Computational Microelectronics HW.3

EECS, 20204003

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1. For $\boldsymbol{\phi(0) = 1, \phi(a) = -1}$ boundaries

1)

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ -1 \end{bmatrix}$$

By solving Ax=b which is the Laplace equation, we can get potential with boundary condition $\boldsymbol{\phi(0) = 1, \phi(a) = -1.}$

Numerical solution and Exact solution which was solved by $-\frac{2}{L}x + 1$ is compared in a below graph.

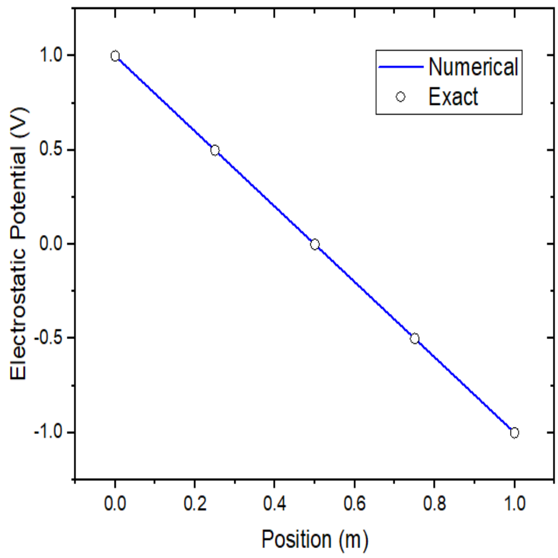

Fig 1. Exact solution and Numerical solution are compared when N=5.

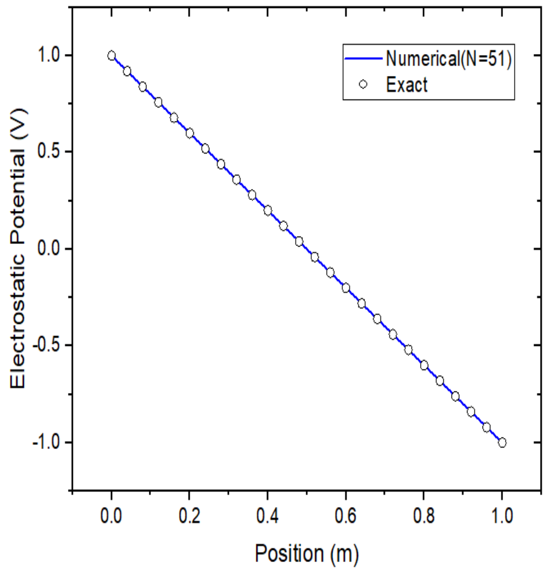

Fig 2. Exact solution and Numerical solution are compared when N=51.

The numerical solved results are consistent with analytical solved results. Moreover, the result graph is quite same no matter of discretization level since it is just simple linear graph.

2. Poisson equation

1) Dirac Delta function

To make Dirac-Delta function there may be two ways which are represented below. ($\Delta x \rightarrow 0$)

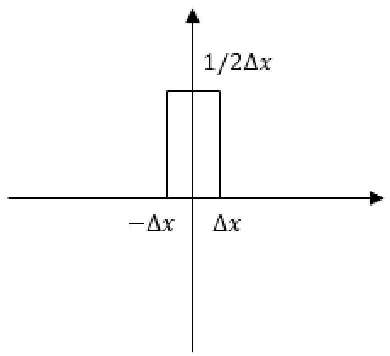

Fig 3. Using Square

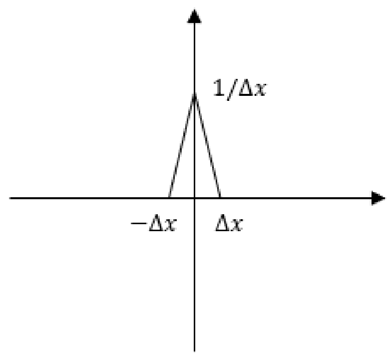

Fig 4. Using Triangle

Both of them can be a model. However, implementing using square type is little more complicated than using triangle one. As a result, I will use triangle version.

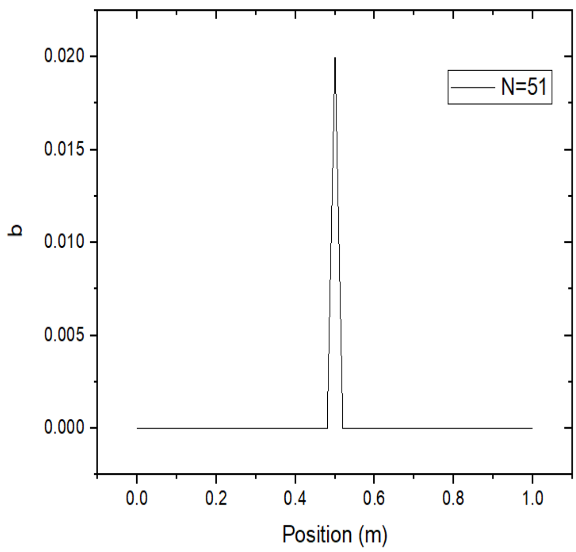

Fig 5. Dirac-Delta function when N=51

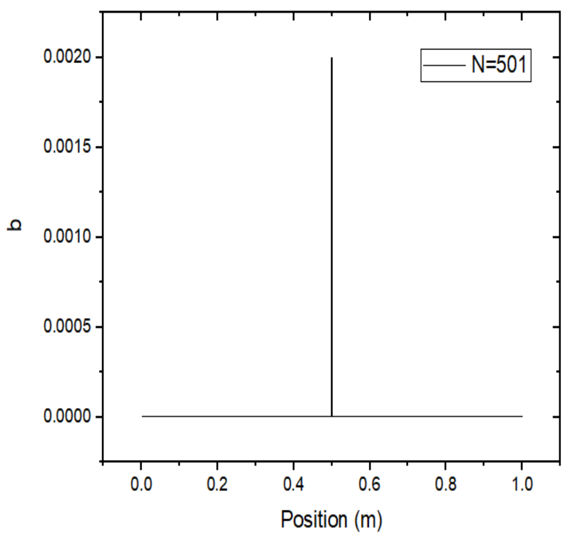

Fig 6. Dirac-Delta function when N=501

As we increase discretization level, its shape becomes closer to Dirac-Delta function that we want to use.

2) Potential Graph.

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = (\Delta x)^2 \begin{bmatrix} 0 \\ \vdots \\ \delta(x - \frac{a}{2}) \\ \vdots \\ 0 \end{bmatrix}$$

Analytical solution is required to compare.

$$\int_0^L \frac{d^2}{dx^2}\phi(x)dx = \frac{d}{dx}\phi(x)\bigg|_{x=L} - \frac{d}{dx}\phi(x)\bigg|_{x=0} = \int_0^L \delta\left(x - \frac{a}{2}\right)dx = 1$$

since the graph is symmetric,

$$\frac{d}{dx}\phi(x)\bigg|_{x=L} = \frac{1}{2}, \quad \frac{d}{dx}\phi(x)\bigg|_{x=0} = -\frac{1}{2}$$

$$\phi\left(\frac{L}{2}\right) = -\frac{L}{4}$$

$$\phi(x) = \begin{cases} -\frac{1}{2}x, & x \le \frac{L}{2} \\ \frac{1}{2}x - \frac{L}{2}, & x \ge \frac{L}{2} \end{cases}$$

The results of numerical solution and exact solution are represented in below graph.

a) L = 1 (m)

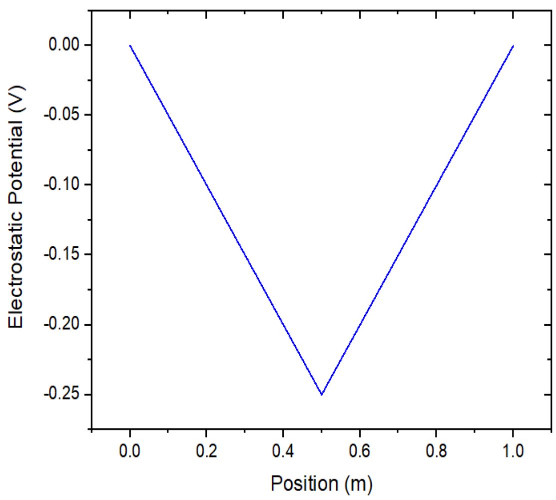

Fig 7. Position vs. Potential graph when N=501 and L=1m.

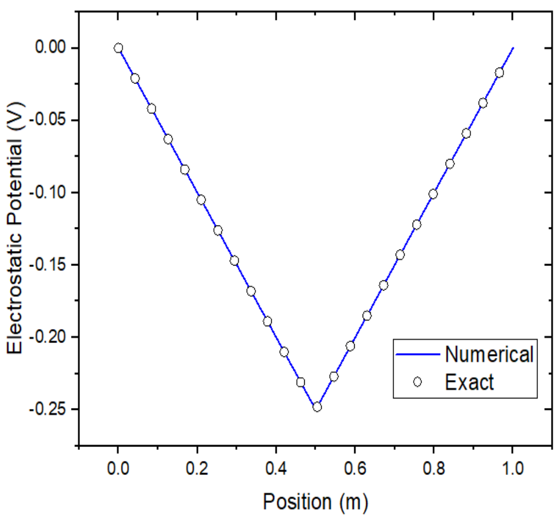

Fig 8. Position vs. Potential graph when N=501 and L=1m.

b) L = 1 (mm)

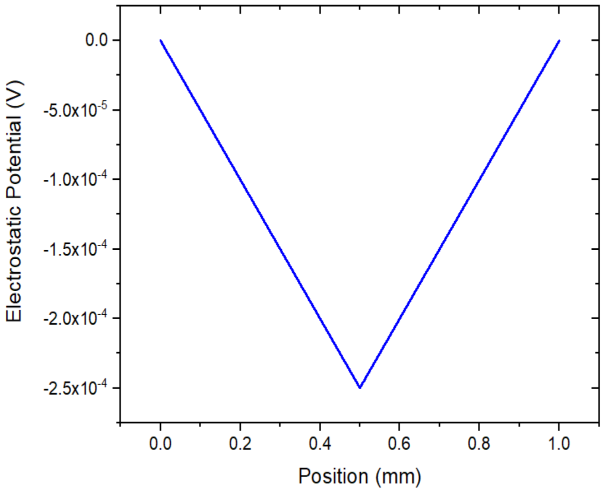

Fig 9. Position vs. Potential graph when N=501 and L=1mm.

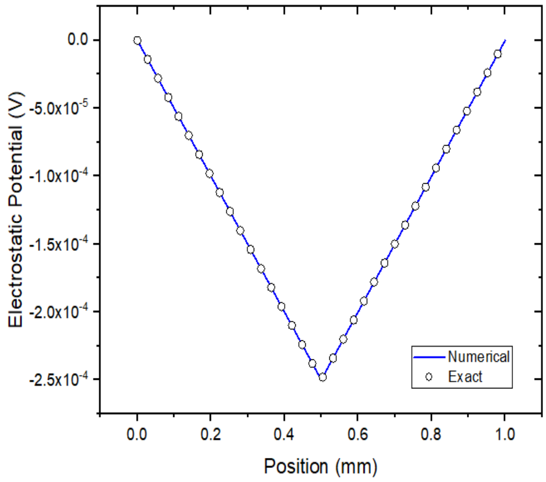

Fig 10. Position vs. Potential graph when N=501 and L=1mm.