Urban Diversity Through an Information-Theoretic Lens

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Introduction

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Central Claim

These two questions are deeply related, and we should approach both using *information theory*.

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No global diversity $\implies$ perfect evenness $\implies$ maximal exposure.

Neighborhoods, Exposure, and the Checkerboard

Claim: Spatial exposure is about the number, scale, and pattern of neighborhoods.

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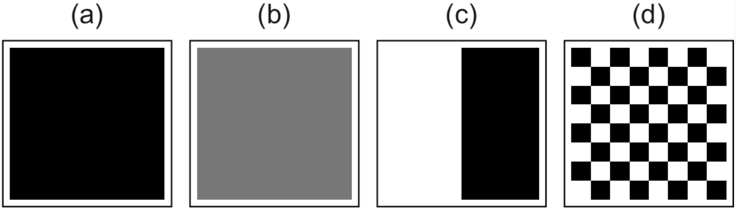

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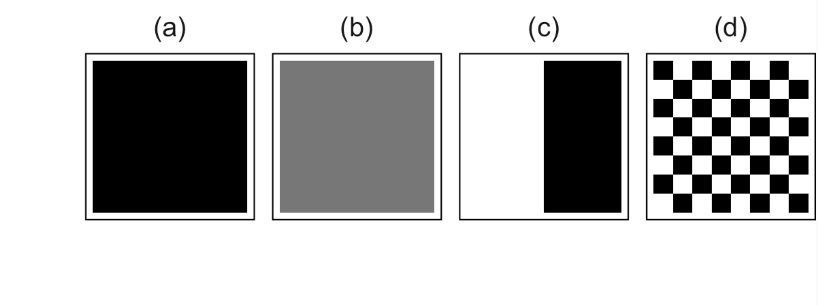

City	Exposure	Neighborhoods
(c)	Low	Few, large
(d)	High	Many, small

Diversity and Information Theory

Summary: Information Theory and Diversity

Entropy Measures Diversity

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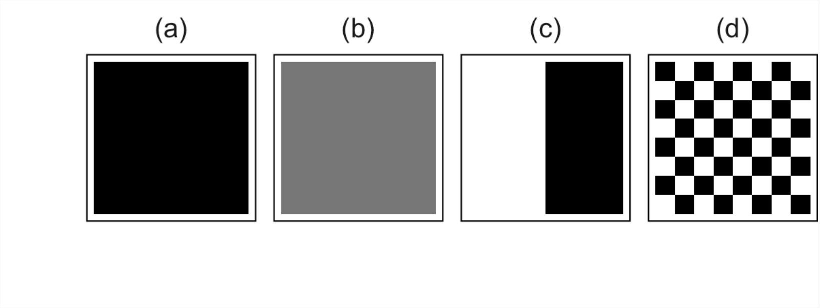
Mean Local Information Measures Exposure

$J(X,Y) \triangleq \mathbb{E}_X[\text{trace } J_Y(X)]$

Back to the checkerboard



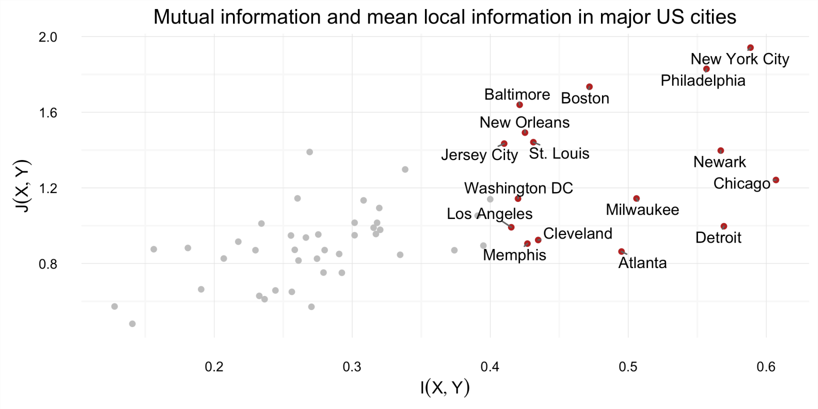
City	$H(Y)$	$I(X, Y)$	$J(X, Y)$
(a)	0.0	0.0	0.0
(b)	0.7	0.0	0.0
(c)	0.7	0.7	0.6
(d)	0.7	0.7	2.7

A Three-Dimensional Characterization of Urban Diversity

City	$H(Y)$	$I(X,Y)$	$J(X,Y)$
Albuquerque	1.08	0.19	0.66
Detroit	1.12	0.57	1.00
Philadelphia	1.31	0.56	1.83

Data accessed from the American Community Survey of the U.S. Census [1].

Visualizing Diversity Profiles in Major Cities



- Philadelphia has greater exposure / more neighborhood structure than Detroit.

Identifying Natural Neighborhoods

Key Property of Mutual Information

Suppose we cluster the locations X into groups with labels C.

“Additive Organizational Decomposability” (Chain Rule)

$$I(X, Y) = I(C, Y) + I(X, Y|C)\ .$$

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We can use this to identify neighborhoods!

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Recipe for Neighborhood Identification

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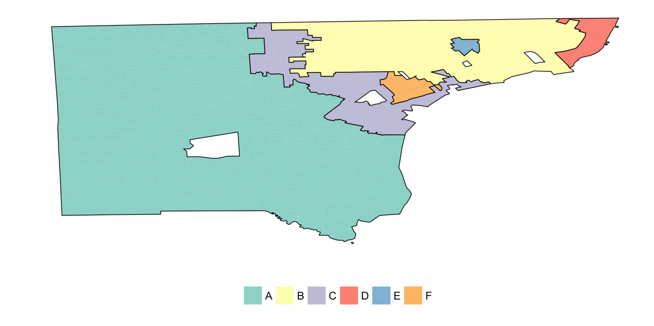
This optimization is hard to do exactly, but we can use a greedy approach based on hierarchical clustering.

Practical Example: Racial Neighborhoods in Detroit



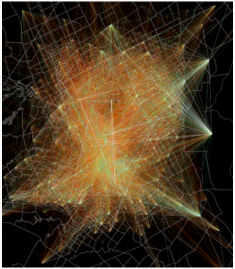
	A	B	C	D	E	F
White	36	2	7	2	1	1
Black	5	29	4			
Hispanic	2		1			2
Asian	2				1	
Other	2	1	1			

Percentage of population by race and neighborhood.

Wrapping Up

Generality and future work

- **Other demographics**: education level, income, occupation type, etc.
- **Other dimensions**: time
    - Evolution of urban demographics (timescale: decades)
    - Dynamics of diversity in daily mobility (timescale: minutes)



Daily commuting flows in Riyad, Saudi Arabia.
Image credit: Shan Jiang and MIT HuMNetLab

Download our tools!

R Package for Information-Theoretic Analysis

https://github.com/PhilChodrow/compx

This project: analysis and presentations

https://github.com/PhilChodrow/spatial_complexity

THANK YOU! Questions? Feedback?

Supplementary Materials

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Relation of local and Fisher informations

Suppose that $p(y|x) > 0$ and that $p(y|x)$ is differentiable as a function of x for all $x, y \in \mathbb{R}^n \times \mathcal{Y}$. Define $B_r \triangleq B_r(x_0) \triangleq \{x \in \mathbb{R}^n \mid |x - x_0| \leq r\}$. Additionally, define the *local mutual information* in B_r as the mutual information between X and Y where X is restricted to B_r:

$$
\begin{aligned}
I_r(x_0) &\triangleq \mathbb{E}_X[D[p(\cdot|X)\|p(\cdot|X \in B_r)]|X \in B_r] &(1)\\
&= \int_{B_r} p(x|X \in B_r) D[p(\cdot|x)\|p(\cdot|X \in B_r)] d^n x\,. &(2)
\end{aligned}
$$

where $D[p\|q] \triangleq \sum_y p(y) \log \frac{p(y)}{q(y)}$ is the Kullback-Leibler divergence of q from p.

Relation of local and Fisher informations

Theorem

Under the stated conditions,

$$\lim_{r \to 0} \frac{I_r(x_0)}{r^2} = \frac{n}{2(n+2)} \operatorname{trace} J_Y(x_0) \ . \tag{3}$$

The Fisher information matrix J_Y is given by

$$J_Y(x) \triangleq \mathbb{E}_Y \left[\nabla_x S_Y(x) \nabla_x S_Y(x)^T \right] \tag{4}$$

$$S_y(x) \triangleq \log p(y|x) \ . \tag{5}$$

Information Measures Check the Boxes

Criterion	$H(Y)$	$I(X, Y)$	$J(X, Y)$
Organizational Equivalence	✔	✔	✔
Size/Density Invariance	✔	✔	✔
Additive Group Decomposability	✔	✔	✔
Additive Spatial Decomposability	NA	✔	*
Scale Interpretability	NA	✔	✔
Boundary Independence	NA	✔	✔
Exchanges	NA	✔	✔

* We contend that [12]'s version is not a workable criterion for explicitly spatial measures.

✔ In the theoretical definition; in practice we are reliant on the data as it is provided.

Some Comparative Clusters of Equal Information