Empiricism, Conservativeness and Quasi-Truth

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A first step is taken towards articulating a constructive empiricist philosophy of mathematics, thus extending van Fraassen's account to this domain. In order to do so, I adapt Field's nominalisation programme, making it compatible with an empiricist stance. Two changes are introduced: (a) Instead of taking conservativeness as the norm of mathematics, the empiricist countenances the weaker notion of quasi-truth (as formulated by da Costa and French), from which the formal properties of conservativeness are derived. (b) Instead of quantifying over space-time regions, the empiricist only admits quantification over *occupied* regions, since this is enough for his or her needs.

1. INTRODUCTION

As is well known, Bas van Fraassen (1980, and 1989) has articulated a new empiricist interpretation of science, called constructive empiricism. According to this view, the aim of science is not truth, but something weaker, empirical adequacy. And its chief claim is that it is possible to provide an account of science without the commitment to unobservable entities and other metaphysically problematic notions (at least from an empiricist perspective), such as possible worlds and objective chance.

Although this proposal constitutes an advance in comparison with earlier empiricist interpretations (providing room, for instance, for more of the theoretical aspects of science), it is not without problems. Foremost among these is the following (see Rosen 1994). According to the semantic approach to science, advocated by van Fraassen, to present a scientific theory is to specify a class of structures, its models (van Fraassen 1980, 64). The latter are, of course, mathematical constructs. Thus, at the very core of constructive empiricism, there is a commitment to mathematical objects, which are arguably unobservable. The immediate question then is how to reconcile this feature with an empiricist ontology. To accomplish this, what is required, in my view, is a constructive empiricist philosophy of mathematics.

The aim of this paper is to develop an aspect of such a philosophy, thus extending van Fraassen's proposal to this domain. In order to achieve this, I shall provide a nominalisation strategy for mathematics (a procedure that allows one not to be committed to mathematical objects (compatible with empiricism. The main idea is to reformulate, in empiricist terms, the fictionalist account put forward by Hartry Field (1980). As we shall see, this requires an appropriate formal framework. In my view, this framework is provided by Newton da Costa's and Steven French's (1989, and 1990) partial structures approach. It has two main pillars: a broader notion of structure (partial structure) and a weaker notion of truth (quasi-truth). And in terms of this setting the "openness" and "incompleteness" of empirical information and scientific practice can be accommodated. My contention is that this framework, in addition to a restatement of Field's approach, can be fruitfully adopted as the basis for the development of a constructive empiricist philosophy of mathematics. I shall start by examining Field's view.

2. CONSERVATIVENESS AND ITS USE

In a series of works, Field (1980, and 1989) provided an ingenious strategy for the nominalisation of science. As opposed to platonist views, in order to explain the usefulness of mathematics in science, Field does not postulate the truth of mathematical theories. In his view, it is possible to explain successful applications of mathematics with no commitment to mathematical objects. Therefore, the main argument for platonism, according to Field, which relies on the (apparent) indispensability of mathematics to science, is resisted. The nominalist component of Field's account derives from the fact that no mathematical objects are assumed to exist; hence, mathematical theories are false.[1] By devising a strategy which shows how to dispense with mathematical objects in the formulation of scientific theories, Field rejects the indispensability argument, and provides strong grounds for the articulation of a nominalist stance.

Field's strategy depends on two interrelated moves. The first is to change the aim of mathematics, which is not taken to be truth, but something different. In his view, the proper norm of mathematics, which will then guide his nominalisation programme, is *conservativeness*. This concept is somewhat stronger than consistency, but it is not weaker than truth (Field 1980, 16-19, and Field 1989, 59). So Field is not countenancing a weaker aim of mathematics, but only a *different* one. According to him, a mathematical theory is conservative if it is consistent with every internally consistent theory about the physical world (where such theories do not involve any reference to, nor quantification over, mathematical objects, such as sets, functions, numbers etc. (Field 1989, 58). And it is precisely because mathematics is conservative that, despite being false, it can be useful. This usefulness can be explained, of course, with no commitment to mathematical entities. According to Field, mathematics is useful because it shorten our derivations. After all, if a mathematical theory M is conservative, then a nominalistic assertion A about the physical world (i.e. an assertion which does not refer to mathematical objects) is implied by a body N of such assertions and M only if it is implied by N alone. That is, provided we have a sufficiently rich body of nominalistic assertions, by using mathematics, we do not obtain any new nominalistic consequences. Mathematics is only a useful instrument to help us in the derivations.

The outcome of this is that conservativeness can only be employed to do the required job if we have nominalistic premises to start with (Field 1989, 129).[2] The second move of Field's strategy is then to provide such nominalistic premises in one important and typical case: Newtonian gravitational theory. Field elaborates on a work which has a respectable tradition: Hilbert's (1971) axiomatisation of geometry. What Hilbert provided was a synthetic formulation of geometry, which dispenses with metric concepts, and therefore does not include any quantification over real numbers. His axiomatisation was based on concepts such as *point*, *betweenness* and

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[1] Strictly speaking, as Field notices, any existential mathematical statement is false, and any universal mathematical statement is (vacuously) true.

[2] As Field points out, it is simply a confusion to argue against his view by claiming that if we add some bits of mathematics to *a body of mathematical claims* (not nominalistic ones), we may obtain new consequences that could not be achieved otherwise (Field 1989, 128). The restriction to *nominalistic* assertions is crucial.

congruence. Intuitively speaking, we say that a point y is *between* the points x and z if y is a point in the line-segment whose endpoints are x and z. Also intuitively, we say that the line-segment xy is *congruent* to the line-segment zw if the distance from the point x to the point y is the same as that from the point z to w. After studying the formal properties of the resulting system, Hilbert proved a representation theorem. He showed, in a stronger mathematical theory, that given a model of the axiom system for space he had put forward, there is a function d from pairs of points onto non-negative real numbers such that the following "homomorphism conditions" are met:

(i) xy is congruent to zw iff $d(x, y) = d(z,w)$, for all points x, y, z, and w.
(ii) y is between x and z iff $d(x, y) + d(y, z) = d(x, z)$, for all points x, y, and z.

As a result, if the function d is taken to represent distance, we obtain the expected results about congruence and betweenness. Thus, although we cannot talk about numbers in Hilbert's geometry (there are no such entities to quantify over), there is a metatheoretic result which associates assertions about distances with what can be said in the theory. Field calls such numerical claims *abstract counterparts* of purely geometric assertions, and they can be used to draw conclusions about purely geometrical claims in a easier way. Indeed, because of the representation theorem, conclusions about space, statable *without* real numbers, can be drawn far more easily than we could achieve by a direct proof from Hilbert's axioms. This illustrates Field's point that the usefulness of mathematics derives from shortening derivations. (For a discussion, see Field 1980, 24-29.)

Roughly speaking, what Field established was how to extend Hilbert's results about space to space-time. Similarly to Hilbert's approach, instead of formulating Newtonian laws in terms of numerical functors, Field showed how they can be recast in terms of comparative predicates. For example, instead of adopting a functor such as "the gravitational potential of x", which is taken to have a numerical value, he employed a comparative predicate such as "the difference in gravitational potential between x and y is less than that between z and w". Relying on a body of representation theorems (which plays the same role as Hilbert's representation theorem in geometry), Field established how several numerical functors can be "obtained" from comparative predicates. But in order to use those theorems, he first showed how to formulate Newtonian numerical laws (such as, Poisson's equation for the gravitational field) only in terms of comparative predicates. The result, as Field points out (1989, 130-131), is the following *extended representation theorem*. Let N be a theory formulated only in terms of comparative predicates (with no recourse to numerical functors), and S be an (arbitrary) model of N, whose domain is constituted by space-time regions. For any model S of N, there are:

(i) a 1-1 spatio-temporal co-ordinate function f (unique up to generalised Galilean transformation) mapping the space-time of S onto quadruples of real numbers;
(ii) a mass density function g (unique up to a positive multiplicative transformation) mapping the space-time of S onto an interval of non-negative reals; and
(iii) a gravitational potential function h (unique up to positive linear transformation) mapping the space-time onto an interval of reals.

Moreover, all these functions "preserve structure", in the sense that the comparative relations defined in terms of them coincide with the comparative relations used in N. Furthermore, if f, g and h are taken as the denotation of the appropriate functors, the laws of Newtonian gravitational theory in their functorial form hold.

Notice that, in quantifying over space-time regions, Field assumes a substantivalist view of space-time, according to which there are space-time regions which are not fully occupied (Field 1980, 34-36, and Field 1989, 171-180). Given this result, the nominalist is allowed to draw nominalistic conclusions from premises involving N plus a mathematical theory T. After all, due to the conservativeness of mathematics, such conclusions can be obtained independently of T. Hence, what Field provided is a nominalisation strategy, and since it reduces ontology, it seems a promising candidate to an empiricist who needs to adopt a nominalist stance *vis-à-vis* mathematics.

3. TWO PROBLEMS

Despite the apparent attractiveness, there are two difficulties with this suggestion. Firstly, the empiricist cannot accept Field's first move of putting forward conservativeness as the norm of mathematics. An important feature of the constructive empiricist's axiology is that the aim of science should be *weaker than truth*, otherwise the grounds for adopting an anti-realist proposal are lost. And in fact this is one of the reasons why empirical adequacy becomes an adequate aim (van Fraassen 1980, 12, 64). However, as we saw, conservativeness is *not* weaker than truth, it is simply a different norm (Field 1989, 59). Moreover, as we shall see, the empiricist needs an openness in his or her account of mathematics that cannot be accommodated by taking conservativeness at the outset. Thus, in extending the empiricist account to mathematics, we cannot assume conservativeness (at least as formulated by Field) as the norm of mathematical discourse.

Secondly, as noted above, a substantivalist view of space-time was assumed when Field quantified over space-time regions in order to establish his extended representation theorem. (Such regions entered in the domain of the models of the theory N.) In other words, in order for Field to be an *anti-realist* in the philosophy of mathematics, he has to adopt a *realist* attitude in the philosophy of science. And of course, despite Field's arguments for substantivalism, unoccupied space-time regions are precisely the sort of entities an empiricist cannot assume. Firstly, it is not at all clear in what sense they are concrete physical objects, nor in what respect they are observable. Moreover, they are not open to change, nor have a mass-energy content. Or, as Malament pointed out, "it is not even clear in what sense they exist *in* space and time" (1982, 532). Furthermore, if Field argues that space-time regions are causal agents (Field 1980, 114, note 23, and Field 1989, 45-48, 181-184), this simply begs the question against an empiricist view, which does not take generally causal discourse to describe features of the world, but only of our models (van Fraassen 1989, 214). This move also begs the question against a relationalist view, which countenances the existence of only *occupied* space-time regions. According to this view, electromagnetic fields, for instance, are taken to be "physical objects", since they are repositories of mass-energy. Thus, as Malament stressed, "instead of saying that space-time points enter into causal interactions and explaining this in terms of the 'electromagnetic properties' of those points, I would simply say that it is the electromagnetic field itself that enters into causal interaction" (1982, 532, note 11).

But unless the empiricist is able to circumvent these two problems, he or she cannot assume Field's nominalisation strategy, and will have to articulate a philosophy of mathematics on distinct grounds. In my view, there is still a way out, provided the empiricist is willing to countenance the formal resources provided by the partial structures approach. Before spelling out this alternative, let me first present the main features of the latter view.

4. PARTIAL STRUCTURES AND QUASI-TRUTH

The partial structures approach relies on three main notions: partial relation, partial structure and quasi-truth.[3] One of the main motivations for introducing this proposal comes from the need for supplying a formal framework in which the “openness” and “incompleteness” of scientific practice and knowledge can be accommodated in a unified way (da Costa and French 1990). This is accomplished by extending, on the one hand, the usual notion of structure, in order to model the partialness of information we have about a certain domain (introducing then the notion of a partial structure), and on the other hand, by generalising the Tarskian characterisation of the concept of truth for such “partial” contexts (advancing the corresponding concept of quasi-truth).

The first step then, in order to introduce a partial structure, is to formulate an appropriate notion of partial relation. When investigating a certain domain of knowledge (, we formulate a conceptual framework which helps us in systematising and organising the information we obtain about (. This domain is then tentatively represented by a set D of objects, and is studied by the examination of the relations holding among D’s elements. However, we often face the situation in which, given a certain relation R defined over D, we do not know whether all the objects of D (or n-tuples thereof) are related by R. This is part and parcel of the “incompleteness” of our information about (, and is formally accommodated by the concept of partial relation. The latter can be characterised as follows. Let D be a non-empty set. An n-place *partial relation* R over D is a triple (R_1,R_2,R_3(, where R_1, R_2, and R_3 are mutually disjoint sets, with R_1(R_2ER_3 = D^n, and such that: R_1 is the set of n-tuples that belong to R, R_2 is the set of n-tuples that do not belong to R, and R_3 is the set of n-tuples for which it is not defined whether they belong or not to R. (Notice that if R_3 is empty, R is a usual n-place relation which can be identified with R_1.)

However, in order to represent appropriately the information about the domain under consideration, we need of course a notion of structure. The following characterisation, spelled out in terms of partial relations and based on the standard concept of structure, is meant to supply a notion which is broad enough to accommodate the partiality usually found in scientific practice. The main work is of course done by the partial relations. We have, thus, the following definition. A *partial structure* S is an ordered pair (D, R_i($_{i(I}$, where D is a non-empty set, and

[3] This approach was first presented in Mikenberg, da Costa and Chuaqui (1986), and in da Costa (1986). Since then it has been extended and developed in several different ways; see, for instance, da Costa and French (1989) and (1990), and Bueno (1997).

$(R_i)_{i(I}$ is a family of partial relations defined over D.[4]

We have now defined two of three basic notions of the partial structures approach. In order to spell out the last, and crucial one (namely, quasi-truth (, we will need an auxiliary notion. The idea is to use, in the characterisation of quasi-truth, the resources supplied by Tarski's definition of truth. However, since the latter is only defined for full structures, we have to introduce an intermediary notion of structure to link it to the former. And this is the first role of those structures which extend a partial structure A into a full, total structure (which are called A-normal structures). Their second role is purely model-theoretic, namely to put forward an interpretation of a given language and, in terms of it, to characterise basic semantic notions.[5] The question then is: how are A-normal structures to be defined? Let A = (D, R_i($_{i(I}$ be a partial structure. We say that the structure B = (D',R'_i($_{i(I}$ is an *A-normal structure* if (i) D=D', (ii) every constant of the language in question is interpreted by the same object both in A and in B, and (iii) R'_i extends the corresponding relation R_i (in the sense that, each R'_i is defined for every n-tuples of objects of its domain).

It should be noticed that, given a partial structure A, there might be *too many* A-normal structures. We need to provide constraints to restrict the acceptable extensions of A. In order to do that, we need first to formulate a further auxiliary notion (Mikenberg, da Costa and Chuaqui 1986). A *pragmatic structure* is a partial structure to which a third component has been added: a set of accepted sentences P, which represents the accepted information about the structure's domain. (Depending on the interpretation of science which is adopted, different kinds of sentences are to be introduced in P: realists will typically include laws and theories, whereas empiricists will tend to add certain laws and observational statements about the domain in question.) A *pragmatic structure* is then a triple A = (D,R_i,P($_{i(I}$, where D is a non-empty set, $(R_i)_{i(I}$ is a family of partial relations defined over D, and P is a set of accepted sentences. The idea, as we shall see, is that P introduces constraints on the ways that a partial structure can be extended.

Our problem now is, given a *pragmatic* structure A, what are the necessary and sufficient conditions for the existence of A-normal structures? We can now spell out one of these conditions (Mikenberg, da Costa and Chuaqui 1986). Let A = (D,R_i,P($_{i(I}$ be a pragmatic structure. For each partial relation R_i, we construct a set M_i of atomic sentences and negations of atomic sentences, such that the former correspond to the n-tuples which satisfy R_i, and the latter to those n-tuples which do not satisfy R_i. Let M be ($_{i(I}M_i$. Therefore, a pragmatic

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[4] If partial relations and partial structures are partial, this is due to the 'incompleteness' of *our knowledge* about the domain under investigation (with further information about this domain, a partial relation may become total. Thus, the partialness modelled by the partial structures approach is not understood as an intrinsic, ontological 'partialness' in the world (an aspect about which an empiricist will be glad to remain agnostic. We are concerned here with an 'epistemic', not an 'ontological' partialness.

[5] For a different formulation of quasi-truth, independent of the notion of an A-normal structure and in terms of quasi-satisfaction, see Bueno and de Souza (1997).

structure A admits an A-normal structure if, and only if, the set M(P is *consistent*.
As this condition makes it clear, the notion of consistency plays a crucial role in the partial structures approach. In fact, the very concept of quasi-truth, since it depends on the existence of A-normal structures, supposes the *consistency* of M and P. This point should be stressed because, as we shall see later, it will be relevant in our discussion of modality.
Having said that, we are finally able to formulate the concept of quasi-truth. A sentence (is *quasi-true* in A according to B if (i) A = (D, R_i($_{i(I}$ is a partial structure, (ii) B = (D',R'_i($_{i(I}$ is an A-normal structure, and (iii) (is true in B (in the Tarskian sense). If (is not quasi-true in A according to B, we say that (is *quasi-false* (in A according to B). Moreover, we say that a sentence (is *quasi-true* if there is a partial structure A and a corresponding A-normal structure B such that (is true in B (according to Tarski's account). Otherwise, (is *quasi-false*.
The idea, intuitively speaking, is that a quasi-true sentence (does not necessarily describe, in an appropriate way, the whole domain to which it refers, but only an aspect of it (the one modelled by the relevant partial structure A. After all, there are several different ways in which A can be extended to a full structure, and in some of these extensions (may not be true. As a result, the notion of quasi-truth is strictly weaker than truth: although every true sentence is (trivially) quasi-true, a quasi-true sentence is not necessarily true (since it may be false in certain extensions of A). This is an important feature of this notion.
It is time now to return to the discussion of conservativeness. As we shall see, it can be addressed in a new way if we explore the formal resources provided by the above framework.

5. WEAK CONSERVATIVENESS, EMPIRICISM AND QUASI-TRUTH

In order to adapt Field's nominalisation strategy to an empiricist setting, I will introduce two changes in the former proposal, corresponding to the two problems we saw above (in section 3). With regard to the first problem, because conservativeness is too strong an aim for mathematics (according to empiricism), I shall countenance a weaker aim, namely quasi-truth. As we saw, quasi-truth is *weaker* than truth, and to this extent it is adequate for the empiricist. But its adequacy derives also from a second reason: in terms of quasi-truth, we can put forward a correspondingly weaker account of conservativeness. A mathematical theory M is *weakly conservative* if it is quasi-true in a partial structure with regard to a consistent body of nominalistic claims. (These claims are viewed as members of the set P of accepted sentences in a given pragmatic structure.) It follows that M is weakly conservative iff M is consistent with *some* internally consistent body of claims about the physical world. So, whereas Field presents conservativeness as consistency with *all* consistent physical theories (invoking thus a *universal* quantification; see Field 1989, 58), the empiricist proposes something weaker: consistency with *some* such theories (depending only on an *existential* quantifier).
As we saw, the crucial feature of conservativeness is the following property C: M is conservative iff given a nominalistic assertion A and a body N of such assertions, if N+M implies A, so does N alone (see Field 1989, 58). It is clear that C is equivalent to Field's formulation of conservativeness presented in the previous paragraph. But although C is *not* equivalent to weak conservativeness in general, they are equivalent in those cases in which N and M are consistent, and this is all the empiricist needs in order to make use of C. But there is also a second strategy for the empiricist to achieve C. We say that a theory T is *necessarily quasi-true* if *for all*

A-normal structures B, T is true in B. Of course, if T is necessarily quasi-true, T will be consistent with every internally consistent body of nominalistic assertions. (Again, these assertions are elements of the relevant sets P of accepted sentences, which are components of the A-normal structures; the latter, in turn, are models of T.) So, if the empiricist is dealing with necessarily quasi-true mathematical theories, the property C is immediately obtained. Therefore, in either case, the use of conservativeness is open for her stance.

One of the main motivations for introducing a weaker norm of mathematics is that it allows for the introduction of more open-ended relationships between mathematical structures. As we saw, a crucial feature of Field's account is to establish representation theorems between nominalist and platonist formulations of given scientific theories. In particular, those theorems establish appropriate "homomorphism conditions" (i.e., structure preserving mappings) between the relevant structures. The empiricist, however, having weakened the norm of mathematics, requires correspondingly weaker mappings: only *partial homomorphisms* will do. The idea is that, in order to accommodate appropriate relationships between mathematical structures, the empiricist only needs to "preserve partially" the structure in question. More formally (see French, Ladyman, and Bueno 1998), let S = $(D,R_i(_{i(I}$ and S' = $(D',R'_i(_{i(I}$ be partial structures. Thus each R_i is of the form $(R_1,R_2,R_3($, and each R'_i, of the form $(R'_1,R'_2,R'_3($. We say that f: $D(D'$ is a *partial homomorphism* from S to S' if for every x and every y in D,

(i) R_1xy ($R'_1f(x)f(y)$,

(ii) R_2xy ($R'_2f(x)f(y)$.

The importance of this point is highlighted when we consider how the empiricist can circumvent the second problem discussed above. As we saw, in order to run his nominalisation programme, Field quantified over space-time regions. Roughly speaking, instead of quantifying over real numbers, Field took space-time regions as concrete surrogates for these abstract objects. But if this is straightforward in a realist view of science, the same is not the case in an empiricist. Now, given that all that the empiricist countenances is quasi-true theories (and not true ones), the "completeness" of the account of mathematics has been relinquished: the relevant structures will be taken to characterise the domain under consideration at best partially, always leaving certain open-ended features. Now, this means that, when establishing representation theorems, the empiricist will only demand *partial homomorphisms* between the relevant structures, allowing the possibility that one of the structures may have "more structure" than the other.

The outcome is that, in order to establish the extended representation theorem for the Newtonian gravitational theory, the empiricist will only require a partial homomorphism between N (the nominalistic reformulation of physical laws in terms of comparative predicates) and the real numbers structure. Hence, this allows for the possibility that the latter structure be richer than the former. Thus, instead of quantifying over space-time regions, the empiricist can quantify only over *occupied* (space-time) regions. The fact that there are "fewer" occupied regions than non-occupied ones, instead of being a problem, is in fact the counterpart of the richness of the real number structure *vis-à-vis* the relationalist space-time. Notice that the empiricist has no qualms with *occupied* regions, since none of the objections presented in section 3 against *un*occupied ones can be applied to those which are *fully* occupied. In any case, the latter mesh nicely with a relationalist account (according to which there are *no* non-occupied regions). And provided the empiricist introduces quasi-truth as the norm of mathematics, this sort of “structural openness” can be readily accommodated.

In this way, with the introduction of the partial structures framework, both problems can be solved, and a first step towards formulating a constructive empiricist account of mathematics has been taken. As a result, the difficulty that Rosen put forward to constructive empiricism can now be circumvented: given the nominalist features of the proposal sketched here,[6] the empiricist will be entitled to use mathematical structures without incurring unacceptable ontological costs.

Finally, together with constructive empiricism, and once it is properly developed, this empiricist philosophy of mathematics supplies a unified account of science; since together they encompass both science’s empirical and non-empirical aspects. This offers an additional argument not only for empiricism (since it extends the empiricist stance to a new domain (, but also for the empiricist account of mathematics (by showing how such an account can be integrated into a larger scheme of interpretation of science.

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[6] If it is complained that the concepts of quasi-truth and partial structures are formulated in set theory, and therefore the present account cannot be nominalist, I would claim that these concepts can be defined in a second-order language. Thus, it becomes clear that they are nominalistically acceptable. However, there is no space to present this reformulation here.

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