# Threshold-Activated Predator Dispersal in Spatially Extended Ecological Systems

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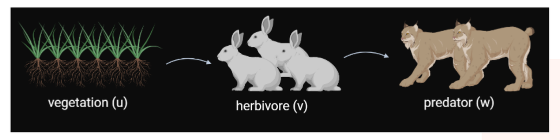
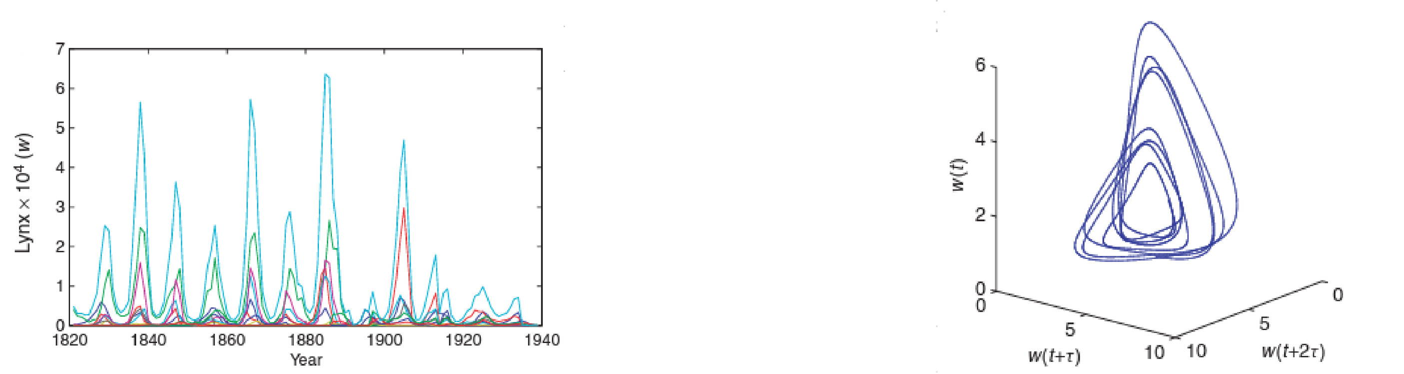
## Tritrophic Food Web Model[1]




The vertical foodweb model with vegetation(u), prey/herbivores(v) and predators(w):

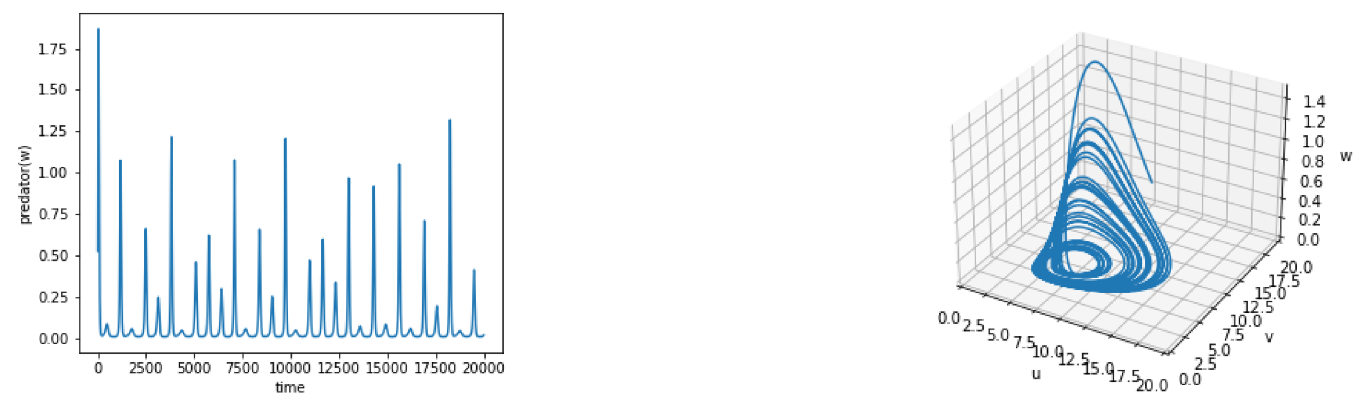
$$\begin{aligned} \dot{u} &= au - \alpha_1 f_1(u,v) \\ \dot{v} &= -bv + \alpha_1 f_1(u,v) - \alpha_2 f_2(v,w) \\ \dot{w} &= -c(w - w_0) + \alpha_2 f_2(v,w) \end{aligned} \tag{1}$$

**Holling-type II** function ($f_i = xy/(1+k_i x)$) - models consumer-resource interactions
**vegetation-herbivore** ($k_1 = 0.05$) - $f_1(u,v) = uv/(1+0.05u)$
**prey-predator** ($k_2 = 0$) - $f_2(v,w) = vw$ (Lotka-Volterra Term).
$a$,$b$ and $c$ - growth rates. $w_0$ - equilibrium level of predator.



(a) Lynx Time Series across Canadian Regions (b) Data-reconstructed Attractor Plot

Figure 1. Population Dynamics of the Canadian Hare-Lynx system



(a) Predator Time Series (b) u-v-w Attractor Plot (simulated)

Figure 2. Population Dynamics of the Tritrophic Food Web Model

- Uniform Phase evolution and Chaotic Amplitude (**UPCA**) observed
- Strikingly similar to the **Canadian Lynx** data

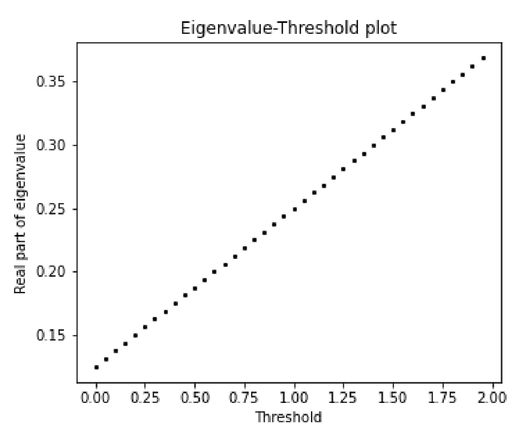
## Stability in the u-v subsystem

2D subsystem of vegetation($u$)-prey($v$) when the predator($w$) is fixed at the threshold value $w_c$:

$$\begin{aligned} f(u,v) &= \dot{u} = au - \alpha_1 \frac{uv}{(1+ku)} \\ g(u,v) &= \dot{v} = -bv + \alpha_1 \frac{uv}{(1+ku)} - \alpha_2 vw \\ w &= w_c \end{aligned} \tag{2}$$

Stability analysis using Jacobian:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{bmatrix} = \begin{bmatrix} a - \frac{\alpha_1 v}{(1+ku)^2} & \frac{-\alpha_1 u}{(1+ku)} \\ \frac{\alpha_1 v}{(1+ku)^2} & -b + \frac{\alpha_1 u}{(1+ku)} - \alpha_2 w_c \end{bmatrix}$$



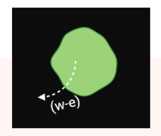

Real part of the $u-v$ Jacobian eigenvalues is positive for all threshold - indicates **unstable spiral** in non-linear systems.

The model is unstable in the $u-v$ subsystem

Extreme events observed in vegetation and prey for low threshold control on predator.

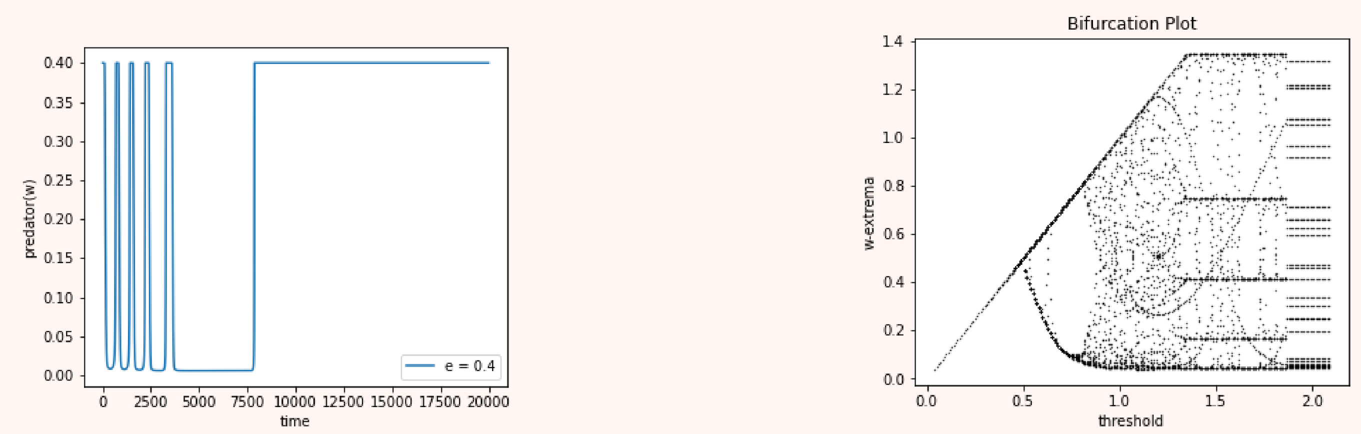
For higher threshold, $u-v$ subsystem is chaotic.

## Threshold Control on a Single Patch



A single food-web patch with a threshold control on the predator population. The excess $(w-e)$ migrates outside the patch to the external world:

- The set of equations (1) are evolved using Euler's method
- If $w(t) > e \implies w(t+1) = e$



(a) Stability by Low Threshold (b) Single Patch Stability Bifurcation

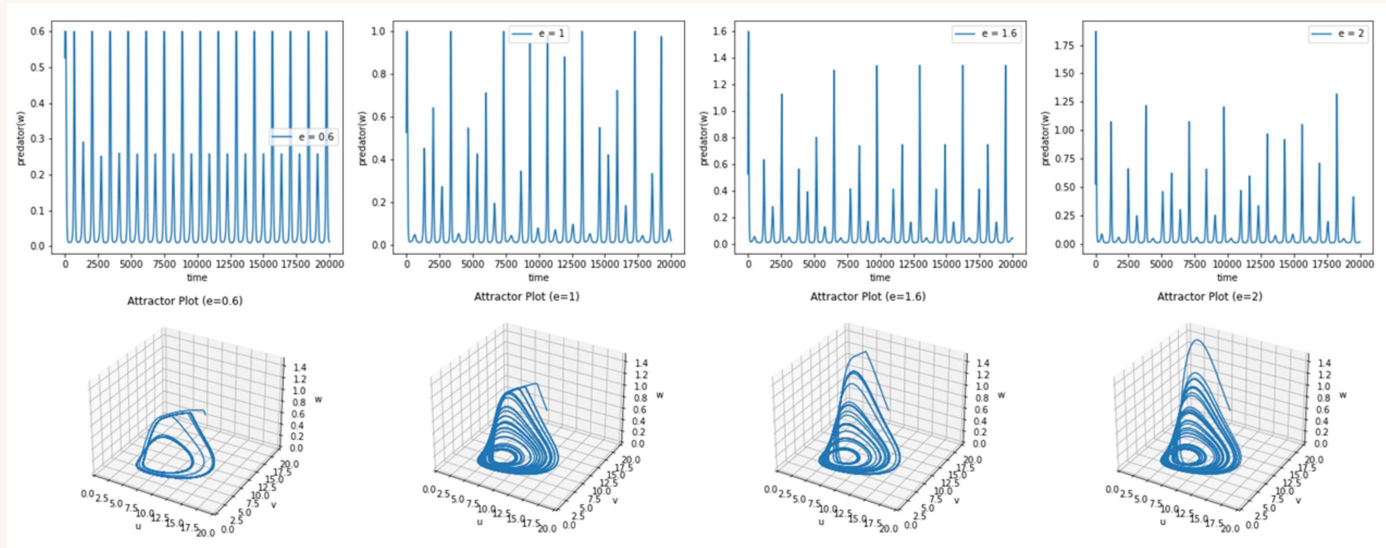


Figure 4. Dynamics across Threshold

- Predator population never crosses threshold value.
- **UPCA dynamics is stabilised by low threshold** ($e < 0.5$).
- Extent of stabilisation decreases with increasing Threshold ($e$).
- Beyond $e \approx 1.8$, threshold effect ceases to exist.

### Vegetation(u)-Prey(v) Dynamics

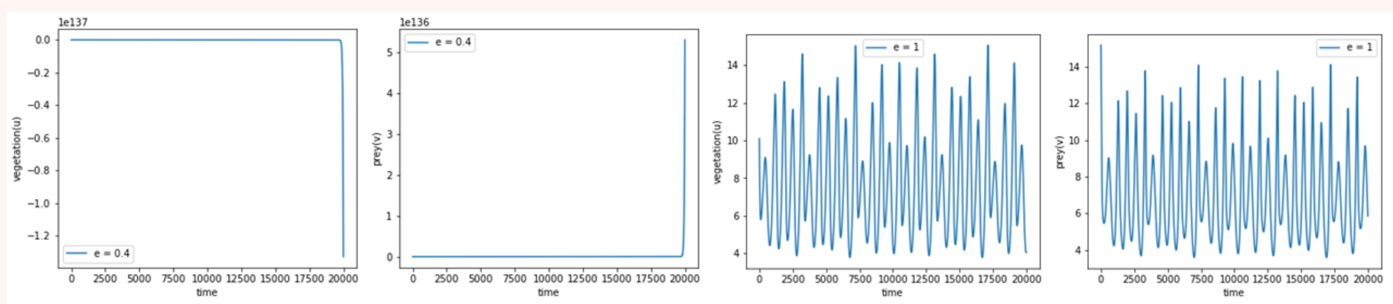
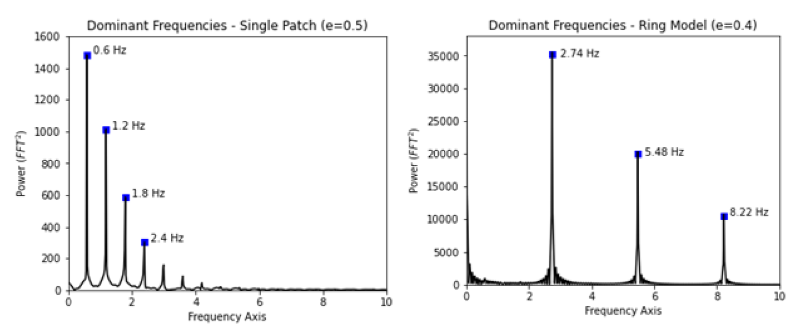


Figure 5. $u-v$ Extreme Events(Low Threshold), Chaotic(High Threshold)

## Frequency Analysis of Predator Oscillations

Using **Fast Fourier Transform (FFT)**, dominant frequency for low threshold (period-2 cycles) analysed for predator oscillations on single patch and patch in a ring.



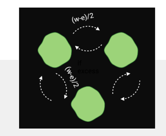

At low threshold, predator oscillates at relatively higher frequency (lower time period) on a patch in a ring compared to single patch.

Dominant frequency peaks for a patch in ring are noisier.

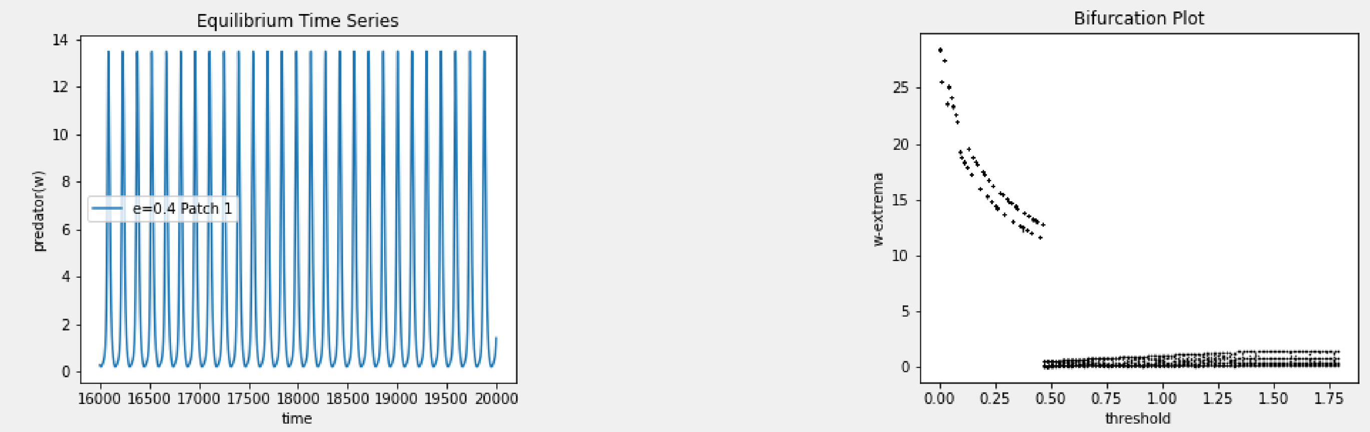
## Ecological Relevance

Threshold-based modeling could give insight into density-dependent dispersal rules owing to habitat saturation, resource availability, etc. The results have potential implications in understanding extinction, extreme events and general patterns in ecological populations connected in discontinuous spaces such as foraging patches with application in conservation studies.

## Threshold Control in a Ring of Patches



A ring of ecological patches ($N = 3$) with bidirectional symmetric dispersal of the predator across the ring. Half of the excess $(w-e)$ migrates to each of the neighbouring patches.




(a) Period-2 cycle by Low Threshold (b) Patch in a Ring Stability Bifurcation

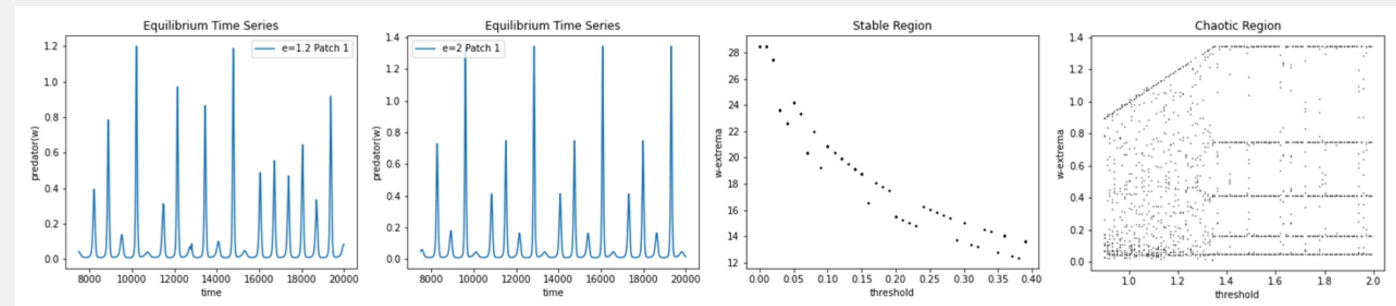


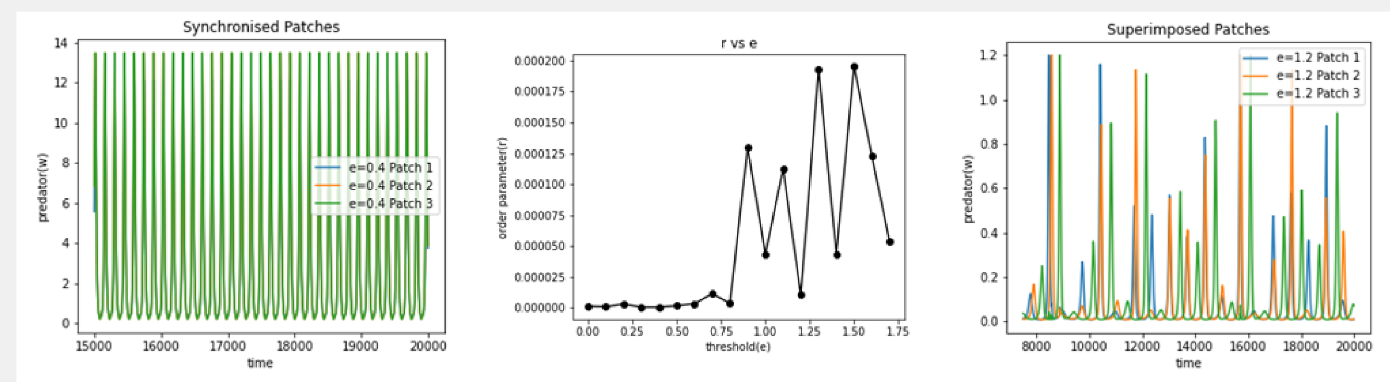

Figure 7. Dynamics across Threshold regions

- Closed-coupling between patches through **low threshold** (upto $e \approx 0.5$) leads to a **period-2 cycle of predator with high amplitudes** (way above threshold value).
- Beyond the critical threshold point ($e \approx 0.5$), chaotic dynamics are observed.
- Threshold effect reduces with increasing threshold but never ceases to exist.

### Synchronisation Order Parameter

Mean Square Deviation ($r$):

$$r = \left(\frac{1}{N}\sum_i (w_i - W_{avg})^2\right)_{tavg} \tag{3}$$




- Patches in a ring synchronise for low threshold with respect to predator population

### Vegetation(u)-Prey(v) Dynamics

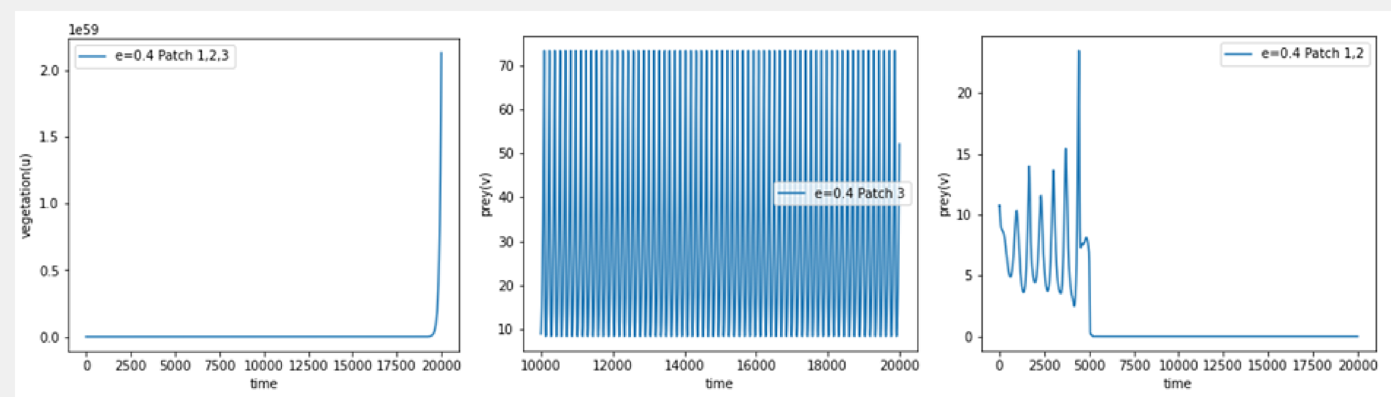


Figure 8. $u-v$ Extreme Events(Vegetation), Patch-Saturation(Prey) for Low Threshold

## References

1. Blasius, B. et al. (1999, May). Complex dynamics and phase synchronization in spatially extended ecological systems. Nature, 399(6734), 354–359.
2. Meena, C., Rungta, P. D., Sinha, S. (2017, August 25). Threshold-activated transport stabilizes chaotic populations to steady states. PLOS ONE, 12(8), e0183251.

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