Lesson #4 : Quadratics

Quadratics Sections/Topics:

1.) Factoring

2.) Completing the Square

3.) Quadratic Formula

4.) Discriminant

5.) Applications

Introduction to Quadratics

Definition → A quadratic equation is any equation that can be written in the form:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Where :

- a = coefficent of x^2 (controls parabola's width + direction)
- b = coefficent of x (controls slope)
- c = constant (y-intercept).

Graph Shape : (Parabola).

- U-Shaped Curve
- Opens Upward if $a > 0$.
- Opens downward if $a < 0$
- The Vertex is the "peak" (if down) or "valley" (if up).

Solutions (Roots): Values of x that make $y = 0$. These are the x-intercepts of the parabola.

- A quadratic can have 1, 2, or 0 real solutions.

Example: $y = x^2 - 4x + 3$

- Opens upward ($a = 1 > 0$)
- Crosses x-axis at $x = 1, 3$
- Vertex: Midway between roots $= x = 2, y = -1$

Solving By Factoring:

- Factoring Works when the quadradic is "nice" (integer solutions).
- Rule: If $(x+p)(x+q) = 0$, then either $x = -p$ or $x = -q$
- Think → "What multiplies to c, and adds to b?"

Steps:

1. Write in standard form $ax^2 + bx + c = 0$
2. If $a = 1$: find two numbers that multiply to c and add to b.
3. Factor into binomials.
4. Solve each factor $= 0$

Example: $x^2 + 5x + 6 = 0$

Two numbers that multiply to 6 and add to 5: 2, 3.

$(x+2)(x+3) = 0 \rightarrow x = -2, -3$

Completing The Square:

- Idea: turn part of the quadratic into a perfect square trinomial.
- This makes it easy to solve by taking square roots.

Steps:

1. Move constant (c) to other side.
2. Take half of coefficent of x (b/2), square it, add both sides.
3. Rewrite as $(x+p)^2$
4. Solve with square root property.

Example:

$$x^2 + 6x + 5 = 0$$
$$x^2 + 6x = -5$$

Half of 6 = 3, square = -9, Add:

$$x^2 + 6x + 9 = -5 + 9$$
$$(x+3)^2 = 4$$
$$x+3 = \pm 2$$
$$x = -1, -5$$

Quadratic Formula:

- Universal Method:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Works when factoring fails.

Discriminant: $b^2 - 4ac$

- >0: 2 real solutions (parabola crosses x-axis twice).
- $=0$: 1 real solution (parabola touches x-axis once, vertex on axis).
- <0: No real solutions (parabola doesn't touch x-axis).

Example: $2x^2 + 3x - 2 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{4}$$

$$= \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$$

$$x = \frac{1}{2}, -2$$

Graphing Quadratics:

- Standard form = $ax^2 + bx + c$
- Vertex Formula:

$$x = -\frac{b}{2a}, \quad y = f\left(-\frac{b}{2a}\right)$$

- Axis of symmetry: Vertical line through vertex
- Roots: Where parabola intersects x-axis (solve quadratic).
- Y-intercept = constant c

Example: $y = x^2 - 4x + 3$

$$x = -\frac{-4}{2(1)} = 2$$

$$y = 2^2 - 4(2) + 3 = -1$$

Vertex = $(2, -1)$

Application of Quadratics:

- Quadratics model real world scenarios with curves.
- Common types:
  - Projectile motion: $h(t) = -16t^2 + v_0 t + h_0$
  - Area Problems: Max area with given parameter.
  - Optimization: Maximizing / minimizing profit / cost.

How to Solve Quadratic Equations by Factoring:

Step 1.) Write in standard form.

Make sure the equation looks like:

$$ax^2 + bx + c = 0$$

- The right-hand side must be 0.
- If its not, rearrange the equation.

Example:

$$x^2 + 5x = -6$$

Bring over -6:

$$x^2 + 5x + 6 = 0$$

Step 2.) Look for a common factor (optional).

Example: $2x^2 + 4x = 0$

Factor out $2x$: $2x(x+2) = 0$

Then Solve:

$$x = 0 \quad \text{or} \quad x = -2$$

Step 3.) Factor the Trinomial

For $x^2 + bx + c$: Find two numbers that multiply to c and add to b.

Why? Because... $(x+p)(x+q) = x^2 + (p+q)x + pq$

Example: $x^2 + 5x + 6 = 0$

Multiply to 6, add to 5: $(2,3)$

$$(x+2)(x+3) = 0$$

Step 4.) Use Zero Product Property

If $(x+p)(x+q)=0$, then

$$x+p=0 \quad \text{or} \quad x+q=0$$

Solve each:

Example: $(x+2)(x+3)=0$

$x+2=0 \rightarrow x=-2$

$x+3=0 \rightarrow x=-3$

Step 5.) Check Your Solution

Plug back into original equation to make sure it works

Example: $x=-2: (-2)^2+5(-2)+6=4-10+6=0$

$x=-3: (-3)^2+5(-3)+6=9-15+6=0$

Simple Breakdown for: $a=1$

- Put into standard form.
- Find two numbers that multiply to c, and add to b.
- Put in form: $(x+p)(x+q)=0$
- Solve for x.

Factoring Quadratics when $a \neq 1$

Step by Step : AC Method

Step 1.) Multiply $a \cdot c$

- Take the coefficent of x^2 (a) and multiply it by the constant (c).

Step 2.) Find two numbers.

Look for two numbers that:

- Multiply to $a \cdot c$
- Add to b.

Step 3.) Split Middle Term

- Rewrite the middle term (bx) using those two numbers.

Step 4.) Factor by Grouping

- Group the first two terms and the last two terms, then factor each group.

Step 5.) Factor Common Binomial

- Pull out the common binomial → gives two factors

Step 6.) Solve each factor.