Entanglement Generation via Locally Mediated Interactions

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Abstract

Two similar proposals have been made for witnessing quantum signatures of gravity by Bose et al.[1] and by Marletto and Vederal [2]; these two proposals are based on the claim:*A classical system mediating interaction between two quantum systems can not entangle the two quantum systems*. Our work is motivated by the above assertion. This work studies the entanglement generation between two quantum systems by interactions locally mediated by a third physical system. We aim to test their claim in a simple analytical model, with interaction mediated by a classical system. We then look at the entanglement dynamically created between the quantum systems by a quantum mediator and study the dependence of entanglement on the mediator's various quantum states: Gaussian and Fock states. We try to identify quantum states which might behave classically in the sense of producing no entanglement.

Model and the Set up:

We consider the model of three coupled harmonic oscillators, the quantum oscillators 1 and 3, which are locally interacting with the mediator: oscillator 2.

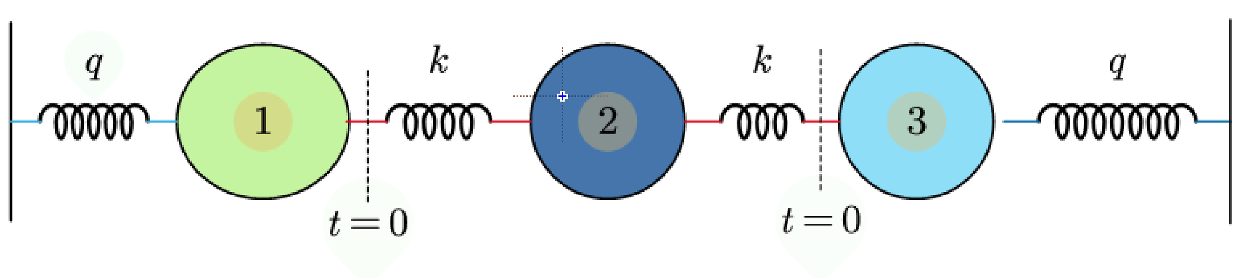


Figure 1. Quantum Oscillators-1 and 3 locally interact with mediating oscillator-2.

At $t = 0$, the oscillators were free harmonic oscillators, and $t \geq 0$; the oscillators become coupled.

The **Hamiltonian** with a classical mediating oscillator (x_2, p_2) are not quantized:

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_3^2}{2m} + \frac{q\hat{x}_1^2}{2} + \frac{q\hat{x}_3^2}{2} + \frac{k(\hat{x}_1 - x_2(t))^2}{2} + \frac{k(\hat{x}_3 - x_2(t))^2}{2} + \frac{p_2^2(t)}{2M} \tag{1}$$

Questions:

The present study investigates the following two primary questions:

- **Question 1:** Can a classical oscillator entangle the two quantum oscillators? Answer: **No!**
- **Question 2:** Are there quantum states of the mediating oscillator producing a decaying trend in entanglement or no entanglement at all? Answer: **Yes!**

Dynamics with classical mediator!

The Hamiltonian 1 can be re-written as:

$$H = \frac{\hat{p}_1^2}{2m} + \frac{1}{2}(k+q)\hat{x}_1^2 - kx_2(t)\hat{x}_1 + \frac{\hat{p}_3^2}{2m} + \frac{1}{2}(k+q)\hat{x}_3^2 - kx_2(t)\hat{x}_3 + \frac{p_2^2(t)}{2M} + kx_2^2(t) \tag{2}$$

Looking at equation 2 from the perspective of quantum systems, this hamiltonian represents two non-interacting, forced harmonic oscillators, with a common forcing function: $x_2(t)$

This makes the time evolution operator a local operator which independently acts on each subsystem.

$$\hat{U}(t, 0) = e^{-i\int_0^t \hat{H} dt'} = \hat{U}_1 \otimes \hat{U}_3 \tag{3}$$

Consider an initial quantum state of oscillators prepared at $t \leq 0$:

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_3\rangle. \tag{4}$$

A system local time evolution operator as in eq. 3 **will not** create entanglement!

Dynamics with a Quantum Mediator!

Now, we quantize our mediator! And consider the fully quantum mechanical Hamiltonian 1 with operators $\hat{x}_2, \hat{p}_2$

Consider an initial state:

$$|\psi\rangle = |0\rangle_1 \otimes |\phi\rangle_2 \otimes |0\rangle_3 \tag{5}$$

We choose various quantum states $|\phi\rangle$ for the mediator: Coherent states, Rotated Squeezed states, Thermal states, and number operator states $|n\rangle$

And study the sensitivity of the entanglement created on ϕ.

The time evolution operator $\hat{U} = \exp(-i\hat{H}t)$ is no longer of the form $\hat{U}_1 \otimes \hat{U}_2 \otimes \hat{U}_3$, i.e., **no longer a local operator**, This means that time evolution ***can*** now create entanglement.

What do we compute and why?

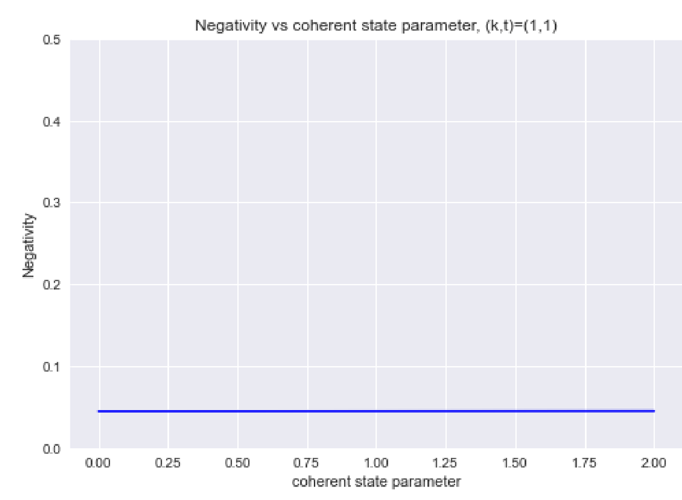
We computed the following as a function of the mediator's initial quantum state parameters.

- **Entanglement Negativity**: We compute entanglement negativity to quantify the entanglement mediated by oscillator-2 between the quantum oscillators 1 and 3.
- **Entanglement Entropy**: We compute the entanglement entropy of oscillator-1 to quantify its entanglement with the rest of the subsystem.

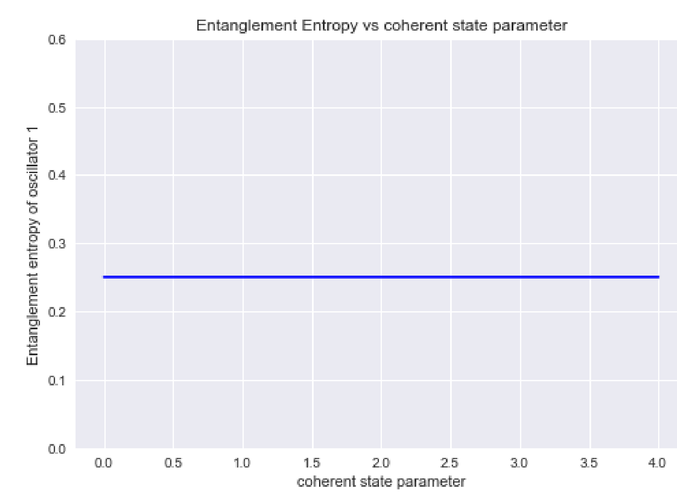
Conclusion and Discussions

- We have seen in a physical model that a classical mediator cannot entangle two quantum systems.
- All coherent state configurations of the mediator entangle equally!
- Entanglement is strongly sensitive to the quantum states of the mediator; much more and much less entanglement can be mediated by squeezing and rotating operations.
- The large $|n\rangle$ state does not seem to reproduce the classical behavior of the oscillator.
- Entanglement between oscillators 1-3 behaves significantly differently than the entanglement between oscillator 1 and the rest.
- A non-zero entanglement between mediated certifies that the mediator is quantum. However, a zero entanglement mediated does not allow us to conclude anything.

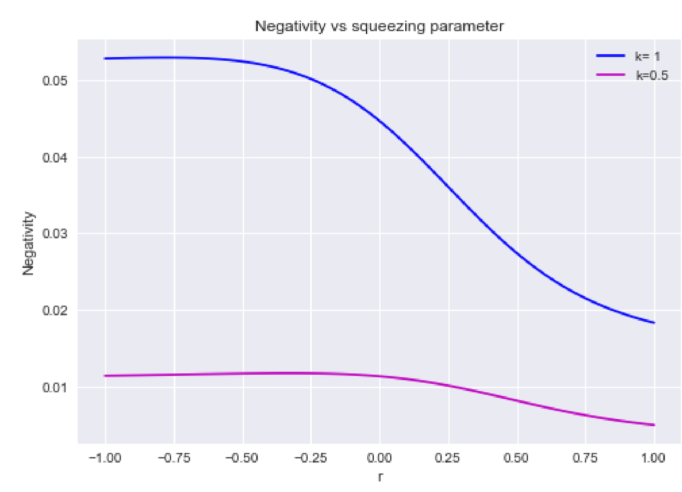
Results: Mediator in Coherent states and General pure Gaussian states



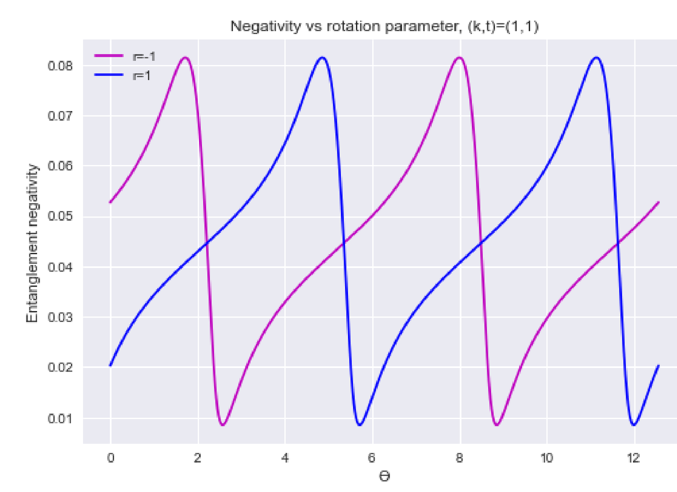
(a) Negativity of Oscillator 1-3 vs. Coherent state parameter, Hilbert space dimension $N = 40^3$



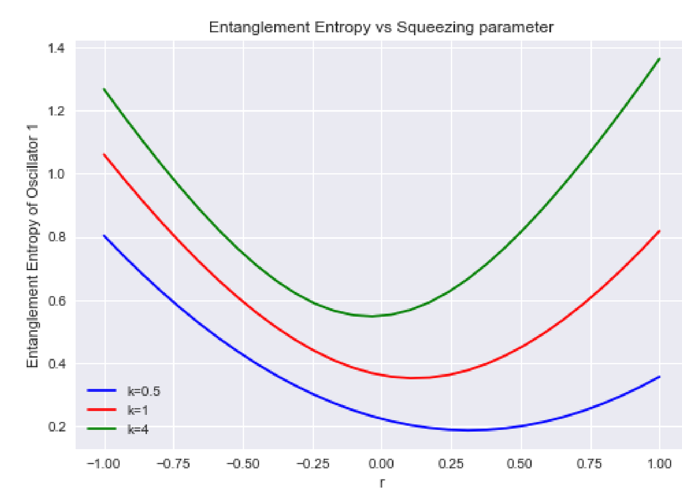
(b) Entanglement entropy of Oscillator-1 vs Coherent state parameter,Hilbert space dimension $N = 40^3$ at $(k, t) = (1, 1)$



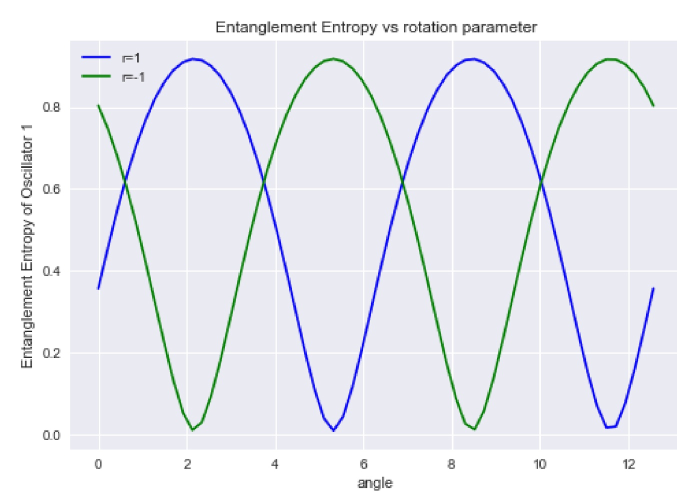
(a) Entanglement negativity, as a function of squeezing parameter, 'r' at $(k, t) = (1, 1)$



(b) Entanglement negativity vs. rotation parameter (in radians) θ

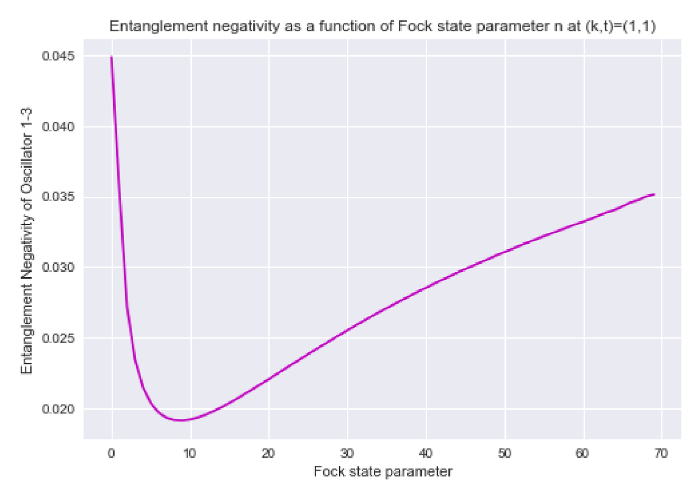


(c) Entanglement entropy of oscillator-1 vs squeezing parameter r, $(k, t) = (1, 1)$

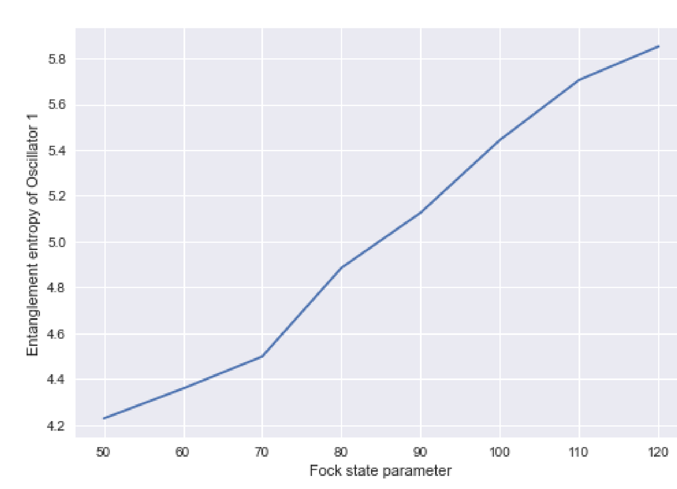


(d) Entanglement entropy of oscillator-1 vs. rotation parameter,$(k, t) = (1, 1)$

Does a large $|n\rangle$ state reproduces the behavior of a classical oscillator?



(a) Entanglement Negativity of Oscillator 1-3 state vs. n, Hilbert space dimension $N = 100^3$

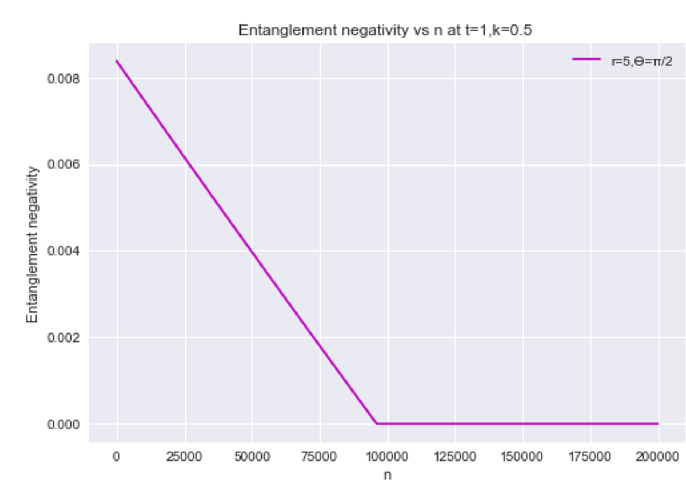


(b) Entanglement entropy of oscillator-1 vs n, Hilbert space dimension $N = 200^3$,$(k, t) = (1, 1)$

Result: Mediator in Mixed Gaussian states

A mixed Gaussian state is a **thermal state** (up to a unitary transformation). Entanglement-mediated drops as the purity of the mediator decreases.

(a) Entanglement negativity vs mean photon number, $\bar{n}$



(b) Entanglement negativity vs mean photon number, $\bar{n}$ for rotated squeezed state

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References

[1] Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A. Geraci, Peter F. Barker, M. S. Kim, and Gerard Milburn.
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Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity.
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