

## Factoring Quadratics when $a \neq 1$ Examples:

Example:  $2x^2 + 7x + 3 = 0$

1. Multiply  $a \times c = 2 \cdot 3 = 6$
2. Numbers that multiply to 6 add to 7.  $\rightarrow (6, 1)$
3. Split Middle Term:

$$2x^2 + 6x + x + 3 = 0$$

4. Group:  $(2x^2 + 6x) + (x + 3) = 0$

5. Factor Each Group:

$$2x(x + 3) + 1(x + 3) = 0$$

6. Factor Common Binomial:

$$(2x + 1)(x + 3) = 0$$

7. Solve:

$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

$$x + 3 = 0 \rightarrow x = -3$$

8. Answer:  $x = -\frac{1}{2}, -3$

Example #2:  $3x^2 - 2x - 8 = 0$

1. Multiply  $a \cdot c = 3 \cdot -8 = -24$

2. Numbers that multiply to  $-24$  and add to  $-2 \rightarrow (-6, 4)$

3. Split Middle Term:

$$3x^2 - 6x + 4x - 8 = 0$$

4. Group:  $(3x^2 - 6x) + (4x - 8) = 0$

5. Factor Each:

$$3x(x - 2) + 4(x - 2) = 0$$

6. Factor Common Binomial

$$(3x + 4)(x - 2) = 0$$

7. Solve:

$$3x + 4 = 0 \rightarrow x = -\frac{4}{3}$$

$$x - 2 = 0 \rightarrow x = 2$$

8. Answer:  $x = -\frac{4}{3}, 2$

## Pratize Drills: Factoring Quadratics

Set A: Easy Case ( $a=1$ )

1.  $x^2 + 7x + 10 = 0$

(5, 2)  $(x+5)(x+2) = 0$

$x^2$

$x+5=0 \rightarrow x=-5$

$x=-5, -2$

$x+2=0 \rightarrow x=-2$

2.  $x^2 + 8x + 15 = 0$

(5, 3)  $(x+5)(x+3) = 0$

$x=-5, -3$

$x+5=0 \rightarrow x=-5$

$x+3=0 \rightarrow x=-3$

3.  $x^2 + 11x + 18 = 0$

(9, 2)  $(x+9)(x+2) = 0$

$x=-9, -2$

4.  $x^2 - 2x - 15 = 0$

(-5, 3)  $(x-5)(x+3) = 0$

$x=5, -3$

5.  $x^2 + 9x + 20 = 0$

(5, 4)  $(x+5)(x+4) = 0$

$x=-5, -4$



Set B: (AC Method,  $a \neq 1$ )

6.  $2x^2 + 7x + 3 = 0$

Multiply to 6 and add to 7  
(6, 1)

$$2x^2 + 6x + x + 3 = 0$$

$$(2x^2 + 6x) + (x + 3) = 0$$

$$2x(x + 3) + 1(x + 3) = 0$$

$$(2x + 1)(x + 3) = 0$$

$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

$$x + 3 = 0 \rightarrow x = -3 \quad x = (-3, -\frac{1}{2})$$

7.  $3x^2 + 10x + 7 = 0$

Multiply to 21 + add to 10  
(7, 3)

$$3x^2 + 7x + 3x + 7 = 0$$

$$(3x^2 + 7x) + (3x + 7) = 0$$

$$x(3x + 7) + 1(3x + 7) = 0$$

$$(3x + 7)(x + 1) = 0$$

$$x = -\frac{7}{3}, -1$$

$$3x + 7 = 0 \rightarrow x = -\frac{7}{3}$$

$$x + 1 = 0 \rightarrow x = -1$$

$$8. \quad 6x^2 - 5x - 6 = 0$$

Multiply to -36, add to -5

$$(-9, 4)$$

$$6x^2 - 9x + 4x - 6 = 0$$

$$(6x^2 - 9x) + (4x - 6) = 0$$

$$3x(2x - 3) + 2(2x - 3) = 0$$

$$(2x - 3)(3x + 2) = 0$$

$$2x - 3 = 0 \rightarrow$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$3x + 2 = 0$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$x = \frac{3}{2}, -\frac{2}{3}$$

$$9. \quad 4x^2 + 11x + 6 = 0$$

Multiply = 24

Add = -11

$$(8, 3)$$

$$4x^2 + 8x + 3x + 6 = 0$$

$$(4x^2 + 8x) + (3x + 6) = 0$$

$$4x(x + 2) + 3(x + 2) = 0$$

$$(x + 2)(4x + 3)$$

$$x = -2$$

$$4x + 3 = 0$$

$$4x = -3$$

$$x = -\frac{3}{4}$$



## Factoring - In Group Step of Solving Quadratics

### 1.) General Rule:

When you group (eg.  $4x^2 + 8x$ )

- Factor out the greatest common factor (GCF).

- If both terms have an  $x$ , take out the lowest power of  $x$ .
- If both have numbers, take out the largest number that divides evenly.
- Whatever you pull out, divide each term by it, and put factor outside.

### 2.) Why It Matters:

- The whole point of grouping is:

→ The two groups must end with the same binomial.  
That is only possible if you pull out the largest common factor in each group.

### 3.) Step By Step Checklist:

1. Look at pair → Example:  $4x^2 + 8x$

2. Find GCF:

- Numbers GCF of 4 and 8 = 4
- Variables: Both terms have at least 1  $x$ .
- GCF =  $4x$

3. Divide each term by GCF, to see whats left.

$$\left. \begin{array}{l} 4x^2 / 4x = x \\ 8x / 4x = 2 \end{array} \right\} \text{ So: } 4x^2 + 8x = 4x(x+2)$$

## Factoring Quadrates Drill Sets:

$$1.) x^2 + 5x + 6 = 0$$

(3, 2)

$$(x+3)(x+2) = 0$$

$$x+3 = 0 \rightarrow x = -3$$

$$x+2 = 0 \rightarrow x = -2$$

$$\underline{x = -3, -2}$$

$$2.) x^2 - 7x + 12 = 0$$

(-4, -3)

$$(x-4)(x-3) = 0$$

$$\underline{x = 4, 3}$$

$$3.) x^2 + 11x + 24 = 0$$

(8, 3)

$$(x+8)(x+3) = 0$$

$$\underline{x = -8, -3}$$

$$4.) x^2 - 3x + 18 = 0$$

(-6, 3)

$$(x-6)(x+3) = 0$$

$$\underline{x = 6, -3}$$

$$5.) x^2 + 8x + 15 = 0$$

(5, 3)

$$(x+5)(x+3) = 0$$

$$\underline{x = -5, -3}$$

$$6.) \quad 2x^2 + 7x + 3 = 0$$

$$\text{Multiply} = 6$$

$$\text{Add} = 7$$

$$(6, 1)$$

Answer:

$$\underline{x = -3, -\frac{1}{2}}$$

$$2x^2 + 6x + 1x + 3 = 0$$

$$(2x^2 + 6x) + 1(x + 3) = 0$$

$$2x(x + 3)$$

$$(x + 3)(2x + 1) = 0$$

$$x + 3 = 0$$

$$x = -3$$

$$2x + 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$7.) \quad 3x^2 - 10x - 8 = 0$$

$$\text{Multiply} = -24$$

$$\text{Add} = -10$$

$$(-12, 2)$$

$$3x^2 - 12x + 2x - 8 = 0$$

$$(3x^2 - 12x) + (2x - 8) = 0$$

$$3x(x - 4) + 1(x - 4) = 0$$

$$(x - 4)(3x + 1) = 0$$

$$x - 4 = 0 \rightarrow x = 4$$

$$3x + 1 = 0 \rightarrow x = -\frac{1}{3}$$

Answer:

$$\underline{x = 4, -\frac{1}{3}}$$



$$8.) 4x^2 + 12x + 5 = 0$$

$$\text{Multiply} = 20$$

$$\text{Add} = 12$$

$$(10, 2)$$

$$4x^2 + 10x + 2x + 5 = 0$$

$$(4x^2 + 10x) + (2x + 5) = 0$$

$$2x(2x + 5) + 1(2x + 5) = 0$$

$$(2x + 5)(2x + 1) = 0$$

$$2x + 5 = 0 \rightarrow x = -\frac{5}{2}$$

$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

Answer:

$$x = -\frac{5}{2}, -\frac{1}{2}$$

$$9.) 5x^2 - x - 6 = 0$$

$$\text{Multiply} = -30$$

$$\text{Add} = -1$$

$$(-6, 5)$$

$$(5x^2 - 6x) + (5x - 6) = 0$$

$$1x(5x - 6) + 1(5x - 6) = 0$$

$$(5x - 6)(1x + 1) = 0$$

$$x = -\frac{6}{5}, 1$$

$$10.) 6x^2 + 13x + 5 = 0$$

$$\text{Multiply} = 30$$

$$\text{Add} = 13$$

$$(10, 3)$$

$$(6x^2 + 10x) + (3x + 5) = 0$$

$$2x(3x + 5) + 1(3x + 5) = 0$$

$$(3x + 5)(2x + 1) = 0$$

$$2x + 1 = 0$$

$$x = -\frac{1}{2}$$

$$3x + 5 = 0 \rightarrow 3x = -5$$

$$x = -\frac{5}{3}$$

Answer:

$$x = -\frac{5}{3}, -\frac{1}{2}$$

## Lesson 7: Completing The Square: Quadratics

### What it means?

Completing the square rewrites a quadratic in the form:

$$ax^2 + bx + c \rightarrow a(x-h)^2 + K \quad \left. \vphantom{ax^2 + bx + c} \right\} \text{ This is called "Vertex Form".}$$

- Where the vertex of the parabola is  $(h, K)$ .

### Steps (for $x^2 + bx + c$ ):

1. Move the constant term to the other side.

Example:  $x^2 + 6x + 5 = 0 \rightarrow x^2 + 6x = -5$

2. Take half of the coefficient of  $x$ , then square it.

- Coefficient of  $x$  is 6
- Half of 6 is 3.
- Square it  $3^2 = 9$

3. Add this square to both sides (if solving) or inside the trinomial (if rewriting).

•  $x^2 + 6x + 9 = -5 + 9$

4. Now factor the trinomial into perfect square

•  $x^2 + 6x + 9 = (x+3)^2$

• So equation becomes:  $(x+3)^2 = 4$

5. Complete Solving:

• Take root:  $x + 3 = \pm 2$

• Roots:  $x = -3 \pm 2 \rightarrow x = -1, -5$



### Special Notes:

- If  $a \neq 1$  (like  $2x^2 + 8x + 3$ ), factor out a first from  $x^2$  and  $x$  terms.
- Completing the square is also how the quadratic formula is derived

### Walk Thru Examples:

Example #1:  $x^2 + 4x - 5 = 0 \rightarrow a = 1$

1. Move constant:  $x^2 + 4x = 5$
2. Half of 4 = 2  $\rightarrow$  Square = 4
3. Add:  $x^2 + 4x + 4 = 5 + 4$
4. Factor:  $(x + 2)^2 = 9$
5. Solve:  $x + 2 = \pm 3 \rightarrow x = -2 \pm 3$   
• Roots:  $x = 1, -5$

Example #2:  $2x^2 + 8x + 3 = 0 \rightarrow a \neq 1$

1. Factor out 2:  $2(x^2 + 4x) + 3 = 0$
2. Inside: half of 4 = 2  $\rightarrow$  Square = 4
3. Add/Subtract inside carefully:

$$2(x^2 + 4x + 4 - 4) + 3 = 0$$

$$2((x + 2)^2 - 4) + 3 = 0$$

$$2(x + 2)^2 - 8 + 3 = 0$$

$$2(x + 2)^2 - 5 = 0$$

4. Solve:  $(x + 2)^2 = \frac{5}{2}$

5. Take square root:  $x + 2 = \pm \sqrt{\frac{5}{2}}$   
• Roots:  $x = -2 \pm \sqrt{\frac{5}{2}}$



Example #3:  $y = x^2 - 6x + 11 \rightarrow$  Rewrite In Vertex Form

1. Group terms:  $y = (x^2 - 6x) + 11$

2. Half of  $-6 : 3 \rightarrow$  Square  $= 9$

3. Add/subtract inside:

$$y = (x^2 - 6x + 9) + 11 - 9$$

4. Factor Trinomial:

$$y = (x - 3)^2 + 2$$

Vertex form:  $y = (x - 3)^2 + 2$ , vertex at  $(3, 2)$ .