Computational Microelectronics HW.7

EECS, 20204003

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1. Newton method

1) Numerical Expression

$$f(\phi^k) = N_{dop}^{+} - n_{int}\exp\left(\frac{\phi^k}{V_T}\right) + n_{int}\exp\left(-\frac{\phi^k}{V_T}\right) = 0$$

$$(k : Newton\ loop\ index)$$

$$\left.\frac{df}{d\phi}\right|_{\phi=\phi^k} == -\frac{n_{int}}{V_T}\exp\left(\frac{\phi}{V_T}\right) - \frac{n_{int}}{\mathrm{V_T}}\exp\left(-\frac{\phi}{V_T}\right)$$

$$\delta\phi^k = -\frac{f(\phi^k)}{\left.\frac{df}{d\phi}\right|_{\phi=\phi^k}}$$

$$\phi^{k+1} = \phi^k + \delta\phi^k$$

HW7 is about solving non-linear equation by Newton method. By solving above equations many times, the numerical solution will be given.

2) Analytic expression

$$N_{dop}^{+} - n_{int}\exp\left(\frac{\phi}{V_T}\right) + n_{int}\exp\left(-\frac{\phi}{V_T}\right) = 0$$

$$n_{int}\sinh\left(\frac{\phi}{V_T}\right) = \frac{N_{dop}^{+}}{2}$$

$$\phi = V_T\sinh^{-1}\left(\frac{N_{dop}^{+}}{2n_{int}}\right)$$

To compare with numerical solution, above equations are used to get analytic solutions.

3) Result

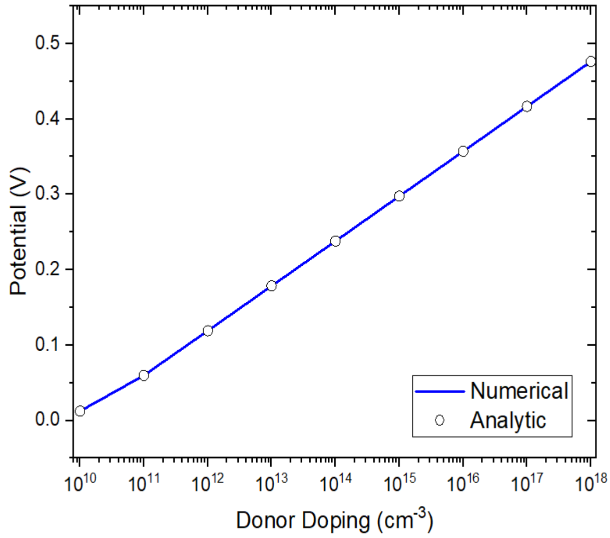

Fig 1. Donor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared.

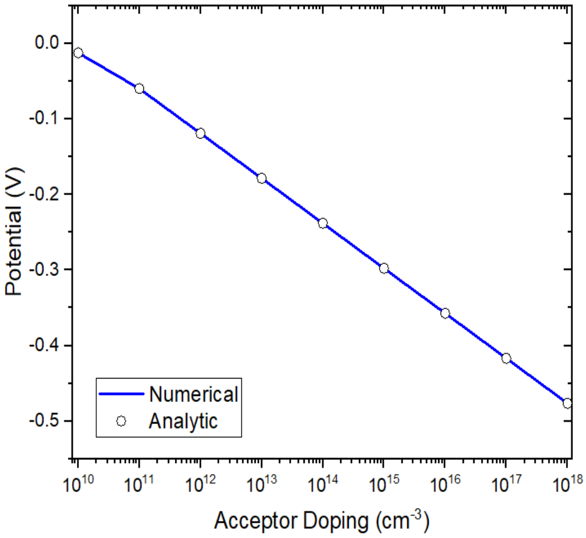

Fig 2. Acceptor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared.

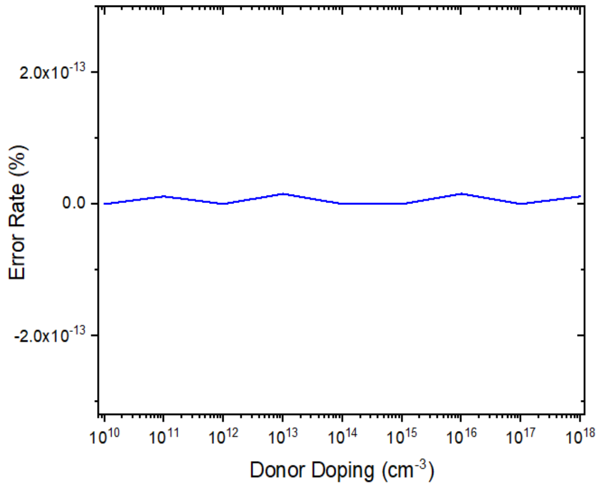

Fig 3. Donor doping vs. Error rate graph. Error rate is calculated by difference between numerical and analytic solutions.

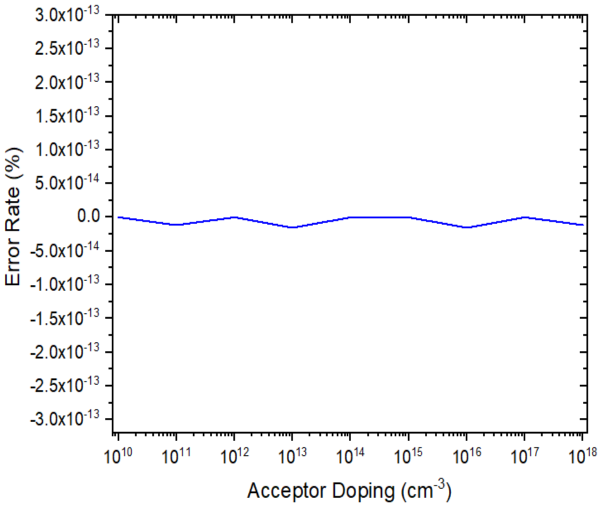

Fig 4. Acceptor doping vs. Error rate graph. Error rate is calculated by difference between numerical and analytic solutions.

Numerical and analytic ways, which are represented by above equations, are implemented to MATLAB. Those results are compared and checked that there are almost no differences. Error rate which is calculated by below equation is almost zero.

$$Error\ rate\ (\%) = \frac{\left|\phi_{Numerical} - \phi_{Analytic}\right|}{\phi_{Analytic}} \times 100$$

If the update vector becomes smaller than 10^{-15}, then the system get out of Newton's loop.

What happens if Newton loop is small and initial solution is far from exact solution? For this case, the maximum iteration is set to 10 times and initial solutions are set $\pm 1V$ and $0V$. Results are represented in below figures.

a) Max iteration=10, initial solution=$\pm 1V$

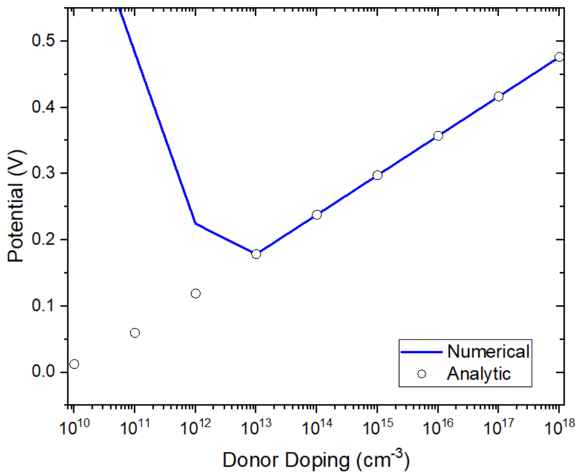

Fig 5. Donor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared when initial solution is 1V.

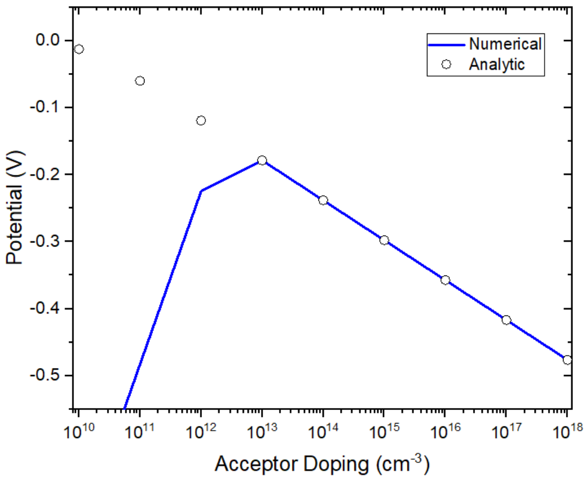

Fig 6. Acceptor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared when initial solution is -1V.

b) Max iteration=10, initial solution=$0V$

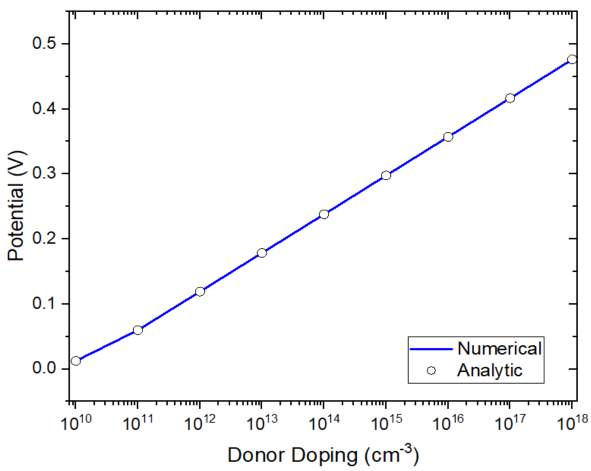

Fig 7. Donor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared when initial solution is 0V.

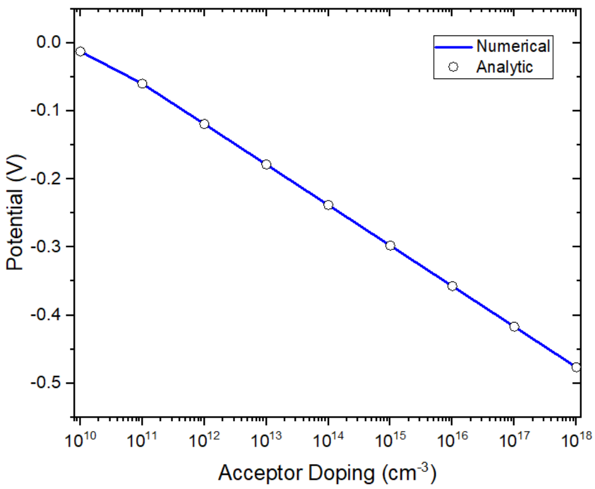

Fig 8. Acceptor doping concentration vs. Potential graph. Numerical solution and analytic solution are compared when initial solution is 0V.

As it can been seen, not only choosing the quantity of iteration is important, but also choosing good initial solution is important. Even in same quantity of iteration, the change of initial solution makes huge differences in results which may cause problems like reduction of accuracy and expansion of the time that takes to get results.