Emergence of classicality in quantum resource theories

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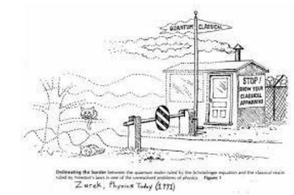
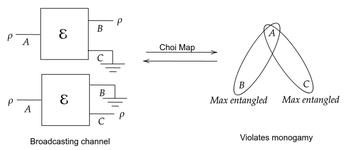
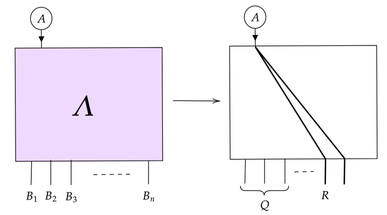
Introduction

- Constraints on information spreading in many-body systems can help us understand the quantum-classical transition. These constraints can be formulated as monogamy of entanglement relations on quantum states and extended to channels by the channel-state duality.
- For a system A that leaks out it's information to a large environment, we can show that except a region of size $O(1)$, the information accessible locally in the environment is classical, and can be obtained by a fixed measurement on A
- We analyze such constraints for the quantum evolutions in resource theories, with a focus on quantum coherence. We look at how imposing such a structure helps in understanding aspects of this classicality.





Bounds on spreading of coherence using DIO channels



For every DIO-state of $A \otimes B_1 \dots B_n$, we can find a region $Q \in B_1, B_2 \dots B_n$ such that every for every region R disjoint from Q such that the reduced state AR satisfies

$$\|\rho_{AR} - \Delta_{AR}\rho_{AR}\|_1 \leq \sqrt{2 \ln d_A |R|/|Q|}$$

For every DIO channel $\mathcal{E} : D(A) \to D(B_1 \otimes B_2 \otimes \dots B_n)$, we can find a region $Q \in B_1, B_2 \dots B_n$ such that every for every region R disjoint from Q such that the reduced channel $\mathcal{E}_R : D(A) \to D(R)$ satisfies

$$\|\mathcal{E}_R - \Lambda^{CBC}_{A\to R}\|_\diamond \leq d_A \sqrt{2 \ln d_A |R|/|Q|}$$

Monogamy of coherence holds for DIO states

$$C(\rho_{AB}) + C(\rho_{AC}) \leq C(\rho_{ABC})$$

Resource theory of Coherence

The resource theory of coherence has the following structure :

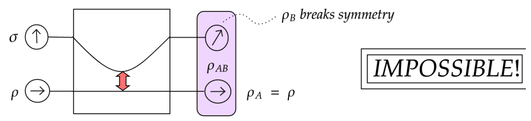
1. The free states satisfy $\Delta\rho = \rho$
2. The free operations satisfy the $\Delta_B \circ \Lambda = \Lambda \circ \Delta_A$.
   These are called Dephasing Covariant Incoherent (DIO) Operations

Coherence breaking channels

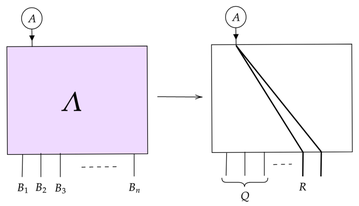
A DIO channel Λ is CBC iff

1. $\Lambda(|i\rangle\langle j|) = 0 \forall i \neq j$
2. $\Lambda(\rho) = \sum_i \mathrm{Tr}(|i\rangle\langle i|\,\rho)\sigma^i$ where $\sigma_i \in I$

No-broadcasting of asymmetry and coherence



Classicality in multi-partite states and channels



Let there be a state $\rho_{AB_1B_2\dots B_n}$ on a multi-partite system $A \otimes B_1 \otimes B_2 \otimes \dots \otimes B_n$. Then excluding a set of size $|Q|$ the reduced states ρ_{AR} on R disjoint from Q satisfy

$$\|\rho_{AR} - \sum_i p_i \tilde{\rho}^i_A \otimes \sigma^i_R\|_1 \leq \sqrt{\Omega d_A^2 |R|/|Q|}$$

for some separable state $\sum_i p_i \tilde{\rho}^i_A \otimes \sigma^i_R$

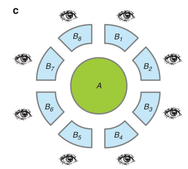
Similarly for quantum channels from $\mathcal{D}(A) \to \mathcal{D}(B_1 \otimes B_2 \otimes \dots \otimes B_n)$, excluding a set of size $|Q|$ in the output, the reduced channels $\mathcal{E}_R : \mathcal{D}(A) \to \mathcal{D}(R)$ with R disjoint from Q satisfy

$$\|\mathcal{E}_R - \Lambda_{MP}\|_\diamond \leq d_A \sqrt{\Omega d_A^2 |R|/|Q|}$$

Applications

Quantum Darwinism

It provides more understanding of **what** POVMs emerge as observables for Quantum Darwinism in DIO channels. These observables are often called "pointer" basis. The typical example of Quantum Darwinism, the $C - X$ fan-out evolution is DIO.



Distribution of timing information

Since the reduced channels are approximately CBC, which only allow information about the diagonal elements of ρ_A, any information encoded in coherences of a quantum state cannot be shared using these channels. Such encodings occur in quantum metrology, and quantum clocks. Hence our theorem shows that such encodings cannot be shared to more than $O(1)$ parties.

Future Work

We have a few ideas for the future :

1. We will try extending these ideas in the resource theories of asymmetry and athermality. We will try to characterize the asymmetry-breaking channels for general symmetry groups.
2. We will look at the state marginal problem, and compatibility of DIO states.

Summary

- We understood the monogamy of coherence relation for quantum states, and showed that it holds for the choi states of DIO channels.
- We used this to prove a emergence of classicality theorem specifically for DIO channels, and states. This allows us to formulate stronger versions of the results of Brandao et al. and Qi et al. for DIO channels.
- We looked at the application of these results to Quantum Darwinism, and distribution of timing information.

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