QM and STR

The combining of quantum mechanics and relativity theory

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"Bohr would always go in for this remark, 'You cannot really explain it in the framework of space and time.' By God, I was determined I was going to explain it in the framework of space and time."

(John Slater, quoted in Cushing (1994), p. 109)

1. Introduction.

The aim is to show that quantum mechanics and special relativity can be consistently combined. This requires that the dynamical evolution of a quantum system be describable in any inertial frame, that transformations from one frame to another be admissible, and that quantum mechanical states and laws remain invariant under these transformations.

Many difficulties stand in the way of achieving this reconciliation. To begin with, the state of a QM system is represented by a vector or a density operator in Hilbert space, or by a wavefunction in configuration space, and the evolution of the quantum state takes place within that arena. But none of these vector spaces is the familiar space-time to which the principles of special relativity apply. The first requirement in bringing together QM and STR is to find a way of describing quantum change not in terms of the evolution of the state vector in Hilbert space, but in terms of something changing in ordinary space and time.

Secondly, collapse is notoriously difficult to picture as a space-time process. A particle passing through a slit undergoes diffraction, and the wavefunction which specifies the probability of locating the particle in different regions of space spreads out. But when a measurement is made the wavefunction collapses instantaneously, e.g. to a darkened region on a photographic plate. Similarly, a measurement performed on one of two particles in an entangled spin-state partially collapses the superposition and changes the state of the other particle. How are these collapses to be pictured in space-time? If instantaneous, relative to which reference frame? What mechanism produces an instantaneous distant effect? The most difficult part of constructing a space-time picture of collapse is dealing with its instantaneous, non-local character.

To be sure, one way of treating collapse is to eliminate it and adopt a no-collapse theory; either (i) Bohmian dynamics (Bohm (1952), Albert (1992), ch. 7, Cushing (1994), (1996)) or (ii) the many-worlds interpretation (DeWitt and Graham (1973)). These no-collapse interpretations, though of great interest and merit, stand somewhat apart from the present work. The difference is that the latter emphasizes the *probabilistic* character of QM. Neither Bohm's dynamics nor the Everett interpretation allows for objective quantum probabilities; Bohm's because it is deterministic (at least in its original form, but see Nelson (1985)), and Everett's because every outcome of every QM experiment is realized on some branch, and consequently there is no sense in speaking of the "probability" of it being realized.

Although in this paper collapse is held on to, and no-collapse theories set aside, no attempt will be made to give a theory of what causes or "triggers" collapse. This might be an act of consciousness (Wigner), or a GRW-type spontaneous localization process, or something else. All that is provided here is a space-time model of collapse which is instantaneous and non-local, and which is compatible with a variety of different theories of what causes it.

2. The space-time arena.

Two questions confront us. First, how is it possible to represent, in space and time, an indeterministic process with more than one possible outcome? Second, what would constitute a space-time description of the evolution of quantum states, i.e. their temporal evolution in ordinary space as distinct from Hilbert space? Answers to these questions will establish the quantum space-time arena in which relativistic transformations apply.

Suppose that a single photon passes through a calcite crystal equipped with detectors. To this stochastic process there are two possible outcomes, "+" and "-". The usual space-time representation of such a process would record only the outcome that actually occurs. The "missing" outcome, the one that was not realized, would not be represented.

An alternative way of representing a two-outcome stochastic process - the one adopted here - is to put in *both* possible outcomes, but to locate them in disjoint space-time regions which exclude one another. From the common region containing the initial conditions two different regions branch off. This is the space-time structure at the start of the experiment, when the photon is in a superposition $|+\rangle + |-\rangle$ of plus and minus polarization states. The collapse of the superposition is represented by the vanishing of the branch containing the unrealized outcome. The other branch remains, bringing it about that the experiment has a unique outcome. Space-time diagrams depicting branching and "branch attrition" of this kind are found in McCall (1994), (1995).

An important advantage of this representation over a non-branching single manifold representation, in which only the actual outcome is represented, is that branching provides a physical basis for probability values. The values are not attached in some unclear way to each branch, i.e. "put in by hand", but are part and parcel of the overall branching structure.

If for example the photon entering the calcite crystal is already prepared in a pure polarization state oriented at 30 degrees to the crystal's axis, it will have a probability of 75% of passing "+" and 25% of passing "-". These probabilities are built into the space-time structure in that 75% of the branches contain the "+" outcome and 25% the "-" outcome. If there is a democracy of branches, with each branch having an equal chance of being selected, the corresponding probabilities of "+" and "-" outcomes will be 0.75 and 0.25 respectively. These are objective single-case probability values, uniquely determined by the structure. The probability of a given outcome X is fixed by the proportion of branches on which X occurs, given that each branch has an equal probability of being selected.

For probability values which are rational numbers, finite sets of branches suffice. Thus an outcome with probability *n/m* will be located on *n* of a total of *m* branches, relative to a given initial segment. But to represent irrational probability values, such as $\cos^2 20^o$, or to represent a stochastic process in which there are infinitely many possible outcomes, as in scattering experiments, radioactive decay times, etc., fixed and definite proportions within infinite sets of branches are needed. For details see McCall (1994), pp. 88-92.

Given the space-time representation of quantum probabilities, the road to defining and representing quantum states is open. What is a quantum state? Unlike a classical state, which assigns a definite value of a physical quantity to a system, a quantum state assigns only an expectation value. More precisely, the quantum state of a system is an exhaustive specification of the probabilities of all possible outcomes of all possible tests that can be performed on the system. It is in Abner Shimony's words a "compendium of possibilities", a precise specification of a system's dispositional powers. What must be seen is how this "compendium" can be represented in space and time, and furthermore how it can be represented as changing and evolving.

Imagine as a concrete example a system S to be an electron moving towards an adjustable Stern-Gerlach apparatus which can be tilted so as to measure the electron's spin at any one of 100 different angles j_1, j_2, ... $j_{100.}$ To each angle j_i there corresponds an observable O_i, i.e. j_i-spin, and for each O_i the quantum state of S assigns a probability to the values "up" and "down" of j_i-spin on S. In four-dimensional space-time, the world-line of the electron S branches to couple with each of the 100 possible Stern-Gerlach instruments available to the experimenter by twisting a dial. There is only one 3-dimensional instrument, and the 4-dimensional representation of the experimental set-up will include one type of branch for each setting.

In the space-time representation of S's quantum state, each branch associated with the measuring-instrument settings divides again, at a time slightly later than that at which the electron enters the magnet. This branching determines, by proportionality, the probability of the different outcomes spin-up and spin-down for each setting, and it is in these proportions that the quantum state of S lies. If, for example, the electron on entering the magnet is in the pure state |x+>, spin-up along the x-axis, then the proportions of "up" and "down" outcomes for each of the 100 observables will take a definite set of values. In branched space-time, the state |x+> of S consists of these proportions. But if the electron is prepared in the state |z+>, or in the mixed state $1/2(P_{x+} + P_{x-})$, the proportion of "up" and "down" outcomes for each of the 100 observables will vary. It is this variation in proportions which constitutes the difference, in 4-dimensional space-time, between the quantum states |x+>, |z+>, and $1/2(P_{x+} + P_{x-})$.

In the space-time representation of a probabilistic process, there is one additional element and that is collapse. In the example just described there will be an infinity of spin-up and spin-down branches for each of the 100 observables. In configuration space, the wavefunction of the electron in the position basis gets correlated with its spin, and exhibits two peaks in the "up" and "down" channels of the apparatus, the amplitudes of which are functions of the quantum state. But in the end one and only one detector is triggered, the wavefunction collapses, and the experiment has a unique outcome. Its uniqueness, in branching space-time, is brought about by branch attrition, consisting of the survival of exactly one branch out of the infinity which existed at the start of the experiment.

To be accurate, there are two "collapses" in the course of the experiment, the first when the experimenter selects the angle of the measuring instrument, and the second when a single branch gets randomly selected from the infinity of "up" and "down" branches. The first of these is "observer's choice" (Heisenberg), the second "nature's choice" (Dirac) (see Bohr (1949), p. 223). In the example considered the role of the observer in selecting the quantity to be measured is an active one, though it is possible to think of situations in which the selection of the measured observable is itself the result of a non-anthropic probabilistic process.

With the identification of a space-time correlate of quantum states, the first and most important step has been taken in constructing a space-time arena in which the histories of quantum systems unfold. The quantum state of a system has been identified with the relative proportionality of sets of branches containing all possible outcomes of all possible tests that can be performed on the system. Defined in this way the evolution of quantum states takes place not within Hilbert space but within four-dimensional space-time.

Continuing with the previous example, suppose an additional option is now available in which the experimenter can route the electron through a magnetic field which changes the orientation of its spin. In Hilbert space, the state vector undergoes a unitary transformation. If now the electron, in its new spin state, is once more confronted with the 100 tests, the up/down probabilities in each case will be altered. This change in the electron's state between t_1 and t_2, i.e. before and after its spin has been rotated, is represented in the space-time arena as follows.

At t_1 there are 101 branching life-line paths in space-time for the electron, and its quantum spin state is determined by the up/down probabilities at the ends of the paths leading to the 100 tests. The 101st path, along which the electron's spin precesses, leads once more to the 100 tests at time t_2, and again the electron's life line branches, but at the end of each branch the proportions of "up" and "down" branches are different from those on the paths that branched off at t_1. In the 3-dimensional world, this situation is described by saying that at t_1 the electron was disposed to pass "up" and "down" in each test with one set of probabilities, whereas at t_2 its probabilities of responding were different. Between t_1 and t_2 the electron's quantum state has changed, and the branching structure permits that change to be described in space-time.

A final point should be made concerning quantum states in Hilbert space and in real space. In Hilbert space there is in principle no limit to the number of different Hermitian operators, for the values of which a given quantum state yields probabilities. But in real space, in any experimental situation, there is at most a finite number of different alternative tests that can be performed on a system. Hence the number of different probability distributions that are built into real space-time will be only a fraction of those generated by the projections of the state vector in Hilbert space. What this implies is that the quantum state of a system in space-time has an empirical dimension that the quantum state in Hilbert space lacks. There is no theoretical limit to the number of tests that can be performed on the system, but they cannot all be performed at once.

3. Quantum states represented in space-time and their relativistic transformations.

In QM dynamics the state vector $v(t_1)$ at time t_1 is transformed into the vector $v(t_2)$ at time t_2 by the Schrodinger equation $v(t_2) = U(t)v(t_1)$, where $t = t_2 - t_1$. Use of the words "at time t_1" and "at time t_2" implies a reference frame. If the state of a particle p is represented by a wavefunction in the position basis which yields probabilities of detecting the particle in different regions of real space, then in space-time the state of p at time t in frame f will consist of the distribution of these probabilities along an equal-time hyperplane in the frame f (hereafter referred to as a "f-hyperplane"). In a different inertial frame f', states of p consist of probability distributions along f'-hyperplanes, slanted with respect to those in f.

A basic assumption of relativity theory is that if an object or event is describable in one inertial frame of reference then it is describable in all frames, and that there should be rules for translating the description from one frame to another. In branching space-time, p's state in frame f is translatable into p's state in frame f' by switching to branch proportions along the new family of f'-hyperplanes, these proportions being built into the space-time structure. If however p's wavefunction collapses at or near the time at which such a translation is attempted, there exist well-known difficulties which make the translation impossible. An example from Fleming (1988) makes the problem clear.

In figure 1, a particle p enters into a state reducing interaction (e.g. a position measurement) in a region R of space-time.

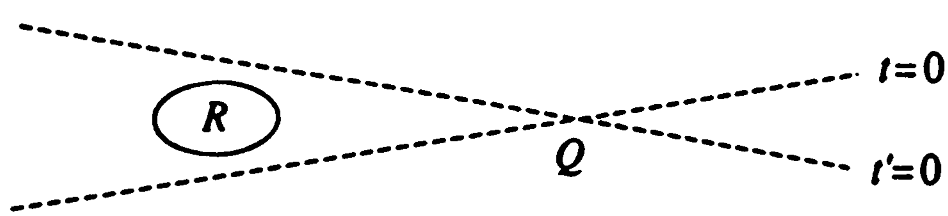

Figure 1

Consider the state Ψ of p along t=0 in f, and its state Ψ' along t'=0 in f'. How is Ψ to be transformed into Ψ'?

Relative to t=0, before the position measurement at R, the wavefunction of p has a non-zero amplitude at Q. Relative to t'=0, however, p's wavefunction either has non-zero amplitude at Q if p is not detected at R, or has zero amplitude if p is detected at R. There is therefore no straightforward way of transforming _ into _', since the transformation depends on the stochastic and unpredictable result of the reduction at R.

Fleming's example shows that no transformation "across a collapse" is possible, i.e. a transformation from a hyperplane relative to which a collapse is future, to one relative to which the same collapse is past. However, if transformations across collapses are avoided and if a whole segment of a system's history in one reference frame, including collapses, is translated into a corresponding segment of its history in another frame, also including collapses, then relativistic covariance is preserved. In each frame f, collapse of a quantum state brought about by a measurement will be instantaneous along an f-hyperplane.

I turn now to a second example in Fleming (1988), discussed extensively in Maudlin (1994), (1996), Fleming (1996), and Wayne (1997). Figure 2 (see Fleming (1988), p.117; Maudlin (1994), p.205) depicts the history of a Bell-EPR experiment using photons, written from the standpoint of two different frames k and k'.

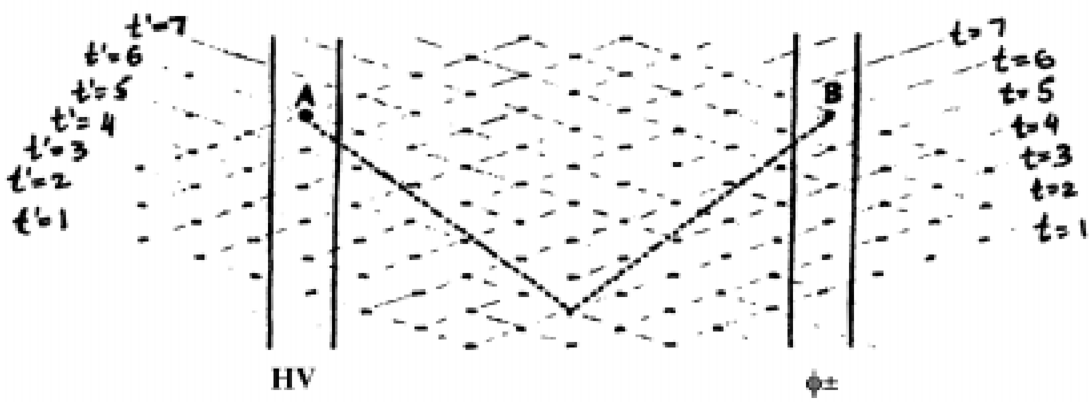

Figure 2

The sequence of k-hyperplanes t=1, t=2, ... represent moments of the system's history in k, and the sequence t'=1, t'=2, ... moments in k'. As Fleming says, we may imagine that k' is the rest-frame of an observer on the ground when the experiment is conducted on a railway flatcar moving to the right, while k is that of a motorcar moving faster than the train in the same direction. In k' the left measurement event A occurs before the right measurement event B; in k it occurs after B.

When emitted, the 2-photon system is in the entangled spin state $|h\rangle|v\rangle + |v\rangle|h\rangle$. Each photon, considered separately, is in a state empirically indistinguishable from the mixed state $1/2(P_h + P_v)$, which is to say that each photon has a probability of 0.5 of passing + or - when its polarization is measured at any angle whatsoever. Suppose that the left polarizer is horizontal/vertical, and the right polarizer is oriented at an angle ϕ to the vertical. In frame k', following the measurement of the left photon, the entangled state partially collapses and the right photon is thrown into a pure spin state. If the left outcome is h, the right photon enters the state |v>, and its probability of being detected in the ϕ+ channel is $\cos^2\phi$. But if the left outcome is v, the right photon enters the state |h>, and its probability of passing ϕ+ is $\sin^2\phi$.

Two questions arise:

(i) How can the right photon, at the moment at enters its analyzer, be both in a mixed state from the standpoint of frame k', and in a pure state from the standpoint of k?

(ii) How, in frame k', can the outcome of the left measurement event affect the state of the right photon, given that the two photons are receding from each other at the speed of light?

The first of these questions is discussed in the next section, and the second in section 6.

4. Hyperplane dependency and covariant state vector reduction.

Fleming's method of dealing with the apparently contradictory states of the right photon - mixed in frame k' and pure in frame k - is to introduce the concept of hyperplane dependency. We cannot speak simply of the "state" of a quantum system but always of its "state relative to a hyperplane". As Maudlin points out, we are familiar with relational states and properties: familiar with Socrates not being "tall" in any absolute or intrinsic sense, but only "tall relative to Theaetetus" (Maudlin (1994) p. 210). The question is, can polarization be a relational property of a photon, analogous to tallness? Or is polarization an intrinsic property, analogous to being snub-nosed?

Fleming (1996) defends the notion of hyperplane dependency without answering the last question explicitly, but Wayne (1997) supports the idea that the polarization states of photons in the EPR experiment are in an important sense relational rather than purely intrinsic. For Wayne, a single photon in a two-photon entangled system
|h>|v> + |v>|h> has "no intrinsic state; instead, the particle pair has an irreducibly relational state" (p. 562). Just as, in STR, the spatial length of an object is a hyperplane dependent property, so the polarization state of each photon is determined by "hyperplane-dependent temporal relations between the photon and the measurement event at the distant detector" (p. 565).

A useful notion in the present context is that of separability. Don Howard introduces the term to describe Einstein's belief in the independent existence of objects located in different regions of space. In Howard's definition physical systems in disjoint regions are "separable" if (a) each possesses its own distinct physical state, and (b) the joint state of the two systems is wholly determined by their separate states (Howard (1989) p. 226). Plainly the notion of hyperplane dependence violates separability. At the same time, in addition to separability Einstein also believed in locality, namely that events in a region X of space-time cannot be affected by events in spacelike separation from X.

The immediate task is to reconcile a description of EPR in one frame, including collapses, with its description in another frame. The great virtue of Fleming's pioneering work in this area is that it presents clearly, in a way that cannot be ignored, the problems of developing a theory of wavefunction collapse in a relativistic context. In the branched model, branching occurs along different criss-crossing hyperplanes in different frames. What happens to a photon in an entangled state when its twin is measured?

In a single space-time manifold it is possible for the right photon to be in a mixed state in a given region, and it is possible for it to be in a pure state, but it is not possible for it to be both pure and mixed in the same region. In the branched model the situation is different. There are many future branches relative to the initial EPR conditions; on some of them the photon enters its analyzer in a mixed state, and in others in a pure state. Since the model branches along different hyperplanes in different frames, there will be sub-models in which the right measurement occurs respectively (i) before, (ii) after, and (iii) simultaneously with the left measurement. On branches in models of types (i) and (iii), the right photon enters its analyzer in a mixed state, and on branches of type (ii) it enters in a pure state. On its own branch each photon's polarization state is intrinsic to it. By putting different possible outcomes on different branches, depending on the frame in which the story of the experiment is unfolding, the model avoids making polarization states relational.

Summing up, figure 2 shows how, based on the branched model, consistent relativistic accounts of the EPR experiment can be given in frames k and k'. "Translation" from one to the other requires going to the sub-models and reading off the respective histories. In neither case are we forced to describe a photon as being in two contradictory polarization states at the same time.

5. Lorentz-invariant probability values.

Across translations from one frame to another, probability values of the possible outcomes of a single quantum event remain invariant. The structure of the model which demonstrates this, and which also guarantees that every stochastic process has a unique outcome, is complex. What follows is a bare outline which may be omitted on first reading, although here as elsewhere "the devil is in the details".

Each "history", or complete path through the model, is a 4-dimensional space-time manifold. Every history contains a discrete set of "choice points", and the model branches along every "slice" (maximal set of spacelike separated choice points). The spacelike hypersurfaces ("branching surfaces") defined by slices are partitioned into parallel families, each of which approximately fits the foliation of equal-time hyperplanes in a frame of reference. At a small interval _t above each branching surface S are located the "outcomes" of its choice points, the probability of each outcome being determined by the proportion of branches which split along S and on which the outcome occurs.

Since each branching surface S belongs to one and only one parallel family, which in turn corresponds to a set of inertial frames which differ only infinitesimally from one another, the probability of an outcome at a choice point is relative to the family to which S belongs. These probabilities (which reflect the law-like character of QM) must be Lorentz-invariant. For the branched model to serve as a means of combining QM and STR it must therefore possess the following symmetry: for all parallel families, the proportions of branches on which a given type of outcome occurs above a given choice point are the same.

Through each choice point C in each history there pass many branching surfaces at different angles, on the branches above each of which are located C's possible outcomes. Branch attrition consists in the selection of one and only one branch B of these. Since B is unique, C has a unique actual outcome. B in turn splits along other branching surfaces containing B's choice points, which again have unique actual outcomes. The actual outcomes of all regions of space-time are "accessible" to one another; they belong to the same history.

6. A space-time mechanism for non-local effects.

The most distinctive and unusual feature of the branched model is that it provides a mechanism which explains non-local connections: how it is possible for the outcomes of two or more stochastic, spacelike separated events to be correlated. In section 3 it was asked how, in frame k', the outcome of the left measurement event in the EPR experiment could affect the state of the right photon, given that the photons are moving in opposite directions at the speed of light. Conversely, in frame k, how could the outcome of the right measurement affect the left photon? An answer to these questions provides the branched solution to the difficult puzzle of non-local connections.

Figure 3 pictures the dynamic behaviour of the two-particle system in frame k'.

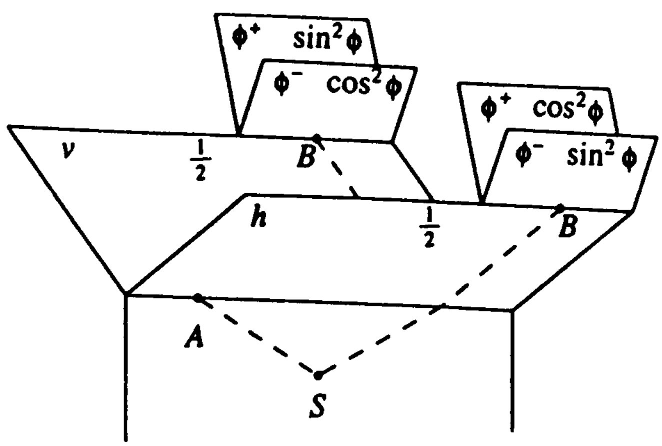

Figure 3

At A the entangled state partially collapses, the relative proportions of h- and v-branches being 0.5. If the outcome of the left measurement is v this means that a v-branch is stochastically selected (with probability 0.5), and on this branch the state of the right photon is |h>. On the other hand if the left outcome is h then an h-branch is selected, and on this branch the state of the right photon is |v>. In frame k', the selection of a single branch, brought about by the left measurement, produces an instantaneous effect on the right photon. The photon starts off in the mixed state $1/2(P_h + P_v)$, and is thrown into state |h> if the selected branch is a v-branch, and into |v> if the selected branch is an h-branch. Branch attrition along k'-hypersurfaces therefore accounts for the instantaneous influence (in frame k') of the left measurement outcome on the polarization state of the right photon. This influence in turn generates the statistical correlations between the outcomes (h,v) on the left and (ϕ+,ϕ-) on the right. If the left outcome is h, then the probability of ϕ+ on the right is $\cos^2\phi$, whereas if the left outcome is v, the probability of ϕ+ on the right is $\sin^2\phi$.

In frame k, where the right measurement precedes the left, a mirror-image space-time structure shows the state of the left photon being instantaneously influenced by the outcome of the right measurement. By the equivalence of inertial frames, neither of these frame-dependent histories is privileged over the other, and therefore there is no uni-directional causal relationship linking the two outcomes. (Causation is necessarily an asymmetric relation: if F is the cause of G, G cannot also be the cause of F.) Nevertheless, although not connected by causation proper, the two outcomes *are* connected. What connects them is a reciprocal, empirical, non-logical relationship of mutual dependence which is instantaneous in whatever frame one chooses to describe it. It's a "non-local connection" deriving from the way event-types are instantiated in branching space-time, and from the fact that the branching extends along spacelike hypersurfaces.

In the description two paragraphs ago of the space-time mechanism which gives rise to non-local effects, the concept of branch attrition played an important part. Thus the right photon is "thrown into state |h> if the selected branch is a v-branch". Certainly branch attrition is required if quantum experiments are to have unique outcomes: out of a range of possible outcomes it is branch attrition which selects the actual outcome. That being said, the existence of non-local connections can be read off, or read into, branching space-time without branch attrition. In figure 4 the world-line of the right photon splits at the point at which it intersects the lowest k'-hypersurface. On some branches above that split the photon is polarized horizontally, whereas in others it is polarized vertically. But the lowest k'-hypersurface is also the hypersurface on which A, the left measurement event lies, and it is precisely on branches where A has outcome v that the right photon is polarized horizontally. Also, it is on branches where A has outcome h that the right photon is polarized vertically. All this information is contained in the branching structure, independently of branch attrition. Branch attrition is needed for collapse, but not for non-locality.

7. 3D/4D equivalence.

Throughout this paper, the concept of branching has played the role of the Prince of Denmark in *Hamlet*. Different possible outcomes of stochastic processes are located on different branches; their probabilities are defined by proportionality relations among the branches on which they are located; quantum states are identified with such proportionalities in future branching structures; etc. However, the ontological burden imposed by branching space-time is considerable, and there may be those who would prefer the coming together of QM and STR at lower cost. Some suggestions in this direction are contained in what follows.

3D/4D equivalence is a philosophical thesis which holds that to describe the world as a set of 3-dimensional objects evolving in time, or alternatively as a set of 4-dimensional objects extended in time, are equivalent. Every description of the first kind can be translated without loss into a description of the second kind, and vice versa. If the thesis of 3D/4D equivalence is correct, a dragonfly may be regarded either as 3-dimensional object which moves jerkily, or equivalently as a 4-dimensional world-line which bends erratically.

As was seen above, a stochastic process may be represented 4-dimensionally in branching space-time by putting each possible outcome on a separate branch, and allowing the actual outcome to be selected by branch attrition. The probability of that outcome is objective, defined by branch proportionality. In 3 dimensions the object undergoing the process behaves stochastically: the state it enters into at the end of the process is its actual state, and the possible but non-actual resultant states are not represented. E.g. an electron passing through a Stern-Gerlach apparatus emerges in one channel or the other, and the outcome which might have resulted but didn't does not enter into the 3D description.

In 4 dimensions, what bestows a unique actual outcome upon a stochastic process is branching, combined with branch attrition. In order for the 3D and 4D descriptions to be perfectly equivalent, it is necessary that the probability of the actual outcome, in 3 dimensions, should be as objectively rooted in the real world as the probability value in 4 dimensions, based on branch proportionality. 3D/4D equivalence entails this, namely that in 3 dimensions the outcome of a probabilistic process is rigidly constrained by objective probabilities, the values of which are facts about the world in the same way that the atomic numbers of aluminum and copper are facts about the world.

Provided this last condition is met, as 3D/4D equivalence implies that it is, it makes no difference whether the world is described 3 dimensionally or 4 dimensionally. Seen in this light branching space-time, and branch proportionality, may be treated as the ladder which is thrown away once we have climbed up the wall. In the context of quantum mechanics, and the reconciling of QM and STR, branching serves to give precise quantum probability values an ontological footing in the space-time world. Those who wish to attach no more fundamental ontological significance to branching can simply assume that this has been done, and that the footing exists. Specifically, objective probability values (including joint probabilities of distant events) are established relative to spacelike hypersurfaces belonging to parallel families, each family representing a set of reference frames. Once this has been accomplished the ladder of space-time branching can be thrown away, and the task of combining QM and STR proceeds within a single Minkowski manifold.

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