DIGITAL SIGNAL PROCESSING

REPORT08

- Name: Võ Phi Trường.
- ID: 1814582.

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1. Exercise 1: Find the all possible signal x(n) that have Z transforms as follows:

a) $x(z) = \frac{1-1.5z^{-1}}{1-1.5z^{-1}+0.5z^{-2}}$

<=> $x(z) = \frac{-1}{1-z^{-1}} + \frac{2}{1-\frac{1}{2}z^{-1}}$

TH1: $x(n) = -1*u[n] + 2*(\frac{1}{2})^n u[n]$ ROC: $|z|>1$

TH2: $x(n) = (-1)^{n+1}u[-n-1]-2*(\frac{1}{2})^n u[-n-1]$ ROC: $|z|<\frac{1}{2}$

TH3: $x(n) = (-1)^{n+1}u[-n-1]+2*(\frac{1}{2})^n u[n]$ ROC: $1>|z|>\frac{1}{2}$

b) $x(z) = \frac{1}{1-z^{-1}+0.25z^{-2}}$

TH1:|z|>0.5

$x(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$

=> $x(n) = (\frac{1}{2})^n u[n]+n(\frac{1}{2})^n u[n]$

TH2:|z|<0.5

$x(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$

=> $x(n) = -(\frac{1}{2})^n u[-n-1]-n(\frac{1}{2})^n u[-n-1]$

c) $x(z) = \frac{1}{3-10z^{-1}+3z^{-2}}$

<=> $x(z) = -\frac{1}{24} * \frac{1}{1-\frac{1}{3}*z^{-1}} + \frac{3}{8} * \frac{1}{1-3z^{-1}}$

TH1: $x(n) = \frac{3}{8}(3)^n$u[n]- $\frac{1}{24}(\frac{1}{3})^n$u[n] ROC: $|z| > 3$

TH2: $x(n) = -\frac{3}{8}(3)^n$u[-n-1]+ $\frac{1}{24}(\frac{1}{3})^n$u[-n-1] ROC: $|z| < \frac{1}{3}$

TH3: $x(n) = -\frac{3}{8}(3)^n$u[-n-1]- $\frac{1}{24}(\frac{1}{3})^n$u[n] ROC: $3 > |z| > \frac{1}{3}$

d) $x(z) = \frac{1}{2-3z^{-1}+z^{-2}}$

<=> $x(z) = 2 * \frac{1}{1-z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$

TH1: $x(n) = 2u[n] - (\frac{1}{2})^n$u[n] ROC: $|z| > 1$

TH2: $x(n) = -2u[-n-1] + (\frac{1}{2})^n$u[-n-1] ROC: $|z| < \frac{1}{2}$

TH3: $x(n) = -2u[-n-1] - (\frac{1}{2})^n$u[n] ROC: $1 > |z| > \frac{1}{2}$

e) $x(z) = -\frac{38}{25}(0.3)^n u[-n-1] - \frac{31}{20}(-0.2)^n u[-n-1] + \frac{25}{12}(3)^n u[-n-1] ROC \not\in |< 0.2$

<=>

f) $x(z) = \frac{2z^2 - 12z}{(z-0.3)(z+0.2)(z-3)}$

<=> $x(z) = \frac{38}{25}\frac{1}{z-0.3} + \frac{31}{20}\frac{1}{z+0.2} - \frac{25}{12}\frac{1}{z-3}$

TH1: $x(z) = \frac{38}{25}(0.3)^n u[n] + \frac{31}{20}(-0.2)^n u[n] - \frac{25}{12}(3)^n u[n] ROC \not\in |> 3$

TH2: $x(z) = -\frac{38}{25}(0.3)^n u[-n-1] + \frac{31}{20}(-0.2)^n u[n] + \frac{25}{12}(3)^n u[-n-1] ROC : 0.3 > |z| > 0.2$

TH3: $x(z) = \frac{38}{25}(0.3)^n u[n] + \frac{31}{20}(-0.2)^n u[n] + \frac{25}{12}(3)^n u[-n-1] ROC : 3 > |z| > 0.2$

TH4: $x(z) = \frac{38}{25}(0.3)^n u[n] + \frac{31}{20}(-0.2)^n u[n] + \frac{25}{12}(3)^n u[-n-1] ROC : 3 > \not\in |> 0.2$

2. Exercise 2: Compute convolution x(n)=x1(n)*x2(n) using Z and Inverse Z transforms, where:

a) $x_1(n)=\{1\uparrow,1,1,1,1\}, x_2(n)=\{1\uparrow,1,1,1\}$

$$x_1(z)=\sum_{k=0}^{4}x(k)z^{-k}=1+z^{-1}+z^{-2}+z^{-3}+z^{-4} ROC: z\neq 0$$

$$x_2(z)=\sum_{k=0}^{3}x(k)z^{-k}=1+z^{-1}+z^{-2}+z^{-3} ROC: z\neq 0$$

$$x(n)=x_1(n)*x_2(n)=1+2z^{-1}+3z^{-2}+4z^{-3}+4z^{-4}+3z^{-5}+2z^{-6}+z^{-7}$$

$$\Rightarrow x(n)=\{1\uparrow,2,3,4,4,3,2,1\}$$

b) $x_1(n)=\{1\uparrow,2,3,4,5\}, x_2(n)=\{1\uparrow,1,1\}$

$$x_1(z)=\sum_{k=0}^{4}x(k)z^{-k}=1+2z^{-1}+3z^{-2}+4z^{-3}+5z^{-4} ROC: z\neq 0$$

$$x_2(z)=\sum_{k=0}^{2}x(k)z^{-k}=1+z^{-1}+z^{-2} ROC: z\neq 0$$

$$x(n)=x_1(n)*x_2(n)=1+3z^{-1}+6z^{-2}+9z^{-3}+12z^{-4}+9z^{-5}+5z^{-6}$$

$$\Rightarrow x(n)=\{1\uparrow,3,6,9,12,9,5\}$$

c) $x_1(n)=(\frac{1}{5})^n u(n), x_2(n)=2^n u(n)$

$$x_1(z)=\frac{1}{1-\frac{1}{5}z^{-1}}, x_2(z)=\frac{1}{1-2z^{-1}}, ROC: |z|>2$$

$$x(z)=-\frac{1}{9}\frac{1}{1-\frac{1}{5}z^{-1}}+\frac{10}{9}\frac{1}{1-2z^{-1}}, ROC: |z|>2$$

$$\Rightarrow x(n)=-\frac{1}{9}(\frac{1}{5})^n u(n)+\frac{10}{9}(2)^n u(n)$$

d) $x_1(n)=nu(n), x_2(n)=2^n u(n-1)$

$$x_1(z)=\frac{z^{-1}}{(1-z^{-1})^2}, x_2(z)=2\frac{z^{-1}}{1-2z^{-1}}, ROC: |z|>2$$

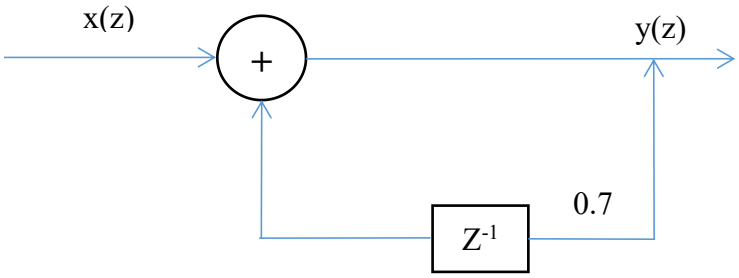
$$x(z)=\frac{2z^{-2}}{(1-z^{-1})^2(1-2z)}=\frac{-2}{1-z^{-1}}+\frac{2}{1-2z^{-1}}-\frac{2z^{-1}}{(1-z^{-1})^2}, ROC: |z|>2$$

$$\Rightarrow x(n)=-2u(n)+2*(2)^n u(n)-2nu(n)$$

3. Given an LTI system represented by the input - output description equation:

$$y(n) = 0.7y(n-1) + x(n) \Rightarrow y(z) = 0.7z^{-1}y(z) + x(z)$$

a) Draw the corresponding block diagram.



b) Determine h(n) using Z and inverse Z transforms.

$$y(z)(1-0.7z^{-1}) = x(z)$$

$$h(z) = \frac{y(z)}{x(z)} = \frac{1}{1-0.7z^{-1}}$$

$$\Rightarrow h(n) = \begin{cases} (0.7)^n u(n), |z| > 0.7 \\ -(0.7)^n u(-n-1), |z| < 0.7 \end{cases}$$

c) Determine y(n) when x(n) = u(n).

$$y(n) = 0.7y(n-1) + u(n)$$

$$y(z)(1-0.7z^{-1}) = \frac{1}{1-z^{-1}}, ROC: |z| > 1$$

$$y(z) = \frac{1}{(1-z^{-1})(1-0.7z^{-1})} = \frac{10}{3}\frac{1}{1-z^{-1}} - \frac{7}{3}\frac{1}{1-0.7z^{-1}}$$

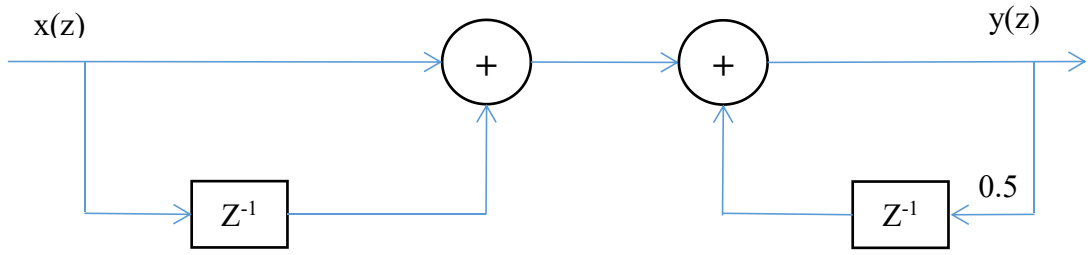
$$\Rightarrow y(n) = \frac{10}{3}u(n) - \frac{7}{3}(0.7)^n u(n), ROC: |z| > 1$$

4. Given an LTI system represented by the input - output description equation:

$$y(n) = 0.5y(n-1) + x(n) + x(n-1)$$

a) Draw the corresponding block diagram.

$$y(z) = 0.5z^{-1}y(z) + x(z) + z^{-1}x(z)$$



b) Determine h(n) using Z and inverse Z transforms.

$$y(z)(1-0.5z^{-1}) = x(z)(1+z^{-1})$$

$$h(z) = \frac{y(z)}{x(z)} = \frac{1+z^{-1}}{1-0.5z^{-1}} = -2 + \frac{3}{1-0.5z^{-1}}$$

$$\Rightarrow h(n) = \begin{cases} -2\delta(n) + 3(0.5)^n u(n), ROC: |z| > 0.5 \\ -2\delta(n) - 3(0.5)^n u(-n-1), ROC: |z| < 0.5 \end{cases}$$

c) Determine y(n) when x(n) = $(2)^n u(n)$

$$x(n) = (2)^n u(n) \Rightarrow x(z) = \frac{1}{1-2z^{-1}}, |z| > 2$$

$$y(n) = h(n) * x(n)$$

$$\Rightarrow y(z) = h(z) * x(z) = \frac{1+z^{-1}}{1-0.5z^{-1}} \frac{1}{1-2z^{-1}} = \frac{-1}{1-0.5z^{-1}} + \frac{2}{1-2z^{-1}}$$

$$\Rightarrow y(n) = 2 * (2)^n u(n) + (-1)(0.5)^n u(n), ROC: |z| > 2$$

5. Use SciLab to find h(n) where:

a) $H(z)=\dfrac{z^{-2}}{1-3z^{-1}+2z^{-2}}, |z|>2$

```
--> s=poly(0,'s')
 s  =

  s

--> h=s^2/(1-3*s+2*s^2)
 h  =


       s²
   ----------
   1 -3s +2s²

--> pfss(h)
 ans  =


       ans(1)


     1
   -----
   -1 +s

       ans(2)


    -0.25
   -------
   -0.5 +s

       ans(3)

  0.5

-->
```

$$h(z)=\frac{1}{-1+z^{-1}}+\frac{-0.25}{-0.5+z^{-1}}+0.5$$

$$h(z)=\frac{-1}{1-z^{-1}}+\frac{0.5}{1-2z^{-1}}+0.5$$

$$h(n)=-u(n)+0.5(2)^2u(n)+0.5\delta(n), ROC: |z|>2$$

b) $H(z)=\dfrac{z}{z^3-6z^2+11z-6},|z|<2$

$\Leftrightarrow H(z)=\dfrac{z^{-2}}{1-6z^{-1}+11z^{-2}-6z^{-3}}$

```
--> h=s^2/(1-6*s+11*s^2-6*s^3)
 h  =


          s²
   ----------------
   1 -6s +11s² -6s³

--> pfss(h)
 ans  =


       ans(1)


   -0.5
   -----
   -1 +s

       ans(2)


     0.5
   -------
   -0.5 +s

       ans(3)


    -0.1666667
   -------------
   -0.3333333 +s

-->
```

$$h(z)=\frac{-0.5}{-1+z^{-1}}+\frac{0.5}{-0.5+z^{-1}}+\frac{-\frac{1}{6}}{-\frac{1}{3}+z^{-1}}$$

$$h(z)=\frac{0.5}{1-z^{-1}}-\frac{1}{1-2z^{-1}}+\frac{\frac{1}{2}}{1-3z^{-1}}$$

$$h(n)=\begin{cases}0.5u(n)+(2)^n u(-n-1)-0.5(3)^n u(-n-1), ROC: 2>|z|>1\\ -0.5u(-n-1)+(2)^n u(-n-1)-0.5(3)^n u(-n-1), ROC: |z|<1\end{cases}$$