Neighborhoods and Segregation: An Information-Theoretic Lens

Phil Chodrow

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MIT Human Mobility and Networks Laboratory & Operations Research Center

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Introduction

Two questions for today:

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Neighborhoods and Diversity

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2. What is a **neighborhood**, and how can we distinguish neighborhoods in principled ways?

Central Claim

These two questions are deeply related, and we should approach both using *information theory*.

Three Dimensions of Urban Diversity

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Spatial exposure *"refers to the extent that members of one group encounter members of another group (or their own group, in the case of spatial isolation) in their local spatial environments."* [3]

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Spatial exposure *“refers to the extent that members of one group encounter members of another group (or their own group, in the case of spatial isolation) in their local spatial environments.”* [3]

No global diversity $\implies$ perfect evenness $\implies$ maximal exposure.

Neighborhoods, Exposure, and the Checkerboard

Claim: Spatial exposure is about the number, scale, and pattern of neighborhoods.

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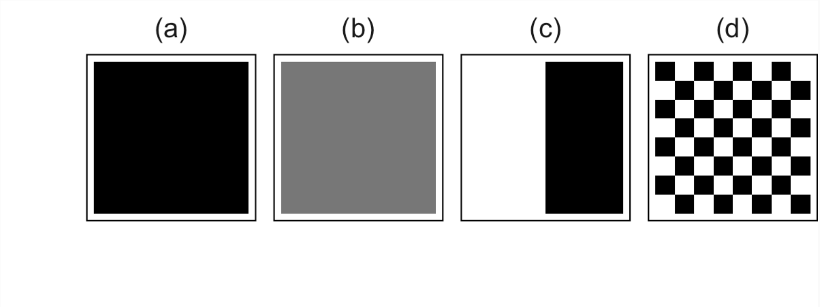

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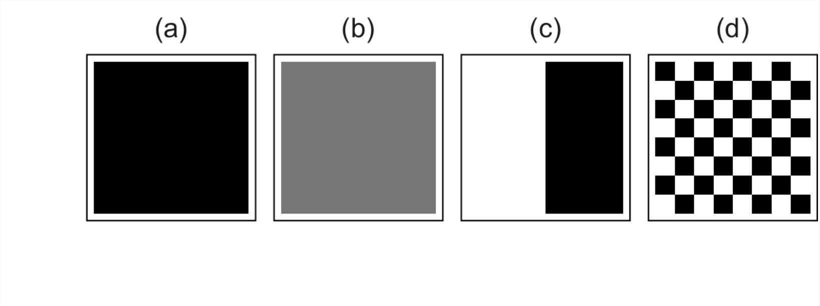
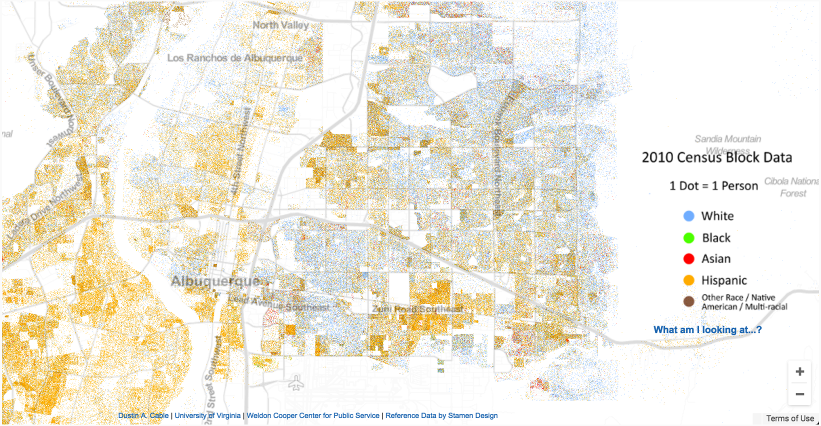


City	Exposure	Neighborhoods
(c)	Low	Few, large
(d)	High	Many, small

Albuquerque: Low diversity, fairly even, high exposure



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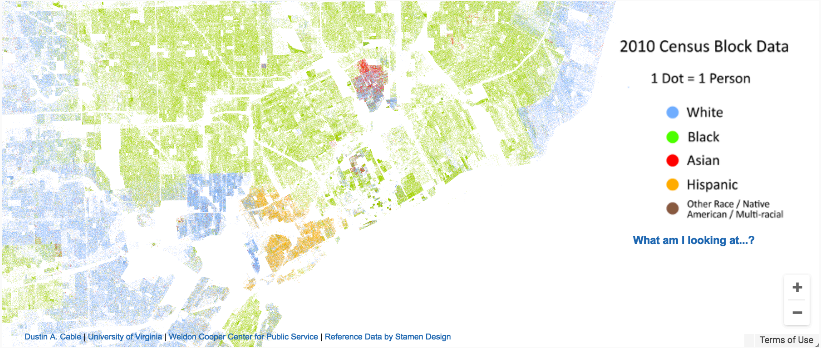
Detroit: High diversity, highly uneven, low exposure



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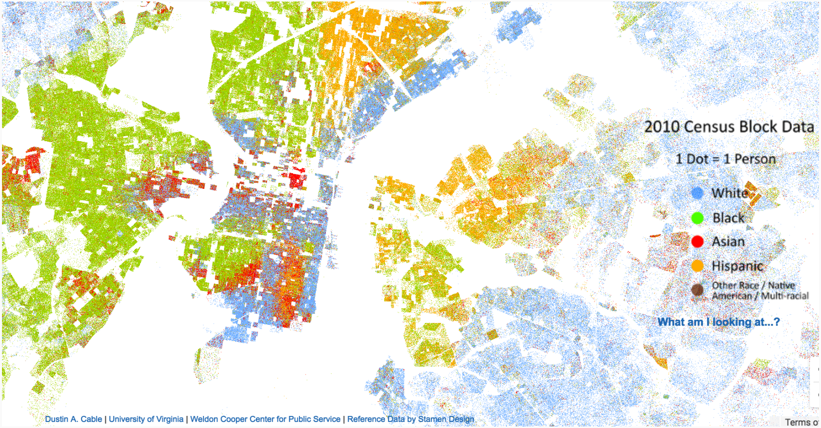
Philadelphia: High diversity, highly uneven, high exposure



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Diversity and Information Theory

Why Information Theory?

Information theory quantifies concepts like uncertainty and predictability. Why use it for studying race?

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Core Insight

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- ...But good prediction is **hard**.

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Core Insight

- Complex systems allow **some** prediction…
- …But good prediction is **hard**.

Examples of prediction:

- If you pick a random person from a city, is it hard to guess their race? "Yes" $\implies$ more diverse city.
- Does it help if you tell me where they live? "Yes" $\implies$ more uneven city.

Entropy as Global Diversity

Let $p(X, Y)$ be the joint distribution of location X and race Y. The marginal distribution of race alone is $p(Y) = \sum_{x \in \mathcal{X}} p(x, Y)$.

Entropy

$$H(Y) \triangleq -\mathbb{E}_Y[\log p(Y)] = -\sum_{y \in \mathcal{Y}} p(Y) \log p(Y)$$

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Entropy measures global diversity: more entropy, more diversity.

Mutual Information as Evenness

The mutual information quantifies the degree of dependence between two random variables.

Mutual Information

$$I(X, Y) \triangleq \sum_{x,y \in \mathcal{X} \times \mathcal{Y}} p(X, Y) \log \frac{p(X, Y)}{p(X)p(Y)} \, .$$

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Mutual information measures evenness: more information, the more uneven the spatial distribution of demographics.

Local Information

Let $I_r(x_0)$ be local the mutual information between X and Y, restricted to a small area of radius r centered at x_0. Think of this as the **spatial variation of racial trends in a small area.**

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Theorem

Under appropriate regularity conditions, for small r,

$$\frac{I_r(x_0)}{r^2} \cong \frac{1}{4}\mathrm{trace}\, J_Y(x_0)\ , \qquad \text{(proof on request)}$$

where $J_Y(x_0)$ is the Fisher information in Y about x_0

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Significance: the local information $I_r(x_0)$ is related to an intrinsic statistical measure $J_Y(x)$ *that is independent of the resolution r.*

Mean Local Information Measures Spatial Exposure

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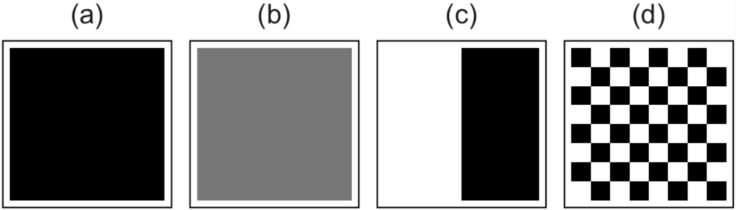
This is one way of "spatializing" the mutual information $I(X, Y)$; for another see [4, 5].

Back to the checkerboard



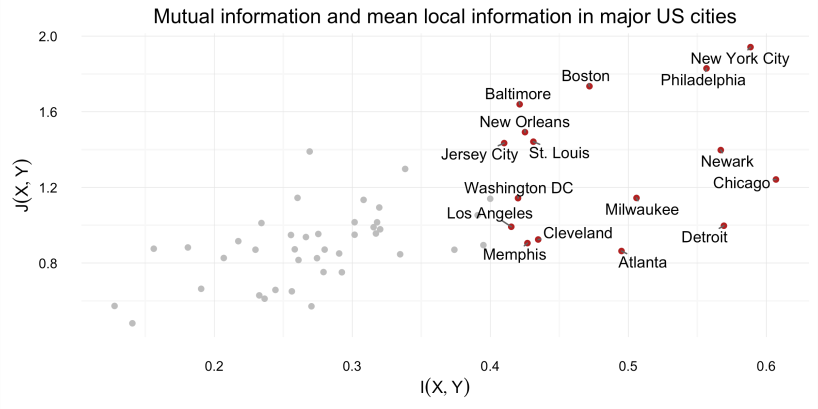
City	$H(Y)$	$I(X,Y)$	$J(X,Y)$
(a)	0.0	0.0	0.0
(b)	0.7	0.0	0.0
(c)	0.7	0.7	0.6
(d)	0.7	0.7	2.7

A Three-Dimensional Characterization of Urban Diversity

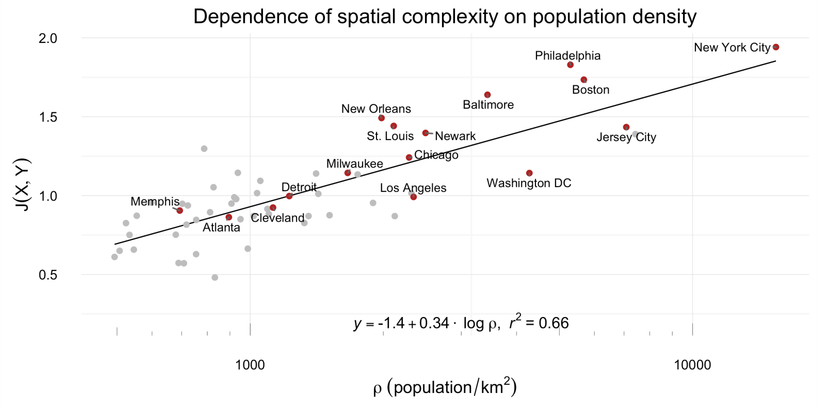
City	$H(Y)$	$I(X,Y)$	$J(X,Y)$
Albuquerque	1.08	0.19	0.66
Detroit	1.12	0.57	1.00
Philadelphia	1.31	0.56	1.83

Data accessed from the American Community Survey of the U.S. Census [1].

Visualizing Diversity Profiles in Major Cities



Urban Density and the Compression of Social Space



Information-Theoretic Identification of Neighborhoods

How can we use this?

$H(Y)$, $I(X, Y)$, and $J(X, Y)$ measure diversity, evenness, and spatial exposure.

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Does can we use these measures to **find** neighborhoods?

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$J(X, Y)$ also measures “neighborhoodiness.”

Does can we use these measures to **find** neighborhoods?

Yes!

- We can use $I(X, Y)$ to find neighborhoods
- We can use $I(X, Y)$ and $J(X, Y)$ to estimate how many we need to preserve key patterns.
- So, we can view $H(Y)$, $I(X, Y)$, and $J(X, Y)$ as measurements of **complexity** in spatiosocial structure.

Key Property of Mutual Information

Suppose we cluster the locations X into groups with labels C.

"Additive Organizational Decomposability" (Chain Rule)

$$I(X, Y) = I(C, Y) + I(X, Y|C)\ .$$

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We can use this to identify neighborhoods!

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Recipe for Neighborhood Identification

$$I(X, Y) = \underbrace{I(C, Y)}_{\text{maximize this}} + \overbrace{I(X, Y|C)}^{\text{minimize this}} .$$

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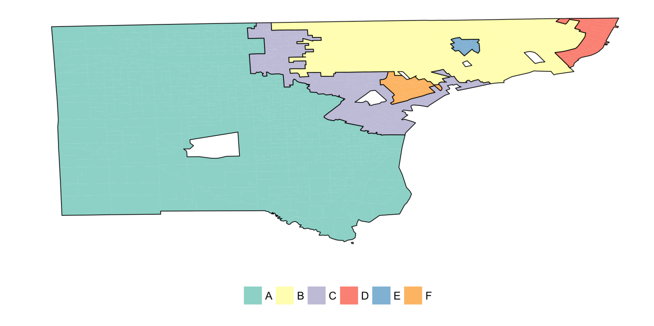
This optimization is hard to do exactly, but we can use a greedy approach based on hierarchical clustering.

Neighborhoods in Detroit



	A	B	C	D	E	F
White	36	2	7	2	1	1
Black	5	29	4			
Hispanic	2		1			2
Asian	2				1	
Other	2	1	1			

Information Measures and "Clusterability"

We would expect that cities with low spatial exposure $J(X, Y)$ are "easy" to cluster. Evaluate using "Area Under the Curve":

Together, $H(Y)$, $I(X, Y)$ and $J(X, Y)$ explain 75% of the variability in the clusterability (AUC) of cities.

Wrapping Up

Learnings

Information theory provides a natural language for the conceptualization and measurement of sociospatial variability.

Open Directions

Let’s learn about...

Thank you!

Questions?

Feedback?

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Relation of local and Fisher informations

Suppose that $p(y|x) > 0$ and that $p(y|x)$ is differentiable as a function of x for all $x, y \in \mathbb{R}^n \times \mathcal{Y}$. Define $B_r \triangleq B_r(x_0) \triangleq \{x \in \mathbb{R}^n \mid |x - x_0| \leq r\}$. Additionally, define the *local mutual information* in B_r as the mutual information between X and Y where X is restricted to B_r:

$$
\begin{aligned}
I_r(x_0) &\triangleq \mathbb{E}_X[D[p(\cdot|X)\|p(\cdot|X \in B_r)]|X \in B_r] &\quad (1)\\
&= \int_{B_r} p(x|X \in B_r) D[p(\cdot|x)\|p(\cdot|X \in B_r)] d^n x \,. &\quad (2)
\end{aligned}
$$

where $D[p\|q] \triangleq \sum_y p(y) \log \frac{p(y)}{q(y)}$ is the Kullback-Leibler divergence of q from p.

Relation of local and Fisher informations

Theorem

Under the stated conditions,

$$\lim_{r \to 0} \frac{I_r(x_0)}{r^2} = \frac{n}{2(n+2)} \operatorname{trace} J_Y(x_0) \ . \tag{3}$$

The Fisher information matrix J_Y is given by

$$J_Y(x) \triangleq \mathbb{E}_Y \left[\nabla_x S_Y(x) \nabla_x S_Y(x)^T \right] \tag{4}$$

$$S_y(x) \triangleq \log p(y|x) \ . \tag{5}$$

Information Measures Check the Boxes

Criterion	$H(Y)$	$I(X,Y)$	$J(X,Y)$
Organizational Equivalence	✓	✓	✓
Size/Density Invariance	✓	✓	✓
Additive Group Decomposability	✓	✓	✓
Additive Spatial Decomposability	NA	✓	*
Scale Interpretability	NA	✓	✓
Boundary Independence	NA	✓	✓
Exchanges	NA	✓	✓

* In the full paper, we argue that [3]'s version is not a workable criterion for spatial measures.

✓ In the theoretical definition; in practice we are reliant on the data as it is provided.

Some Comparative Clusters of Equal Information

Example clusters

Atlanta: I(C,Y) = 0.27

	A	B	C	D	E
White	32	3			
Other	3	1			
Hispanic	7	2			
Black	9	38			
Asian	5				

Boston: I(C,Y) = 0.27

	A	B	C	D	E
White	30	6	3	1	3
Other	2		1	2	1
Hispanic	4	2	3	5	9
Black	4		3	12	1
Asian	6				1

Chicago: I(C,Y) = 0.27

	A	B	C	D	E
White	37	2	1	2	
Other	2	1			
Hispanic	10	2	8	5	
Black	5	14	4		
Asian	6				

Detroit: I(C,Y) = 0.27

	A	B	C	D	E
White	44	3	2		
Other	3	1			
Hispanic	5				
Black	9	29			
Asian	2	1			