

Computational Microelectronics HW.10

EECS, 20204003

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1. 2D nonlinear Poisson Equation

1) Used Model

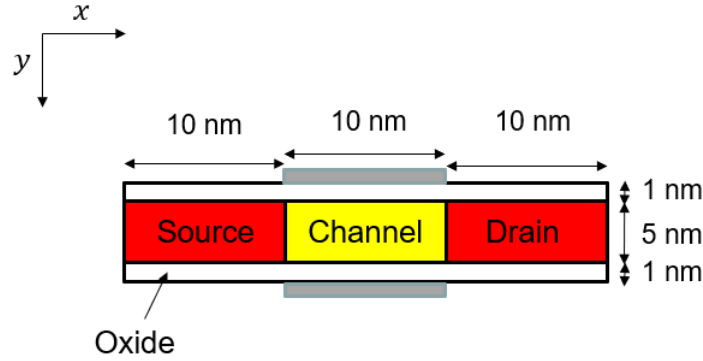


Fig. 1 Double gate model

위 그림은 이번 과제에서 사용한 Double gate model이며, Source와 Drain은 Donor 도핑이 10^{20} cm^{-3} 로 되어 있고 Channel은 intrinsic 상태이다. Source와 Drain 양 옆단에는 contact이 있다고 가정하며, 이 부분은 Dirichlet 경계조건을 사용하여, 초기해가 들어 가게 한다. X 축에 대한 mesh는 61개, Y축에 대한 mesh는 36개를 사용하였다. (간격 $\Delta x = 0.5 \text{ nm}$, $\Delta y = 0.2 \text{ nm}$)

초기해를 구하는 방법은 analytic 하게 구한 값을 사용하였으며, 다음과 같다.

$$N_{dop}^+ - n_{int} \exp\left(\frac{\phi}{V_T}\right) + n_{int} \exp\left(-\frac{\phi}{V_T}\right) = 0$$

$$n_{int} \sinh\left(\frac{\phi}{V_T}\right) = \frac{N_{dop}^+}{2}$$

$$\phi = V_T \sinh^{-1}\left(\frac{N_{dop}^+}{2n_{int}}\right)$$

아래는 알맞은 행렬을 구성하기 위해 나타낸 수식이다.

(a)For left side Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (\phi_{i+1,j} - \phi_{i,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (0.5\phi_{i,j+1} - \phi_{i,j} + 0.5\phi_{i,j-1}) = 0$$

(b)For right side Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (-\phi_{i,j} + \phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (0.5\phi_{i,j+1} - \phi_{i,j} + 0.5\phi_{i,j-1}) = 0$$

(c)For top side Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (0.5\phi_{i+1,j} - \phi_{i,j} + 0.5\phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (\phi_{i,j+1} - \phi_{i,j}) = 0$$

(d) For bottom side Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (0.5\phi_{i+1,j} - \phi_{i,j} + 0.5\phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (-\phi_{i,j} + \phi_{i,j-1}) = 0$$

(e) For left top corner Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (0.5\phi_{i+1,j} - 0.5\phi_{i,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (0.5\phi_{i,j+1} - 0.5\phi_{i,j}) = 0$$

(f) For right top corner Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (-0.5\phi_{i,j} + 0.5\phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (0.5\phi_{i,j+1} - 0.5\phi_{i,j}) = 0$$

(g) For left bottom corner Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (0.5\phi_{i+1,j} - 0.5\phi_{i,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (-0.5\phi_{i,j} + 0.5\phi_{i,j-1}) = 0$$

(h) For right bottom corner Neumann boundary (Oxide)

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (-0.5\phi_{i,j} + 0.5\phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (-0.5\phi_{i,j} + 0.5\phi_{i,j-1}) = 0$$

(i) For contact Dirichlet boundary

$$\phi_{i,j} = V_{\text{applied}} \text{ for gate}$$

$$\phi_{i,j} = \phi_{\text{initial guess}} \text{ for Drain, Source}$$

(j) For interface Oxide-Silicon

$$0.5 \frac{\epsilon_{ox} + \epsilon_{si}}{(\Delta x)^2} (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + \frac{1}{(\Delta y)^2} (\epsilon_{si}\phi_{i,j+1} - (\epsilon_{ox} + \epsilon_{si})\phi_{i,j} + \epsilon_{ox}\phi_{i,j-1}) + 0.5q(N_{dop}^+ - n_{int}e^{\frac{\phi_{i,j}}{V_T}}) + n_{int}e^{\left(-\frac{\phi_{i,j}}{V_T}\right)} = 0$$

(k) For interface Silicon-Oxide

$$0.5 \frac{\epsilon_{ox} + \epsilon_{si}}{(\Delta x)^2} (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + \frac{1}{(\Delta y)^2} (\epsilon_{ox}\phi_{i,j+1} - (\epsilon_{ox} + \epsilon_{si})\phi_{i,j} + \epsilon_{si}\phi_{i,j-1}) + 0.5q(N_{dop}^+ - n_{int}e^{\frac{\phi_{i,j}}{V_T}}) + n_{int}e^{\left(-\frac{\phi_{i,j}}{V_T}\right)} = 0$$

(l) For the rest of points

$$\frac{\epsilon_{ox}}{(\Delta x)^2} (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + \frac{\epsilon_{ox}}{(\Delta y)^2} (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) = 0 \text{ (for Oxide)}$$

$$\frac{\epsilon_{si}}{(\Delta x)^2} (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + \frac{\epsilon_{si}}{(\Delta y)^2} (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) + q(N_{dop}^+ - n_{int}e^{\frac{\phi_{i,j}}{V_T}}) + n_{int}e^{\left(-\frac{\phi_{i,j}}{V_T}\right)} = 0$$

(for Silicon)

위의 수식들은 각 위치에서 residual 값이고, 이를 알맞은 항에 따라 미분을 하여 Jacobian 행렬을 구성하여 이전 과제들과 같이 풀면 된다.

2) Results

a) Electrostatic Potential when Gate voltage is 0V.

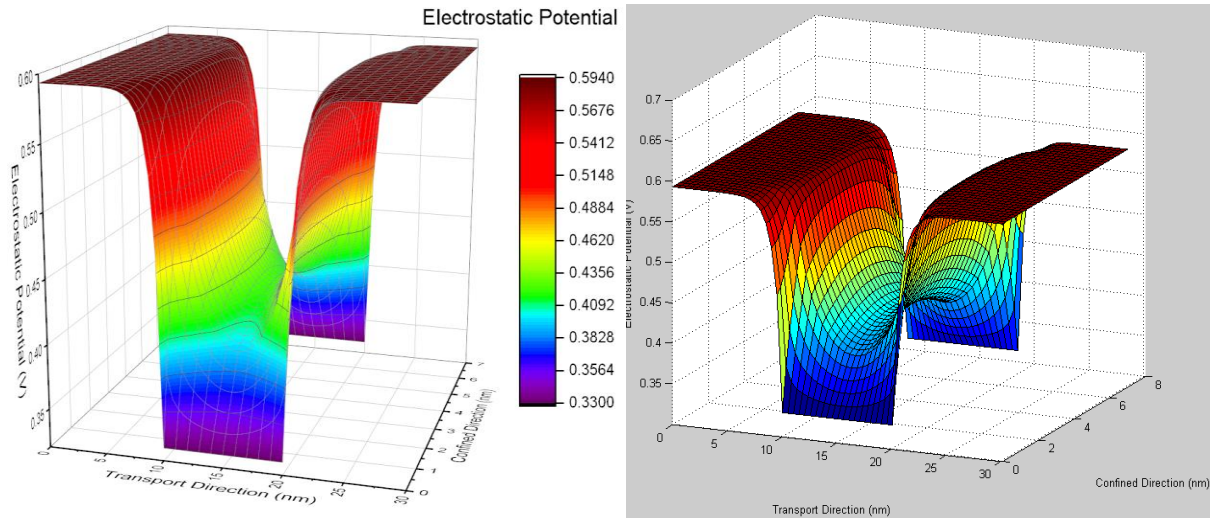


Fig 2. Potential 3D graph when Gate voltage is 0V. Left figure is drawn by Origin, and right figure is drawn by MATLAB.

b) Electrostatic Potential when Gate voltage is 1.1V.

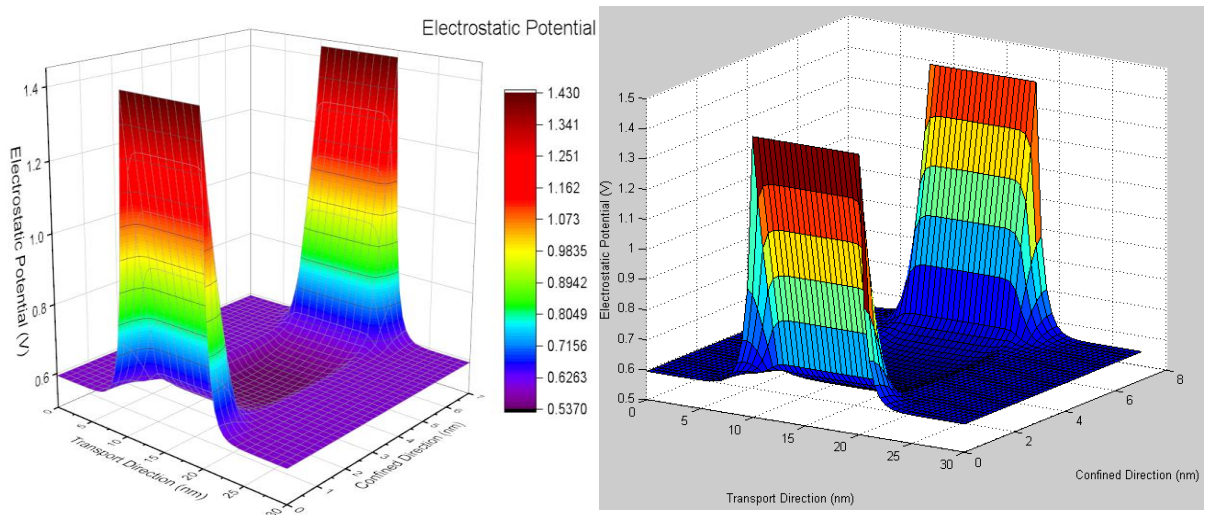


Fig 3. Potential 3D graph when Gate voltage is 1.1V. Left figure is drawn by Origin, and right figure is drawn by MATLAB.

c) Maximum Update Vector at $V_g=0V$ and $1.1V$.

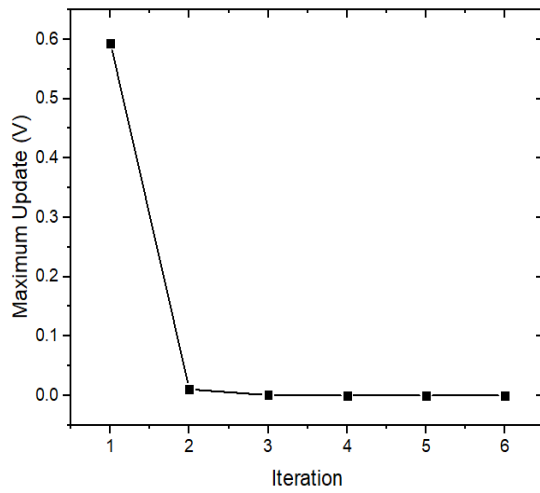


Fig 4. Maximum updated value graph at each iteration when gate voltage is 0V

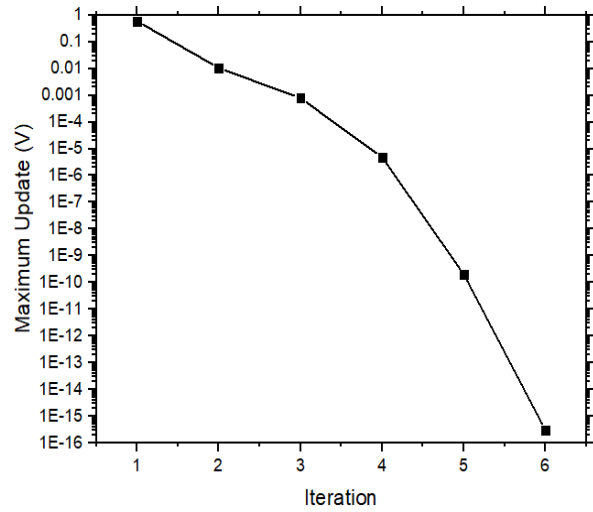


Fig 5. Log scaled Maximum updated value graph at each iteration when gate voltage is 0V

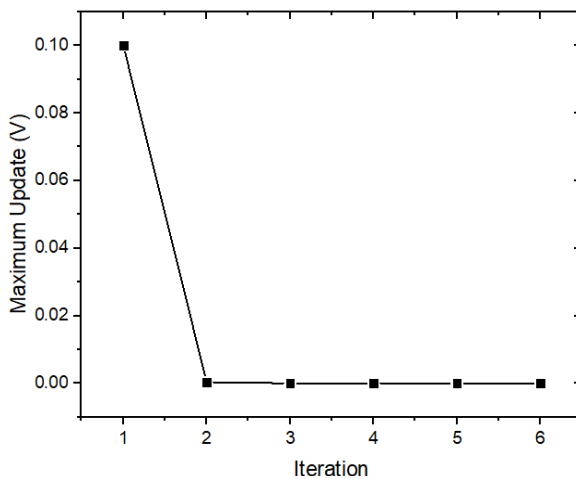


Fig 6. Maximum updated value graph at each iteration when gate voltage is 1.1V

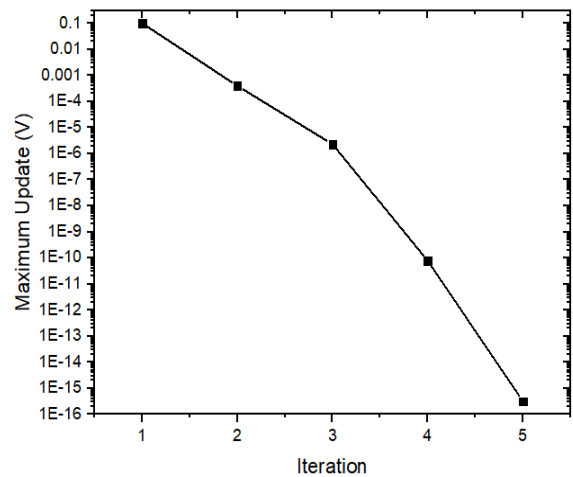


Fig 7. Log scaled Maximum updated value graph at each iteration when gate voltage is 1.1V

Gate 전압이 0V와 1.1V일 때 Potential 그래프와 Maximum Updated value를 나타냈다. Gate 전압이 상승함에 따라 Gate 쪽 Potential이 증가한 것을 확인할 수 있었다. Update vector를 확인해보면, Gate 전압이 0V일 때 첫번째 iteration값이 0.1로 0.59정도로 매우 큰데, 이유는 Oxide potential 값이 0V이다가 약 0.59V로 변하기 때문에 크게 나타났다. 1.1V일 때 첫번째 iteration에서 0.1V 나타나는 이유는 gate voltage increasement를 0.1V로 설정하였기 때문이다. 두 경우 모두 6번안에 Maximum update value가 $10^{-16}V$ 정도로, 수렴하는 것을 확인할 수 있다.