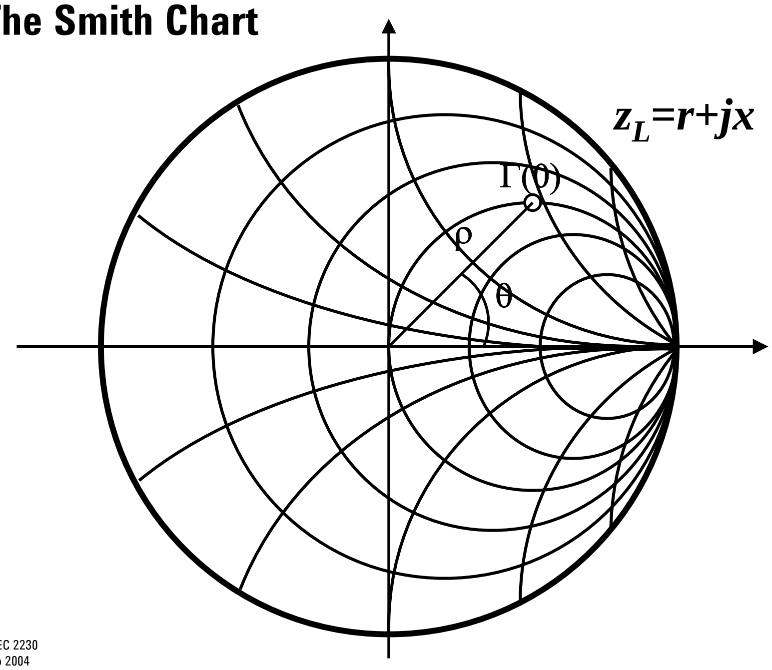
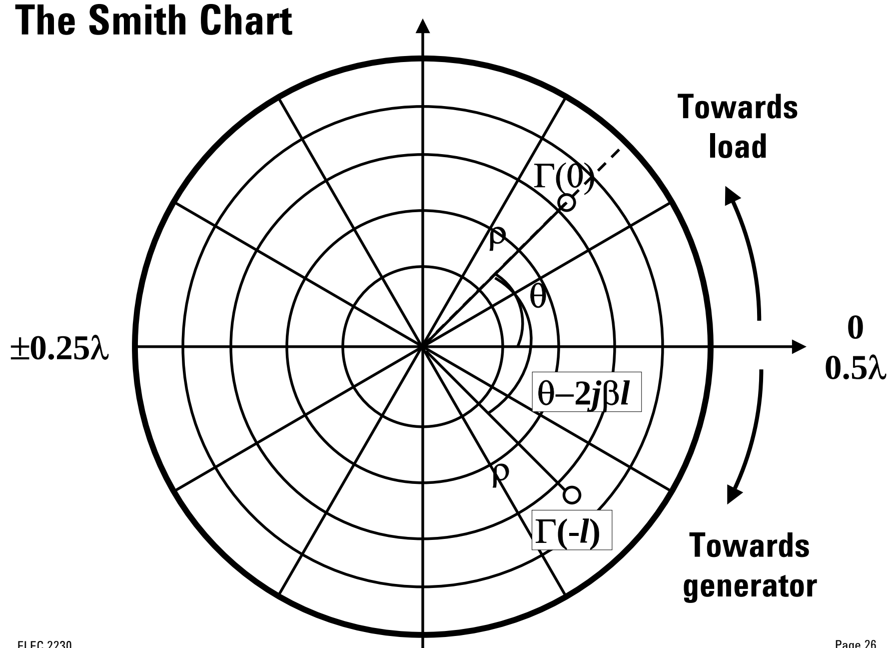
The Smith Chart



The Smith Chart



The Smith Chart