Topologically fault-tolerant measurement based quantum computation

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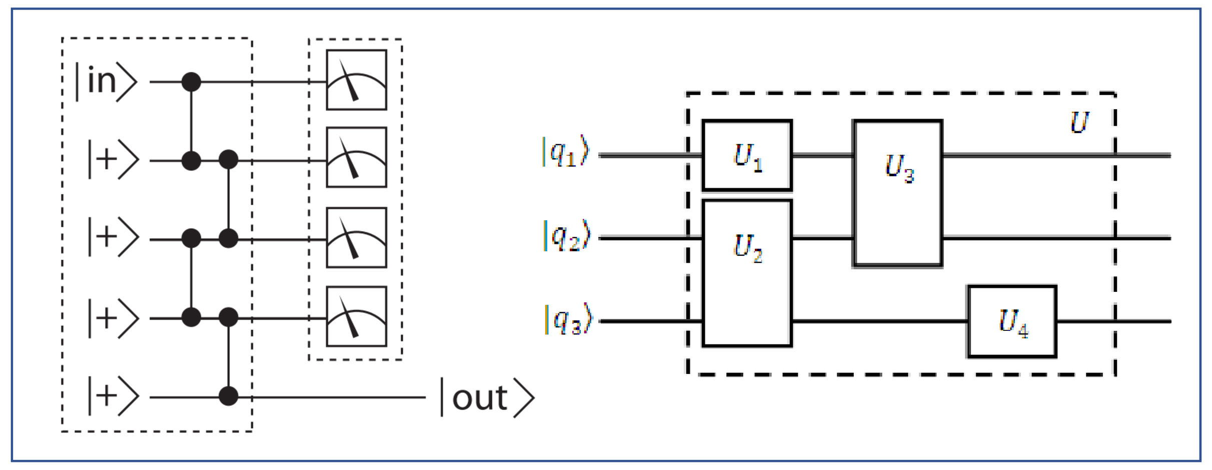
Introduction:

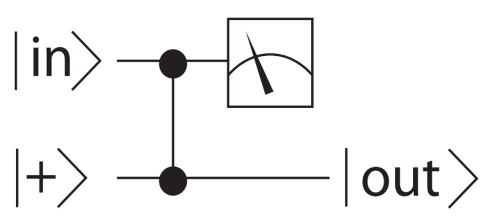
- Measurement-Based Quantum Computation (MBQC) is an model of quantum computation in which computation on a qubit is performed by performing measurements on one of the entangled qubits to teleport the qubit with some operation applied to it to other qubits and hence doing the computation in the process.
- MBQC is a universal computational model same as the circuit model of quantum computation.



MBQC Circuit model

- Two reasons Measurement based quantum computation is advantageous than quantum circuit model of computation.
1. Compared to quantum gates, single-qubit measurements are easier to implement in practice with higher fidelity
2. the resource states in MBQC can be independent of specific computational tasks
- Surface code is an encoding with a very high code distance.
- If both schemes can be used simultaneously then resulting computational scheme will be highly useful.
- In this poster we will explore the same idea.

MBQC: Teleportation with a twist



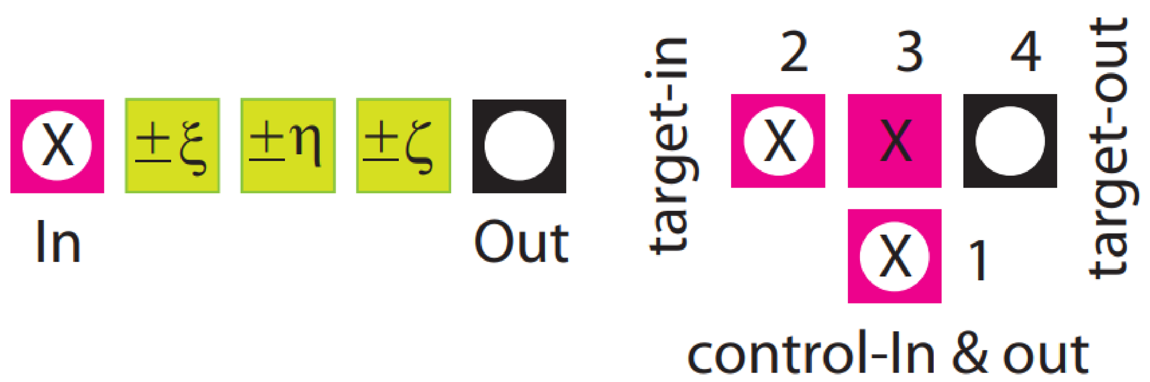
- Entangling the input state with ancilla qubit in |+> state and measuring the input qubit in the basis,

$$O(\varphi_1) = \cos\varphi_1\, X + \sin\varphi_1\, Y, \quad -\frac{\pi}{2} < \varphi \leq \frac{\pi}{2}$$

- Other qubit state transform into,

$$|\mathrm{out}\rangle = He^{i\phi Z/2} Z^s |\mathrm{in}\rangle$$

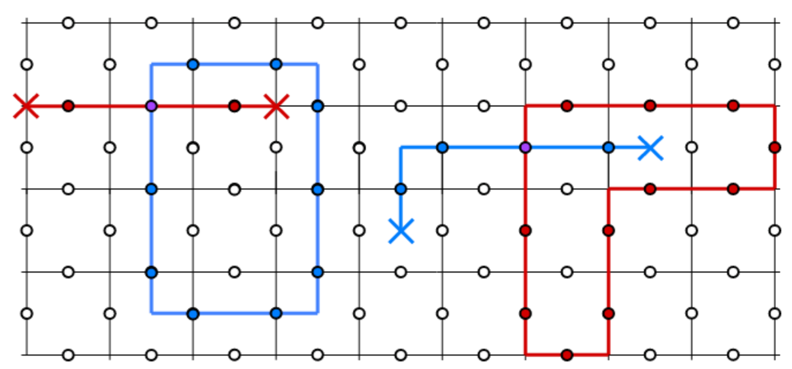
- Repeating this operation iteratively rotation around Z and X-axis can be performed, hence a general one-qubit gate can be performed.
- One-qubit general rotation and CNOT gate form a universal set of gates, measurement pattern for them can be seen below,



General one-qubit rotation CNOT gate

Surface code:

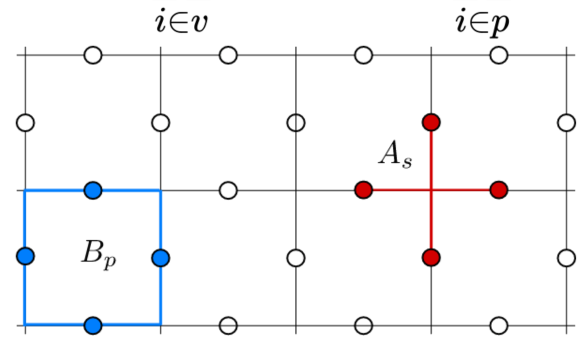
- Consider qubits arranged on the lattice as shown.
- Two types of error can affect a qubit:
1. Bit-flip(Pauli X)
2. Phase-flip(Pauli Y)



Stabilizer operators

- Define two stabilizer operators:

$$A_v = \prod_{i\in v} X_i, \quad B_p = \prod_{i \in p} Z_i$$

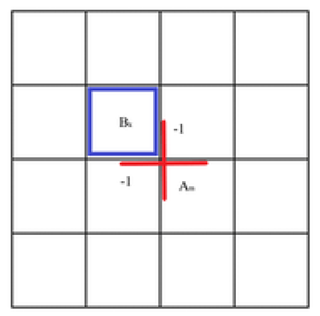


- A_V is known as Vertex operator and B_p is known as plaquette operator.

Code state

- Code state is defined as +1 eigenstate of both operators on all vertices and plaquettes,

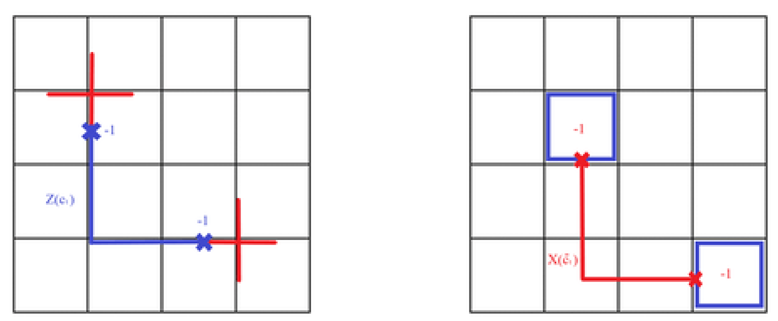
$$A_m|\Psi\rangle = |\Psi\rangle, \quad B_k|\Psi\rangle = |\Psi\rangle, \forall f_m \in F \;\&\; \forall v_k \in V$$



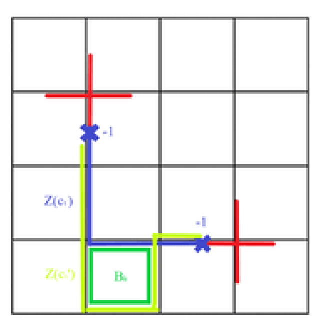
- Commutativity of plaquette and vertex operators is obvious when they don't share any edges, but when they share edges those are even times so -1 cancel out so plaquette and star operators commute.

Error syndrome

- Z and X-error chains can be detected by star and plaquette operators respectively

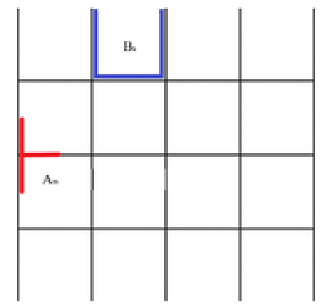


- Error chains ending on the same points are indistinguishable



1. Planar code:

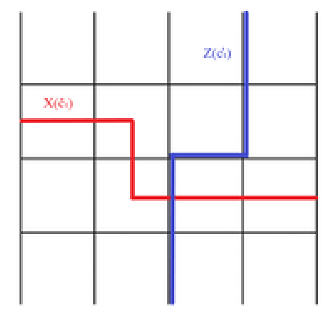
- A quantum state can be encoded into lattice with boundary condition
- Two types of boundaries:
1. Rough Edge (there are no qubits on the edge)
2. Smooth Edge (there are qubits on the edge)



- n Qubits create the Hilbert space of dimension 2^n
- Only +1 eigenspace of the each stabilizer operator reduces dimension by 2^{-1}
- So we have $2^{\text{no. of qubits} - \text{no. of stabilizers}} = 2$ dimensional Hilbert space in planar code equivalent to one encoded qubit.

Logical operators:

- Notice when Z-chain ends on rough boundary, there is no vertex operator there, so it produces no syndrome, there is still in the code space but different than the original state, same is true for X-chain ending on smooth boundary.

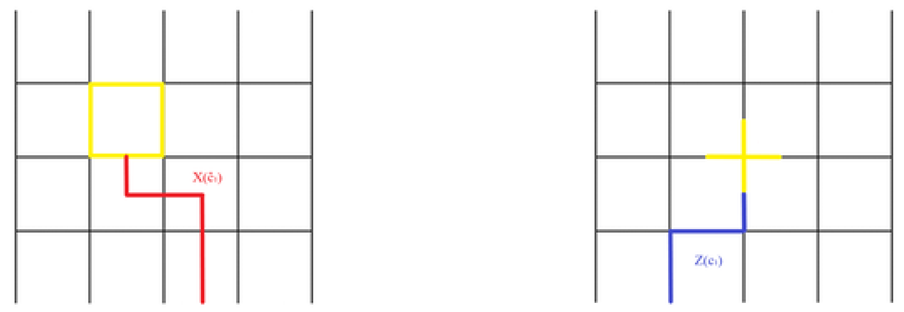


Logical operators for planar code

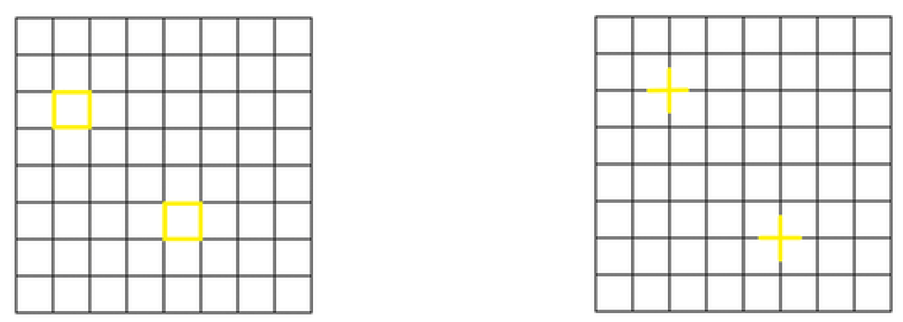
- So we define Z-chains ending on opposite rough boundaries as Z-operator on encoded qubit, X-operator can be defined similarly.

2. Defect based qubits:

- Two observations:
1. If we remove a stabilizer then respective logical operator chain can end at the site of absent stabilizer.

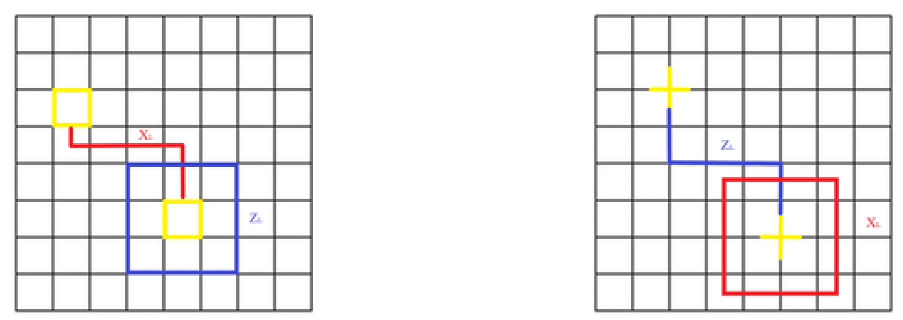


2. Removing one stabilizer increases dimension of Hilbert space by 1, by removing two stabilizers we can encode a qubit.



Primal defect qubit Dual defect qubit

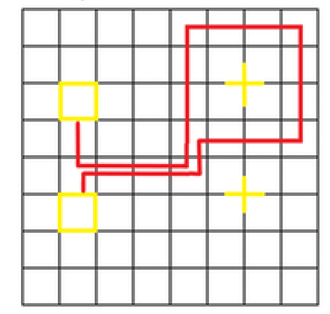
- Logical operators for respective qubits can be defined as shown below,



Logical operators for defect pair qubits

CNOT gate:

- If we braid one of defect around other defect and bring defects in the initial configuration then a loop is left around other defect.
- Hence applying a logical operation in process.



Braiding

- CNOT gate is defined as below, which can be performed by braiding,

$$CNOT^\dagger I_p \otimes X_d CNOT = I_p \otimes X_d$$

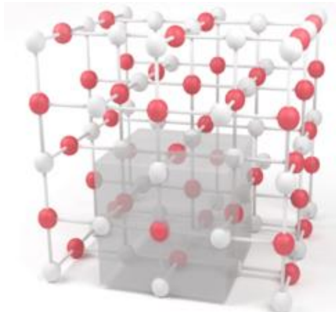
$$CNOT^\dagger X_p \otimes I_d CNOT = X_p \otimes X_d$$

$$CNOT^\dagger I_p \otimes Z_d CNOT = Z_p \otimes Z_d$$

$$CNOT^\dagger Z_p \otimes I_d CNOT = Z_p \otimes I_d$$

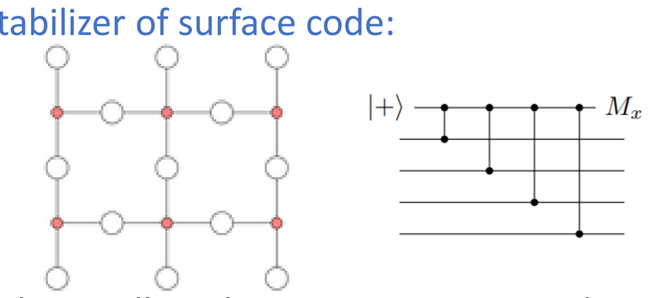
Amalgamation of MBQC with surface code:

- Consider RHG lattice in the diagram, It has two different type of slices



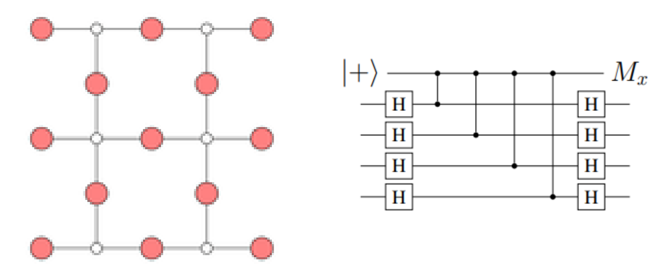
RHG lattice

1. Odd slice - Z stabilizer of surface code:



- by measuring the ancilla qubit we measure Z-syndrome on all four qubits simultaneously, i.e. plaquette operator $B_p = \prod_{i\in p} Z_i$.
- Data qubits get teleported to next layer with operator |out⟩ = H|in⟩.

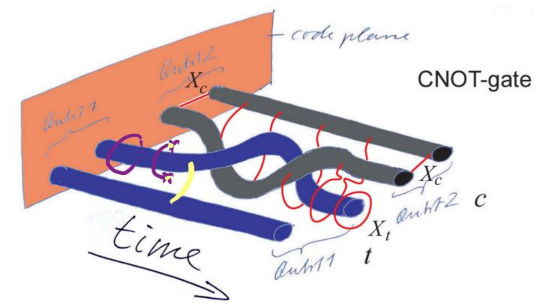
2. Even slice - X stabilizer of surface code:



- by measuring the ancilla qubit we measure X-syndrome on all four qubits simultaneously, i.e. vertex operator $A_v = \prod_{i\in v} X_i$.

Topological fault-tolerant measurement based quantum computation

- As we encoded a qubit in planar code we can similarly encode a qubit in slices of 3d-cluster state. Here in the diagram defect pair qubit world line is shown. Defects are braided to perform CNOT gate.



Resources and Image credits:

1. Surface codes: Towards practical large-scale quantum computation, A. G. Fowler et al
2. Fault-Tolerant Quantum Computation with High Threshold in Two Dimensions, Robert Raussendorf et al.
3. Quantum computation by local measurement, Robert Raussendorf et al.
4. Quantum Computation with Topological Codes: from qubit to topological fault tolerance, Keisuke Fujii
5. Lattice Surgery on the Raussendorf Lattice, Daniel Herr