



# A FLEXIBLE APPROACH TO PREDICTIVE BIOMARKER DISCOVERY

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## Background

This is the background.

## Variable Importance Parameter

Consider  $n$  identically and independently distributed (i.i.d) full-data random vectors  $X = (W, A, Y^{(0)}, Y^{(1)}) \sim P_X$ .

- $W$ : A  $p$ -length random vector of centered pretreatment biomarkers with nonzero variance.
- $A$ : A random binary indicator of treatment assignment.
- $Y^{(0)}, Y^{(1)}$ : Continuous potential outcomes under assignment to the control and treatment allocations, respectively.

Our causal variable importance parameter is  $\Psi^F(P_X) = (\Psi_1^F(P_X), \dots, \Psi_p^F(P_X))$ , where

$$\Psi_j^F(P_X) \equiv \frac{\mathbb{E}_{P_X} \left[ \left( Y^{(1)} - Y^{(0)} \right) W_j \right]}{\mathbb{E}_{P_X} \left[ W_j^2 \right]}.$$

Given access instead to  $n$  i.i.d. censored random observations  $O = (W, A, Y)$  where  $Y = AY^{(1)} + (1 - A)Y^{(0)}$ ,  $\Psi^F(P_X)$  is identified under the assumptions of unmeasured confounding and positivity by  $\Psi(P_0) = (\Psi_1(P_0), \dots, \Psi_p(P_0))$ . Here,

$$\Psi_j(P_0) \equiv \frac{\mathbb{E}_{P_0} \left[ \left( \bar{Q}_0[A = 1, W] - \bar{Q}_0[A = 0, W] \right) W_j \right]}{\mathbb{E}_{P_0} \left[ W_j^2 \right]},$$

where  $\bar{Q}_0[A, W] = \mathbb{E}_{P_0}[Y|A, W]$ .

## Inference

Let  $g_0(W) = \mathbb{P}[A = 1|W]$ , and let  $\hat{g}$  and  $\hat{\bar{Q}}$  be estimators of  $g_0$  and  $\bar{Q}_0$ , respectively. Define the Augmented Inverse Probability Weighted outcome difference as

$$T(O) \equiv \left( \frac{I(A = 1)}{\hat{g}(W)} - \frac{I(A = 0)}{1 - \hat{g}(W)} \right) \left( Y - \hat{\bar{Q}}(A, W) \right) + \hat{\bar{Q}}(1, W) - \hat{\bar{Q}}(0, W).$$

We derive from the efficient influence function of  $\Psi_j(P_0)$ ,  $D_j(O)$ , the double-robust one-step estimator

$$\hat{\Psi}_j(P_n) \equiv \frac{\sum_{i=1}^n T(O_i) W_{ij}}{\sum_{i=1}^n W_{ij}^2},$$

where  $\sum_i W_{ij} = 0$  for all  $j$  and  $P_n$  is the empirical distribution.

If  $\hat{g}$  and  $\hat{\bar{Q}}$  are trained using sample splitting techniques, and we assume that  $\|\hat{g} - g_0\|_2 \|\hat{\bar{Q}} - \bar{Q}_0\|_2 = o_p(n^{-1/2})$ , then

$$\sqrt{n} \left( \hat{\Psi}_j(P_n) - \Psi_j(P_0) \right) \xrightarrow{D} N \left( 0, \mathbb{V}_{P_0} [D_j(O)] \right).$$

## Simulation Study

Here are the results of our simulation study.

## Clinical Trial Application

IMmotion 151 heatmaps.

## Conclusions

Here are the important takeaways.

## References

List of references.

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