Non-equilibrium transport in quantum chains under quasiperiodic potentials

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Motivation

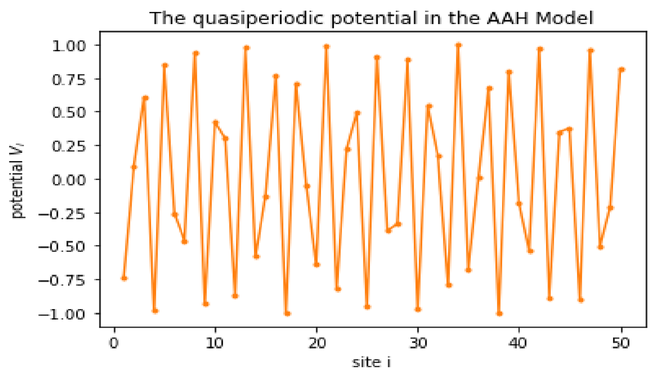
Non-trivial transport properties, localisation transitions and fractal properties appearing in one-dimensional quasiperiodic potentials have kept physicists (and mathematicians as well) interested for quite some time. With novel experimental tools and present-day technology allowing us to have extreme control over quantum systems, this interest has resurfaced. In this thesis, we numerically study an interpolating model between the two most paradigmatic one-dimensional quasiperiodic models, the Aubre-André-Harper (AAH) model and the Fibonacci model and study the non-equilibrium transport in an open and closed system setup.

Aubry-André-Harper Model

The Hamiltonian is given by the tight-binding Hamiltonian with on site potential,

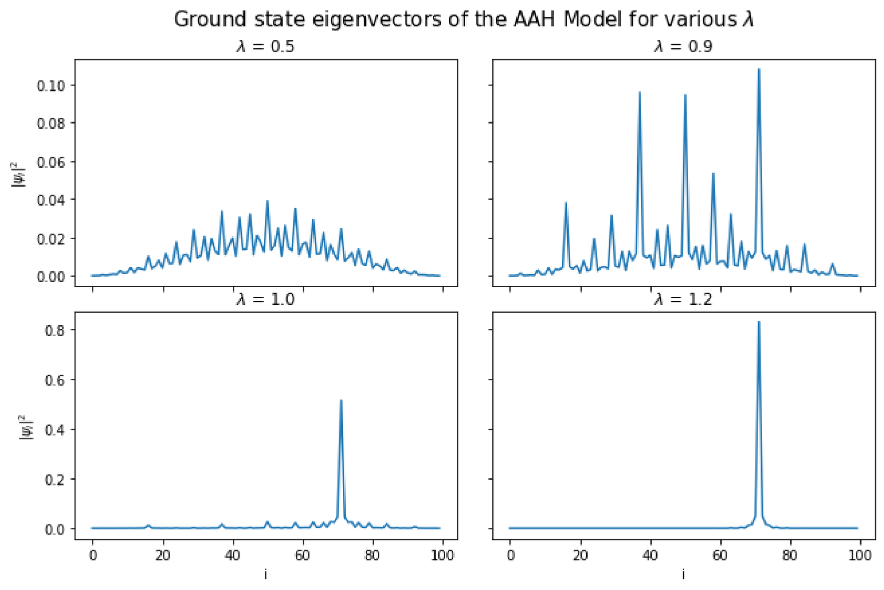
$$\mathcal{H} = -J\sum_{i=1}^{L-1}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \sum_{i=1}^{L} V_i c_i^\dagger c_i$$

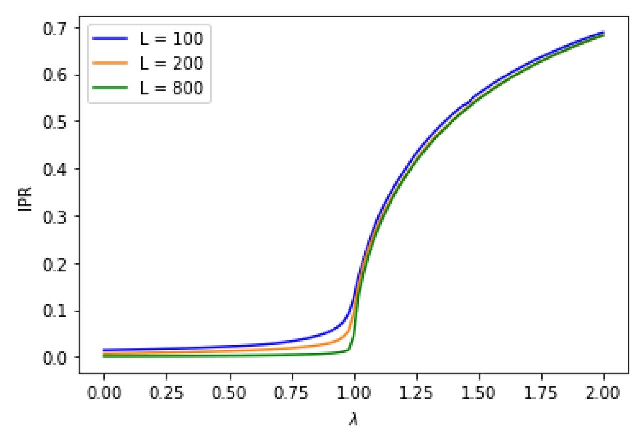
$$V_i = 2\lambda\cos(2\pi b i + \theta), \quad i = 1, \ldots, L$$

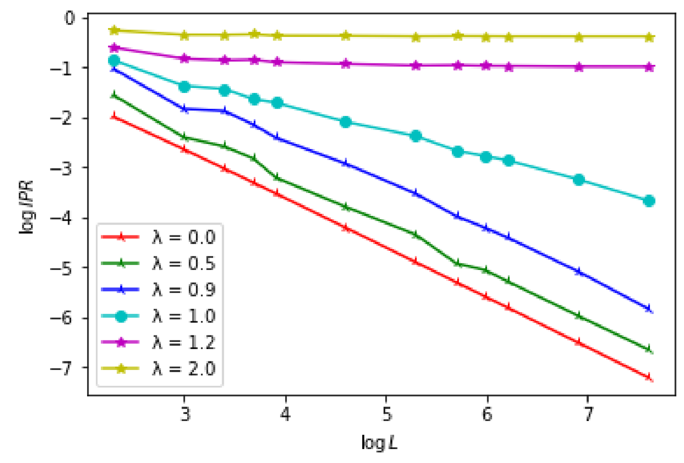


- λ is the potential strength, b is an irrational constant which we set to the golden ratio φ and θ is an arbitrary global phase.
- When b is irrational the potential never repeats itself on the lattice.
- It undergoes a localisation transition at λ = 1.0 (which is a multi-fractal state).
- The Inverse Participation Ratio (IPR) is one of the measures used to quantify localisation of states

$$\mathrm{IPR}(\psi) = \sum_{n=1}^{L} |\psi_n|^4$$



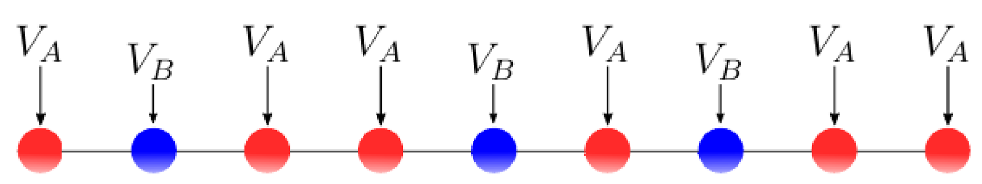




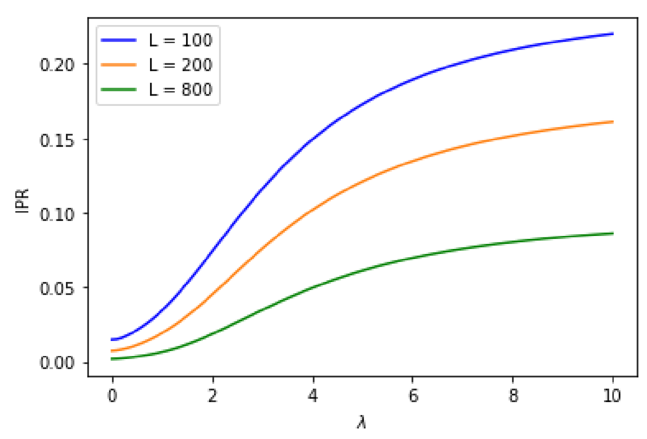
Fibonacci Model

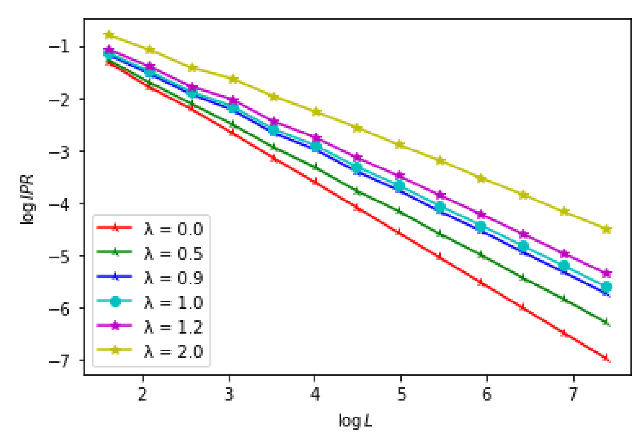
The Hamiltonian is given by the tight-binding Hamiltonian with on site potential,

$$V_i = \lambda\left(\left[\frac{i+1}{\varphi}\right] - \left[\frac{i}{\varphi}\right]\right)$$



- λ is the potential strength and φ is the golden ratio.
- This potential is motivated by the sequence called "*Fibonacci word*".





Interpolating Aubry-André Fibonacci Model

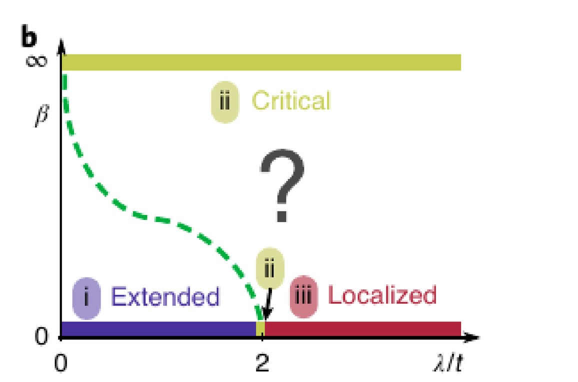
The Hamiltonian is given by the tight-binding Hamiltonian with on site potential,

$$V_j(\beta) = -\frac{\tanh\left[\beta\cos\left(2\pi b j + \phi\right) - \beta\cos\left(\pi b\right)\right]}{\tanh\beta}$$

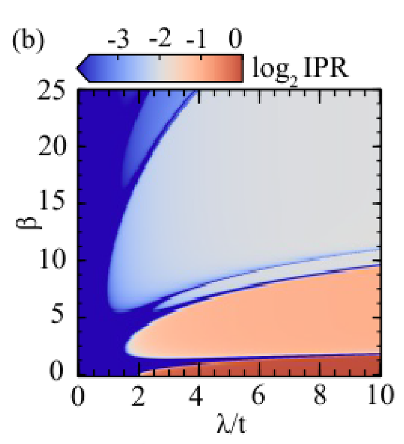
β –> 0 is the AAH limit
β –> ∞ is the Fibonacci limit

The QR code on the right shows the animation of the potential going from the AAH limit to the Fibonacci limit.

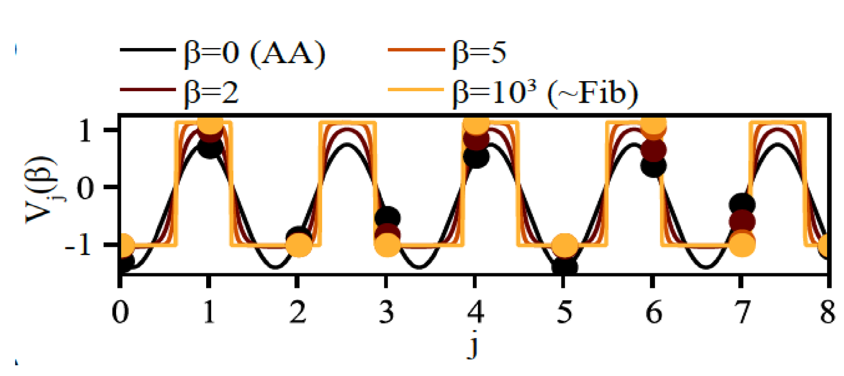
Animation: IAAF Potential



Source: Fig 1.b DOI:10.1038/s41567-020-0908-7



Source: Fig 2.b arXiv:2106.13841v2



Source: Fig 1.a arXiv:2106.13841v2

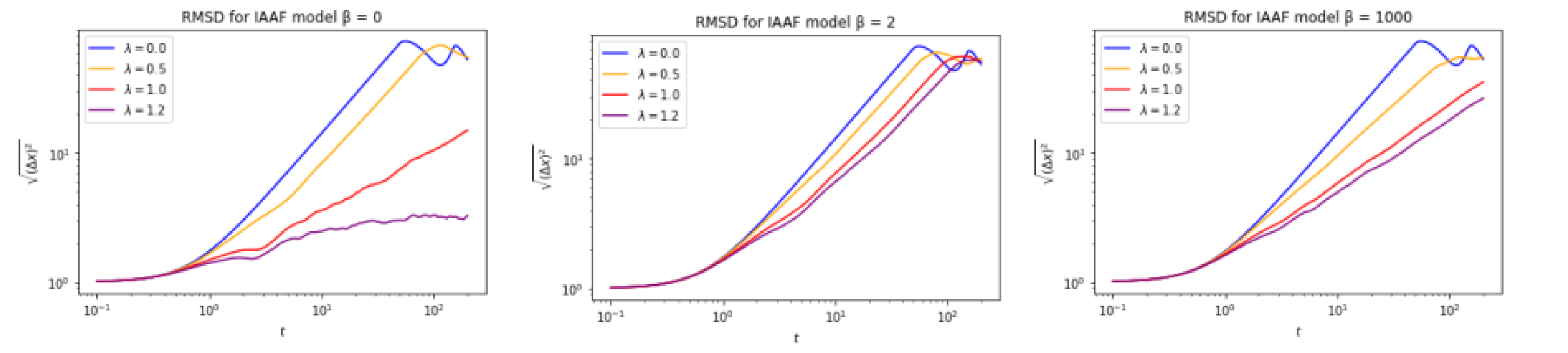
Unitary Evolution

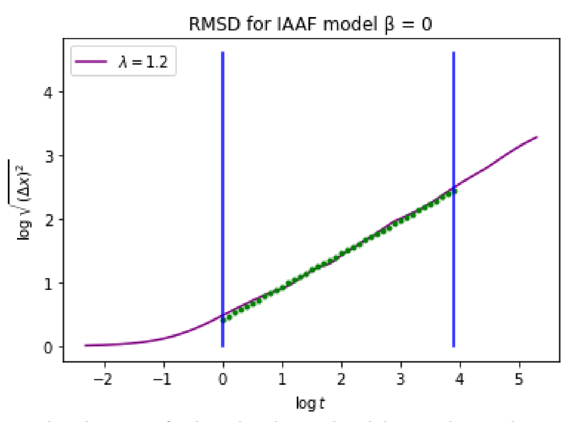
A particle (delta wavefunction) was initialised at the central site of the chain and was let to evolve unitarily. The animations of the unitary evolution of a gaussian wavefunction and a delta wavefunction are given below.

Animation: Gaussian **Animation: Delta**

We use the Root Mean Squared Displacement (RMSD) from the central site to measure the spread.

$$\sqrt{(\Delta x)^2} = \sqrt{\sum_{n=1}^{L}(n-n_0)^2|\psi_n|^2} \qquad \sqrt{(\Delta x)^2} \sim t^\alpha$$

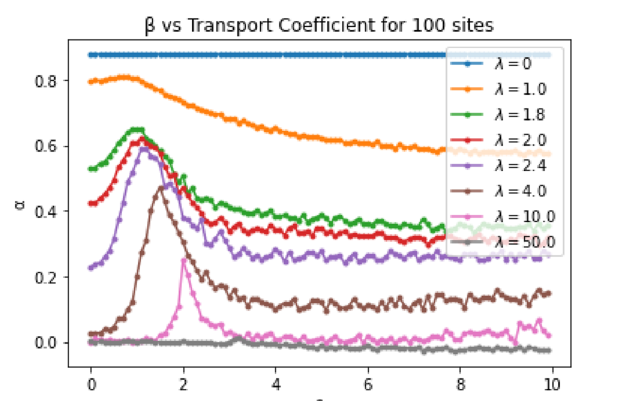




The interpolation is done in this region where the scaling is linear.

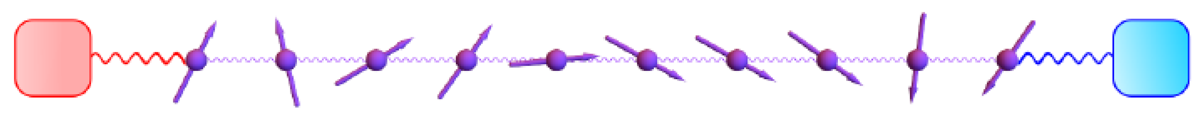
Based on the scaling, we get different transport regimes.

Transport regime	Transport coefficient
Localized	$\alpha = 0$
Ballistic	$\alpha = 1$
Diffusive	$\alpha = \frac{1}{2}$
Subdiffusive	$\alpha < \frac{1}{2}$



The transport coefficient was plotted against β for various λ

Boundary-Driven Chain



A current (J) flows through the boundary-driven chain as the spin baths give a bias.

$$J \sim \frac{1}{L^\nu}$$

Transport regime	Transport coefficient
Ballistic	$\nu = 0$
Superdiffusive	$0 < \nu < 1$
Diffusive	$\nu = 1$
Subdiffusive	$\nu > 1$
Localized	$\nu = \infty$

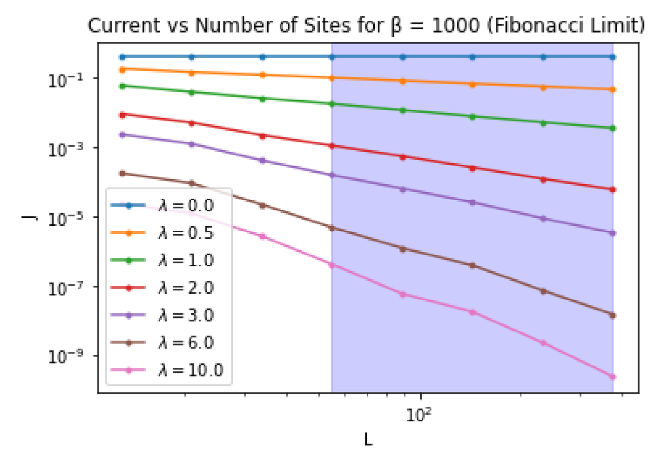
In many cases, α can be compared with ν as follows,

$$\alpha = \frac{1}{1+\nu}$$

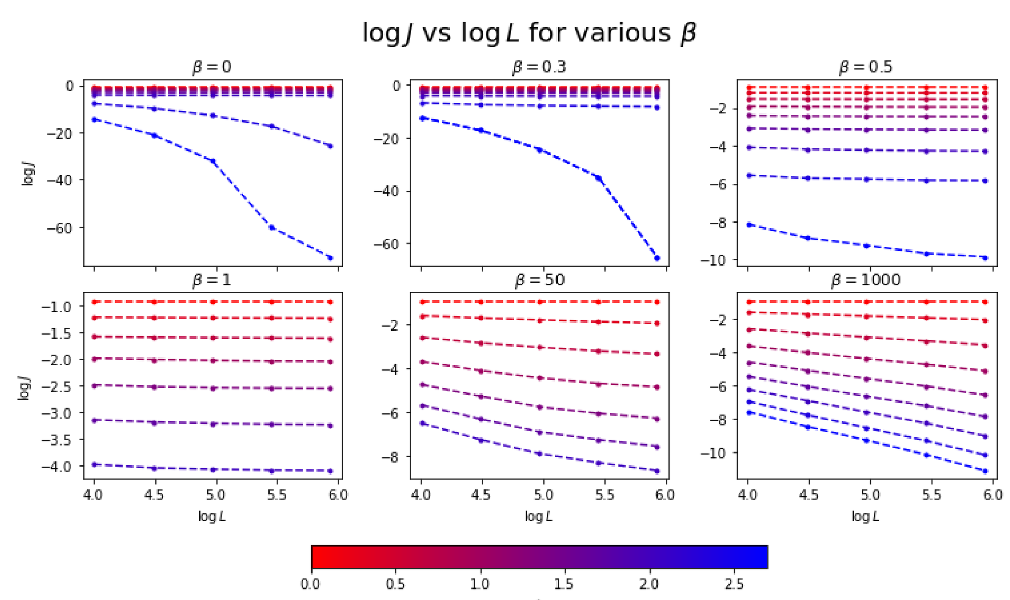
J scales with L as below and depending on ν we get transport regimes.

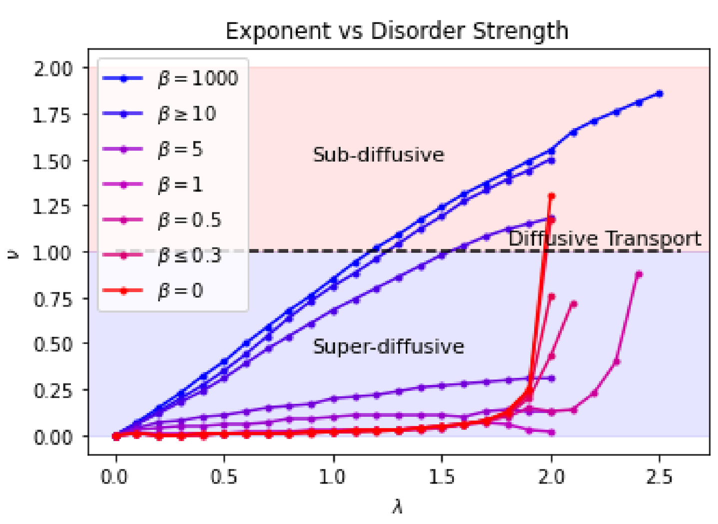
Using the covariance formalism in the Lindblad master equation with appropriate dissipators (we use the Local Master Equations (LMEs) approach), we get the Lyapunov Equation $WC + CW^\dagger = F$

with $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_L)$ and $F = \mathrm{diag}(\gamma_1 f_1, \gamma_2 f_2, \ldots, \gamma_L f_L)$ and $W = iH + \frac{\Gamma}{2}$



The largest five systems sizes are used for the linear regresion.





The transport coefficient shoots up to ∞ after the critical point for AAH model and till for up to some β_c. After that, it goes on to become a Fibonacci model. This β_c gives a model that is in-between AAH and Fibonacci models.

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