1. $y = 3x + 2$ and $2x + y = 11$

$2x + (3x + 2) = 11$

$2x + 3x + 2 = 11$

$5x + 2 = 11$

$5x = 9$

$x = \frac{9}{5}$

$y = 3\left(\frac{9}{5}\right) + \frac{2}{1}$

$\frac{27}{5} + \frac{10}{5} = \frac{37}{5}$

$y = \frac{37}{5}$

$\left(\frac{9}{5}, \frac{37}{5}\right)$

2. $x = 2y - 1$ and $3x + y = 12$

$3(2y - 1) + y = 12$

$6y - 3 + y = 12$

$7y - 3 = 12$

$7y = 15$

$y = \frac{15}{7}$

$x = 2\left(\frac{15}{7}\right) - 1$

$\frac{30}{7} - \frac{7}{7}$

$x = \frac{23}{7}$

$\left(\frac{23}{7}, \frac{15}{7}\right)$

Rule of Thumb : Tip

"$+(\ldots)$" → Just copy everything inside

"$-(\ldots)$ → Flip all signs inside

"number$(\ldots)$" → Multiply through

3. $y = -x + 4$ and $2x + y = 7$

$2x + (-x + 4) = 7$

$2x - x + 4 = 7$

$1x + 4 = 7$

$1x = 3 \quad x = \frac{3}{1}$

$y = -\left(\frac{3}{1}\right) + 4$

$\frac{-3}{1} + \frac{4}{1} = 1$

$y = 1$

$\left(\frac{3}{1}, 1\right)$

4. $x = y + 3$ and $4x - 2y = 10$

$4(y+3) - 2y = 10$

$4y + 12 - 2y = 10$

$2y + 12 = 10$

$2y = -2$

$y = -1$

$x = -1 + 3$

$x = 2$

$(2, -1)$

5. $y = \frac{1}{2}x - 1$ and $x + y = 7$

$x + \left(\frac{1}{2}x - 1\right) = 7$

$2\left(\frac{x}{1} + \frac{1}{2}x - 1 = 7\right)$

$\frac{3}{2}x - 1 = 7$

$\frac{3}{2}x = 8 \quad \frac{3}{2} \div \frac{8}{1} \quad \frac{3}{16}$

$x = \frac{3}{16}$

$y = \frac{1}{2}\left(\frac{3}{16}\right) - 1$

$\frac{4}{32} - \frac{1}{1}$

$\frac{4}{32} - \frac{32}{32} = \frac{-28}{32}$

$\left(\frac{3}{16}, \frac{-28}{32}\right)$

Substitution: Systems of Equations Final Drills:

1. $y = 2x + 1$ / $3x + y = 10$

$3x + (2x+1) = 10$

$3x + 2x + 1 = 10$

$5x + 1 = 10$

$5x = 9$

$x = \frac{9}{5}$

$y = 2\left(\frac{9}{5}\right) + 1$

$\frac{18}{5} + \frac{5}{5} = \frac{23}{5}$

$y = \frac{23}{5}$

Solution $= \left(\frac{9}{5}, \frac{23}{5}\right)$

2. $x = y - 2$ / $4x + y = 10$

$4(y-2) + y = 10$

$4y - 8 + y = 10$

$4y + y = 18$

$5y = 18$

$y = \frac{18}{5}$

$x = \frac{18}{5} - 2$

$\frac{18}{5} - \frac{2}{1}$

$\frac{18}{5} - \frac{10}{5} = \frac{8}{5}$

$x = \frac{8}{5}$

Solution: $\left(\frac{8}{5}, \frac{18}{5}\right)$

3. $y = -x + 7$ / $2x + y = 9$

$2x + (-x + 7) = 9$

$2x - x + 7 = 9$

$x + 7 = 9$

$x = 2$

$y = -(2) + 7$

$y = -2 + 7$

$y = 5$

Solution: $(2, 5)$

4. $x = \frac{1}{2}y$ / $3x + y = 15$

$3(\frac{1}{2}y) + y = 15$ $\frac{3}{2}y + y = 15$ $\frac{3}{2}y + \frac{2}{2}y = \frac{5}{2}y = 15$

$(3, 6)$

5. $y = \frac{3}{4}x - 5$ / $2x + y = 7$

$2x + (\frac{3}{4}x - 5) = 7$

$\frac{2x}{1} + \frac{3}{4}x - 5 = 7$

$\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$

$\frac{11}{4}x - 5 = 7$

$\frac{11}{4}x = 12$

$x = 12 \cdot \frac{4}{11} = \frac{48}{11}$

$x = \frac{48}{11}$

$y = \frac{3}{4}(\frac{48}{11}) - 5$

$y = -\frac{19}{11}$

Solution:

$(\frac{48}{11}, \frac{-19}{11})$

6. $y = -2x + 4$ / $x + y = 1$

$x + (-2x + 4) = 1$

$x - 2x + 4 = 1$

$-x + 4 = 1$

$-x = -3$

$x = -3$

$y = -2(-3) + 4$

$6 + 4$

$y = 10$

$(-3, 10)$

7. $x = \frac{2}{3}y$ / $x + y = 20$

$\frac{2}{3}y + y = 20$

$\frac{2}{3} + \frac{3}{3}\ \frac{5}{3}y = 20 \cdot \frac{3}{5}$

$y = 12$

$x = \frac{2}{3}(12)$

$\frac{2}{3} \cdot \frac{12}{1}$

$x = \frac{24}{3}$

$\left(\frac{24}{3}, 12\right)$

8. $y = \frac{1}{2}x + 6$ / $3x - y = 12$

$3x - \left(\frac{1}{2}x + 6\right) - y = 12$

$3x - \frac{1}{2}x + 6 - y = 12$

$\frac{3}{1} - \frac{1}{2}$

$\frac{6}{2} - \frac{1}{2} = \frac{5}{2}$

$\frac{5}{2}x + 6 = 12$

$\frac{5}{2}x = 6 \cdot \frac{2}{5} = \frac{12}{5} = x$

$y = \frac{1}{2}\left(\frac{12}{5}\right) + 6$

$y = \frac{6}{5} + \frac{6}{1}$

$\frac{6}{5} + \frac{30}{5}\ \ \frac{36}{5}$

$\left(\frac{12}{5}, \frac{36}{5}\right)$

9. $x = y + 5$ / $2x - 3y = 9$

$2(y+5) - 3y = 9$

$2y + 10 - 3y = 9$

$2y - 3y = -1$

$-1y = -1 \quad y = 1$

$x = (1) + 5$

$x = 6$

$(6, 1)$

10. $y = \frac{-3}{2}x + 8$ / $x + 2y = 10$

$x + 2(\frac{-3}{2}x + 8) = 10$

$\frac{2}{1} \cdot \frac{-3}{2} =$

$x + -3x + 16 = 10$

$-2x + 16 = 10$

$-2x = -6$

$x = 3$

$y = \frac{-3}{2}(3) + 8$

$\frac{-3}{2} \cdot \frac{3}{1}$

$\frac{-9}{2} + \frac{8}{1}$

$\frac{-9}{2} + \frac{16}{2} = \frac{7}{2}$

$(3, 7/2)$

Systems of Equations: Elimination Method Concept Notes

1. What is "Elimination?"

- A method for solving systems of equations by adding or subtracting equations to eliminate one variable.
- After one variable is eliminated, solve for the other, then back-substitute

2. General Process:

1. Line up equations (variables and constants stacked vertically).

$$a_1x + b_1y = c_1 \quad a_2x + b_2y = c_2$$

2. Choose a variable to eliminate - Look for coefficients that already match (or are easy to match).
3. Multiply one or both equations if necessary so the coefficients of that variable are equal (or opposite).
4. Add or Subtract the equations to eliminate that variable.
5. Solve the remaining variable.
6. Back-substitute into either original equation to find the other variable

3. Key Tricks:

- Signs matter: Sometimes you add equations, sometimes you subtract them.
- $0 = 5 \rightarrow$ No Solution
- $0 = 0 \rightarrow$ Infinite Solutions.

4. Example #1: (Basic Match).

Solve: $2x + y = 10$ $3x - y = 5$

Step 1: Lines up:

- $2x + y = 10$
- $3x - y = 5$

Step 2: Notice $+y$ and $-y$ cancel if added.

Step 3: Add equations

$$(2x + 3x) + (y - y) = 10 + 5$$

$$5x = 15$$

$$x = 3$$

Step 4: Substitute into $2x + y = 10$

$$2(3) + y = 10$$

$$6 + y = 10$$

$$y = 4$$

Doesn't matter which equation we plug the solution into just pick simplest one.

Solution: $(3, 4)$

Example #2: Multiplication

Problem: $2x + 3y = 12$ $4x + 5y = 20$

Step 2: Choose variable to eliminate

- Coefficients of x are 2 and 4
- Double the first equation, both will have 4x.

Step 3: Multiply the first equation by 2.

$$(2x + 3y = 12) \cdot 2 \rightarrow 4x + 6y = 24$$

Step 4: Eliminate x

$$(4x - 4x) + (6y - 5y) = 24 - 20$$

$$0x + y = 4 \qquad y = 4$$

Step 5: Back-Substitution

plug $y = 4$ into $2x + 3y = 12$

$$2x + 3(4) = 12$$

$$2x + 12 = 12$$

$$2x = 0$$

$$x = 0$$

Solution = $(x, y) = (0, 4)$.

Elimination Coefficient Rules:

Allowed Moves:

1. Multiply or divide the entire equation by a non zero constant.
   - eg. $2x + 4y = 10 \rightarrow$ Divide by $2 \rightarrow x + 2y = 5$
   - This is safe because every term is scaled equally.
2. Add or subtract one entire equation from another.

   eg. $(2x + y = 10)$ and $(3x - y = 5)$

   Adding gives: $5x = 15$

Not Allowed:

- Changing just one coefficient by itself.

  Example: changing $2x + y = 10$ into $5x + y = 10$ without multiplying the whole equation