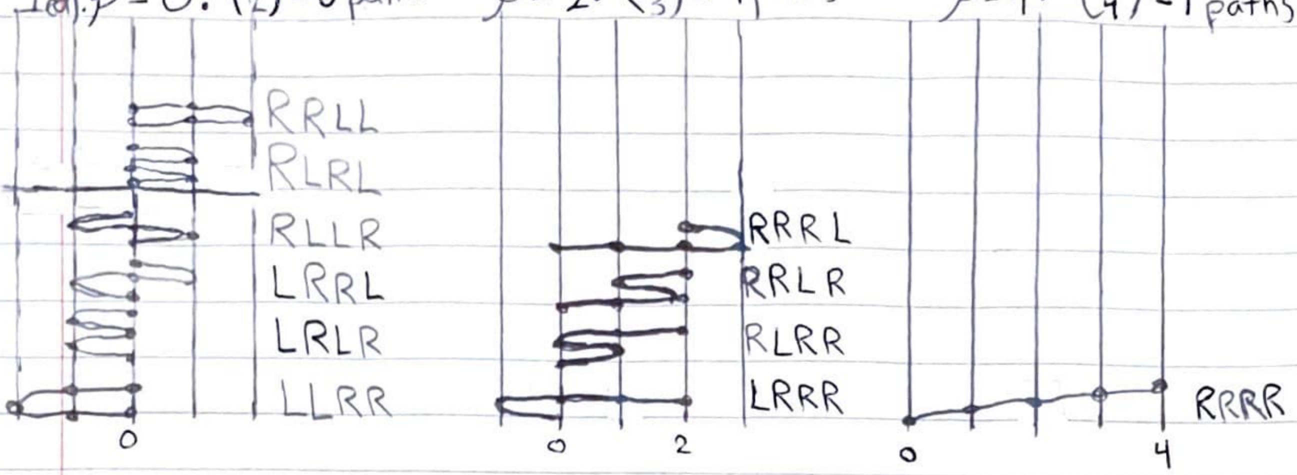
Philip Nicoll
2/12/24
Section: only

# Mat 4860 Homework #2

1(a). $p=0$: $\binom{4}{2}=6$ paths   $p=2$: $\binom{4}{3}=4$ paths   $p=4$: $\binom{4}{4}=1$ paths




$p=1,3$ are impossible to get to when $N=4$, resulting in 0 paths

$p=(N+m)/2$

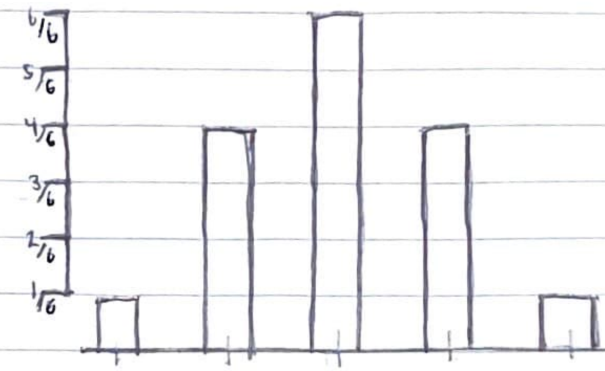
(b) $w(-4,4) = \binom{4}{0}/2^4 = \frac{1}{16}$

$w(-2,4) = \binom{4}{1}/2^4 = \frac{4}{16}$

$w(0,4) = \binom{4}{2}/2^4 = \frac{6}{16}$

$w(2,4) = \binom{4}{3}/2^4 = \frac{4}{16}$

$w(4,4) = \binom{4}{4}/2^4 = \frac{1}{16}$




This makes sense due to symmetry and higher probabilities towards the center.

2. (a) $w(m,N) = \underbrace{\binom{N}{p}}_{\text{number of paths}} \cdot \underbrace{(\lambda)^{N-p}}_{\text{chance of picking left}} \cdot \underbrace{(\mu)^p}_{\text{chance of picking right}}$

$N-p$: # of steps left   $p$: # of steps right

(i) $\binom{N}{p}\left(\frac{1}{2}\right)^{N-p}\left(\frac{1}{2}\right)^p = \binom{N}{p}\left(\frac{1}{2}\right)^{N-p+p} = \binom{N}{p}\left(\frac{1}{2}\right)^N = \binom{N}{p}/2^N$ ✓

(ii) $\sum_{p=0}^{N} w(m,N) = \sum_{p=0}^{N} \binom{N}{p}(\lambda)^{N-p}(\mu)^p$ must equal 1

From Binomial Theorem $\rightarrow \sum_{p=0}^{N}\binom{N}{p}(\lambda)^{N-p}(\mu)^p = (\lambda+\mu)^N = 1^N = 1$

$\mu + \lambda = 1 \rightarrow \lambda = 1 - \mu$

b) $G(u) = \sum_{p=0}^{N} w(m,N) \cdot u^p = \sum_{p=0}^{N} \binom{N}{p} (\lambda)^{N-p} (\mu)^p (u)^p$

$= \sum_{p=0}^{N} \binom{N}{p} (\lambda)^{N-p} (\mu \cdot u)^p \Rightarrow$ Binomial Theorem $\rightarrow (\lambda + \mu \cdot u)^N = G(u)$

c) $G'(u) = (N)(\lambda + \mu \cdot u)^{N-1} (\mu) = (N\mu)(\lambda + \mu \cdot u)^{N-1}$

$G'(1) = (N\mu)(\lambda + \mu \cdot 1)^{N-1} = (N\mu)(1)^{N-1} = N \cdot \mu = E[p]$

$E[m] = E[2p - N] = 2 \cdot E[p] - N = 2N \cdot \mu - N$
$= (2\mu - 1) \cdot N$

This makes sense. If $\mu$ is $< 0.5$, then the expected m value will be to the left. Otherwise, it will be to the right. It also lines up when plugging in test values like $\mu = 1$ or $\mu = -1$ or even $\mu = .75$.