Cosmological Data Fitting

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Abstract

The main aim of this project is to explore cosmological data contained in the Pantheon SN1a sample, a collection of 1048 supernovae combining Pan-STARRS (PS1) medium deep survey, SDSS, SNLS, and various low-z and HST data samples.

When constraining the sample to a wCDM cosmological model using chi-square analysis in the form of either grid search sweeping or direct search simplex method we can estimate the best fit parameters and compare them with the original analysis as given by Scolnic et al (2018).

Furthermore, the same analysis was performed on the Pantheon+SH0ES data sample, and Principal Component Analysis was performed on both samples in order to reduce the parameter space.

Review of Cosmology

We begin with the Luminosity distance as a function of the Density Parameters:

$$d_{L,th}(z) = \frac{(1+z)c}{H_0\sqrt{|\Omega_{k0}|}} S\left(\sqrt{|\Omega_{k0}|}\int_0^z \frac{dz'}{E(z')}\right) \tag{1}$$

Where c, H_0, z and Ω_{k0} are the speed of light, Hubble Parameter, redshift and Curvature density parameters respectively. S is defined as,

$$S(x) = \begin{cases} \sin x \text{ for } \Omega_{k0} > 0 \\ x \text{ for } \Omega_{k0} = 0 \\ \sinh x \text{ for } \Omega_{k0} < 0 \end{cases} \tag{2}$$

And $E(z)$ is,

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}(1+z)^{3(1+w(z))} + \Omega_{k0}(1+z)^2} \tag{3}$$

Where Ω_{m0}, $\Omega_{\Lambda 0}$ and $w(z)$ are the Matter density parameter, Dark Energy density parameter, and the Dark Energy equation of state parameters respectively.

Luminosity distance can also be defined as a function of the apparent B-magnitude:

$$d_{L,obs} = 10^{\frac{(m_B - M_B)}{5} - 5} \text{ Mpc} \tag{4}$$

Thus, there are multiple equations that describe Luminosity distance, and optimization is needed to equate these distances.

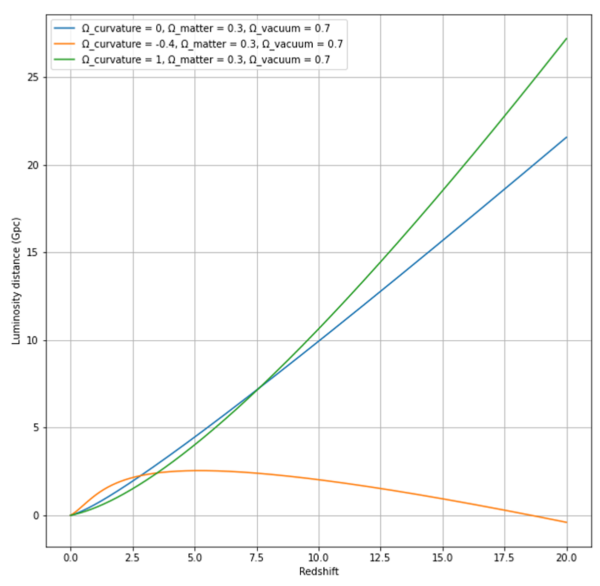


Figure 1. Evolution of the universe (Luminosity Distance, Gpc vs Redshift) according to different Density Parameters: Blue - Flat. Green - Open. Yellow - Closed.

Research objectives

The purpose of the project is to optimize the Pantheon and Pantheon+SH0ES datasets with the parameters of the ΛwCDM Model of Cosmology.

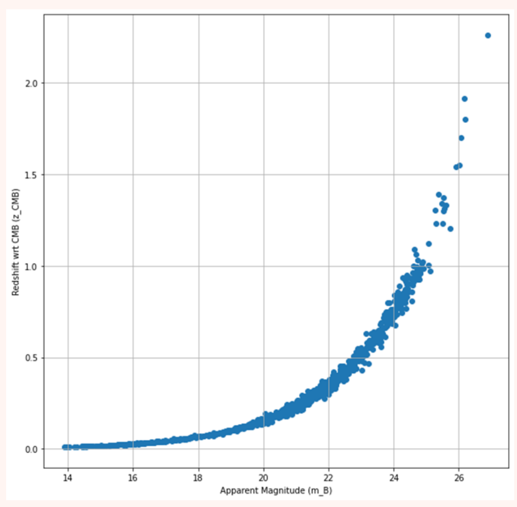


Figure 2. Plot of Redshift (y-axis) vs Apparent Magnitude (x-axis) for Pantheon dataset

We also prove some standard results, including a preference for flat cosmology, and that the Taylor series expansion around $a = 1$ for the equation of state parameter w yields vanishing contributions from higher derivatives.

Parametrization of the equation of state parameter w was performed as:

$$w(z) = w_0 + w_1\left(\frac{z}{1+z}\right) + w_2\left(\frac{z}{1+z}\right)^2 + \cdots \tag{5}$$

Study methodology

The data sets contain two relevant data coordinates: apparent B-magnitude and redshift. We wish to then constrain M_B, H_0, Ω_{m0}, $\Omega_{\Lambda 0}$, Ω_{k0}, and $w(z)$. This was done using the framework of Chi-square analysis, given below:

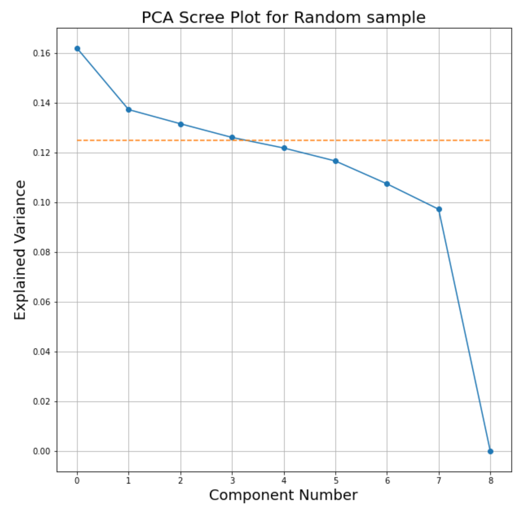
$$\chi^2 = \sum_{i=1}^{N} \frac{(d_{L,obs,i} - d_{L,th,i})^2}{d_{L,th,i}} \tag{6}$$

However, the parameter set must first be reduced in order of priority, which was done by evaluating χ^2 1000 times over random values of the parameters, then performing Principal Component Analysis (PCA) to figure out how many parameters can be dropped without significant decrease in variability and which parameters should be kept.

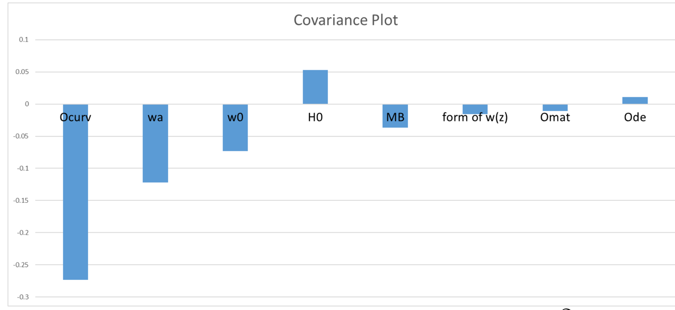
The data set was then optimized for the reduced parameter set using a Hyperparameter Grid Search, as well as using the Nelder-Mead/Downhill Simplex method, a direct search algorithm.

Grid Search allows for 2 Dimensional Chi-square contours to be plotted, while Nelder-Mead method owing to a $O(n)$ complexity compared to Grid search's $O(C^n)$ complexity allows for more than 3 parameters to be tuned simultaneously for significantly faster run times at the cost of convergence, which is quite poor.

Constraining the parameter space and $w(z)$ parametrizations



(a) Parameter space Scree Plot

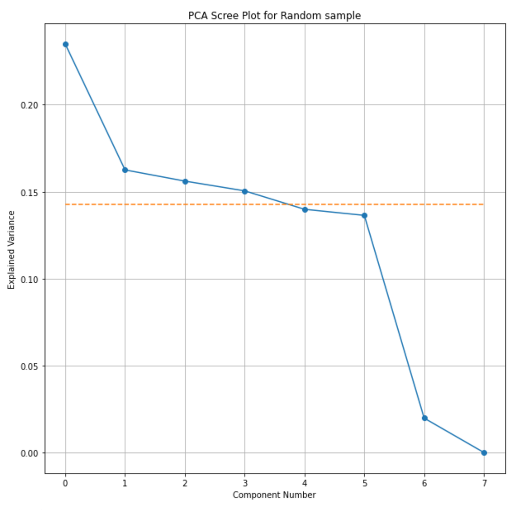


(b) Covariance(Parameter,χ^2)

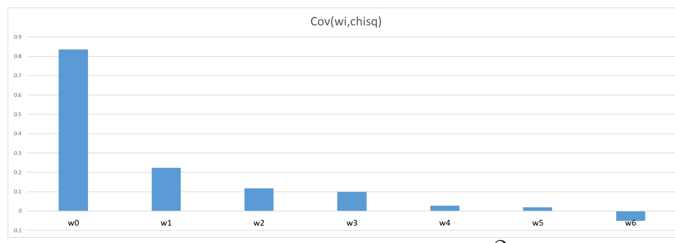
Figure 3. PCA Plots for Parameter Space, taking 1000 randomly generated values for each parameter.

Based on the above figure, we can see that the parameter with the least amount of variance contribution is $\Omega_{\Lambda 0}$, or the Dark Energy Density parameter. Curvature Density Ω_{k0} has the highest variance contribution, however it is negative and thus highly biased towards flat/open models.

Below are the same plots when applied to a Taylor series expansion of $w(z)$:



(a) $w(z)$ coefficient Scree Plot

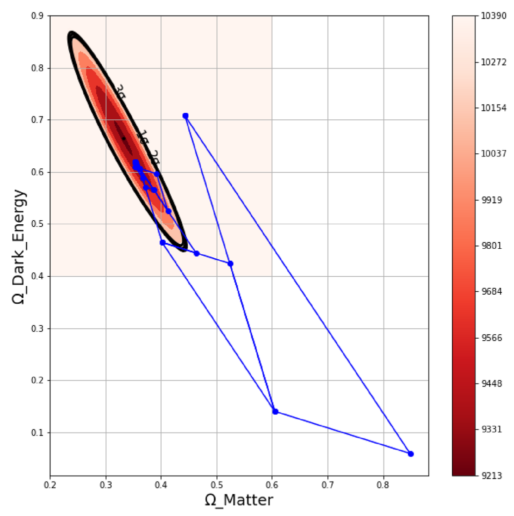


(b) Covariance(w_i,χ^2)

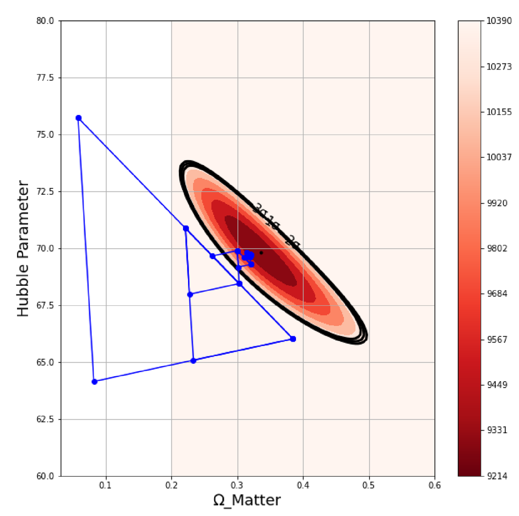
Figure 4. PCA Plots for Taylor Series Expansion of $w(z)$

Optimization

Below are some plots showing optimization (minimization) of χ^2 using both Grid Search Algorithm (Contour plot) and the Nelder-Mead Simplex method:



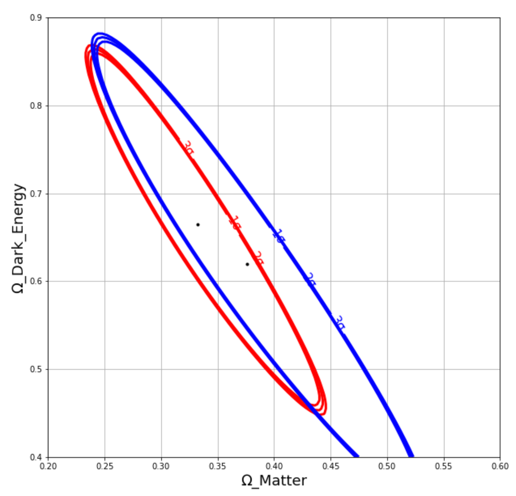
(a) Ω_{m0} vs $\Omega_{\Lambda 0}$



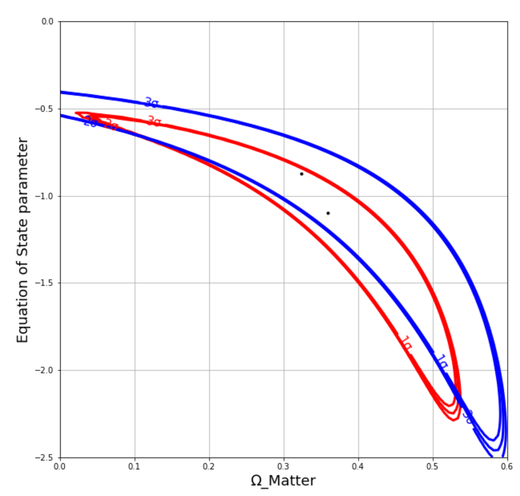
(b) Ω_{m0} vs H_0

Figure 5. Confidence Contour Plots for Parameter vs Parameter. The 1, 2 and 3σ contour lines are in black, the values within the contours are in red, and the blue triangles are the "guesses" made by the Nelder-Mead algorithm

Below are some more plots comparing the confidence contours for both Pantheon (red) and Pantheon+SH0ES (blue) datasets:



(a) Ω_{m0} vs $\Omega_{\Lambda 0}$



(b) Ω_{m0} vs w_0

Figure 6. Confidence Contour Plots for Parameter vs Parameter. The 1, 2 and 3σ contour lines for the Pantheon dataset are in red, while the confidence contour lines for Pantheon+SH0ES are in blue

Results and discussion

Our analysis of both Pantheon and Pantheon+SH0ES yields the following values for each Cosmological parameter:

1. Equation of state parameter, $w = -1.02 \pm 0.163$
2. Matter density parameter, $\Omega_{m0} = 0.345 \pm 0.031$
3. Dark Energy density parameter, $\Omega_{\Lambda 0} = 0.655 \pm 0.031$
4. Curvature density parameter, $\Omega_{k0} = -0.007 \pm 0.045$
5. Hubble parameter, $H_0 = (70.3 \pm 1.6)$ km/s/Mpc

Nelder-Mead has a convergence rate of $(37.0 \pm 5.8)\%$

Conclusions

- Flat models or models with a very slightly negative value of curvature density are most preferred.
- The first coefficient in the parametrization of equation of state parameter w_0, is the most important in the Taylor series expansion, with higher order coefficients having lower contribution.

References

- Barreira A., Avelino P.. (2011)
- De Souza R. S. (2022)
- Scolnic D. M. (2018)
- Jolliffe I. T., Cadima J. (2016)

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