Computational Microelectronics HW.4

EECS, 20204003

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1. Heterostructure

1)

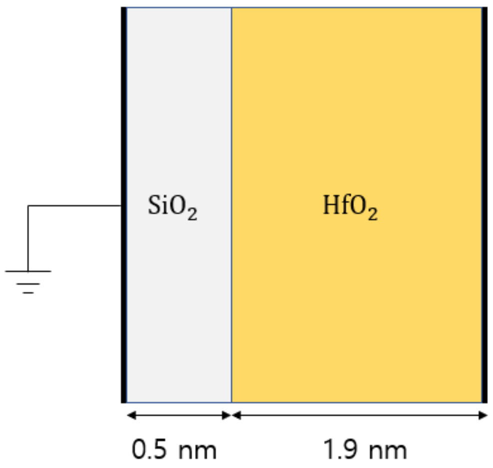

Fig. 1 Capacitor with two dielectrics

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \epsilon_1 & -2\epsilon_1 & \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & -\epsilon_2-\epsilon_1 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_2 & -2\epsilon_2 & \epsilon_2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},\quad \mathbf{b}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

HW4 is similar to HW3 which the only difference is using permittivity or not.

For Silicon oxide, 3.9 is used for relative permittivity. For Hafnium oxide, 22.0 is used for relative permittivity.

By solving Ax=b equation, electrostatic potential is given. With this potential we can get Displacement by equation below.

$$D \approx -\epsilon(x)\frac{\phi_i - \phi_{i-1}}{\Delta x}$$

Capacitance can be obtained using Displacement field. ρ_s is surface charge ($\frac{\mathrm{C}}{\mathrm{cm}^2}$)

$$D(0^+) = \rho_s(0^+)$$

$$D(L^-) = -\rho_s(L^-)$$

$$CV = Q = A\int \rho\, dx \ = D(L^-) \qquad (\text{unit of C is } \frac{\mathrm{F}}{\mathrm{cm}^2})$$

$$C = \frac{D}{V}$$

2) Analytic expression

Exact solution of potential should be obtained to compare with numerical solution.

$$V(x) = \begin{cases} Ax & , \quad 0 \le x \le 0.5\,nm \\ C(x - 1.9nm) + V_{Applied}, & 0.5\,nm \le x\ 2.4\,nm \end{cases}$$

$$D(0 \le x \le 0.5\,nm) = \epsilon_{SiO_2} A \quad = \quad D(\,0.5\,nm \le x \le 2.4\,nm) = \epsilon_{HfO_2} C$$

$$V(x = 0.5\,nm) = A\ \times 0.5\,nm = C(\,0.5\,nm - 1.9\,nm) + V_{Applied}$$

$$V(x) = \begin{cases} \dfrac{V_{Applied}}{0.5\,nm + \frac{\epsilon_{SiO_2}}{\epsilon_{HfO_2}}\,2.4\,nm} x & , 0 \le x \le 0.5\,nm \\ \dfrac{V_{Applied}}{\frac{\epsilon_{HfO_2}}{\epsilon_{SiO_2}}\,0.5\,nm +\ 2.4\,nm}(x - 2.4\,nm) + V_{Applied} & , 0.5\,nm \le x\ 2.4\,nm \end{cases}$$

Capacitor with two dielectrics can be calculated separately, for each of them as in Fig. 2

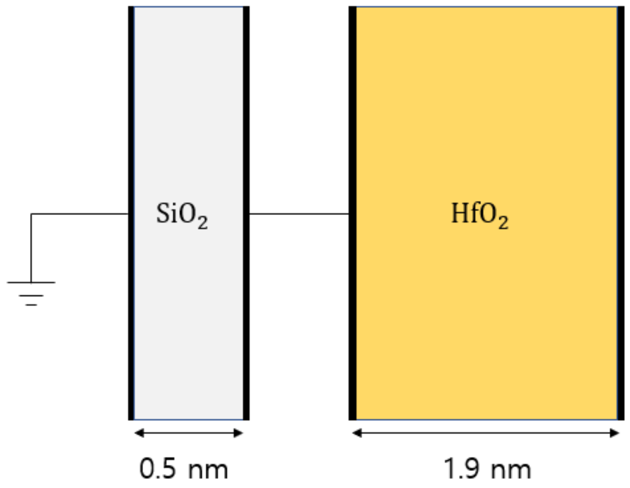

Fig. 2 Two Capacitors with different dielectrics

They are calculated in series connection.

$$\frac{1}{C_{total}} = \frac{1}{C_{SiO_2}} + \frac{1}{C_{HfO_2}}$$

$$C_{SiO_2} = \frac{\epsilon_{SiO_2}}{t_{SiO_2}} \qquad C_{SiO_2} = \frac{\epsilon_{HfO_2}}{t_{HfO_2}}$$

3) Results

a) Electrostatic Potential

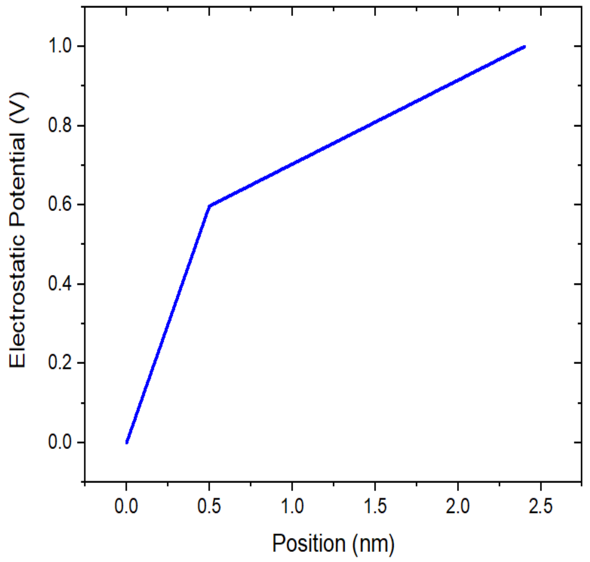

Fig 3. Position vs. Potential graph which is solved numerically.

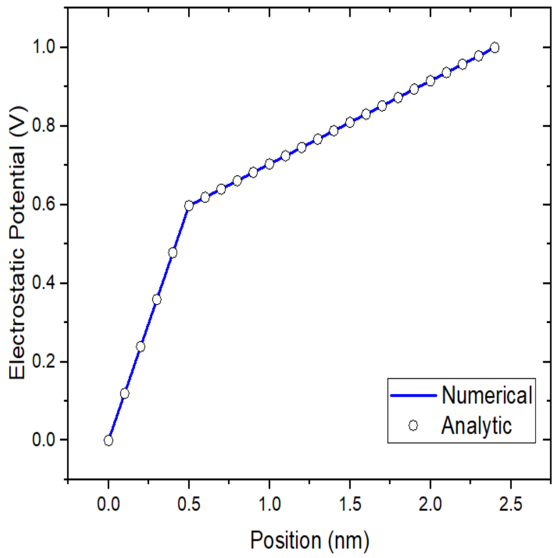

Fig 4. Exact solution and numerical solution are compared in this figure.

b) Displacement Field

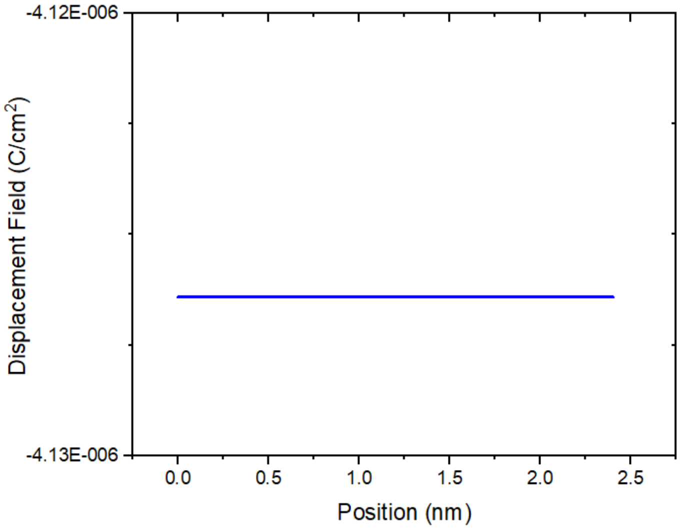

Fig 5. Position vs. Displacement Field

Since there are no charges in dielectrics, displacement field should remain same.

c) Capacitance

	Numerical	Analytical	Error(%)
Capacitance($\frac{F}{cm^2}$)	4.1264×10^{-6}	4.1264×10^{-6}	~0