Bell's Theorem, Non-Separability and Space-Time Individuation in Quantum Mechanics

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Abstract. We first examine Howard's analysis of the Bell factorizability condition in terms of 'separability' and 'locality' and then consider his claims that the violations of Bell's inequality by the statistical predictions of quantum mechanics should be interpreted in terms of 'non-separability' rather than 'non-locality' and that 'non-separability' implies the failure of space-time as a principle of individuation for quantum-mechanical systems. And I find his arguments for both claims to be lacking.

1. Introduction

Don Howard has claimed that Bell's theorem and its (meta-)physical implications can be fruitfully understood by way of the 'Separation Principle' found in Einstein's own incompleteness argument (Howard 1985; cf. Einstein 1948). Howard gives an interpretive analysis of the 'Separation Principle' in terms of two logically independent, conceptually distinct principles he calls 'separability' and 'locality' and shows that the Bell factorizability condition is itself a consequence of these two principles (Howard 1989 and 1992). On this basis he argues that the violations of the Bell inequalities by the statistical predictions of quantum mechanics and the confirmation of those predictions in experiment can (and should) be interpreted as exhibiting 'non-separability' rather than 'non-locality'. And such 'non-separability', he claims, implies the failure of space-time itself as a principle of individuation for quantum-mechanical systems generally. In the following I will develop and criticize his arguments for each of these claims and find his conclusions less than compelling.

2. Analysis of the Bell Factorizability Condition

We begin with Howard's notions of 'separability' and 'locality'. Howard states the 'separability principle' "asserts that any two spatially separated systems possess their own separate real states", while the 'locality principle' "asserts that all physical effects are propagated with finite subluminal velocities, so that no effects can be communicated between systems separated by a space-like interval" (Howard 1985, 173). Regarding the 'separability principle' in particular, which will be of primary importance to our discussion, Howard states that it

> asserts that the contents of any two regions of space-time separated by a nonvanishing spatio-temporal interval constitute separable physical systems, in the sense that [(i)] each possesses its own, distinct physical state, and [(ii)] the joint state is wholly determined by these separate states. In other words, the separability principle asserts that the presence of a non-vanishing spatio-temporal interval is a *sufficient condition* for the individuation of physical systems and their associated states.... (Howard 1989, 225–6)

For this principle to be applicable, some interpretation of 'state' is required. Howard defines the 'state' $p_\lambda(x\,|m)$ λ of a physical system S primarily in terms of a (marginal) conditional probability measure for possible outcomes x of measurements on S given measurement context m, $p_\lambda(x\,|m)$ (Howard 1989, 226, n. 2; cf. 1992, 310). According to this definition, the quantum state of a physical system would represent a definite property of the system if and only if $p_\lambda(x\,|m) = 1$, that is, if and only if λ is an eigenstate and x is the corresponding eigenvalue of the observable X, which is precisely the orthodox ‘Eigenstate-Eigenvalue Rule’.

Now, beginning with Jarrett's analysis of the Bell factorizability condition into two distinct independence conditions, outcome independence and parameter independence, Howard aims to show that the factorizability condition is also equivalent to the conjunction of what he calls the 'separability condition', which he claims follows from his separability principle, and the 'locality condition', which he claims follows from his locality principle (Howard 1989, 1992). Let $A\,(B)$ be the respective outcome for an EPR-Bell type correlation experiment on $S_1\,(S_2)$, $a\,(b)$ be the respective apparatus setting, and λ the joint state of the composite system. Then the Jarrett (1984) analysis of the factorizability condition,

$$p_\lambda^{S_{12}}(A,B|\,a,b) = p_\lambda^{S_1}(A|a)\,p_\lambda^{S_2}(B|b)\,,$$

(1)

yields the sets of following sets of conditions — outcome and parameter independence, respectively:

$$p_{\lambda}^{S_1}(A|a,b,B) = p_{\lambda}^{S_1}(A|a,b)$$
$$p_{\lambda}^{S_2}(B|a,b,A) = p_{\lambda}^{S_2}(B|a,b)$$

(2)

and

$$p_{\lambda}^{S_1}(A|a,b) = p_{\lambda}^{S_1}(A|a)$$
$$p_{\lambda}^{S_2}(B|a,b) = p_{\lambda}^{S_2}(B|b).$$

(3)

Howard's analysis of the factorizability condition (1) begins by making the following identifications:

$$p_{\lambda_1}^{S_1}(A|a,b) = p_{\lambda_{12}}^{S_1}(A|a,b)$$
$$p_{\lambda_2}^{S_2}(B|a,b) = p_{\lambda_{12}}^{S_2}(B|a,b).$$

(4)

Now, *given* these identifications (4), Howard's 'locality condition' is just the statement of parameter independence (3) with λ_{12}, λ_1 and λ_2 replacing λ where appropriate —

$$p_{\lambda_1}^{S_1}(A|a,b) = p_{\lambda_1}^{S_1}(A|a)$$
$$p_{\lambda_2}^{S_2}(B|a,b) = p_{\lambda_2}^{S_2}(B|b).$$

(5)

Howard claims that (5) follows from his locality principle, and this claim appears unproblematic (but we will return to it below). So, his analysis requires further only that the outcome independence condition (2) be shown to be equivalent to his 'separability condition', which he defines as follows:

> Two physical systems S_1 and S_2 are 'separable' if there exist 'separate' states λ_1 and λ_2 for S_1 and S_2, respectively, such that
>
> $$p_{\lambda_{12}}^{S_{12}}(A,B|a,b) = p_{\lambda_1}^{S_1}(A|a,b)\,p_{\lambda_2}^{S_2}(B|a,b),$$
>
> (6)
>
> where λ_{12} represents the (complete) joint state of the system S_{12} composed of S_1 and S_2.

Howard claims also that this separability condition (6) follows from his separability principle, and given his definition of 'state' this claim is unobjectionable.

Howard's argument to show that (2) and (6) are equivalent runs as follows. First, to show that (6) implies (2), we apply (6) and (4) successively to the following definition of conditional probability:

$$
\begin{aligned}
p^{S_1}_{\lambda_{12}}(A|a,b,B) &\equiv p^{S_{12}}_{\lambda_{12}}(A,B|a,b) \big/ p^{S_2}_{\lambda_{12}}(B|a,b) \\
&= p^{S_1}_{\lambda_1}(A|a,b)\, p^{S_2}_{\lambda_2}(B|a,b) \big/ p^{S_2}_{\lambda_{12}}(B|a,b) \\
&= p^{S_1}_{\lambda_{12}}(A|a,b)\, p^{S_2}_{\lambda_{12}}(B|a,b) \big/ p^{S_2}_{\lambda_{12}}(B|a,b) \\
&= p^{S_1}_{\lambda_{12}}(A|a,b).
\end{aligned}
$$

Thus, (6) implies (2). To show that (2) implies (6), we again begin with the definition of conditional probability, but instead apply successively (2) and (4):

$$
\begin{aligned}
p^{S_{12}}_{\lambda_{12}}(A,B|a,b) &\equiv p^{S_1}_{\lambda_{12}}(A|a,b,B)\, p^{S_2}_{\lambda_{12}}(B|a,b) \\
&= p^{S_1}_{\lambda_{12}}(A|a,b)\, p^{S_2}_{\lambda_{12}}(B|a,b) \\
&= p^{S_1}_{\lambda_1}(A|a,b)\, p^{S_2}_{\lambda_2}(B|a,b).
\end{aligned}
$$

Thus, (2) implies (6). Therefore, the argument concludes, (2) and (6) are equivalent and, hence, outcome independence is equivalent to the separability condition.

Although the mathematics here is surely correct, one may question whether Howard's argument constitutes *strict* proof that the outcome independence condition (2) and the separability condition (6) are *equivalent*. Crucial to the argument is the identifications (4), the assumption of which Howard himself does not justify. Now, the argument establishes the *equivalence* of (2) and (6) *only if* (4) is *independent* of *both* (2) *and* (6) and, moreover, only if (4) is as unquestionable here as the definition of conditional probability. Regarding the first point, note that the very statement of the identifications (4) implicitly assumes that there are *already* separate states λ_1 and λ_2 for the spatially separated systems S_1 and S_2, respectively, and, hence, takes for granted (at least) clause (i) of the separability principle which postulates the existence of such states. Thus, neither the identifications (4) nor the locality condition (5), the statement of which assumes (4), are fully independent of the separability principle. Howard does recognize this dependence of the locality condition (5) upon clause (i) of the separability principle (Howard 1989, 227), but fails to see the same dependence regarding the identifications (4), the latter of which being relevant here. The question now arises whether the identifications (4) is independent of clause (ii) of the separability principle, which is crucial for the statement of the separability condition (6). Stipulating that $p_{|_{1}}^{S_1}(A|a,b) = p_{|_{12}}^{S_1}(A|a,b)$ and $p_{|_{2}}^{S_2}(B|a,b) = p_{|_{12}}^{S_2}(B|a,b)$ assumes effectively that the marginal outcome probabilities for S_1 and S_2 are independent of conditionalizing upon the *states* of S_2 and S_1, respectively. Given Howard's definition of the 'state' of a system in terms of a marginal conditional outcome-probability measure, this amounts to the assumption that the respective states of S_1 and S_2 are each independent of the states of the other system, which is just what the separability condition (6) asserts. Thus, (4) is *not* independent of (6). So, while assuming the identifications (4) when showing that the separability condition (6) implies the outcome independence condition (2) remains unproblematic, making the same assumption when showing the converse threatens circularity. It appears, then, that the most Howard can prove here is that the separability condition (6) implies the outcome independence condition (2), *not* that the two conditions are *equivalent*. However, that is in fact all that is actually required to derive implications for the separability principle from violations of the Bell inequality and, hence, the failure of the Bell factorizability condition (1). For if one gives up the outcome independence condition (2) rather than the parameter independence condition (3), as Howard argues we should, then one must still also give up the separability condition (6), which would consequently imply that the separability principle itself fails in some respect. We'll return to this below.

Laudisa (1995) has also called into question Howard's argument that the separability condition (6) is equivalent to the outcome independence condition (2). He also focuses on the identifications (4), but criticizes them on somewhat different grounds. He claims that the identifications (4) are implausible, for they implicitly conflate two distinct notions of 'state' — namely, the notions of 'value state' and 'dynamical state' — which are generally distinguished within the 'modal' interpretation. While the value state of a physical system is specified by which observables for the system have a definite value at a given time and what those definite values are, the dynamical state is specified by how the system evolves over time, where the prediction of measurement outcome probabilities belongs primarily to the dynamical state. In the context of quantum mechanics, the dynamical state just is the quantum state, while the value state is assigned on the basis of some property rule; and in the orthodox interpretation, which Howard implicitly adopts, a value state exists if and only if the quantum state is an eigenstate of some quantum-mechanical observable (the Eigenstate-Eigenvalue Rule), in which case the value state is effectively represented by the quantum state itself, but which is not the case in general. Laudisa claims that, *given* such a distinction, what is required of a 'separable' state-description is *only* that it assign spatially separated systems S_1 and S_2 separate (i.e., distinct) *value* states, *not* separate dynamical states (in the sense of the separability condition). Thus, Howard's separability condition (6), which requires separate dynamical states, demands too much, and, hence, the identification of *dynamical* states in (4) *need not* hold for a state-description that is separable in terms of value states. And, in that case, without the crucial identifications (4), the separability condition (6) is *not equivalent* to the outcome independence condition (2) (Laudisa 1995, 318).

One could further develop this criticism as follows. The separability principle, inasmuch as it concerns the space-time individuation of systems, should be characterized generally in terms of value, rather than dynamical, states (i.e., in terms of definite properties rather than probabilities). But, in order to maintain the equivalence of the separability condition (6) and the outcome independence condition (2), the separability condition must be defined in terms of dynamical states. Thus, whereas separate states in the separability *principle* would refer to the *value* states of the spatially separated systems S_1 and S_2, the separability *condition* (6) would refer to the *dynamical* states I_1 and I_2 of S_1 and S_2, respectively; and, because separate (i.e., distinct) value states does *not necessarily* imply separate dynamical states (in the sense of the separability condition), the inference from the separability principle to the separability condition would be rendered *non sequitur*, for the latter could fail (with respect to dynamical states) while the former holds (with respect to value states).

The above criticism is valid *given* a general value/dynamical state distinction. Such a distinction, of course, is foreign to Howard's orthodox interpretation of ‘state’. This makes it clear, then, that Howard's claims that the separability principle implies the separability condition and that the separability condition (6) is equivalent to the outcome independence condition (2) are specific to an orthodox interpretation of 'state' and thus need not hold generally. This point will be of importance when assessing the implications of giving up the separability condition.

3. Locality versus Separability

Leaving aside such objections until the next section, we turn next to assessing Howard's claim that we *must* (or, at least, *should*) give up the separability condition (4) in the face of violations of the Bell inequality by the statistical predictions of quantum mechanics. His claim rests upon the argument that violation of the locality condition (5) is incompatible with both the special theory of relativity and reasonable methodological constraints on theory construction and confirmation. We will consider these arguments in turn.

Howard's argument that violation of the locality condition is incompatible with the special theory of relativity begins with the claim *only* that the separability condition is *suspect* regarding violation of the Bell inequality, for violation of the locality condition "threatens" special-relativistic locality constraints because it "could" be used to signal super-luminally, which raises a question of only a *possible* incompatibility between violations of locality and the special theory of relativity. But, in the summary of the argument, his characterization of the modality of such incompatibility suddenly changes. Whereas before his claim was only that violations of the locality condition present a possible violation of special-relativistic locality constraints, Howard now claims that violations of the former *imply* violations of the latter; he thus concludes that the *only possible* way to explain the violation of the Bell inequality by the statistical predictions of quantum mechanics in a way that is compatible with the special theory of relativity is to deny the separability condition (Howard 1992, 306–7, 312–13).

Crucial to this sudden shift of modality in Howard's argument is the claim that violation of the locality condition *implies* violation of 'special relativistic locality constraints'. Regarding what such locality constraints entail, Howard uses this phrase throughout the argument to refer consistently to "constraints on superluminal signaling" (Howard 1992, 306). Thus, the crucial claim here is just that violation of the locality condition or parameter independence *implies* the possibility of super-luminal signaling. But this claim, as it stands, is incorrect; for parameter dependence *by itself* does *not necessarily* allow for signaling, super-luminal or otherwise. Signaling is possible via violations of parameter independence *only if* the joint (complete) state l is *controllable* (Shimony 1984). Therefore, Howard's claim notwithstanding, the locality condition does *not* simply "embody" special-relativistic constraints on super-luminal signaling; it is a stronger assumption than is needed to rule out super-luminal signaling (i.e., non-controllability will do). So, his conclusion here is simply *non sequitur*.

This also opens up an objection to his claim that the locality condition follows from the locality principle, the latter of which explicitly prohibits super-luminal signaling. Because violations of the locality condition do not necessarily permit signaling at all and, hence, do not necessarily imply failure of the locality principle, one could well argue that the locality principle does *not by itself* imply the locality condition. Something more — a Principle of Common Cause, say — is still needed. Combined with the objection concerning the connection between the separability principle and separability condition made in the previous section, this would undercut completely Howard's initial claim, namely, that the Bell factorizability condition is a consequence his separability and locality principles; for although the factorizability condition *does* follow from the locality and separability *conditions* (5) and (6), respectively (given the identifications (4)), there appears to be no necessary connection of each respective condition to its associated principle.

Again leaving this objection aside, the second argument that Howard marshals in favor of giving up the separability condition rather than the locality condition is that abandoning the locality principle (as a consequence of giving up the locality condition) would be methodologically unreasonable. Here he offers glosses on Einstein's remarks concerning the relation of separability and locality to the formulation and testing of theories (cf. Einstein 1948). The separability principle is necessary to the formulating of physical theory because, if separability were to fail so that we could not individuate systems spatio-temporally, then the only system to which theoretical state-descriptions could refer unambiguously would be the entire universe itself. And the locality principle is necessary to the testing of physical theory because, if locality were to fail so that we could not screen off distant influences, we could not be sure of the reliability of measurement results (Howard 1989, 245–6). Where I disagree with Howard concerning the methodological role of separability and locality in formulating and testing theories is primarily in regard to which should be privileged in this respect when a choice between them is forced.

Howard privileges the locality principle over the separability principle, and for this he appeals to Einstein's own distinction between 'principle' and 'constructive' theories (Einstein 1919): principle theories provide high-level empirical-regulative principles which, though neither *a priori* certain nor uniquely determined by the phenomena, serve as (contingently) necessary conditions that guide and constrain the formulation of constructive theories, which articulate the models (or “build up a picture”) of physical systems that (in Einstein's view) are necessary for understanding phenomena. Einstein gives the law of the conservation of energy and the second law of thermodynamics as examples of such empirical-regulative principles and statistical mechanics and the kinetic theory of gases as examples of constructive theories: whatever mechanical model one might construct to explain the collective behavior of gases, such a model must obey the law of conservation of energy and the second law of thermodynamics. Now, Einstein himself considered the special theory of relativity to be a principle theory and thus took the 'special principle of relativity' to be one of those high-level empirical-regulative principles; and, while Einstein himself never makes any characterization of his own ‘Separation Principle’ in such categories, Howard places the separability principle in the constructive class. Howard then argues:

> Like Einstein, I believe that ultimate understanding is provided only by a constructive theory; but, also like Einstein, I believe that any particular constructive hypothesis should bow to the authority of regulative principles that, like the locality principle, enjoy considerable empirical substantiation. And so I would argue that the locality principle ought to be given the benefit of the doubt. The burden of proof should fall on those who prefer nonlocality to nonseparability. (Howard 1989, 247)

While appeal to authority and proof by default is no argument, we will challenge this claim.

First, it is not clear that Einstein himself did believe that a given constructive hypothesis should *always* conform to *any* well-established empirical-regulative principle. For in his view, it is *constructive* theories, not principle theories, that comprise the “most important class of theories” (Einstein 1919); and the primacy of constructive theories is due to their essential role in fulfilling the aim of physical theory, namely, understanding the phenomena. Second, even if Einstein would agree with Howard that a given constructive principle should always conform to any well-established empirical-regulative principle, it is Howard, not Einstein himself, who characterizes separability as a *constructive* principle. But separability has all the marks of an empirical-regulative principle of theory construction; for it does not make any prescription for filling in the content a specific physical model of a certain phenomenon, but rather asserts a generic constraint on all physical models that would employ a space-time representation to comprehend any phenomenon. In this respect separability plays a role in theory construction similar to that of the principle of conservation of energy and the second law of thermodynamics. And, third, it is *not* obvious that Howard's locality principle is itself an empirical-regulative principle, for it does not necessarily follow from what Einstein calls the 'special principle of relativity'. The latter for Einstein consists precisely of the two original postulates — namely, the invariance of the speed of light in all inertial frames and the covariance of the laws of physics under all Lorentz transformations (Einstein 1919). As Maudlin (1994) has shown, these special-relativistic constraints *by themselves* do *not necessarily* imply that "no effects can be communicated between systems separated by a space-like interval", which is what Howard's locality principle asserts. Thus, violation of the locality principle (via, e.g., superluminal signaling) need not imply violation of the 'special principle of relativity'. So, the supposed privileged status of the locality principle as an empirical-regulative principle cannot be justified on the basis of its being a strict consequence of the ‘special principle of relativity’ and is thus open to question. Such considerations aside, I think one has here two empirical-regulative principles and not, as Howard suggests, one empirical-regulative and one constructive principle, in which case one must make a further argument for privileging one over the other.

Taking both separability and locality as empirical-regulative principles, one should ask to begin with whether they are *equally necessary* to the formulation and testing of physical theory. First, consider locality; is it, as both Einstein and Howard (1989, 246) agree, an *absolutely* necessary condition of the possibility of testing physical theory? Assume that it is, that if locality failed and the world were in fact 'non-local', then *any* physical theory would be untestable *in principle*. And suppose that Bohmian mechanics is, in fact, *true*; because Bohmian mechanics is 'non-local' in the sense that it fails to satisfy the locality condition, this would effectively be to assume that the world itself is non-local in that sense. But, if the world were non-local, then Bohmian mechanics would *not* be testable, strictly speaking. Now, Bohmian mechanics is empirically equivalent to standard quantum mechanics in that it makes all the same statistical predictions as standard quantum mechanics; and insofar as such statistical predictions are in fact well-confirmed experimentally, they are so with respect to *both* standard quantum mechanics *and* Bohmian mechanics *alike*. Thus, in this sense at least, Bohmian mechanics is just as testable as standard quantum mechanics. But, then, if Bohmian mechanics is testable, then the world itself can*not* be non-local, for by hypothesis locality is a *necessary* condition of the testability of *any* physical theory. And if the world itself is local, then Bohmian mechanics must be false. So, if locality is a necessary condition of the possibility of testing physical theories, then we have the following conclusion: if Bohmian mechanics is true, then it is not testable; if it is testable, then it is not true. The claim, then, is effectively that no non-local physical theory can be both true and testable *in principle*, and that seems clearly too strong a constraint on physical theory construction, for it would rule out *a priori* all viable (i.e., logically consistent, empirically adequate) non-local theories.

So, we must ask again whether locality is an *absolutely* necessary condition for the possibility of testing physical theories. And, I think, it is not. Bohmian mechanics is evidently testable *in practice*, at least to the extent that it makes all the same statistical predictions as standard quantum mechanics, which are experimentally confirmed. This suggests that what is minimally required for testing physical theory is *only* that physical systems be *effectively* 'localized' or 'localizable', such that distant influences can be sufficiently screened off to allow the interpretation of experimental results to be as unambiguous as is ever possible or practically necessary. To make the claim that locality is an *absolutely*, and *not* merely *practically*, necessary condition of the possibility of testing physical theory in general, one must show that such effective 'localizability' is *impossible* under *any* conditions for *any* physical theory that represents physical systems via state-descriptions that fail to satisfy the locality condition; or to make the claim that a particular non-local theory, such as Bohmian mechanics, is untestable *in principle*, one must show that the specific way in which the state-descriptions of the theory fail to satisfy those conditions implies that physical systems as represented by the theory can*not* be *effectively* 'localized' under *any* conditions. Neither Einstein nor Howard have shown that such is the case in general or in particular. And I take the evident *in-practice* testability of Bohmian mechanics to strongly suggest, if not imply, that the claim of the *in-principle* non-testability of non-local physical theories is simply incorrect.

Let us suppose, though, that Einstein and Howard are correct in claiming that locality is a strictly necessary condition of the possibility of testing physical theories. Is locality then distinguished from separability in this respect such that locality is privileged over separability? Einstein himself did not think so. For in that regard he made a claim for separability similar to that he made for locality: "Neither does one see [if separability were to fail] how physical laws could be formulated and tested without such clean separation [of spatially distant physical systems]" (Einstein 1948, 321, my translation). But Howard, while acknowledging that "some scheme for individuating systems is necessary in order to formulate and test scientific theories", goes on to diverge from Einstein's (supposed) view that the separability principle is itself necessary. And the reason he gives for disagreeing with Einstein's view here is not unlike the one we have just given for disagreeing with both Einstein and Howard regarding the necessity of locality for testing physical theory:

> Agreeing with [Einstein] on this last point would entail one's declaring the quantum theory, which violates the separability principle, to be, in effect, a fundamentally incoherent theory.... But this is a step I do not feel compelled to take. (Howard 1989, 246)

So, separability and locality do appear after all to be on equal footing with regard to the formulation and testing of physical theories.

Where, then, does this leave us regarding the relative methodological status of the separability and locality principles? Privileging either principle over the other is ultimately a matter of theory selection, one decided by normative criteria, not by logic or experiment. That is, to privilege one principle over the other is effectively to choose one theory over another, a theory which satisfies that principle but not the other. Einstein himself made a comparison of constructive and principle theories according to their respective virtues: "The advantages of the constructive theory are completeness, adaptability, and clearness, those of the principle theory are logical perfection and security of the foundations." (Einstein 1919). This suggests that in his view — if strict adherence to both a well-established empirical-regulative principle and a currently-accepted constructive hypothesis precludes the construction of a coherent and adequate model or 'picture' (and, hence, understanding) of certain phenomena — one can be warranted in either privileging a well-established empirical-regulative principle over a given constructive principle, or vice-versa; and that would cohere with his view that *all* theoretical scientific concepts and principles (including both constructive and empirical-regulative principles) are ultimately conventional and so always revisable and provisional (Einstein 1933 and 1936).

Of course, which choice is appropriate in any given context would depend upon one's relative evaluation of the respective virtues or advantages of the available options for theory modification, considered with respect to both the empirical evidence and the aims of physical theory (e.g., causal-explanatory understanding). In the present case, to privilege locality over separability, as Howard claims we should, would be to choose, say, standard quantum mechanics over Bohmian mechanics, which are equally well-supported by the available empirical evidence; and the ultimate reason for doing so would lie not in the individual merits of locality relative to separability, but rather in the appraisal of standard quantum mechanics relative to Bohmian mechanics with respect to certain normative criteria of theory selection. Indeed, Howard reveals himself on this point by remarking that the potential fertility of giving up the separability principle is chief among the reasons for his preference (Howard 1989, 232).

4. Non-Separability and the Space-Time Individuation of Quantum-Mechanical Systems

Finally, let us now suppose along with Howard that it is indeed the separability condition (6), rather than the locality condition (5), that fails in the violations of the Bell inequality by the statistical predictions of quantum mechanics and, hence, that it is the separability principle, rather than the locality principle, that must be sacrificed. The question at hand, then, is what implications follow for the separability principle *as a consequence* of the violation of the separability condition, concerning particularly the possibility of the space-time individuation of quantum-mechanical systems.

We begin by noting that the denial of the separability principle presents (at least) a two-fold ambiguity. First is the non-separability of *states* — either there are no separate (i.e., distinct) states for spatio-temporally separated systems or the joint state of spatio-temporally separated systems is not completely determined by their separate states; second is the non-separability of *systems* — spatio-temporal separation is not a sufficient condition for individuating systems. Now, regarding the implication of the violation of the separability condition for the separability principle, it is clear that violation of the separability condition implies the non-separability of *states* (given, of course, Howard's definition of 'state'). Whether it implies further the non-separability of *systems* is *not* clear, and Howard defers this question by way of a footnote, but never answers it directly. He does, though, express his view that the non-separability of systems is also implied by the violation of the separability condition, both by his promotion of an ontology of non-separable systems and, in particular, by the following remark: "We confront here a radical physical holism at odds with our classical intuitions about the individuation of *systems and states*..." (Howard 1989, 228, emphasis added). In any case, whether that is his express view or not, it is this question that is at stake here. For violation of the Bell factorizability condition, assuming the locality condition is not violated, implies the impossibility of the space-time individuation of quantum-mechanical systems *only if* the non-separability of *systems* is implied by the violation of the separability condition.

First, note that the non-separability of *states*, which *is* implied by the violation of the separability condition, itself presents a further two-fold ambiguity; for one could deny the separability of states in either of two senses: *either* there are no separate (i.e., distinct) states for spatio-temporally separated systems *or* the joint state of spatio-temporally separated systems is not completely determined by their separate states. One could deny the separability of states in the second sense without denying it in the first. In this case, there would be separate or distinct states for each system, but such states would not determine uniquely the joint state of the composite system they comprise when represented as a single system. The 'modal' interpretation of quantum mechanics might be taken as an example of a physical theory which denies separate states in the second sense but not in the first. For, as mentioned above, the 'modal' interpretation introduces a general distinction between the value and dynamical state of a system; by denying the 'only if'-clause of the orthodox Eigenstate-Eigenvalue Rule, the components of composite systems prepared in a joint quantum or dynamical state are *always* assigned distinct value states — namely, definite values for observables defined in terms of the basis vectors of the bi-orthogonal representation of the quantum or dynamical state (cf. Kochen 1985 and Dieks 1989) — but such distinct value states do not in general determine uniquely the joint quantum or dynamical state (except in the case of a factorizable quantum state), thereby violating clause (i) of the separability principle (as formulated by Howard in terms of dynamical states). Thus, it is *only* the failure of the separability of states of spatially separated systems in the *second* sense — namely, the failure of such separate or distinct (value) states to determine uniquely the joint (dynamical) state of a composite system — that follows *necessarily* from the violation of the separability condition. This leaves open the possibility of the existence of separate or distinct states for spatially separated systems, even if the separability condition is violated.

Of course, *if* one accepts Howard's orthodox definition of 'state' in terms of a conditional probability measure *and* his interpretation of 'separate states' in the sense of determining a joint probability measure as the product of the two separate measures, then to have separate states would just be to have a joint product state; in that case, to deny the separability of states would be to deny it in both senses at once, which would eliminate the ambiguity (by effectively denying a general value/dynamical state distinction). But, one surely *need not* accept Howard's orthodox view *here*; and if one does not, then the non-separability of states is left open to interpretation regarding its significance for the space-time individuation of quantum-mechanical systems. In particular, it seems evident that the further inference to the non-separability of *systems* — which is required to infer the impossibility of the space-time individuation of quantum-mechanical systems — is warranted *only if* the non-separability of *states* is denied in the first sense — namely, the non-existence of separate or distinct states for spatially separated systems — as well as in the second sense. For as long as separate or distinct states for spatially separated systems exist, or at least are possible, then the space-time individuation of such systems in terms of those states also remains possible. Therefore, because the violation of the separability condition does not necessarily imply the non-existence of separate or distinct states for spatially separated systems (i.e., non-separability of states in the first sense), the violation of the separability condition *by itself* does *not necessarily* (i.e., independently of a given interpretation) imply the non-separability of systems and, *a fortiori*, does not necessarily imply the impossibility of the space-time individuation of quantum-mechanical systems.

Again, though, let us deny a general value/dynamical state distinction and, hence, grant that separate or distinct states for spatially separated systems would *not* exist if the separability condition were violated. Does such an assumption imply the non-separability of *systems* and, hence, the impossibility of the space-time individuation of quantum-mechanical systems? The answer, I think, is still 'no'. For what has been assumed here is at most the non-existence of separate or distinct *quantum* states for spatially separated systems, which is surely the case for 'entangled' systems (e.g., pairs of spin–1/2 particles prepared in a singlet state). To take the non-separability of states (in both senses) to imply the non-separability of systems is to assume that composite quantum-mechanical systems can be 'separable' *only* with respect to their joint *quantum* state. And such an assumption presupposes that the quantum state-description is *complete in principle*; otherwise, if the quantum state-description were *in*complete, then one could have a supplemental-variable state-description of such systems which includes the non-separable quantum state yet nonetheless assigns distinct supplemental-variable states (i.e., 'classical' variables defined independently of the quantum state) to each system in terms of which they could be individuated. And one can*not* appeal here to Bell's theorem for support; for violation of the Bell inequality *alone* implies *at most* that any (hypothetical) supplemental-variable state-description compatible with the statistical predictions of the quantum state cannot reproduce factorizable probabilities, not that distinct supplemental-variable states for such systems cannot exist.

So, the conclusion that Bell's theorem implies, via the violation of the separability condition, the impossibility of the space-time individuation of quantum-mechanical systems requires, over and above granting methodological privilege to the locality principle over the separability principle, that one assume further both that spatially separated systems having a non-separable joint dynamical or quantum state fail to possess separate or distinct value states for quantum-mechanical observables (or that such value states do not exist in general) and that the quantum state-description is complete in principle. Thus, the conclusion does not hold generally, but rather is peculiar to an orthodox interpretation. From the point of view of the Bell factorizability condition under Howard's analysis, then, the space-time individuation of quantum-mechanical systems remains an open possibility for any viable interpretation that denies (at least) one of these three claims.

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