```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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%%%%%%%%%%% Date: 02/13/2024
%%%%%%%%%%% Title: ECE 711 - HW2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc
clear
close all
syms theta No Lo
```

a)

```
Ps = sym(2);
Pr = sym(2);

Nsa = (No/Ps)*(cos(Ps/sym(2)*theta));
Nsb = (No/Ps)*(cos(Ps/sym(2)*theta-sym(2*pi/3)));
Nsc = (No/Ps)*(cos(Ps/sym(2)*theta-sym(4*pi/3)));

Nra = (No/Pr)*sign(cos(Pr/sym(2)*theta));
Nrb = (No/Pr)*sign(cos(Pr/sym(2)*theta-sym(2*pi/3)));
Nrc = (No/Pr)*sign(cos(Pr/sym(2)*theta-sym(4*pi/3)));

% No = 1;

% self inductance
Las = Lo*int(Nsa^2,0,2*pi)
```

Las =

$$\frac{\pi\,\mathrm{Lo}\,\mathrm{No}^2}{4}$$

```
Lar = Lo*int(Nra^2,0,2*pi)
```

Lar =

$$\frac{\pi\,\mathrm{Lo}\,\mathrm{No}^2}{2}$$

```
% fundamental self inductance
as = fund_extract(Nsa,Ps,theta);
Lms = Lo*(int(as^2,0,2*pi))
```

Lms =

$$\frac{\pi\,\mathrm{Lo}\,\mathrm{No}^2}{4}$$

```
ar = fund_extract(Nra,Ps,theta);
Lmr = Lo*(int(ar^2,0,2*pi))
```

Lmr =

$$\frac{4\,\mathrm{Lo}\,\mathrm{No}^2}{\pi}$$

```
Lls = Las - Lms;
Llr = Lar - Lmr;

% getting estimated values
Lasev = sym(300);
Lmsev = vpa(Lms*Lasev/Las,6)
```

Lmsev = 300.0

```
Llsev = vpa(Lls*Lasev/Las,6)
```

Llsev = 0.0

```
Larev = vpa(Lar*Lasev/Las,6)
```

Larev = 600.0

```
Lmrev = vpa(Lmr*Lasev/Las,6)
```

Lmrev = 486.342

```
Llrev = vpa(Llr*Lasev/Las,6)
```

Llrev = 113.658

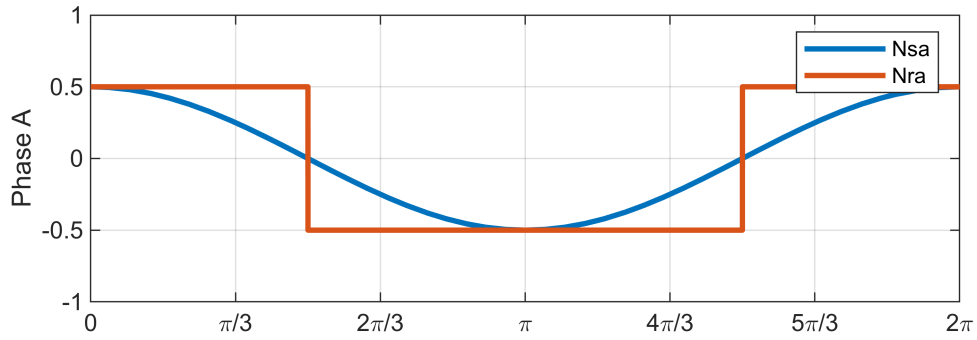
```
N_ratio = sqrt(Lmr/Lms)
```

N_ratio =

$$\frac{4}{\pi}$$

```
%%
No = 1;
figure(1)
subplot(211);
fplot(subs(Nsa),[0 2*pi],'LineWidth',2,'DisplayName','Nsa')
hold on;grid on;ylabel('Phase A');ylim([-No No]);
fplot(subs(Nra),[0 2*pi],'LineWidth',2,'DisplayName','Nra')
set(gca,'XTick',0:pi/3:2*pi);set(gca,'XTickLabel',{'0','\pi/3','2\pi/3','\pi','4\pi/
3','5\pi/3','2\pi'})
```

```
legend
```



b)

```
syms No theta_r t
assume(t,"real")
assume(theta_r,"real")

% for computational issue, assigning theta_r to stator winding function is
% much faster
Nsa = (No/Ps)*(cos(Ps/sym(2)*theta+theta_r));
Nsb = (No/Ps)*(cos(Ps/sym(2)*theta+theta_r-sym(2*pi/3)));
Nsc = (No/Ps)*(cos(Ps/sym(2)*theta+theta_r-sym(4*pi/3)));

Nra = (No/Pr)*sign(cos(Pr/sym(2)*theta));
Nrb = (No/Pr)*sign(cos(Pr/sym(2)*theta-sym(2*pi/3)));
Nrc = (No/Pr)*sign(cos(Pr/sym(2)*theta-sym(4*pi/3)));

% Mutual indutances
Lasbs = Lo*int(Nsa*Nsb,0,2*pi);
Larbr = Lo*int(Nra*Nrb,0,2*pi);
Lasar = Lo*int(Nsa*Nra,0,2*pi);
Lasbr = Lo*int(Nsa*Nrb,0,2*pi);
```

```
Lascr = Lo*int(Nsa*Nrb,0,2*pi);
Lar = Lo*int(Nra*Nra,0,2*pi);

% getting estimated values
Lasev = sym(300);
Larev = Lar*Lasev/Las;
Lasbsev = Lasbs*Lasev/Las;
Larbrev = Larbr*Lasev/Las;
Lasarev = Lasar*Lasev/Las;
Lasbrev = Lasbr*Lasev/Las;
Lascrev = Lascr*Lasev/Las;

% set up constraints
w = sym(377);
Im = sym(4);

% Ldi/dt = v
ias = Im*sin(w*t)
```

ias = $4\sin(377\,t)$

```
vas = vpa(Lasev/1000*diff(ias,t),6)
```

vas = $452.4\cos(377.0\,t)$

```
vbs = vpa(Lasbsev/1000*diff(ias,t),6)
```

vbs = $-226.2\cos(377.0\,t)$

```
vcs = vpa(Lasbsev/1000*diff(ias,t),6)
```

vcs = $-226.2\cos(377.0\,t)$

```
var = vpa(Lasarev/1000*diff(ias,t),6)
```

var = $576.014\cos(377.0\,t)\cos(\theta_r)$

```
vbr = vpa(Lasbrev/1000*diff(ias,t),6)
```

vbr = $-576.014\cos(377.0\,t)\ (0.5\cos(\theta_r) + 0.866025\sin(\theta_r))$

```
vcr = vpa(Lasbrev/1000*diff(ias,t),6)
```

vcr = $-576.014\cos(377.0\,t)\ (0.5\cos(\theta_r) + 0.866025\sin(\theta_r))$

```

```

c)

```

```

```
Labcr_s = [Lasarev, Lasbrev, Lasbrev;Lasbrev, Lasarev, Lasbrev;Lasbrev, Lasbrev,
Lasarev]/1000
```

```
Labcr_s =
```

$$\begin{pmatrix} \frac{6\cos(\theta_r)}{5\pi} & \sigma_1 & \sigma_1 \\ \sigma_1 & \frac{6\cos(\theta_r)}{5\pi} & \sigma_1 \\ \sigma_1 & \sigma_1 & \frac{6\cos(\theta_r)}{5\pi} \end{pmatrix}$$

where

$$\sigma_1 = -\frac{6\left(\frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{5\pi}$$

```
Labcr_r = [Larev, Larbrev, Larbrev;Larbrev, Larev, Larbrev;Larbrev, Larbrev, Larev]/
1000
```

```
Labcr_r =
```

$$\begin{pmatrix} \frac{3}{5} & -\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} & -\frac{1}{5} \\ -\frac{1}{5} & -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$$

```
Labcs_s = [Lasev, Lasbsev, Lasbsev;Lasbsev, Lasev, Lasbsev;Lasbsev, Lasbsev, Lasev]/
1000
```

```
Labcs_s =
```

$$\begin{pmatrix} \frac{3}{10} & -\frac{3}{20} & -\frac{3}{20} \\ -\frac{3}{20} & \frac{3}{10} & -\frac{3}{20} \\ -\frac{3}{20} & -\frac{3}{20} & \frac{3}{10} \end{pmatrix}$$

```
Labcs_r = [Lasarev, Lasbrev, Lasbrev;Lasbrev, Lasarev, Lasbrev;Lasbrev, Lasbrev,
Lasarev]/1000
```

```
Labcs_r =
```

$$\begin{pmatrix} \frac{6\cos(\theta_r)}{5\pi} & \sigma_1 & \sigma_1 \\ \sigma_1 & \frac{6\cos(\theta_r)}{5\pi} & \sigma_1 \\ \sigma_1 & \sigma_1 & \frac{6\cos(\theta_r)}{5\pi} \end{pmatrix}$$

where

$$\sigma_1 = -\frac{6\left(\frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{5\pi}$$

```
% lambda abcr should be zero
% iabcr = - inv(Labcr_r)*Labcr_s*iabcs

iabcs = [ias;0;0]
```

iabcs =

$$\begin{pmatrix} 4\sin(377\,t) \\ 0 \\ 0 \end{pmatrix}$$

```
iabcr = -inv(Labcr_r)*Labcr_s*iabcs
```

iabcr =

$$\begin{pmatrix} 4\sin(377\,t)\left(\frac{3\left(\frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{\pi} - \frac{3\cos(\theta_r)}{\pi}\right) \\ \sigma_1 \\ \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = 4\sin(377\,t)\left(\frac{9\left(\frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{2\pi} - \frac{3\cos(\theta_r)}{2\pi}\right)$$

```
iar = vpa(iabcr(1,1),6)
```

iar = $-4.0\sin(377.0\,t)\ (0.477465\cos(\theta_r) - 0.826993\sin(\theta_r))$

```
ibr = vpa(iabcr(2,1),6)
```

ibr = $4.0\sin(377.0\,t)\ (0.238732\cos(\theta_r) + 1.24049\sin(\theta_r))$

```
icr = vpa(iabcr(3,1),6)
```

icr = $4.0 \sin(377.0\,t)\ (0.238732 \cos(\theta_r) + 1.24049 \sin(\theta_r))$

```
Lambda_abcs_r = Labcs_r*iabcr
```

Lambda_abcs_r =

$$\begin{pmatrix} \dfrac{24 \sin(377\,t) \cos(\theta_r) \left(\dfrac{3\,\sigma_3}{\pi} - \dfrac{3 \cos(\theta_r)}{\pi}\right)}{5\,\pi} - \dfrac{48 \sin(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} \\ \sigma_1 \\ \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{24 \sin(377\,t) \cos(\theta_r)\,\sigma_2}{5\,\pi} - \frac{24 \sin(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} - \frac{24 \sin(377\,t)\,\sigma_3 \left(\dfrac{3\,\sigma_3}{\pi} - \dfrac{3 \cos(\theta_r)}{\pi}\right)}{5\,\pi}$$

$$\sigma_2 = \frac{9\,\sigma_3}{2\,\pi} - \frac{3 \cos(\theta_r)}{2\,\pi}$$

$$\sigma_3 = \frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}$$

```
Lambda_abcs_s = Labcs_s*iabcs
```

Lambda_abcs_s =

$$\begin{pmatrix} \dfrac{6 \sin(377\,t)}{5} \\ -\dfrac{3 \sin(377\,t)}{5} \\ -\dfrac{3 \sin(377\,t)}{5} \end{pmatrix}$$

```
Lambda_abcs = Lambda_abcs_s + Lambda_abcs_r
```

Lambda_abcs =

$$\begin{pmatrix} \frac{6\sin(377\,t)}{5} - \frac{48\sin(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} + \frac{24\sin(377\,t)\cos(\theta_r)\left(\frac{3\,\sigma_3}{\pi} - \frac{3\cos(\theta_r)}{\pi}\right)}{5\,\pi} \\ \sigma_1 \\ \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{24\sin(377\,t)\cos(\theta_r)\,\sigma_2}{5\,\pi} - \frac{24\sin(377\,t)\,\sigma_3\left(\frac{3\,\sigma_3}{\pi} - \frac{3\cos(\theta_r)}{\pi}\right)}{5\,\pi} - \frac{24\sin(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} - \frac{3\sin(377\,t)}{5}$$

$$\sigma_2 = \frac{9\,\sigma_3}{2\,\pi} - \frac{3\cos(\theta_r)}{2\,\pi}$$

$$\sigma_3 = \frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}$$

```
Lambdaabcs = diff(Lambda_abcs,t)
```

```
Lambdaabcs =
```

$$\begin{pmatrix} \frac{2262\cos(377\,t)}{5} - \frac{18096\cos(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} + \frac{9048\cos(377\,t)\cos(\theta_r)\left(\frac{3\,\sigma_3}{\pi} - \frac{3\cos(\theta_r)}{\pi}\right)}{5\,\pi} \\ \sigma_1 \\ \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{9048\cos(377\,t)\cos(\theta_r)\,\sigma_2}{5\,\pi} - \frac{9048\cos(377\,t)\,\sigma_3\left(\frac{3\,\sigma_3}{\pi} - \frac{3\cos(\theta_r)}{\pi}\right)}{5\,\pi} - \frac{9048\cos(377\,t)\,\sigma_3\,\sigma_2}{5\,\pi} - \frac{1131\,c}{}$$

$$\sigma_2 = \frac{9\,\sigma_3}{2\,\pi} - \frac{3\cos(\theta_r)}{2\,\pi}$$

$$\sigma_3 = \frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}$$

```
vas = simplify(Lambdaabcs(1,1),6)
```

```
vas =
```

$$\frac{2262\cos(377\,t)\,\left(6\,\sqrt{3}\,\cos\left(2\,\theta_r+\frac{\pi}{6}\right)+\pi^2-18\right)}{5\,\pi^2}$$

```
vbs = simplify(Lambdaabcs(2,1),6)
```

vbs =

$$\frac{1131\cos(377\,t)\,\left(27\cos(2\,\theta_r)+3\,\sqrt{3}\,\sin(2\,\theta_r)-\pi^2-18\right)}{5\,\pi^2}$$

```
vcs = simplify(Lambdaabcs(3,1),6)
```

vcs =

$$\frac{1131\cos(377\,t)\,\left(27\cos(2\,\theta_r)+3\,\sqrt{3}\,\sin(2\,\theta_r)-\pi^2-18\right)}{5\,\pi^2}$$

```
% vas = Las*dias/dt + Nr/Ns*Lms*[d(iar*cos(theta_r))/dt
```

d) and e)

```
ias = Im*sin(w*t)
```

ias = $4\sin(377\,t)$

```
ibs = Im*sin(w*t-2*sym(pi)/3)
```

ibs =

$$4\sin\left(377\,t-\frac{2\,\pi}{3}\right)$$

```
ics = Im*sin(w*t-4*sym(pi)/3)
```

ics =

$$4\sin\left(377\,t-\frac{4\,\pi}{3}\right)$$

```
a = exp(j*2*pi/3)
```

a = -0.5000 + 0.8660i

```
Iabcs = simplify(2/3*(ias+a*ibs+a^2*ics))
```

Iabcs = $4\sin(377\,t)-4\cos(377\,t)\,\mathrm{i}$

```
Lambdaabcs = simplify((Lasev - Lasbsev)*Iabcs)/1000
```

Lambdaabcs =

$$\frac{9\sin(377\,t)}{5}-\frac{9\cos(377\,t)\,\mathrm{i}}{5}$$

```
Lambdabcr = expand((Lasarev - Lasbrev)*Iabcs/1000,'ArithmeticOnly',true)
```

Lambdabcr =

$$\frac{36\sin(377\,t)\cos(\theta_r)}{5\,\pi}+\frac{12\,\sqrt{3}\,\sin(377\,t)\sin(\theta_r)}{5\,\pi}-\frac{36\cos(377\,t)\cos(\theta_r)\,\mathrm{i}}{5\,\pi}-\frac{12\,\sqrt{3}\,\cos(377\,t)\sin(\theta_r)\,\mathrm{i}}{5\,\pi}$$

```
vabcs = diff(Lambdaabcs,t)
```

vabcs =

$$\frac{3393\cos(377\,t)}{5}+\frac{3393\sin(377\,t)\,\mathrm{i}}{5}$$

```
vabcr = vpa(diff(Lambdabcr,t),4)
```

vabcr = $864.0\cos(377.0\,t)\cos(\theta_r)+498.8\cos(377.0\,t)\sin(\theta_r)+864.0\sin(377.0\,t)\cos(\theta_r)\,\mathrm{i}+498.8\sin(377.0\,t)\sin$

```
vas = vpa(real(vabcs),4)
```

vas = $678.6\cos(377.0\,t)$

```
vas_check = vpa(real(vabcs),4)
```

vas_check = $678.6\cos(377.0\,t)$

```
vbs = vpa(real(vabcs*a^2),4)
```

vbs = $587.7\sin(377.0\,t)-339.3\cos(377.0\,t)$

```
vcs = vpa(real(vabcs*a),4)
```

vcs = $-339.3\cos(377.0\,t)-587.7\sin(377.0\,t)$

```
var = vpa(real(vabcr),4)
```

var = $864.0\cos(377.0\,t)\cos(\theta_r)+498.8\cos(377.0\,t)\sin(\theta_r)$

```
vbr = vpa(real(vabcr*a^2),4)
```

vbr = $748.3\sin(377.0\,t)\cos(\theta_r)-249.4\cos(377.0\,t)\sin(\theta_r)-432.0\cos(377.0\,t)\cos(\theta_r)+432.0\sin(377.0\,t)\sin(t$

```
vcr = vpa(real(vabcr*a),4)
```

vcr = $-432.0\cos(377.0\,t)\cos(\theta_r)-249.4\cos(377.0\,t)\sin(\theta_r)-748.3\sin(377.0\,t)\cos(\theta_r)-432.0\sin(377.0\,t)\sin$

f)

```
iabcs = [ias;ibs;ics]
```

iabcs =

$$\begin{pmatrix} 4\sin(377\,t) \\ 4\sin\left(377\,t-\frac{2\,\pi}{3}\right) \\ 4\sin\left(377\,t-\frac{4\,\pi}{3}\right) \end{pmatrix}$$

```
iabcr = -inv(Labcr_r)*Labcr_s*iabcs
```

iabcr =

$$\begin{pmatrix} 4\sin(377\,t)\,\sigma_1+\sigma_3+\sigma_2 \\ 4\sin(377\,t)\,\sigma_4+4\sin\left(377\,t-\frac{2\,\pi}{3}\right)\sigma_1+\sigma_2 \\ 4\sin(377\,t)\,\sigma_4+4\sin\left(377\,t-\frac{4\,\pi}{3}\right)\sigma_1+\sigma_3 \end{pmatrix}$$

where

$$\sigma_1=\frac{3\,\sigma_5}{\pi}-\frac{3\cos(\theta_r)}{\pi}$$

$$\sigma_2=4\sin\left(377\,t-\frac{4\,\pi}{3}\right)\sigma_4$$

$$\sigma_3=4\sin\left(377\,t-\frac{2\,\pi}{3}\right)\sigma_4$$

$$\sigma_4=\frac{9\,\sigma_5}{2\,\pi}-\frac{3\cos(\theta_r)}{2\,\pi}$$

$$\sigma_5=\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}$$

```
iar = iabcr(1,1)
```

iar =

$$4\sin(377\,t)\left(\frac{3\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{\pi}-\frac{3\cos(\theta_r)}{\pi}\right)+4\sin\left(377\,t-\frac{2\,\pi}{3}\right)\sigma_1+4\sin\left(377\,t-\frac{4\,\pi}{3}\right)\sigma_1$$

where

$$\sigma_1=\frac{9\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{2\,\pi}-\frac{3\cos(\theta_r)}{2\,\pi}$$

```
ibr = iabcr(2,1)
```

```
ibr =
```

$$4\sin(377\,t)\,\sigma_1+4\sin\left(377\,t-\frac{2\,\pi}{3}\right)\left(\frac{3\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{\pi}-\frac{3\cos(\theta_r)}{\pi}\right)+4\sin\left(377\,t-\frac{4\,\pi}{3}\right)\sigma_1$$

where

$$\sigma_1=\frac{9\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{2\,\pi}-\frac{3\cos(\theta_r)}{2\,\pi}$$

```
icr = iabcr(3,1)
```

```
icr =
```

$$4\sin(377\,t)\,\sigma_1+4\sin\left(377\,t-\frac{4\,\pi}{3}\right)\left(\frac{3\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{\pi}-\frac{3\cos(\theta_r)}{\pi}\right)+4\sin\left(377\,t-\frac{2\,\pi}{3}\right)\sigma_1$$

where

$$\sigma_1=\frac{9\left(\frac{\cos(\theta_r)}{2}+\frac{\sqrt{3}\,\sin(\theta_r)}{2}\right)}{2\,\pi}-\frac{3\cos(\theta_r)}{2\,\pi}$$

```
i0r = simplify(iar+ibr+icr)
```

```
i0r = 0
```

```
Lambda_abcs_r = Labcs_r*iabcr;
Lambda_abcs_s = Labcs_s*iabcs;
Lambda_abcs = Lambda_abcs_s + Lambda_abcs_r
```

```
Lambda_abcs =
```

$$\begin{pmatrix} \frac{6\sin(377\,t)}{5} - \frac{3\,\sigma_{11}}{5} - \frac{3\,\sigma_{10}}{5} + \frac{6\cos(\theta_r)\,\sigma_7}{5\,\pi} - \sigma_3 - \sigma_2 \\ \frac{6\,\sigma_{11}}{5} - \sigma_1 - \frac{3\,\sigma_{10}}{5} + \frac{6\cos(\theta_r)\,\sigma_6}{5\,\pi} - \sigma_4 - \sigma_2 \\ \frac{6\,\sigma_{10}}{5} - \frac{3\,\sigma_{11}}{5} - \sigma_1 + \frac{6\cos(\theta_r)\,\sigma_5}{5\,\pi} - \sigma_4 - \sigma_3 \end{pmatrix}$$

where

$$\sigma_1 = \frac{3\sin(377\,t)}{5}$$

$$\sigma_2 = \frac{6\,\sigma_{13}\,\sigma_5}{5\,\pi}$$

$$\sigma_3 = \frac{6\,\sigma_{13}\,\sigma_6}{5\,\pi}$$

$$\sigma_4 = \frac{6\,\sigma_{13}\,\sigma_7}{5\,\pi}$$

$$\sigma_5 = \sigma_8 + 4\,\sigma_{10}\,\sigma_9 + 4\,\sigma_{11}\,\sigma_{12}$$

$$\sigma_6 = \sigma_8 + 4\,\sigma_{11}\,\sigma_9 + 4\,\sigma_{10}\,\sigma_{12}$$

$$\sigma_7 = 4\sin(377\,t)\,\sigma_9 + 4\,\sigma_{11}\,\sigma_{12} + 4\,\sigma_{10}\,\sigma_{12}$$

$$\sigma_8 = 4\sin(377\,t)\,\sigma_{12}$$

$$\sigma_9 = \frac{3\,\sigma_{13}}{\pi} - \frac{3\cos(\theta_r)}{\pi}$$

$$\sigma_{10} = \sin\left(377\,t - \frac{4\,\pi}{3}\right)$$

$$\sigma_{11} = \sin\left(377\,t - \frac{2\,\pi}{3}\right)$$

$$\sigma_{12} = \frac{9\,\sigma_{13}}{2\,\pi} - \frac{3\cos(\theta_r)}{2\,\pi}$$

$$\sigma_{13} = \frac{\cos(\theta_r)}{2} + \frac{\sqrt{3}\,\sin(\theta_r)}{2}$$

```
Lambdaabcs = Lambda_abcs;
vabcs = diff(Lambdaabcs,t);
vas = simplify(vabcs(1,1))
```

vas =

$$-\frac{3393\cos(377\,t)\,\left(6\cos\left(2\,\theta_r-\frac{\pi}{3}\right)-\pi^2+6\right)}{5\,\pi^2}$$

```
vbs = simplify(vabcs(2,1))
```

vbs =

$$-\frac{10179\cos(\sigma_1)-20358\cos(377\,t)-20358\cos(377\,t+2\,\theta_r)+20358\,\sqrt{3}\,\sin(377\,t)+10179\,\sqrt{3}\,\sin(\sigma_1}{10\,\pi^2}$$

where

$$\sigma_1 = 377\,t-2\,\theta_r$$

```
vcs = simplify(vabcs(3,1))
```

vcs =

$$\frac{20358\cos(377\,t)+20358\cos(377\,t-2\,\theta_r)-10179\cos(\sigma_1)+20358\,\sqrt{3}\,\sin(377\,t)+10179\,\sqrt{3}\,\sin(\sigma_1)-}{10\,\pi^2}$$

where

$$\sigma_1 = 377\,t+2\,\theta_r$$

```
vabcs = simplify(2/3*(vas+vbs*a+vcs*a))
```

vabcs =

$$-\frac{1131\cos(377\,t)\,\left(9\cos(2\,\theta_r)+9\,\sqrt{3}\,\sin(2\,\theta_r)-3\,\pi^2+18-9\sin(2\,\theta_r)\,\mathrm{i}-6\,\sqrt{3}\,\mathrm{i}+\sqrt{3}\,\pi^2\,\mathrm{i}-3\,\sqrt{3}\,\mathrm{c}\right.}{5\,\pi^2}$$

```
iabcr = simplify(2/3*(iar+ibr*a+icr*a))
```

iabcr =

$$-\frac{3\sin(377\,t)\,\left(3\cos(\theta_r)+\sqrt{3}\,\sin(\theta_r)-\sin(\theta_r)\,\mathrm{i}-\sqrt{3}\,\cos(\theta_r)\,\mathrm{i}\right)}{\pi}$$

```
vas_recovered = real(vabcs)
```

vas_recovered =

$$-\frac{1131\cos(377\,t)\,\left(9\cos(2\,\theta_r)+9\,\sqrt{3}\,\sin(2\,\theta_r)-3\,\pi^2+18\right)}{5\,\pi^2}$$

```
err = simplify(vas-vas_recovered)
```

err = 0

```
%% Appendix
% verification between matrix method and complex vector method (e)
ias = Im*sin(w*t)
```

ias = $4\sin(377\,t)$

```
ibs = Im*sin(w*t-2*sym(pi)/3)
```

ibs =

$$4\sin\left(377\,t-\frac{2\,\pi}{3}\right)$$

```
ics = Im*sin(w*t-4*sym(pi)/3)
```

ics =

$$4\sin\left(377\,t-\frac{4\,\pi}{3}\right)$$

```
iabcs = [ias;ibs;ics]
```

iabcs =

$$\begin{pmatrix} 4\sin(377\,t) \\ 4\sin\left(377\,t-\frac{2\,\pi}{3}\right) \\ 4\sin\left(377\,t-\frac{4\,\pi}{3}\right) \end{pmatrix}$$

```
Lambda_abcs_s = Labcs_s*iabcs
```

Lambda_abcs_s =

$$\begin{pmatrix} \frac{6\sin(377\,t)}{5} - \sigma_2 - \sigma_1 \\ \frac{6\sin\left(377\,t - \frac{2\,\pi}{3}\right)}{5} - \frac{3\sin(377\,t)}{5} - \sigma_1 \\ \frac{6\sin\left(377\,t - \frac{4\,\pi}{3}\right)}{5} - \sigma_2 - \frac{3\sin(377\,t)}{5} \end{pmatrix}$$

where

$$\sigma_1 = \frac{3\sin\left(377\,t - \frac{4\,\pi}{3}\right)}{5}$$

$$\sigma_2 = \frac{3\sin\left(377\,t - \frac{2\,\pi}{3}\right)}{5}$$

```
Lambda_abcs = Lambda_abcs_s
```

Lambda_abcs =

$$\begin{pmatrix} \frac{6\sin(377\,t)}{5} - \sigma_2 - \sigma_1 \\ \frac{6\sin\left(377\,t - \frac{2\,\pi}{3}\right)}{5} - \frac{3\sin(377\,t)}{5} - \sigma_1 \\ \frac{6\sin\left(377\,t - \frac{4\,\pi}{3}\right)}{5} - \sigma_2 - \frac{3\sin(377\,t)}{5} \end{pmatrix}$$

where

$$\sigma_1 = \frac{3\sin\left(377\,t - \frac{4\,\pi}{3}\right)}{5}$$

$$\sigma_2 = \frac{3\sin\left(377\,t - \frac{2\,\pi}{3}\right)}{5}$$

```
Lambdaabcs = diff(Lambda_abcs,t)
```

Lambdaabcs =

$$\begin{pmatrix} \frac{2262\cos(377\,t)}{5} - \sigma_2 - \sigma_1 \\ \frac{2262\cos\left(377\,t - \frac{2\,\pi}{3}\right)}{5} - \frac{1131\cos(377\,t)}{5} - \sigma_1 \\ \frac{2262\cos\left(377\,t - \frac{4\,\pi}{3}\right)}{5} - \sigma_2 - \frac{1131\cos(377\,t)}{5} \end{pmatrix}$$

where

$$\sigma_1 = \frac{1131\cos\left(377\,t - \frac{4\,\pi}{3}\right)}{5}$$

$$\sigma_2 = \frac{1131\cos\left(377\,t - \frac{2\,\pi}{3}\right)}{5}$$

```
vas_temp = vpa(Lambdaabcs(1,1),6)
```

vas_temp = $452.4\cos(377.0\,t) - 226.2\cos(377.0\,t - 4.18879) - 226.2\cos(377.0\,t - 2.0944)$

```
vbs = vpa(Lambdaabcs(2,1),6)
```

vbs = $452.4\cos(377.0\,t - 2.0944) - 226.2\cos(377.0\,t - 4.18879) - 226.2\cos(377.0\,t)$

```
vcs = vpa(Lambdaabcs(3,1),6)
```

vcs = $452.4\cos(377.0\,t - 4.18879) - 226.2\cos(377.0\,t - 2.0944) - 226.2\cos(377.0\,t)$

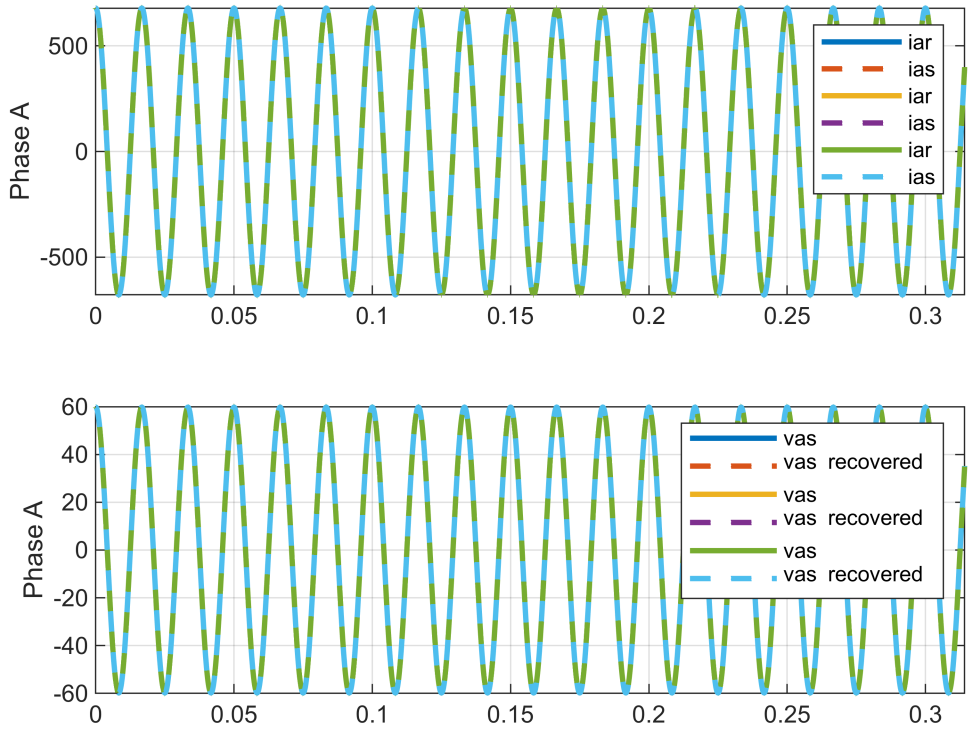
```
theta_r = 0
```

theta_r = 0

```
No = 1;
figure(3)
subplot(211);
fplot(subs(vas_check),[0 pi/10],'LineWidth',2,'DisplayName','iar')
hold on;grid on;ylabel('Phase A');
fplot(subs(vas_temp),[0 pi/10],'--','LineWidth',2,'DisplayName','ias')
% set(gca,'XTick',0:pi/3:2*pi);set(gca,'XTickLabel',{'0','\pi/3','2\pi/
3','\pi','4\pi/3','5\pi/3','2\pi'})
legend

% recovered signal from f)
subplot(212);
fplot(subs(vas),[0 pi/10],'LineWidth',2,'DisplayName','vas')
hold on;grid on;ylabel('Phase A');
```

```
fplot(subs(vas_recovered),[0 pi/10],'--','LineWidth',2,'DisplayName','vas _
recovered')
```




```
% set(gca,'XTick',0:pi/3:2*pi);set(gca,'XTickLabel',{'0','\pi/3','2\pi/
3','\pi','4\pi/3','5\pi/3','2\pi'})
legend
%% functions
% fundamental
```

```
function out = fund_extract(Nsa,Ps,theta)
    % fundamental self inductance
    C_pos1 = 1/2/sym(pi)*int(Nsa*exp(-j*theta*2/Ps),0,2*pi);
    C_neg1 = 1/2/sym(pi)*int(Nsa*exp(j*theta*2/Ps),0,2*pi);
    k = sqrt((C_pos1+C_neg1)^2+(j*C_pos1-j*C_neg1)^2);
    if C_neg1+C_pos1 ==0
        if subs(j*(C_pos1-C_neg1),No,1)>0
            phi = pi/2;
        else
            phi = -pi/2;
        end
    else
        phi = atan(j*((C_pos1-C_neg1)/(C_neg1+C_pos1))); %%% phi = -pi/2
    end
    out = k*sin(theta+phi);
end
```