The Analysis of Singular Spacetimes*

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Abstract

Much controversy surrounds the question of the proper definition of 'singularity' in general relativity, and the question of whether the prediction of such entities constitutes a crisis for the theory. I argue that a definition in terms of curve incompleteness is adequate, and that the idea that singularities correspond to 'missing points' faces insurmountable problems. I conclude that singularities *per se* pose no serious problem for the theory, but their analysis does bring into focus several problems of interpretation at the foundation of the theory often ignored in the philosophical literature.

1 Introduction

I suspect that, for many, talk of a singularity in the context of general relativity conjures up the image of something like a rent in the fabric of spacetime. This metaphor, evocative as it may be, perhaps misleads: a web of cloth exists in space and time, and one naturally would rely (implicitly, at least) upon this fact were one to define what one meant in saying the cloth were rent—one would say that points were missing were from the cloth, a notion made precise by the embedding of the cloth in physical space. When thinking of a singular spacetime,[1] though, one does not have the luxury of imagining it embedded in

*This paper began life as a small criticism of a few points John Earman makes in chapter 2 of his book *Bangs, Crunches, Whimpers and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*, and grew as I grew to realize more fully the complexity and subtlety of the issues involved. I shall not always point out where I am in agreement or disagreement with Earman, much less always discuss why this is so, though I shall try to regarding the most important points. The reader ought to keep in mind, though, that Earman's book is the constant foil lurking in the background.

[1]By 'spacetime', I shall always mean a smooth, 4-dimensional, connected, paracompact, Hausdorff manifold with a fixed smooth pseudo-Riemannian metric of Lorentz signature.

any physically meaningful way in a larger space with respect to which one can try to define what one means by saying there are points missing from spacetime.

On a manifold endowed with a positive-definite Riemannian metric, one can give a precise characterization of what it is for there to be missing points that accords quite well with common intuitions. The manifold has no missing points if and only if it is Cauchy complete as a metric space. On a manifold with a pseudo-Riemannian metric of Lorentz signature, such as a spacetime in general relativity, there is of course no natural way to construct a metric that measures the distance between points of the manifold, so one cannot employ this technique to test whether a spacetime has missing points.

By the Hopf-Rinow theorem, the manifold in the Riemannian case is Cauchy complete with respect to the constructed metric if and only if it is geodesically complete with respect to the Riemannian metric.[2] This naturally suggests that we define a spacetime to have missing points if and only if it is geodesically incomplete with respect to the spacetime pseudo-Riemannian metric. Because, however, there is no natural way to take a Cauchy-like completion of the spacetime manifold, in order to give substance to the idea that there really are points that in some sense ought to have been included in the spacetime in the first place, this criterion can tell one only that there are *some* missing points; it can in no way allow one to construct particular missing points, or even to 'localize where they ought to have been', had they been there.

The usual tack taken at this point in the physics literature is simply to bracket the question of missing points and define a spacetime to be singular if and only if it contains incomplete, inextendible curves of a certain specified type and the spacetime manifold itself satisfies a few collateral conditions. More precisely, the commonly accepted schema for fixing a rigorous definition of a singular spacetime is:

> A spacetime $(\mathcal{M}, g_{ab})$ satisfying ____ is *singular* if and only if there exists a curve γ incomplete in the sense that ____.[3]

Such a conception of singular structure actually has a lot to say for itself, as capturing the idea that singular structure is somehow physically *outré*, even if one is not able to hook it up cleanly to an idea of missing points: on most accounts, an observer traversing such an incomplete curve would be able to experience only a finite total amount of proper time.

This paper has several concrete aims: to investigate particular ways that have been proposed to fill in the blanks of the schematic definition with an eye to determining whether they capture the spirit of the idea that an incomplete curve corresponds to singular structure; to argue that the idea of missing points ought not be central in thought about singular structure; and to argue that the reasons most often given for eschewing singular structure as unphysical do not withstand scrutiny. It also has one overarching, more inchoate aim: to try to

[2] See Spivak (1979, ch. 9) for a precise statement and proof of the theorem.

[3] See, for example, Clarke (1993, p. 10), Joshi (1993, pp. 161–2), Wald (1984, pp. 212–6) and Hawking and Ellis (1973, pp. 256–61).

give a sense of the marvellous philosophical riches still waiting to be mined from thorough investigation of the foundations of general relativity—which is to say, a sense of how little we still comprehend of and about this astounding theory, and how much we stand in need of that comprehension if we wish to understand the world.

2 Curve Incompleteness

The path-breaking work of the mid-1960's demonstrating the existence of singular structure in generic solutions to the Einstein field equations used timelike or null geodesic incompleteness as a sufficient condition for classifying a spacetime as singular, in so far as timelike and null geodesics represent possible world-lines of particles and observers and it appears *prima facie* physically suspect for an observer or a particle to be allowed to pop in or out of existence right in the middle of spacetime, so to speak.[4] There was, however, no consensus on what ought to count as a necessary condition.

Geroch (1968b) gave the first extended discussion of the difficulty of framing a satisfactory definition of a singular spacetime. He settled provisionally on simple geodesic incompleteness as the criterion for singular structure, conceding that the definition is perhaps overly inclusive, but better to brand 10 innocents than to allow one guilty man unmarked. The possible innocents include spacetimes that are timelike and null geodesically complete but possess incomplete spacelike geodesics (null and timelike incomplete and spacelike complete, for short). Spacelike incompleteness (in the absence of the other two types of incompleteness) sets off no serious alarms, for an incomplete spacelike geodesic seems to represent structure of the spacetime that is not physically accessible to any observer (I will discuss this matter more thoroughly in §4 below). Moreover, not only does geodesic incompleteness finger a few possible innocents, but, as Geroch proceeded to show, it almost certainly fails to nab a few clever guilty parties, for a spacetime can be geodesically complete and yet possess an incomplete timelike curve of bounded total acceleration—that is to say, an inextendible curve traversable by a rocket with a finite amount of fuel, along which an observer could experience only a finite amount of proper time.

Because of these problems, null and timelike geodesic incompleteness continued to be used as a sufficient condition for branding a spacetime singular, but was considered inadequate as a definition. To analyze the structure of non-geodetic curves in the search for a necessary condition, a method is required to characterize their completeness. Schmidt (1971) appears to have been the first to propose using so-called generalized affine parameters to define the completeness of general curves. Any curve of unbounded proper length automatically has an unbounded generalized affine parameter, but not vice-versa—any inextendible timelike curve of unbounded total acceleration and finite total proper time in Minkowski space, for example, has an unbounded generalized affine parameter. A spacetime in which every inextendible curve has an unbounded

[4] *Cf.*, *inter alia*, Penrose (1965), Hawking (1965), Geroch (1966) and Hawking (1967).

generalized affine parameter will be referred to as *b-complete*.[5] Thus, one has what Earman (1995, p. 36) refers to as the "semioffical view": a spacetime is singular if and only if it is b-incomplete.[6] This definition is more general than geodesic completeness, in that it implies, but is not implied by, the latter, as Geroch's example suggests.

It is difficult to think of a more comprehensive criterion of completeness than b-completeness, and I suspect its popularity arises therefrom, but that it sits comfortably with some of the intuitions that drove the search for a definition of singular structure in the first place is not so clear on reflection. For the moment, I shall accept b-incompleteness as the definition of singular structure—when I refer to 'incomplete curves', unless I explicitly state otherwise, I shall mean b-incomplete, inextendible curves. I shall return to some of these questions below in §4.

3 Missing Points

We now have a precise definition of a singular spacetime, but, as Earman notes, "it is not true to an idea that is arguably a touchstone of singularities in relativistic spacetimes: spacetime singularities correspond to missing points."(Earman 1995, p. 40) For those who would argue that missing points ought to be such a touchstone, Earman sketches what seems to me the most (initially) promising position, that, though the idea of missing points and that of curve incompleteness lead to *prima facie* different concepts of singular structure, they are extensionally equivalent in all physically reasonable singular spacetimes, and so the two concepts are for all practical purposes in agreement.(Earman 1995, p. 42) I shall argue with this: missing points ought not be a touchstone of discussion of singular structure in relativistic spacetimes.

Missing points, could they be defined, would correspond to a boundary for a singular spacetime—actual points of an extended spacetime at which incomplete curves would terminate.[7] My argument therefore will alternate between speaking of missing points and speaking of boundary points, with no difference of sense intended. Before I begin examining the primary attempts to define boundary points for singular spacetimes,[8] it is well to note an oddity of the situation: compact spacetimes can contain incomplete, inextendible geodesics, as

[5] 'b' for 'bundle': with this construction one tacitly defines a natural (basis-dependent) Riemannian metric on the bundle of frames of the spacetime manifold to define curve completeness. See Schmidt (1971) for details.

[6] Strictly speaking, this is not the standardly accepted definition, since I have not mentioned anything about the maximality of the spacetime in question, whether, that is, it can be embedded in (thought of as merely a part of) a larger spacetime in such a way as to make previously incomplete, inextendible curves extendible. I shall take up this issue in §4.

[7] Strictly speaking, such a space would not be a manifold in the usual sense of the term, but a manifold with boundary. See Spivak (1979).

[8] I shall not consider in this paper the 'ideal-point' boundary construction of Geroch, Kronheimer, and Penrose (1972), as it requires the singular spacetime to be past- and future-distinguishing, a fairly strong causality condition. I intend to sidestep all questions about the physical plausibility or necessity of such conditions.

shown by a simple example due to Misner (1963). In a sense that can be made precise, Hausdorff compact sets, from a topological point of view, 'contain every point they could possibly be expected to contain',[9] one consequence of which is that a compact manifold cannot be embedded as an open submanifold of any other manifold, a necessary pre-requisite for attaching a boundary to a singular spacetime—a manifold-with-boundary minus its boundary is embeddable by the identity map as an open submanifold into itself. This already suggests that, even were one able to come up with a satisfactory definition of missing points in the context of Lorentzian metrics, it might not be extensionally equivalent to the existence of incomplete curves, unless we were willing to swallow unpalatable topological structure.

Schmidt (1971) produced the most well-known boundary construction for singular spacetimes, the so-called b-boundary based on the b-completeness criterion. The relativity community at first embraced Schmidt's contruction with enthusiasm, to judge by the remarks in chapter 8 of Hawking and Ellis's canonical work *The Large Scale Structure of Space-Time*. Shortly thereafter, however, Bosshard and Johnson independently showed that the b-boundary had undesirable properties in the most physically relevant spacetimes known, the Friedmann-Robertson-Walker spacetimes, which to a quite high degree of approximation accurately model the large scale structure of the actual universe, and the Schwarzschild spacetimes, which represent the neighborhood of spherically symmetric isolated bodies, such as stars.[10] For closed Friedmann-Robertson-Walker spacetimes, the b-boundary consists of a single point (the same for the big bang as for the big crunch) that is not Hausdorff-separated from any point in the interior of the spacetime. Not only does the universe contract to the same point from which it itself originally expanded, but that point is, in a certain sense, arbitrarily near every single spacetime event! Similarly, the b-boundary of a Schwarzschild spacetime consists of a single point not Hausdorff-separated from any interior point of the spacetime. This certainly will not do for the advocates of missing points.[11]

A second method of constructing a boundary for singular spacetimes due to Geroch (1968a) fares much better with physically relevant spacetimes. In this construction, the so-called *g-boundary*, geodesic incompleteness rather than b-incompleteness defines singular structure, and one defines a boundary point to be an equivalence class of incomplete geodesics under the equivalence relation 'approach arbitrarily close to each other' (in a certain technical sense). The set of boundary points can be given a topology and, in many cases of physical

[9] See Geroch (1985, §30) for a discussion of this precise sense.

[10] *Cf.* Bosshard (1976) and Johnson (1977).

[11] The reactions to these problems vary widely. Clarke (1993) still embraces the b-boundary contruction, and defines a singularity to be a point on the b-boundary of a singular spacetime (§3.4). He barely mentions these problems, noting only in passing that the topological structure of the singular spacetime with boundary can be "very strange," (p. 40) which I do not think an adequate address. Wald (1984), on the other hand, does not like the b-boundary construction precisely because of these problems (*cf.* pp. 213–4), and Joshi (1993) does not even mention the possibility of attaching boundaries to singular spacetimes, speaking only of incomplete curves.

interest, can even be given a differentiable and metric structure, so that one can locally analyze the structure of spacetime at a 'singularity' rather than mess around with troublesome limits along incomplete curves.[12] The g-boundary construction, moreover, yields the boundaries one might have expected on physical grounds in spacetimes of particular physical interest: the g-boundary of a Schwarzschild spacetime is a spacelike 3-surface, topologically $S^2 \times \Re$, and that of a closed Friedmann-Robertson-Walker spacetime is the disjoint union of two spacelike S^3's. Pathological topology rears its head here as well, though, in the case of Taub-NUT spacetime: the g-boundary of this spacetime contains a point that again is not Hausdorff-separated from any point in the interior of the spacetime.[13]

The advocate of missing points may at this point retort that Taub-NUT spacetime is hardly a physically relevant spacetime for other reasons, namely that it violates strong causality, which is to say that it contains causal curves that come arbitrarily close to intersecting themselves. While I do not think this reply carries much weight,[14] I have a better example at hand. Geroch, Can-bin, and Wald (1982) construct a geodesically incomplete spacetime with no causal pathology for which a very large class of boundary constructions, including the b- and the g-boundary, will yield pathological topology in the completed spacetime (the conditions that a boundary construction must satisfy to fall prey to this example are quite weak). The advocate of missing points may point out that the example appears artificial and contrived, with closed sets excised here and conformal factors plastered on there, and in short has no physical relevance. I would reply that this judgment has its roots in the schooling our intuitions have received from our contemplation of well-worked out examples of physical theories, which by and large tend to include mathematical structures that strike us as 'simple' and 'natural'. This ought not escape our notice: most such examples of physical theories are demonstrably false (Newtonian mechanics and classical Maxwell theory) or have at the moment insuperable problems of interpretation (quantum mechanics) or experimental accessibility (general relativity). We should beware of relying too much on intutions trained in such schools—especially when one also recalls how much of our contemplation of those theories involves models of systems with physically unrealistic perfect symmetries and vaguely justified approximations and simplifications. It may turn out, for all we know, that spacetime instantiates just such topological structure as $\Re^4$ with closed sets excised.

I refer those unmoved by this sermon to a remark that Geroch, Can-bin, and Wald (1982, p. 435) make: "The purpose of [a boundary] construction, after all, is merely to clarify the discussion of various physical issues involving singular space-times: general relativity as it stands is fully viable with no precise notion of 'singular points.'" When we contemplate potential phenomena that

[12] In certain *outré* examples, there is an ambiguity in choice of topology for the g-boundary, but I shall waive this concern for the sake of argument. I have bigger fish to fry.

[13] *Cf.* Hawking and Ellis (1973, §5.3) for a thorough account of Taub-NUT spacetime.

[14] See Earman (1995, chs. 6–7) for arguments that a violation of strong causality *simpliciter* does not constitute an argument that a spacetime is unphysical.

we have little or no observational access to, I submit that the standards for what can count as a *physical* account of a situation ought to be priggishly severe, if we are not unwittingly to degenerate into pure mathematical discourse.[15] A construction that yields topological pathology, and contains no precise criteria for what ought to count as a 'physically relevant' spacetime, does nothing to clarify discussion of the physical issues involved in analyzing singular spacetimes.

My final point against the idea of missing points as touchstones in the investigation of singular spacetimes is a simple one: the definition of singular spacetimes by incomplete curves is logically prior to the construction of missing points for singular spacetimes. All the missing point constructions I know of, and all the ways I can more or less easily imagine trying to concoct a new one, rely on probing the spacetime with curves of some sort to discover where points may be thought of as missing, just as in the Riemannian case one cannot complete a manifold until one knows which Cauchy sequences do not have a limit point, or equivalently which geodesics are incomplete. One, however, does not need any conception of a missing point, much less a definition of such, to define and investigate the existence of incomplete curves on a manifold. I therefore disagree with the gist of much of the discussion of Earman (1995, ch. 2), wherein he suggests that unclarity plagues the semi-official definition of a singular spacetime, in terms of b-incompleteness, in so far as, on the face of it, one does not know how it relates to the idea of missing points. Incomplete curves seem to me a fine definition of singular structure on their own.

I believe Geroch, Can-bin, and Wald (1982, p. 435) deserve the last word on this topic: "Perhaps the localization of singular behavior will go the way of 'simultaneity' and 'gravitational force.'"

4 The Finitude of Existence

I turn now to examine whether singularities as characterized are objectionable on physical or interpretive grounds, and whether one is forced to or ought to take them as indicating the 'breakdown' of classical general relativity, as some would have it. In the process, I shall examine whether b-completeness is wholly consistent with some of the explicit sentiments behind using curve incompleteness as a criterion for singular structure.

Two types of worries, one psychological, the other physical, give rise to the dissatisfaction with the existence of incomplete curves in relativistic spacetimes. Trying to imagine the experience of an observer traversing one of the incomplete curves provokes the psychological anxiety, for that observer would, of necessity, be able to experience only a finite amount of proper time's worth of observation, even were he, in Earman's evocative conceit, to have drunk from the fountain of youth. The physical worry arises from the idea that particles could pop in

[15] R. Geroch stressed this point to me in a conversation in which he also dismissed the adequacy of his own g-boundary construction *merely because* it gave unphysical results in the admittedly contrived example of Geroch, Can-bin, and Wald (1982). It gives very nice results in almost all other known types of examples.

and out of existence right in the middle of a singular spacetime, and spacetime itself could simply come to an end with no warning, though no fundamental physical mechanism or process is known that could produce such effects. These two types of worries are not always clearly distinguished from each other in discussions of singular structure, but I think it important to keep in mind that in fact there are two distinct types of problems envisaged for incomplete curves, requiring to some degree two separate sorts of responses.

The existence of incomplete spacelike curves is often felt not to be so objectionable as that of incomplete timelike or null curves, on the grounds that it represents structure beyond the experience of any observer.[16] I submit that, on this criterion, neither ought one be so bothered by the existence of incomplete timelike or null curves, *for an observer traversing such a curve will never experience the fact that he has only a finite amount of proper time to exist*—there is no spacetime point, no event in spacetime, that corresponds to the observer's ceasing to exist. This is not to say that the person traversing this worldline cannot surmise the fact that he has only a finite amount of time to exist, rather that there will never be an instant when the observer experiences himself dissipating, popping out of existence as it were.

In a paper on the foundations of quantum mechanics, discussing the lack of an explicit representation in general relativity of our experience of a privileged instant in our history, the 'now', Stein (1984, p. 645) makes a remark most *à propos* to the present case: "...although relativity does not give us a *representation* of that experience, there is no *incompatibility* between the experience and the theory: a gap is not a contradiction."[17] There is a gap between the raw materials the theory provides us and the rich content of our experience to be explained—but it is no flaw of general relativity that it does not illuminate the experience it predicts for an observer, no more than Newtonian mechanics fell short in so far as it did not show why I understand by certain irritations of my eardrum from perturbations in the ambient air pressure the import of the spoken word 'gap'. It cannot be an argument against general relativity that it predicts phenomena we find it difficult to envisage.

These considerations suggest a tension between the definition of singular structure by b-incompleteness and the intuitions that drove some to look to incomplete curves as marks of singular structure in the first place. Only the finitude of proper time matters so far as the experience of a possible observer goes—a generalized affine parameter has no clear physical significance—but, while a curve's being b-incomplete implies that the curve is of finite total proper time, the converse is not true: timelike curves of unbounded total acceleration in Minkowski space can be of finite total proper time and yet be b-complete. I would even say that such a curve should be more disturbing on reflection to those with such intuitions than an incomplete null geodesic, for the concept of 'proper time' does not apply to null curves at all, even though they are the possible paths of massless particles.

[16] See, *e.g.*, Hawking and Ellis (1973, §8.1).

[17] The italics are Stein's. The point Stein makes with this remark is somewhat different than the point I wish to extract from it, but they are akin enough for my profitable use of it.

I speculate, with no hard evidence, that people have not wanted to count such curves as constituting singular structure because of vague worries about energy conservation: it seems initially plausible that no observer nor particle could traverse such a path, as it would require an unlimited amount of work to do so. In general relativity, however, there is no 'energy conservation'—there is not even a general, rigorous, invariant definition of 'energy'![18] There is thus no *a priori* reason to suspect that anything in the structure of general relativity excludes a particle's getting shot out asymptotically 'to infinity' in finite total proper time, having started from perfectly regular (in whatever sense of that term one likes) initial data. An example of a spacetime that was b-complete for all timelike curves of bounded total acceleration but not for timelike curves of unbounded total acceleration would clarify some of these issues, and I conjecture that examples of such spacetimes exist. Those who would not want to count such a spacetime as singular would be forced to give up b-incompleteness as the criterion for singular structure—which, given the lack of a clear physical interpretation of b-incompleteness in general, as opposed to incompleteness with respect to total proper time, I would not mind. Of course, if timelike curves of unbounded total acceleration and finite total proper time were to constitute singular structure, then every solution to Einstein's field equations would be singular. Many would reject this conclusion out of hand, but it does not seem so unbearable to me. Singular structure would simply be one more type of global structure that all spacetimes necessarily had, along with, *e.g.*, paracompactness. Once so much was settled, then one could further classify spacetimes, according to the needs of the project at hand, by satisfaction of various more restrictive types of completeness.

On physical grounds, the prediction of curve incompleteness has been objected to because it seems to imply that particles could be 'annihilated' or 'created' right in the middle of spacetime, with no known physical force or mechanism capable of pulling off such a feat.[19] The demand that a spacetime be maximal, *i.e.* have no proper extension, often rests on similar considerations: Clarke (1975, pp. 65–6) and Ellis and Schmidt (1977, p. 920) conjecture that maximality is required by the lack of a physical process that could 'cause' spacetime to draw up short, as it were, and not continue on as it could have, were it to have had an extension. This sort of argument, though, relies (implicitly) on a picture of physics that does not sit comfortably in general relativity: that of the dynamical evolution of a system. From a natural point of view in general relativity, spacetime does not evolve at all. It just sits there, sufficient unto itself, very like the Parmenidean One. From this point of view, the question of a physical mechanism capable of 'causing' the spacetime manifold not to have all the points it could have had, as it were, becomes less poignant, perhaps even misleading. Of course, an opponent of this point of view could argue that such a move could foreclose the possibility of deterministic physics, to which I would whole-heartedly agree, for we already know that general relativity does

[18] See Curiel (1998).

[19] *Cf.*, *e.g.*, Hawking (1967, p. 189).

not guarantee deterministic physics: there may be no Cauchy surface in our spacetime, or there may even be so-called naked singularities.[20]

Perhaps a more serious worry is that such a viewpoint would seem to deny that certain types of potentially observable physical phenomena require explanation, when on their face they would look puzzling, to say the least. Were we to witness particles popping in and out of existence, the mettle of physics surely would demand an explanation. I would contend in such a case, however, that a perfectly adequate explanation was at hand: we would be observing singular structure. If there were no curvature pathology around, such a response might appear to be ducking the real issue, *viz.* why is there this anomalous singular structure when all our strongest intuitions and metaphysical principles tell us it should be impossible? Far from ducking the issue, the viewpoint I advocate is the only one I know of that gives a toehold on looking for precise answers to such questions—or, more precisely, on making such questions precise in the first place. Those who balk at this viewpoint ought be as equally troubled by the big bang singular structure as they are by the example under discussion, for it just as surely 'lacks an explanation'. From the viewpoint I advocate, questions about what happened 'before' the big bang, or why the universe 'came into being', can come from their present nebulosity into sharper definition, for they become questions about the presence of a certain global structure in the spacetime manifold, in principle no different from paracompactness, connectedness or the existence of an affine connection, and one can at least envisage possible forms of an answer to the question, 'Are there any factors that necessitated spacetime's having such and such global structure?'. And were we actually to observe particles popping in and out of existence, we could formulate and begin trying to answer the analogous questions.

The most serious problem I can imagine for the viewpoint I advocate is that of representing our subjective experience, experience that seems inextricably tied up with ideas of evolution and change. I suggest that this problem is not an idiosyncracy of the viewpoint I advocate, but in fact arises from the character of general relativity itself: 'dynamical evolution' and 'time' are subtle and problematic concepts in the theory no matter what viewpoint one takes, as attested by the most notorious and seemingly intractable problem in the drive to 'quantize' gravity, the so-called problem of time.[21] My viewpoint has the virtue of calling attention to this very fact, that, to judge by the preponderant mass of literature in both physics and philosophy, is easily overlooked: general relativity stands in as much need of an 'interpretation' as does quantum mechanics,[22] and in its own way is at least as counter-intuitive and marvelous, as reflection on the nature of singular structure reveals.[23]

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[20] See Earman (1995, ch. 3) for a discussion of these phenomena.

[21] See Kuchař (1992) for a thorough discussion of this problem.

[22] Belot (1996) reaches a similar conclusion, from quite different considerations.

[23] I thank R. Geroch and D. Malament for stimulating conversations on all these topics. I am also grateful to M. Dorato for writing a review of Earman's book that made me realize the need to reread it and think more about singular structure.

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