

**ARE GRW TAILS AS BAD AS THEY SAY?**

**Alberto Cordero  
Graduate Center & Queens College  
City University of New York**

## ARE GRW TAILS AS BAD AS THEY SAY?

**SUMMARY:** GRW models of the physical world are severely criticized in the literature for involving wave function 'tails', the allegation being that the latter create fatal interpretive problems and even compromise standard arithmetic. I find such objections both unfair and misguided. But not all is well with the GRW approach. The complaint I articulate in this paper does not have to do with tails as such but with the specific way in which past physical structures linger forever in the total GRW wave function. I argue that this feature, which is an artifact of a particular and ultimately optional genre of collapse mechanisms, yields a total picture that is too close to the "Many Worlds" type to deserve clear methodological acceptability, particularly in light of the effective empirical equivalence between the two objectivist approaches.

Presently, the most influential model of wave function reduction in foundational quantum mechanics is an appealing proposal originally introduced by G.C. Ghirardi, A. Rimini, and T. Weber (GRW)<sup>1</sup>. Over the last decade, Ghirardi and a team of collaborators have transformed this model into a detailed theory of spontaneous collapse of the wave function, making significant advances toward both a proper relativistic version and a mass-density field formulation of the theory<sup>2</sup>. Although the resulting approach owes much to the efforts of many additional scholars, for simplicity's sake I'll continue to refer to it as "GRW".

GRW postulates a stochastic mechanism of spontaneous state reduction that operates in the position basis. In a single microscopic particle, the stochastic process is extremely weak, which makes its dynamical evolution practically indistinguishable from the one dictated by the standard linear theory (QT<sub>0</sub>). The difference is that, every 10<sup>16</sup> seconds (10<sup>9</sup> years), on average, the particle's state is overtaken by a sudden contraction in coordinate representation. Systems with more degrees of freedom and elementary particles develop these kicks proportionately. An ordinary laboratory pointer, for example, containing about 10<sup>20</sup> atoms, undergoes a kick every 10<sup>-4</sup> seconds on average. So, what was previously an odd and inexplicable difference of behavior between standard microscopic and macroscopic systems follows naturally from GRW dynamics. The new theory does not change the time-independent part of QT, which means that the possibility structure furnished by the standard

time-independent algorithm is preserved.

## THE CRITICAL RECEPTION OF THE THEORY

GRW is encouragingly agreeable from an objectivist point of view. Its fundamental dynamics is free of inauspicious references to consciousness or measurement; its portrayal of the ordinary macroscopic world tells us about an empirically adequate stochastic succession of sharply concentrated wave functions in coordinate space. Furthermore, the intended ontology of the theory is clearly centered on the wave function as a physical field. The proposal does not tally well with ordinary intuition, and this has been regarded by some critics as a fault<sup>3</sup>. But surely this cannot compromise the worth of a contemporary physical theory<sup>4</sup>. Unfortunately, however, there are other difficulties.

One obvious concern is that it would be imprudent to accept GRW as a correct description, given the theory's present lack of experimental force. This is, of course, a fair methodological caveat; the same point applies, however, to all current attempts to improve the standard theory. A second line of objection stresses the weirdness of the GRW world, where a cat that ends up dead in a Schrödinger-cat experiment may conceivably come back to life, because the amplitude  $\langle \text{Cat alive} | \Psi_{\text{Total}} \rangle$  will remain nonzero forever —whatever the number of intervening GRW kicks. But the probability of a cat's ever gaining resurrection through this "loophole" is demonstrably remote —well below the chance of having a cup of hot coffee spoiled by Maxwell's demon. This response is admittedly "for all practical purposes" only (FAPP). But it seems good FAPP, because the instrumentalism involved affects only theories known to be approximately correct over restricted domains (such theories as classical mechanics and 'common sense metaphysics'). GRW is not compromised by implications like the noted ones; no one is trying to save a realistically interpreted theory by pretending that it is only approximately correct. On the contrary, what GRW submits to instrumentalist interpretation is some standard beliefs about cats and such, not quantum theory.

A more serious concern focusses on the "external" coherence of GRW, specifically on the generalizability of the proposal into a proper Lorentz-invariant theory, in spite of Ghirardi et al.'s valuable advances in that direction<sup>5</sup>. As independent studies reveal, the tension between special relativity and stochastic collapse is not as simple and redeemable a feature of GRW as was once imagined. The issues involved here are extraordinarily delicate, since the question whether the

tensions encountered with special relativity are terminal depends on which level of Lorentz invariance one considers appropriate to associate with relativity theory. I shall not deal with this difficulty here<sup>6</sup>.

## TAILS AND INTERPRETIVE COHERENCE

Seemingly more damaging are objections of a fourth type, focused on the internal conceptual coherence of GRW. Their center of attention is the infinite tails of GRW's wave functions in coordinate representation. Such tails have, for some time now, been feared to be flatly incompatible with a proper concept of 'having a position'. The charges here have matured over the years into four seemingly major objections to the GRW theory<sup>7</sup>: (1) That GRW's relaxation of the standard orthonormal rule (SOR) leads to significant problems in the interpretation of QM. (2) That, accordingly, GRW cannot account for the determinate locations of macroscopic objects. (3) That the only imaginable measure of "being in the box" (BIB) is  $BIB_{SOR}$ . (4) That the previous difficulties compromise standard arithmetic. Since these charges have some initial credibility, it is good to consider them in some detail.

Clearly, what the GRW dynamics delivers is not the "expected" reduction to a proper mixture but the very same pre-measurement correlation, with one of the initial components now vastly dominating over the others. That said, however, how damaging are really the said charges? Their key note is that, by relaxing SOR, GRW tails create more problems than solutions. Consider the mutually exclusive states of affairs of a marble placed inside a box and outside it respectively, where superpositions like the following provide the interesting case<sup>8</sup>:

$$|\Psi\rangle = (1/2)^{1/2} \{ |in\rangle + |out\rangle \} \quad [1]$$

*In terms of SOR*, a marble in such a state is clearly neither in the box nor outside the box, nor 'not both inside and outside the box, nor neither inside nor outside the box'. Now, in GRW, the stochastic part of the dynamics drives states like [1] very quickly into a "reduced" state in configuration space which, in simplified form, can be sketched as being of either of the two following forms:

$$|\Psi\rangle = a|in\rangle + b|out\rangle \quad [2]$$

or

$$|\Psi\rangle = a|out\rangle + b|in\rangle \quad [3]$$

with  $|a|^2 \gg |b|^2 > 0$ . Unquestionably, neither of these reduced states is an eigenstate for the location

of the marble. Still, in GRW [2] and [3] are meant to represent mutually exclusive states of affairs, even though the corresponding vectors are not orthogonal. Let us analyze the noted objections in terms of this case.

1. If it is accepted that a marble is in the box when its state is merely close to the proper eigenstate, then one can construct states like [4] below.

$$|\Psi\rangle_{\text{all}} = \prod_{i=1}^n (a|in\rangle_i + b|out\rangle_i) \quad [4]$$

It is states like this which, critics maintain, defy interpretation. They note that, for a large  $n$ , [4] cannot correspond to a state of all  $n$  marbles being in the box, because the eigenstate of all  $n$  marbles being in the box is:

$$|in\rangle_{\text{all}} = \prod_{i=1}^n a|in\rangle_i \quad [5]$$

And the proximity of  $|\Psi\rangle_{\text{all}}$  to the eigenstate in [5] is given by:

$$|\langle\Psi|in\rangle_{\text{all}}|^2 = |a|^{2n} \quad [6]$$

But  $|a|^{2n}$  is slightly less than 1, and so, as  $n$  tends to infinity  $|\Psi\rangle_{\text{all}}$  approaches orthogonality to the eigenstate of all  $n$  marbles being in the box. One cannot claim, therefore, that  $|\Psi\rangle_{\text{all}}$  is in a state in which all  $n$  marbles are in the box on the basis of the proximity of  $|\Psi\rangle_{\text{all}}$  to  $|in\rangle_{\text{all}}$ . Furthermore, the probability that all  $n$  marbles will be found in the box approaches zero for large  $n$ . So, for  $n$  large, state [5] cannot be one in which all  $n$  marbles are in the box, it cannot be a state in which it is not the case that all  $n$  marbles are in the box, and it cannot be a state in which there is no matter of fact about whether or not all  $n$  marbles are in the box. But, critics state, these three options exhaust the possibilities for interpreting [5]. Hence, it appears, relaxing SOR entails that there are states which cannot be interpreted.

But, can this be right? I suggest that the above charge equivocates or begs the question. On GRW terms, state [5] is a state in which all the marbles are in the box. In GRW terms, the predicate "being in the box" (BIB) is simply not the same as the 'corresponding' SOR-predicate.  $\text{BIB}_{\text{GRW}}$  is a fuzzy predicate whose translation into the "corresponding" two-valued predicate in ordinary language can only be approximate (and then, via the stipulation of some adequate measure of negligibility). Come to think about it,  $\text{BIB}_{\text{GRW}}$  departs from  $\text{BIB}_{\text{SOR}}$  even for  $n=1$ . Given a particle in the box in the GRW sense (Eq. 2): (i) the probability of finding that particle outside the box is not zero; accordingly, (ii) for a series of  $N$  position measurements on a particle in state [2], the probability that the particle

will be found outside the box approaches 1 as  $N$  becomes sufficiently large. Simply put,  $BIB_{SOR} \neq BIB_{GRW}$ . Also, the state  $|BIB_{GRW}\rangle$  is not orthogonal to the state  $|\text{not-}BIB_{GRW}\rangle$ , but relaxing SOR does not entail that states like [5] defy interpretation. (At most, they lack SOR-interpretation, but that is nothing from a GRW perspective). Again, it is just unfair to claim that if a marble is in the box only when its state is precisely the eigenstate  $|in\rangle$ , then GRW cannot explain the fact that marbles have determinate locations. As most critics appreciate, the world of GRW does not contain particles but only a peculiar matter-field which, in configuration space, develops objective blob-like structures through well-defined reduction processes. Accordingly, the way  $BIB_{GRW}$  gets to be two-valued is simply by construction, specifically by stipulation of a value of  $|b|$  that is small enough to secure close proximity between  $BIB_{GRW}$  and  $BIB_{SOR}$  over a large, but ultimately limited region of the GRW world.

2. The standard objection about the determinate locations of macroscopic objects is sometimes furthered by noting that, since GRW dynamics entails that there is no determinate number of marbles in the box, the theory fails to account for the determinate locations of macroscopic objects (made of very many elementary particles). Surely, however, the determinate locations that need to be accounted for are only the ones experienced at the phenomenal level, not any metaphysical locations—let alone strictly sharp classical locations. If so, GRW does the required job here. According to GRW, not only  $BIB \neq BIB_{SOR}$ , but the chances of ever finding a GRW-localized marble outside the box are experientially negligible to an astonishing degree. If so, to ask for more would seem to merely beg the question against GRW. Failure to entail that a state like [5] corresponds to no determinate number of marbles in the box does not, automatically, constitute a failure of solving the measurement problem, for no conceptual incoherence is involved, and no actual appearances being really compromised.

3. The third noted charge against GRW is that  $BIB_{SOR}$  is the only imaginable measure of 'being in the box'. I find this, again, biased. Not only is  $BIB_{SOR}$  not the only imaginable measure of 'being in the box'; in fact, it is not even the measure actually used in SQT. According to the latter,  $BIB_{SOR}$  is not even a realizable measure in nature, as no state has or can physically have finite support in configuration space. Recall how SQT actually describes a marble whose CM is, say, in the middle of a square box of side  $2a$ : the radial part of its wave function is proportional to an exponential function of the form  $\exp(-kr)$ . So, what does the state of a system of  $n$  marbles variously located in the said

box look like according to SQT? As with [5], by choosing a large enough value of  $n$ , we can make the above measure as close to zero as we please. The point is that, thanks to such quantum features as tunneling and the dynamical impossibility of having states with strict finite support, quantum marbles can never be placed in a state of "classical" confinement, whatever popular "intuition" recommend.

4. The fourth mentioned objection is that arithmetic is compromised by problem (1). There seem to be several levels of difficulties with this line of argumentation. For starters, how unproblematic is the contention that counting stones, marbles and such is "the foundation of arithmetic"? Nevertheless, for the sake of argument, let us so assume; and let us further assume that counting paradigmatically applies to medium-size physical objects like marbles and apples. I suggest that both of these assumptions are compatible with the existence in nature of a numerical upper bound for standard counting. On theories like GRW (and SQT), the physical counting gives way to a softer rule for extremely large values of  $n$ :  $1, 2, \dots, n$ , many<sub>1</sub>, many<sub>2</sub>, ... But, then, if what is wanted is to have actual counting as the foundation of arithmetic, the objection under consideration would seem unable to get off the ground.

The lesson that I think we should draw from the GRW description of systems like the  $n$  independent marbles considered above is completely different. As I see it, its deepest message (along with that of recent studies of the foundations of quantum mechanics) has to do with *how unexpectedly odd the physical world turns out to be*—so odd that conservative circumspection seems to neither help nor work when it comes to choosing between competing theories in the quantum domain.

The charges reviewed so far simply bring out the odd character of some aspects of the proposed theory and methodology. Arguably, however, GRW confronts deeper difficulties.

## **OF TAILS AND AUTONOMOUS BRANCHES**

In my view, the disturbing aspect of the GRW theory is neither its failure to "truly" recover the classical world at the ordinary macroscopic level, nor its failure to preserve a (really mythical) value-eigenvalue link of SQT, nor its use of merely approximate talk at the level of 'common-sense metaphysics'. My objection is that GRW *unwittingly carries in its bosom too many of the conceptual peculiarities of the "many decohering worlds" approach*—an approach that many find unacceptable,

including Ghirardi et al. This objection, I will suggest, compromises GRW from a methodological point of view. I will articulate my case in three related steps.

It is not probability tails per se that I find problematic but the methodological burden that these put on the ontic-realist makeup of GRW. The difficulty I see has to do with physical fields which export *internal structure* (given by the sub-state  $\phi$ ) across space and time. In the GRW wave function, this takes the basic form:

$$\frac{\alpha^{3/4}}{\pi} e^{-\frac{\alpha(x-x_o)^2}{2}} \Psi(x_{cm}) \phi(\xi) \quad (7)$$

Local environments in conjunction with the standard linear dynamics push such a wave function into interesting parallel histories *a la Many Decohering Worlds* approach. Here it will not help to insist that similar tails are also a feature of the standard theory. That reply would merely highlight the inadequacy of the GRW approach.

Although defenders of GRW overlook this feature of the proposal, the GRW dynamics makes wave function tails the loci of much physical activity. Take, for example, a Schroedinger's cat situation in which the cat's destiny is determined by the resolution of a superposition initially carried by an incoming particle P, so that, at time  $t_0$ :

$$|P(t_0)\rangle = 1/\sqrt{2}|P_1\rangle + 1/\sqrt{2}|P_2\rangle,$$

where  $P_1$  corresponds to a trajectory (say, upwards) that triggers the mechanism responsible for killing the cat, and  $P_2$  corresponds to a trajectory that misses that mechanism. Let us represent the states of the mechanism (M), cat (C), environment (E), and observing scientist (O) in the usual ways. After letting the GRW dynamics operate on the system, we get the customary pre-GRW von Neumann correlation, only now severely damped in favor of one of the two partial states:

$$\begin{aligned} & \text{GRW}(t) |P(t_0)\rangle |M(t_0)\rangle |C(t_0)\rangle |E(t_0)\rangle |O(t_0)\rangle \approx \\ & |P_1(t)\rangle |M_1(t)\rangle |C_1(t)\rangle |E_1(t)\rangle |O_1(t)\rangle + \epsilon |P_2(t)\rangle |M_2(t)\rangle |C_2(t)\rangle |E_2(t)\rangle |O_2(t)\rangle \end{aligned}$$

where  $\epsilon$  is extremely close to zero. The chief point here is that, according to the dynamics of GRW, the subsequent development of the above " $\epsilon$ -branch" will be no less physical or real than that of the



"dominant" branch, because, on this theory, the linear evolution does not depend on amplitude. This is so because the GRW dynamics does not suspend the  $QT_0$  dynamics but merely adds to it. The continuing dynamical evolution at the two initial branches must, therefore, be taken seriously by supporters of the theory's ontology.

But, can we properly talk about branches here? It seems inescapable that, being subject to the standard linear dynamics, local interactions with the environment quickly lead to the formation of many decohering wave fronts. If so, *all the lessons from decoherence studies apply here*. Specifically, as with systems involving a ordinary macroscopic object, here the total wave function decomposes into a dominant wave function field plus myriads of mini fields (not just the immediate post-GRW  $\epsilon$ -branch), each largely driven by its own internal dynamics (including its corresponding local environment). So, the physical history followed each of the low-amplitude branches quickly develops into an autonomous line (at least to an astonishingly high degree of approximation), becoming effectively independent of the history of either the 'dominant' GRW branch or any other branch.

Now, to the extent that GRW supporters take the wave function seriously, they cannot dismiss the dynamics of tails on the grounds that their amplitudes are small. For it is the theory itself that tells us that wave function amplitude has nothing to do with the quality and variety of dynamical development: the vitality of consistent and varied developments in the tails is not different from what is found in the main (dominant) history generated by the GRW dynamics. From a practical point of view, dynamical developments on  $\epsilon$ -branches are supremely unlikely to change the dominant component of the wave function of any ordinary macroscopic object in phase space. Nevertheless, from a conceptual point of view, the evolutions that unfold in the  $\epsilon$ -branches does seem to reveal a Many-Worlds ontology at the heart of theories like both the standard theory and GRW. Unlike Bohm's theory, GRW admits only one kind of physical stuff: the wave function. The structures carried by the dominant front are, according to the theory, as real as those carried by the tails. This point is sadly neglected by the literature, but I think it is an important one.

Notice that the difficulty here is *not* answered by the latest mass density formulation of GRW. From the original model of instantaneous discontinuous localization in coordinate representation, Ghirardi et al. have moved to more elegant and powerful proposals in which a continuous diffusion process in Hilbert space replaces the discontinuous 'jumps' of the original theory<sup>9</sup>. In a recent version

of the theory, the description of the world proceeds in terms of the values taken by a mass density function defined in ordinary configuration space as<sup>10</sup>:

$$\mathbf{M} = \langle \Psi(t) | \mathbf{M}(\mathbf{r}) | \Psi(t) \rangle,$$

where  $|\Psi(t)\rangle$  is the normalized state vector describing an individual system  $S$  at time  $t$  (Hilbert space  $H^{(S)}$ );  $\mathbf{M}(\mathbf{r})$  is defined as:

$$\mathbf{M}(\mathbf{r}) = \sum_k m_k N^{(k)}(\mathbf{r}),$$

$m_k$  being the mass of particles of type  $k$ , and  $N$ , their respective number operator:

$$N^{(k)}(\mathbf{r}) = \left(\frac{\alpha}{2\pi}\right)^{3/2} \sum_s \int d\mathbf{q} e^{-\frac{\alpha}{2}(\mathbf{q}-\mathbf{r})^2} a_k^+(\mathbf{q},s) a_k(\mathbf{q},s). \quad (8)$$

Here  $a^+$  and  $a$  are the creation and annihilation operators of a particle of type  $k$  at point  $\mathbf{q}$  with spin component  $s$ . Notice from the exponential factor that wave function tails are also a feature of the new formulation.

Notice, however, that none of this helps in a Schroedinger's cat situation, for two reasons. (a) As the above expressions make plain, the mass density function respects the internal architecture of ordinary macroscopic objects, and (b) the center of mass density function continues to be 'tailed' —as one might have expected.

My point is that, if the GRW wave function is interpreted realistically, then it becomes extremely difficult to understand why the histories that develop in the tails should not be considered as real as either the wave function itself or the history associated with its peaks in coordinate representation. This leads, then, to my main complaints, which are not about ontological weirdness.

## BAD SMELL

The remaining two complaints on my list are the ones that bother me the most. One is the way in which conceptual proximity to MDW tends to be covered up by auxiliary criteria within GRW. The other is the fact that, after all, GRW is put forward as an alternative to MDW within the realist camp.

In the GRW theory the 'inconvenient' ontological surplus posed by the tails is hidden from

view by what, at first sight, looks like a harmless revision of the concept of 'being at position X', by relating 'being at X' with 'having a wave function sharply concentrated around X'. By the new criterion, it becomes either false or meaningless to say that, after the experiment, the cat is in any state other than  $|C_1\rangle$ . This seems satisfactory for one level of attribution. But, can one use the new criterion to drop  $\epsilon$ -branches from the world-picture? Clearly not, for GRW itself provides a detailed description of the world of these 'insignificant branches', and they branches are extremely rich in significant developing structures, including human-like ones. In our cat experiment, for example, the  $\epsilon$ -branch (dead cat outcome) leads to all sorts of histories, including politically correct ones (with O in jail and fired from his job for messing the cat). Can those stories be considered "real"? I suggest the interpretative question here is of the kind encountered in the traditional "other minds" problem.

I suggest that the proposed revision of the notion of 'having a property', when arbitrarily extended, screens off a good deal of the physical reality that the theory opens up for view. The resulting situation has been linked by Ghirardi et al. to the contention that the mind selects for wave function amplitudes of order one<sup>11</sup>. But there are difficulties here. If the contention is that the mind somehow filters out information rooted in any tails, then this extra assumption will give the GRW project an unexpected (but seemingly harmless) idealist twist. On the other hand, if the contention is that a natural disposition of the mind entitles us to deny reality to  $\epsilon$  branches, then the assumption is no better warranted than other populist challenges to sophisticated theories (say, relativity). Whatever property ascriptions or ontological selections our minds may be biologically programmed to make, scientific inquiry now provides us with reasons for transcending such pre-scientific selections. Only if an instrumentalist stance were taken toward the wave function would one be entitled to ignore the rich life and dynamical evolution of tails. That, however, would make the GRW theory just another proposal about observables rather than the theory about beables (in Bell's sense) that its authors set out to produce<sup>12</sup>. In that case would be hard to see what scientific or philosophic purpose is served by the GRW program.

My second objection, then, is that, at a 'convenient point' in the theory's exploration of the world, the interpretation of the wave function shifts form a realist proposal (about 'beables') to an instrumentalist one (about observables). This move seems, however, counterproductive. Given the extraordinary empirical adequacy and comparative simplicity of the standard theory, it is difficult to

see what interest may served by developing, let alone using, an instrumentalist version of the GRW theory.

I think a better stance for the GRW approach would be to fully embrace the physical world that its own framework reveals, without attempting to hide or disregard any aspect of the ontology generated by the theory's core. One obvious way of biting the 'ε bullet' is to stick to a physicalist interpretation of the wave function and acknowledge that the GRW proposal is really a variant of the MDW approach—one in which, most of the time, there is a well-defined 'dominant' world-branch (a branch which is adequately stable for ordinary macroscopic systems). On this variant, the history of the universe is extremely rich in ε branches, which are all but guaranteed to remain 'ε'. The result would be an odd but not necessarily incoherent world-picture. The picture would, however, raise the question of whether the GRW approach has more plausibility than the approaches it seeks to replace.

Now to my third critical observation. One clear advantage of GRW is that it yields states with almost perfectly stable dominant branches in the position basis. However, the highlighted connections with MDW raise doubts about the preferability of GRW to either SQT or MDW. This is relevant because, if the above considerations are correct, then the comparative virtues of the GRW program will have to be reconsidered. The ontological circumspection that originally made the latter preferable to such approaches as MDW and the causal theory would be gone. If so, then the difficulty noted earlier about the tension between empirically adequate notions of wave function collapse and special relativity would become more pressing, especially to the extent that MDW relaxes that tension.

And so, if the evaluation presented in this paper is correct, then why not simply return to SQT and the old projection postulate, or accept the MDW approach? One might do just that. Or, in a more daring mood, one might try to further Ghirardi et al.'s courageous quest for an objectivist theory of wave function collapse by exploring other, perhaps less 'promiscuous' mechanisms.

## NOTES

1. Ghirardi, Rimini & Weber (1986).

2. See especially Ghirardi, Rimini & Weber (1988), and Ghirardi (1991). Ghirardi, Grassi & Pearle (1990), Ghirardi, Grassi & Pearle (1991), Ghirardi & Pearle (1992), Ghirardi, Grassi, Butterfield & Fleming (1992). See also Gisin (1989). For the mass-density formulation of the approach, see Ghirardi, Grassi & Benatti (1995).
3. See, for example, Albert & Vaidman (1989) and Shimony (1991).
4. Aicardi et al. (1991).
5. See, for example, Ghirardi, Grassi, Butterfield & Fleming (1992), Ghirardi & Grassi (1994).
6. Papers of lasting value on this topic are found in Cushing & McMullin (1989). For major recent analyses of the tension between quantum mechanics and relativity, see, in particular, Fleming & Bennett (1989), and Maudlin (1995).
7. This way of thinking has found a vitally compelling voice in Peter Lewis; see Lewis (1997). Precursor charges are found in, for example, Albert & Loewer (1990).
8. This case is taken from Lewis (1997).
9. Ghirardi, Grassi & Rimini (1990), Ghirardi, Pearle & Rimini (1990), Ghirardi, Grassi & Pearle (1990), Ghirardi, Grassi & Pearle (1991).
10. Ghirardi, Grassi & Benatti (1995).
11. Aicardi et al. (1991), Ghirardi, Grassi & Benatti (1995).
12. Ghirardi, Rimini & Weber (1986), Bell (1987), Ghirardi (1991).

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