

System of Equations - Substitution Method

1. General Idea:

- Solve one equation for one variable (isolate x or y).
- Substitute that expression into the other equation.
- Solve step by step to find exact values.
- Plug back in to find the second variable.

2. Steps:

- #1: Pick one equation. Solve for one variable.
- #2: Substitute that expression into other equation.
- #3: Solve for the single variable.
- #4: Substitute back to find the other variable.
- #5: Check solution in both equations.

3. Example #1: Clean Numbers

Solve: $y = 2x + 1$ and $x + y = 7$

Step 1: First equation already solved for y .

Step 2: Substitute into second equation:

$$x + (2x + 1) = 7$$

$$3x + 1 = 7 \rightarrow 3x = 6 \rightarrow x = 2$$

Step 3: Plug into first: $y = 2(2) + 1 = 5$

Solution = $(2, 5)$.

4. Example 2: Fractions Appear

Solve: $2x + y = 5$ and $x - y = 1$

Step 1: Solve second for x : $x = y + 1$

Step 2: Substitute into first: $2(y + 1) + y = 5$

$$2y + 2 + y = 5 \rightarrow 3y + 2 = 5 \rightarrow y = 1$$

Step 3: Plug into $x = y + 1$: $x = 2$

Solution: $(2, 1)$

5. Special Cases

- If substitution simplifies to a true statement like $(0, 0)$, that means infinite solutions (same line)
- If it simplifies to a false statement (like $0 = 5$), no solution (parallel lines).

6. Additional Notes:

How to choose which equation to begin with?

- Pick the easier equation - the one where a variable already has coefficient 1, or -1, and can be isolated easily.

System of Equations: Substitution Drills

1.) $y = 3x + 2$ and $2x + y = 11$

$$2x + (3x + 2) = 11$$

$$5x + 2 = 11$$

$$5x = 9 \quad x = \frac{9}{5}$$

$$\text{Solution} = \left(\frac{9}{5}, \frac{37}{5}\right)$$

$$y = 3\left(\frac{9}{5}\right) + 2$$

$$\frac{27}{5} + \frac{2 \times 5}{1 \times 5}$$

$$\frac{27}{5} + \frac{10}{5} = \frac{37}{5}$$

$$y = \frac{37}{5}$$

2. $x - y = 4$ and $2x + 3y = 7$

$$x = y + 4 \Rightarrow 2(y + 4) + 3y = 7 \Rightarrow 2y + 8 + 3y = 7 \Rightarrow 5y + 8 = 7$$

$$x = \frac{-1}{5} + 4$$

$$x = \frac{-1}{5} + \frac{4 \times 5}{1 \times 5}$$

$$x = \frac{-1}{5} + \frac{20}{5}$$

$$x = \frac{19}{5}$$

$$2y + 8 + 3y = 7 \Rightarrow 5y + 8 = 7$$

$$5y + 8 = 7$$

$$5y = -1$$

$$y = \frac{-1}{5}$$

$$\left(\frac{19}{5}, -\frac{1}{5}\right)$$

3. $y = 2x + 5$, and $2x - y = -5$

$$2x - (2x + 5) = -5$$

$$2x - 2x - 5 = -5$$

$$-5 = -5$$

True Statement

Solution: Infinite
Solutions
(All points on
one line.)

4. $y = x + 4$ and $2x + y = 10$

$$y = 2 + 4$$

$$y = 6$$

$$\text{Solution: } (2, 6)$$

$$2x + (x + 4) = 10$$

$$2x + x + 4 = 10$$

$$3x + 4 = 10$$

$$3x = 6$$

$$x = 2$$

5. $x + y = 6$ and $y = 2x - 1$

$$x + (2x - 1) = 6$$

$$x + 2x - 1 = 6$$

$$\frac{3x}{3} = \frac{7}{3} \quad x = \frac{7}{3}$$

$$\text{Solution: } \left(\frac{7}{3}, \frac{11}{3}\right)$$

$$y = 2\left(\frac{7}{3}\right) - 1$$

$$y = \frac{14}{3} - \frac{1}{1}$$

$$y = \frac{14}{3} - \frac{3}{3} = \frac{11}{3}$$

6. $3x - y = 7$ and $y = x + 1$

$$3x - (x + 1) = 7$$

$$3x - x - 1 = 7$$

$$2x - 1 = 7$$

$$2x = 8$$

$$x = 4$$

$$y = 4 + 1$$

$$y = 5$$

$$\text{Solution } (4, 5)$$

7. $2x + y = 8$ and $y = -x + 5$

$$2x + (-x + 5) = 8$$

$$y = -(2) + 5$$

$$2x - 1 + 5 = 8$$

$$y = 3$$

$$2x + 4 = 8$$

$$\text{Solution: } (2, 3)$$

$$2x = 4$$

$$x = 2$$

8. $x - 2y = -4$ and $y = 3x - 1$

$$x - 2(3x - 1) = 4$$

$$y = 3\left(-\frac{2}{5}\right) - 1$$

$$x - 6x + 2 = 4$$

$$y = \frac{-6}{5} - \frac{1}{1}$$

$$-5x + 2 = 4$$

$$y = \frac{-6}{5} - \frac{5}{5} = \frac{-11}{5}$$

$$-5x = 2$$

Solution

$$x = \frac{-2}{5}$$

$$\left(-\frac{2}{5}, -\frac{11}{5}\right)$$

$$y = \frac{-11}{5}$$

1. $y = 3x + 2$ and $2x + y = 11$

$$2x + (3x + 2) = 11$$

$$2x + 3x + 2 = 11$$

$$5x + 2 = 11$$

$$5x = 9$$

$$x = \frac{9}{5}$$

$$y = 3\left(\frac{9}{5}\right) + 2$$

$$\frac{27}{5} + \frac{10}{5} = \frac{37}{5}$$

$$y = \frac{37}{5}$$

$$\left(\frac{9}{5}, \frac{37}{5}\right)$$

2. $x = 2y - 1$ and $3x + y = 12$

$$3(2y - 1) + y = 12$$

$$6y - 3 + y = 12$$

$$7y - 3 = 12$$

$$7y = 15$$

$$y = \frac{15}{7}$$

$$x = 2\left(\frac{15}{7}\right) - 1$$

$$\frac{30}{7} - \frac{7}{7}$$

$$x = \frac{23}{7}$$

$$\left(\frac{23}{7}, \frac{15}{7}\right)$$

Role of Thumb: Tip

"+" (...) → Just copy everything inside

"-" (...) → Flip all signs inside

"number(...)" → Multiply through

$$3. \quad y = -x + 4 \quad \text{and} \quad 2x + y = 7$$

$$2x + (-x + 4) = 7$$

$$2x - x + 4 = 7$$

$$1x + 4 = 7$$

$$1x = 3 \quad x = \frac{3}{1}$$

$$y = -\left(\frac{3}{1}\right) + 4$$

$$-\frac{3}{1} + \frac{4}{1} = 1$$

$$y = 1$$

$$\left(\frac{3}{1}, 1\right)$$

$$4. \quad x = y + 3 \quad \text{and} \quad 4x - 2y = 10$$

$$4(y + 3) - 2y = 10$$

$$4y + 12 - 2y = 10$$

$$2y + 12 = 10$$

$$2y = -2$$

$$y = -1$$

$$x = -1 + 3$$

$$x = 2$$

$$(2, -1)$$

$$5. \quad y = \frac{1}{2}x - 1 \quad \text{and} \quad x + y = 7$$

$$x + \left(\frac{1}{2}x - 1\right) = 7$$

$$2 \cdot \frac{x + \frac{1}{2}x - 1}{1} = 7$$

$$\frac{3}{2}x - 1 = 7$$

$$\frac{3}{2}x = 8$$

$$\frac{3}{2} \div \frac{3}{1} = \frac{3}{1} \cdot \frac{2}{3}$$

$$x = \frac{3}{16}$$

$$y = \frac{1}{2}\left(\frac{3}{16}\right) - 1$$

$$\frac{4}{32} - \frac{1}{1}$$

$$\frac{4}{32} - \frac{32}{32} = \frac{-28}{32}$$

$$\left(\frac{3}{16}, \frac{-28}{32}\right)$$

Substitution: Systems of Equations Final Drills:

1. $y = 2x + 1$ / $3x + y = 10$

$$3x + (2x + 1) = 10$$

$$3x + 2x + 1 = 10$$

$$5x + 1 = 10$$

$$5x = 9$$

$$x = \frac{9}{5}$$

$$y = 2\left(\frac{9}{5}\right) + 1$$

$$\frac{18}{5} + \frac{5}{5} = \frac{23}{5}$$

$$y = \frac{23}{5}$$

$$\text{Solution} = \left(\frac{9}{5}, \frac{23}{5}\right)$$

2. $x = y - 2$ / $4x + y = 10$

$$4(y - 2) + y = 10$$

$$4y - 8 + y = 10$$

$$4y + y = 18$$

$$5y = 18$$

$$y = \frac{18}{5}$$

$$x = \frac{18}{5} - 2$$

$$\frac{18}{5} - \frac{2}{1}$$

$$\frac{18}{5} - \frac{10}{5} = \frac{8}{5}$$

$$x = \frac{8}{5}$$

Solution:

$$\left(\frac{8}{5}, \frac{18}{5}\right)$$

$$3. \quad y = -x + 7 \quad / \quad 2x + y = 9$$

$$2x + (-x + 7) = 9$$

$$2x - x + 7 = 9$$

$$x + 7 = 9$$

$$x = 2$$

$$\text{Solution: } (2, 5)$$

$$y = -(2) + 7$$

$$y = -2 + 7$$

$$y = 5$$

$$4. \quad x = \frac{1}{2}y \quad / \quad 3x + y = 15$$

$$3\left(\frac{1}{2}y\right) + y = 15 \quad \frac{3}{2}y + y = 15 \quad \frac{3}{2}y + \frac{2}{2}y = \frac{5}{2}y = 15$$

$$(3, 6)$$

$$5. \quad y = \frac{3}{4}x - 5 \quad / \quad 2x + y = 7$$

$$2x + \left(\frac{3}{4}x - 5\right) = 7$$

$$\frac{2x}{1} + \frac{3}{4}x - 5 = 7$$

$$\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

$$\frac{11}{4}x - 5 = 7$$

$$\frac{11}{4}x = 12$$

$$x = 12 \cdot \frac{4}{11} = \frac{48}{11}$$

$$x = \frac{48}{11}$$

$$y = \frac{3}{4}\left(\frac{48}{11}\right) - 5$$

$$y = -\frac{19}{11}$$

Solution:

$$\left(\frac{48}{11}, -\frac{19}{11}\right)$$

$$6. \quad y = -2x + 4 \quad / \quad x + y = 1$$

$$x + (-2x + 4) = 1$$

$$x - 2x + 4 = 1$$

$$-x + 4 = 1$$

$$-x = -3$$

$$x = -3$$

$$y = -2(-3) + 4$$

$$6 + 4$$

$$y = 10$$

$$(-3, 10)$$

$$7. \quad x = \frac{2}{3}y \quad / \quad x + y = 20$$

$$\frac{2}{3}y + y = 20$$

$$\frac{2}{3} + \frac{3}{3} \quad \frac{5}{3}y = 20 \cdot \frac{3}{5}$$

$$y = 12$$

$$\left(\frac{24}{3}, 12\right)$$

$$x = \frac{2}{3}(12)$$

$$\frac{2}{3} \cdot \frac{12}{1}$$

$$x = \frac{24}{3}$$

$$8. \quad y = \frac{1}{2}x + 6 \quad / \quad 3x - y = 12$$

$$3x - \left(\frac{1}{2}x + 6\right) - y = 12$$

$$3x - \frac{1}{2}x + 6 - y = 12$$

$$\frac{3}{1} - \frac{1}{2}$$

$$\frac{6}{2} - \frac{1}{2} = \frac{5}{2}$$

$$\frac{5}{2}x + 6 = 12$$

$$\frac{5}{2}x = 6 \cdot \frac{2}{5} = \frac{12}{5} = x$$

$$y = \frac{1}{2}\left(\frac{12}{5}\right) + 6$$

$$y = \frac{6}{5} + \frac{6}{1}$$

$$\frac{6}{5} + \frac{30}{5} \quad \frac{36}{5}$$

$$\left(\frac{12}{5}, \frac{36}{5}\right)$$

$$9. \quad x = y + 5 \quad / \quad 2x - 3y = 9$$

$$2(y + 5) - 3y = 9$$

$$2y + 10 - 3y = 9$$

$$2y - 3y = -1$$

$$-1y = -1 \quad y = 1$$

$$x = (1) + 5$$

$$x = 6$$

$$(6, 1)$$

$$10. \quad y = -\frac{3}{2}x + 8 \quad / \quad x + 2y = 10$$

$$x + 2\left(-\frac{3}{2}x + 8\right) = 10$$

$$\frac{2}{1} \cdot \frac{-3}{2} =$$

$$x + -3x + 16 = 10$$

$$-2x + 16 = 10$$

$$-2x = -6$$

$$x = 3$$

$$y = -\frac{3}{2}(3) + 8$$

$$-\frac{3}{2} \cdot \frac{3}{1}$$

$$-\frac{9}{2} + \frac{8}{1}$$

$$-\frac{9}{2} + \frac{16}{2} = \frac{7}{2}$$

$$(3, 7/2)$$