OLD EVIDENCE AND NEW EXPLANATION III

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Abstract. Garber (1983) and Jeffrey (1991, 1995) have both proposed solutions to the old evidence problem. Jeffrey's solution, based on a new probability revision method called *reparation*, has been generalized to the case of uncertain old evidence and probabilistic new explanation in Wagner 1997, 1999. The present paper reformulates some of the latter work, highlighting the central role of Bayes factors and their associated *uniformity principle*, and extending the analysis to the case in which an hypothesis bears on a countable family of evidentiary propositions. This extension shows that no Garber-type approach is capable of reproducing the results of generalized reparation.

1. Introduction

1.1 *Old Explanation and New Evidence.* A basic principle of scientific inference asserts that if hypothesis H in known to imply the less-than-certain proposition E, the subsequent discovery that E is true confirms (i.e., raises the probability of) H. There is a straightforward Bayesian account of such confirmation, for from $p(E|H) = 1 > p(E)$ it follows immediately that $p(H|E) > p(H)$. This *hypothetico-deductive principle* extends in a natural way to the case of probabilistic old explanation and uncertain new evidence:

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Theorem 1. *If H is p-positively relevant to E and new evidence prompts a revision of p to q by probability kinematics on $\{E, \bar{E}\}$, with $q(E) > p(E)$, then $q(H) > p(H)$.*

Proof: Since $q(H) = q(E)p(H|E) + q(\bar{E})p(H|\bar{E})$ and $p(H) = p(E)p(H|E) + p(\bar{E})p(H|\bar{E})$, it follows that $q(H) - p(H) = (q(E) - p(E))(p(H|E) - p(H|\bar{E})) > 0$, the first factor being positive by assumption and the second by the p-positive relevance of H to E (hence, of E to H). □

1.2 *Old Evidence and New Explanation* . Suppose that we first attain certainty regarding E and subsequently discover, quite apart from this certainty, that H implies E. Just as it does when explanation preceeds observation, this explanation of the previously known fact E by the hypothesis H ought to raise the probability of H, but how ? This problem was first posed by Glymour (1980), who called it the *old evidence problem*. As Glymour noted, conditioning the prior p here on E is otiose since $p(E) = 1$, and so $p(H|E) = p(H)$.

One proposed solution to this problem, due to Garber (1983), extends the algebra on which probabilities are defined to include the proposition $H \vdash E$ that H implies E, and then conditions on $H \vdash E$. Elaborating Garber's proposal, Jeffrey (1983) showed that from $p(E) = 1$, along with certain additional assumptions about p, it follows that $p(H|H \vdash E) > p(H)$. A critique of these assumptions appears in Eells (1990) and Earman (1992).

More recently, Jeffrey (1991, 1995) has proposed a different solution to the old evidence problem, one that retains the original algebra, but revises probabilities by an entirely new method called *reparation*. A key feature of Jeffrey's approach is the imaginative reconstruction of a probability distribution that predates both our certainty regarding E and our discovery that H implies E. The explanation-based revision of this *ur-distribution* then serves as a paradigm for all explanation-based revisions.

In Wagner 1997, 1999 Jeffrey's new solution was shown to generalize in a natural way to cases in which observation raises our confidence in E without rendering it certain and the subsequent explanation afforded E by H is probabilistic rather than implicational. The present paper reformulates some of this work, highlighting the central role of Bayes factors and their associated *uniformity principle*, and extending the analysis to the case in which an hypothesis bears on a countable family of evidentiary propositions. This extension shows that no Garber-type approach is capable of reproducing the results of generalized reparation.

The notational conventions of this paper are as follows: If q_2 is a revision of the probability distribution q_1, and A_1 and A_2 are propositions, the *Bayes factor* $\beta_{q_2,q_1}(A_1 : A_2)$ is the ratio

$$\beta_{q_2,q_1}(A_1 : A_2) := \frac{q_2(A_1)}{q_2(A_2)} \bigg/ \frac{q_1(A_1)}{q_1(A_2)} \tag{1.1}$$

of new-to-old odds. A special case of the foregoing, the *q-likelihood ratio of hypothesis H on evidence E* , denoted $\lambda_q(H, E)$, arises when $q_1 = q$, $q_2(\cdot) = q(\cdot|E)$, $A_1 = H$ and $A_2 = \bar{H}$. In this case, Bayes' rule yields the simple formula $\lambda_q(H, E) = q(E|H)/q(E|\bar{H})$.

The revision q_2 is typically (and, in all the cases considered here) based on old as well as new evidence, which may be observational, theoretical, or some combination thereof. Bayes factors express what is learned from the new evidence alone, with the prior q_1 factored out. Hence, such factors, arising in the revision of a particular prior, are reasonably used in the revision of other priors when what is learned from the new evidence is the same. This *uniformity principle* plays a key role in what follows.

2. Reparation Generalized

The generalization of reparation in Wagner 1997, 1999 takes as its starting point a probability distribution p on the algebra $\mathcal{A}$ generated by H and E. Empirical investigation (*observation*) has contributed to a certain measure of confidence in E, as reflected in the value $p(E)$. We subsequently discover, quite apart from such observation, theoretical considerations that indicate that the truth of H would confer probability u on E, and its falsity would confer probability v on E (*explanation*). How should p be revised in light of this theoretical discovery?

Adapting a key feature of Jeffrey's approach, we resurrect a notional *ur-distribution* p_0 predating both observation and explanation, where $p_0(A) > 0$ for all $A \in \mathcal{A}^* := \{HE, H\bar{E}, \bar{H}E, \bar{H}\bar{E}\}$. The assumption is that p has come from p_0 by probability kinematics (Jeffrey 1983, 1988) on $\{E, \bar{E}\}$.

Suppose that we made the aforementioned theoretical discovery in the conceptual state captured by p_0, and that this discovery left the ur-probability of H unchanged. This would warrant the revision of p_0 to the unique distribution p_1 satisfying
(i) $p_1(E|H) = u$, (ii) $p_1(E|\bar{H}) = v$, and (iii) $p_1(H) = p_0(H)$.

The conceit is that the explanatory learning (logico-mathematical in the case treated by Jeffrey, probabilistic in the present case) is the same in the ur-conceptual state as it is in the state in which it actually occurs. Since what is learned is the same in both cases, the *uniformity principle*, described above in §1.2, dictates that the appropriate explanation-based revision q of p should be determined by the Bayes factor identity

$$(2.1) \qquad \beta_{q,p}(A_1 : A_2) = \beta_{p_1,p_0}(A_1 : A_2), \quad \forall A_1, A_2 \in \mathcal{A}^*$$

or, equivalently, by

$$(2.2) \qquad q(A)/p(A) \propto p_1(A)/p_0(A), \quad \forall A \in \mathcal{A}^*.$$ [1]

The distribution q may also be shown to come from p_1 by probability kinematics on $\{E, \bar{E}\}$, where $\beta_{q,p_1}(E : \bar{E}) = \beta_{p,p_0}(E : \bar{E})$. So the appropriate explanation-based revision of the observation-based revision p of p_0 coincides with the appropriate observation-based revision of the explanation-based revision p_1 of p_0. [2] The credentials of the uniformity principle are strengthened by its entailment of this intuitively desirable commutativity of observation- and explanation-based revisions.

In effect, Jeffrey treated the special case of the above in which $p(E) = 1, p_1(E|H) = 1$, and $p_1(E|\bar{H}) = p_0(E|\bar{H})$. In that case it turns out that $q(H) > p(H)$, i.e., that H is always confirmed. In the general case it is of course not necessarily true that $q(H) > p(H)$, and so it is of interest to identify conditions sufficient to ensure that H is confirmed. In the next section we describe two such conditions originally appearing in Wagner 1997, 1999, but formulated here in a way that highlights the central role of the Bayes factor $\beta_{p,p_0}(E : \bar{E})$. Under the assumption of the ur-independence of hypothesis H and evidence E, these conditions coalesce, furnishing an exact old evidence/new explanation analogue of Theorem 1.

3. Confirmation

It is useful to formulate the confirmation of H in terms of the inequality $q(H)/q(\bar{H}) > p(H)/p(\bar{H})$ rather than the equivalent inequality $q(H) > p(H)$. The most salient formulas for these odds are

$$\text{(3.1)} \qquad \frac{q(H)}{q(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})} \frac{[(\beta - 1)p_1(E|H) + 1]}{[(\beta - 1)p_1(E|\bar{H}) + 1]}$$

and

$$\text{(3.2)} \qquad \frac{p(H)}{p(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})} \frac{[(\beta - 1)p_0(E|H) + 1]}{[(\beta - 1)p_0(E|\bar{H}) + 1]},$$

where

$$\text{(3.3)} \qquad \beta := \beta_{p,p_0}(E : \bar{E}).$$ [3]

Theorem 2. *If $\beta > 1$ and either 1° $p_1(E|H) > p_0(E|H)$ and $p_1(E|\bar{H}) \leq p_0(E|\bar{H})$ or 2° $p_1(E|H) \geq p_0(E|H)$ and $p_1(E|\bar{H}) < p_0(E|\bar{H})$, then H is confirmed.*

Proof: Obvious $\square$

Note that (3.1) and (3.2) imply, respectively, that

$$\lim_{\beta\to\infty} \frac{q(H)}{q(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})}\lambda_{p_1}(H,E) \tag{3.4}$$

and

$$\lim_{\beta\to\infty} \frac{p(H)}{p(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})}\lambda_{p_0}(H,E). \tag{3.5}$$

Theorem 3. *If $\lambda_{p_1}(H,E) > \lambda_{p_0}(H,E)$, then H is confirmed for sufficiently large values of β.*

Proof: Obvious $\square$

It is worth noting that (3.1) and (3.2) imply that the Bayes factor $\beta_{q,p}(H:\bar{H})$ depends only on the p_0- and p_1-likelihoods of H and $\bar{H}$ on evidence E, and the Bayes factor $\beta = \beta_{p,p_0}(E:\bar{E})$. Assuming, as Good (1950,1983) and others have argued, [4] that direct assessment of β is possible, it follows that the issue of H's confirmation can be settled without a complete specification of p and p_0. On the further assumption that H and E are ur-independent, [5] things are even simpler:

Theorem 4. *If H and E are p_0-independent, then $\beta_{q,p}(H:\bar{H})$ is a function of the p_1-likelihoods $p_1(E|H)$ and $p_1(E|\bar{H})$ and the Bayes factor β alone, with H being confirmed whenever $\beta > 1$ and $p_1(E|H) > p_1(E|\bar{H})$.*

Proof: If H and E are p_0-independent then by (3.2), $p(H) = p_0(H)$. With (3.1) this implies that

$$\beta_{q,p}(H:\bar{H}) = \frac{(\beta-1)p_1(E|H)+1}{(\beta-1)p_1(E|\bar{H})+1}\,, \tag{3.6}$$

from which the asserted results follow immediately. $\square$

Theorem 4 is an exact old evidence/new explanation analogue of Theorem 1. In each case the confirmation of H follows from theoretical results establishing a relation of positive relevance between H and E, along with observation issuing in a ratio of new-to-old odds on E greater than 1.

4. Finer Evidentiary Partitions

The above approach may be extended to the case in which the hypothesis H bears on a countable family $\{E_i\}$ of mutually exclusive, exhaustive evidentiary propositions. Here p comes from the ur-distribution p_0 by probability kinematics on $\{E_i\}$, and the explanation-based revision p_1 of p_0 is defined by the conditions (i) $p_1(H) = p_0(H)$, (ii) $p_1(E_i|H) = u_i$, and (iii) $p_1(E_i|\bar{H}) = v_i$, where $\sum u_i = \sum v_i = 1$. Let $\mathcal{A}$ be the σ-algebra generated by H and $\{E_i\}$ and $\mathcal{A}^*$ the set of atomic propositions HE_i and $\bar{H}E_i$ of $\mathcal{A}$.

As in §2, we again define the explanation-based revision q of p by means of the uniformity principle:

$$\beta_{q,p}(A_1 : A_2) = \beta_{p_1,p_0}(A_1 : A_2), \quad \forall A_1, A_2 \in \mathcal{A}^*. \tag{4.1}$$

In order for q to be well-defined by (4.1), however, we need to postulate here that

$$\sum_{A \in \mathcal{A}^*} p(A)p_1(A)/p_0(A) < \infty, \tag{4.2}$$

for setting $A_1 = A$ and $A_2 = HE_1$, say, in (4.1) yields $q(A) = p(A)p_1(A)q(HE_1)p_0(HE_1)/p_0(A)p(HE_1)p_1(HE_1)$, and since we must have $\sum_{A \in \mathcal{A}^*} q(A) = 1$, this implies (4.2). It is straightforward to show that (4.2) is equivalent to

$$\sum_i p(E_i)p_1(E_i)/p_0(E_i) < \infty. \tag{4.3}$$

Hence, in particular, q is well-defined if

$$\exists\, B\ \forall i : p(E_i)/p_0(E_i) \le B, \quad \text{or} \tag{4.4}$$

$$\exists\, B\ \forall i : p_1(E_i)/p_0(E_i) \le B, \quad \text{or} \tag{4.5}$$

$$\exists\, B\ \forall i : \frac{p_1(E_i|H)}{p_0(E_i|H)} \le B \text{ and } \frac{p_1(E_i|\bar H)}{p_0(E_i|\bar H)} \le B. \tag{4.6}$$ [6]

As in §2, q comes here from p_1 by probability kinematics on $\{E_i\}$ with

$$\beta_{q,p_1}(E_i : E_j) = \beta_{p,p_0}(E_i : E_j), \quad \forall\, i, j\,, \tag{4.7}$$

and so explanation-and observation-based revisions also commute in this more general setting.

The generalizations here of formulas (3.1) and (3.2) are

$$\frac{q(H)}{q(\bar H)} = \frac{p_0(H)\sum_i p_1(E_i|H)p(E_i)/p_0(E_i)}{p_0(\bar H)\sum_i p_1(E_i|\bar H)p(E_i)/p_0(E_i)}\,, \quad \text{and} \tag{4.8}$$

$$\frac{p(H)}{p(\bar H)} = \frac{p_0(H)\sum_i p_0(E_i|H)p(E_i)/p_0(E_i)}{p_0(\bar H)\sum_i p_0(E_i|\bar H)p(E_i)/p_0(E_i)}\,, \tag{4.9}$$

or, equivalently,

$$\frac{q(H)}{q(\bar H)} = \frac{p_0(H)\sum_i \beta_i\, p_1(E_i|H)}{p_0(\bar H)\sum_i \beta_i\, p_1(E_i|\bar H)}\,, \quad \text{and} \tag{4.10}$$

$$\frac{p(H)}{p(\bar H)} = \frac{p_0(H)\sum_i \beta_i\, p_0(E_i|H)}{p_0(\bar H)\sum_i \beta_i\, p_0(E_i|\bar H)}, \tag{4.11}$$

where $\beta_i := \beta_{p,p_0}(E_i : E_1)$.

The generalization of Theorem 2 is

Theorem 5. *Let I be a nonempty, proper subset of the set of positive integers indexing $\{E_i\}$. If (i) $\forall\, i \in I\ \forall j \notin I : \beta_i > \beta_j$, (ii) $\forall\, i \in I : p_1(E_i|H) > p_0(E_i|H)$ and $p_1(E_i|\bar{H}) \le p_0(E_i|\bar{H})$, and (iii) $\forall\, i \notin I : p_1(E_i|H) \le p_0(E_i|H)$ and $p_1(E_i|\bar{H}) \ge p_0(E_i|\bar{H})$, then $q(H) > p(H)$.*

Proof: Show that a) the numerator of the right-hand side of (4.10) is greater than its counterpart in (4.11) and b) the denominator of the right-hand side of (4.10) is no greater than its counterpart in (4.11). Assertion a) is equivalent to

$$-\sum_{i \notin I} \beta_i \delta_i < \sum_{i \in I} \beta_i \delta_i, \tag{4.12}$$

where $\delta_i := p_1(E_i|H) - p_0(E_i|H)$. Now $\forall i \notin I : \delta_i \le 0$ and $\forall i \in I : \delta_i > 0$, and

$$-\sum_{i \notin I} \delta_i = \sum_{i \in I} \delta_i > 0. \tag{4.13}$$

Let

$$m := \sup_{i \notin I} \{\beta_i\}. \tag{4.14}$$

If $\exists\, i \in I : \beta_i > m$, then

$$-\sum_{i \notin I} \beta_i \delta_i \le -m \sum_{i \notin I} \delta_i = m \sum_{i \in I} \delta_i < \sum_{i \in I} \beta_i \delta_i. \tag{4.15}$$

If $\forall\, i \in I : \beta_i = m$, then $\forall\, i \notin I : \beta_i < m$. But by (4.13) , $\exists\, i \notin I : \delta_i < 0$. So

$$-\sum_{i \notin I} \beta_i \delta_i < -m \sum_{i \notin I} \delta_i = m \sum_{i \in I} \delta_i = \sum_{i \in I} \beta_i \delta_i. \tag{4.16}$$

The proof of assertion b) is similar. □

There is an obvious generalization of Theorem 4 here, which is left as an exercise. There appears to be a generalization of Theorem 3 here only when q is well-defined in virtue of (4.6) :

Theorem 6. *Suppose that q is well-defined in virtue of (4.6), and let I be a nonempty, finite, proper subset of the set of positive integers indexing $\{E_i\}$. If $E := \cup_{i\in I} E_i$ and*

$$\forall_{i,j\in I}: \quad \lambda_{p_1}(H, E_i) > \lambda_{p_0}(H, E_j), \tag{4.17}$$

then $q(H) > p(H)$ for $p(E)$ sufficiently large.

Proof: Let

$$t := \sum_{i\in I} p_1(E_i|H)p(E_i)/p_0(E_i), \tag{4.18}$$

$$\epsilon := \sum_{i\notin I} p_1(E_i|H)p(E_i)/p_0(E_i), \tag{4.19}$$

$$w := \sum_{i\in I} p_1(E_i|\bar{H})p(E_i)/p_0(E_i), \quad \text{and} \tag{4.20}$$

$$\delta := \sum_{i\notin I} p_1(E_i|\bar{H})p(E_i)/p_0(E_i), \tag{4.21}$$

with t', ϵ', w' and δ' denoting the respective results of replacing p_1 by p_0 in these formulas. By (4.8) and (4.9), $q(H) > p(H)$ if and only if

$$(t+\epsilon)/(w+\delta) > (t'+\epsilon')/(w'+\delta') \tag{4.22}$$

To show that (4.22) holds for $p(E)$ sufficiently large, it suffices to show that

$$t\,/(w+\delta)\, >\, (t'+\epsilon')/\,w',$$

i. e., that

$$tw' - wt' > \delta t' + w\epsilon' + \delta\epsilon' \tag{4.23}$$

for $p(E)$ sufficiently large. Now

$$tw' - wt' = \sum_{i,j\in I} \left[p_1(E_i|H)p_0(E_j|\bar{H}) - p_1(E_i|\bar{H})p_0(E_j|H)\right] \frac{p(E_i)p(E_j)}{p_0(E_i)p_0(E_j)}. \tag{4.24}$$

Let m denote the minimum of the bracketed expressions in the above sum. By (4.17), $m > 0$, and for $p(E) \geqslant 1/2$, say, $tw' - wt' \geqslant m/4$. On the other hand, the quantities on the right-hand side of the inequality (4.23) can be made as small as we like by taking $p(E)$ sufficiently large. This follows from the fact that δ and $\epsilon' \to 0$ as $p(E) \to 1$, and t' and w are uniformly bounded above for all p:

Consider first the term $p_1(E_i|\bar{H})p(E_i)/p_0(E_i)$ in the sum δ. Multiplying by $p_0(E_i|\bar{H})/p_0(E_i|\bar{H})$, replacing $p_0(E_i|\bar{H})/p_0(E_i)$ by $p_0(\bar{H}|E_i)/p_0(\bar{H})$, and $p_0(\bar{H}|E_i)$ by $p(\bar{H}|E_i)$, and applying (4.6) yields $\delta \leqslant Bp(\bar{H}\bar{E})/p_0(\bar{H})$. Similar arguments show that $\epsilon' = p(H\bar{E})/p_0(H)$, $t' = p(HE)/p_0(H) \leqslant 1/p_0(H)$, and $w \leqslant Bp(\bar{H}E)/p_0(\bar{H}) \leqslant B/p_0(\bar{H})$. From these inequalities the asserted facts follow. $\square$

It is natural to ask if Garber's approach to the old evidence problem might be extended to the case of uncertain old evidence and probabilistic new explanation. This would involve extending the algebra $\mathcal{A}$ to include a (very complex) proposition R expressing the newly discovered relevance relations between H and the E_i's and $\bar{H}$ and the E_i's, and conditioning an extension of the probability distribution p on R. If the aim is to produce in this way the same explanation-based revision q of p effected by generalized reparation, then this aim cannot always be attained, not even in a purely formal way, as the following analysis shows.

By a well-known result of Diaconis and Zabell (1982, Theorem 2.1), q, as defined by (4.1), can come from p by "superconditioning" p on such a proposition R if and only if

$$\exists\, B \geq 1\ \forall A \in \mathcal{A}^*\ :\ q(A)\ \leq\ B\, p(A). \tag{4.25}$$

Consider the case of a countably infinite partition $\{E_i\}$, where

	HE_1	$\bar{H}E_1$	HE_i	$\bar{H}E_i$
$p:$	$5/12$	$5/12$	$1/2^{2i}$	$1/2^{2i}$
$p_0:$	$11/24$	$11/24$	$1/2^{2i+1}$	$1/2^{2i+1}$
$p_1:$	$1/4$	$3/8$	$1/2^{i+1}$	$1/2^{i+2}$
$q:$	$5/29$	$15/58$	$22/29 \cdot 2^i$	$22/29 \cdot 2^{i+1}$. [7]

The Diaconis-Zabell superconditioning criterion (4.25) fails to hold here since, for example, $q(HE_i)/p(HE_i) = 22 \cdot 2^i/29$ for $i \geq 2$. Hence no Garber-type derivation of q from p is possible. [8] Note that H is confirmed here by the new probabilistic explanation of the E_i in terms of H and $\bar{H}$, since $q(H) = 16/29 > 1/2 = p(H)$.

Notes

1. See Wagner 1997, Theorem 2. The symbol $\propto$ denotes proportionality and (2.2) yields the exact formula

$$q(A) = \frac{p(A)p_1(A)}{p_0(A)} \Big/ \sum_{A \in \mathcal{A}^*} \frac{p(A)p_1(A)}{p_0(A)}.$$

2. It seems reasonable *prima facie* that the appropriate observation-based revision q of p_1 should come from p_1 by probability kinematics on $\{E, \bar{E}\}$, where $\beta_{q,p_1}(E : \bar{E}) = \beta_{p,p_0}(E : \bar{E})$, but we need not rely on this intuition. For under the assumption that what is learned from observation is the same, whether before or after explanation, the uniformity principle would dictate that the appropriate observation-based revision q of p_1 be determined by

(i) $$\beta_{q,p_1}(A_1 : A_2) = \beta_{p,p_0}(A_1 : A_2), \quad \forall A_1, A_2 \in \mathcal{A}.$$

But from the fact that p comes from p_0 by probability kinematics on $\{E, \bar{E}\}$, along with (i), it follows that q comes from p_1 by probability kinematics on $\{E, \bar{E}\}$, with $\beta_{q,p_1}(E : \bar{E}) = \beta_{p,p_0}(E : \bar{E})$. For further elaboration of the uniformity principle, with additional applications, see Wagner 2000.

3. Formulas (3.1) and (3.2) are algebraic variants of formulas (2) and (3) in Wagner 1997.

4. See, e.g., the Appendix of Richard Jeffrey's "Probabilistic Epistemology", at http:// www.princeton.edu/ ˜bayesway .

5. Since p_0 is assumed, *inter alia*, to predate the discovery of a certain relevance relation between H and E, it is natural to explore the consequences of assuming that p_0 predates knowledge of *any* such relevance relation.

6. As in §2, (4.1) is equivalent to the formula $q(A)/p(A) \propto p_1(A)/p_0(A)$, $\forall A \in \mathcal{A}^*$. The equivalence of (4.2) and (4.3) follows from the fact that

$$\sum_{A \in \mathcal{A}^*} p(A)p_1(A)/p_0(A) = \sum_i p(E_i)p_1(E_i)/p_0(E_i),$$

which follows from the fact that $p(HE_i)/p_0(HE) = p(\bar{H}E_i)/p_0(\bar{H}E_i) = p(E_i)/p_0(E_i)$, since p comes from p_0 by probability kinematics on $\{E_i\}$. It is obvious that (4.4) and (4.5) each imply (4.3). Finally, (4.6) implies (4.5) since $p_1(H) = p_0(H)$.

7. Here $p(H|E_i) = p_0(H|E_i) = 1/2$ for all i, and so p comes from p_0 by probability kinematics on $\{E_i\}$. Also, $p_1(H) = p_0(H) = 1/2$, $\sum_{i \geq 1} p_1(E_i|H) = 1/2 + \sum_{i \geq 2} 1/2^i = 1$, and $\sum_{i \geq 1} p_1(E_i|\bar{H}) = 3/4 + \sum_{i \geq 2} 1/2^{i+1} = 1$. Moreover,

$$\sum_{A \in \mathcal{A}^*} p(A)p_1(A)/p_0(A) = 5/22 + 15/44 + \sum_{i \geq 2} 1/2^i + 1/2^{i+1} = 29/22,$$

and so the well-definedness condition (4.2) holds.

8. Of course, even when q can be derived from p by superconditioning, e.g., whenever the partition $\{E_i\}$ is finite, the conditioning proposition R is purely abstract, and would require a detailed elaboration to furnish a Garber-type derivation of q.

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