Singularities and Scalar Fields. Matter Theory and General Relativity.

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Abstract

Philosophers of physics should be more attentive to the role energy conditions play in GR. I review the changing status of energy conditions for quantum fields – presently there are no singularity theorems for semiclassical GR. So we must re-evaluate how we understand the relationship between GR, QFT and singularities. Moreover, on our present understanding of what it is to be a "reasonable" field, the standard energy conditions are violated classically. Thus the singularity theorems are unavailable for classical GR. Our understanding of singularities in GR turns on delicate issues of what it is to be a matter field – issues distinct from the content of the theory.

Introduction

Since the earliest days of General Relativity[1] (GR) it has been clear that the theory admits singular solutions. For a discussion of this state of affairs, see for example Earman[4]. For many years the consensus in the physics community was that these singular solutions were in some sense spurious. All known solutions involving singularities had been constructed by using exact symmetries that were expected to be absent in any physically plausible spacetime. Things changed dramatically in 1965 when Penrose[16] proved the first singularity theorem that did not rely on exact initial symmetries and showed that, in generic cases of gravitational collapse, singular behavior is to be expected.

Penrose's proof established that the following assumptions are inconsistent:

(i) Spacetime M is a time-orientable 4-manifold with non-singular, Lorentzian metric;

(ii) M is null-geodesically complete;

(iii) M possesses a (non-compact[2]) Cauchy surface;

(iv) Everywhere in M, for all timelike vectors t^a, $T_{ab}t^at^b \geq 0$ where T_{ab} is the stress-energy tensor[3];

(v) M contains a closed, trapped 2-surface T.

One notices immediately that, expressed in this form, (i) is essentially GR without Einstein's Field Equations (EFEs). The addition of the EFEs will produce (v) in regions of high matter density. Thus (ii) - (iv) are the parts of the theorem that, strictly speaking, go beyond GR. Condition (iii) is of a special character because we normally think of Cauchy surfaces as essential to determinism, and we take determinism to be a key desirable feature of adequate physical theories. So the real conflict is between conditions (ii) and (iv). I.e. given GR and Cauchy surfaces, if $T_{ab}t^at^b \geq 0$ (for timelike t^a — the weak energy condition) then

[1] For my purposes, GR is the theory of a "nice" 4-manifold with Lorentz metric satisfying the Einstein Field Equations.

[2] As Hawking and Ellis[11] note, this requirement is eliminable.(p. 265)

[3] Actually, what we really need from this condition is that $R_{\mu\nu}t^\mu t^\nu \geq 0$. Since this allows us to infer that the geodesics encounter caustics as they are extended from the closed-trapped surface — the focusing of nearby null-vectors. $R_{\mu\nu}t^\mu t^\nu \geq 0$ follows from either the strong or the weak energy condition. The energy conditions are discussed below

spacetime is not null-geodesically complete.

Many subsequent discussions of the status of singularities in GR tend to downplay the role of energy conditions in the singularity theorems. Those that mention them invoke claims similar to Wald's[25] that all "physically reasonable classical matter" (p. 218) satisfies these conditions, and so to insist on them does little to go beyond GR.

I will argue that philosophers of physics especially should be very interested in the role that energy conditions play in GR, and that they should think carefully about what it is to be a "reasonable" matter field. In §1 I review the changing status of energy conditions in quantum field theory (QFT). In §2 I evaluate their current status and point out that, presently, there are no singularity theorems for semiclassical GR. I suggest that such theorems are not forthcoming, but I argue that we must, in any case, re-evaluate how we understand the relationship between GR, QFT and singularities. In §3 I argue that, on our present understanding of what it is to be a "reasonable" field, all of the energy conditions are violated *classically.* Thus the singularity theorems are unavailable for classical GR. Again it is clear that our understanding of the status of singularities in GR is incomplete. I conclude with mention of some obvious ways that our understanding of singularities in GR turns on delicate issues of what it is to be a matter field. I argue that these issues are distinct from the content of the theory, and thus GR does not "predict the presence of singularities."

1 Energy Conditions

After the singularity theorems began to appear, two distinct positions developed[4]. 1. There are real singularities in our universe. There are geodesics that just *end.* There are points that are in some sense "nearby" that are also on the very boundary of the universe. In the case of the initial singularity, the universe was once confined to a single point — whatever that might mean. GR is the correct theory of classical matter, warts and all. This position exemplifies what Penrose dismisses as the "I'm alright Jack," response to singularities

[4]This presentation of the dichotomy is parallel to that of Earman[4] and Belot, Earman and Ruetsche[2]. A much more complete discussion may be found in the former.

— a position espoused by Misner[14] who, considering the initial singularity, suggested that "the Universe is meaningfully infinitely old because infinitely many things have happened since the beginning."(186) It isn't entirely clear what Misner means by this. But it is an intriguing remark, and he does make it clear that he isn't worried about the universe begining in a singular state a finite time ago. 2. GR is incomplete or incorrect. The presence of singular points is an indication that the theory is invalid over its presumptive domain of applicability. The assumption is that GR will have to be replaced with a better theory or augmented, with QM for example, to correct its defects. A full theory of quantum gravity will rule out singularities in our universe. To do this, quantum gravity will have to modify Einstein's equation in some way (or change some of the assumptions about the nature of the manifold), or violate the energy conditions. Absent a full theory of quantum gravity, the prospects for the former route to eliminating singularities cannot be assessed. However, considerable work has been done exploring the status of the energy conditions in semiclassical GR.

I here recall the entire gamut of pointwise energy conditions of interest to General Relativists[5], comment on their failure to hold in semiclassical GR, and outline some proposals for more general replacements that still allow the demonstration of singularity theorems in semiclassical GR:

- Trace Energy Condition (TEC): $T_a^a \geq 0$
- Strong Energy Condition (SEC): $(T_{ab} - \frac{T}{2} g_{ab}) t^a t^b \geq 0$ for all timelike vectors t^a
- Null Energy Condition (NEC): $T_{ab} t^a t^b \geq 0$ for all null vectors t^a
- Weak Energy Condition (WEC): $T_{ab} t^a t^b \geq 0$ for all timelike vectors t^a
- Dominant Energy Condtion (DEC): $T_{ab} t^a t^b \geq 0$ and $T_{ab} t^a$ is not spacelike for all timelike vectors t^a

The first of these conditions was known to be violated by quantum fields as early as 1961[29]. Since then a variety of models of quantum field theory has been shown to violate, in one way or another, all the pointwise energy condtions. Indeed, two months before Penrose submitted his proof of the first singularity

[5]These conditions and their significance can be found in any reference to GR, I use the conventions of Visser[23]. Also I include for completeness the TEC, so my list is that of Visser and Barceló[24].

theorem, Epstein, Glaser and Jaffe[6] had submitted a paper showing that the energy condition used in that proof cannot always be satisfied for quantum fields.

In 1973, Parker and Fulling[15] constructed a class of solutions to the EFEs, using quantized matter, that was similar to standard big-bang cosmologies but did not necessarily display singular behavior.

Then, in 1975, a decade after the demonstration of the principle singularity theorems and their generalization by Hawking and Penrose[12], Hawking[10] showed that black holes were not so black as (or perhaps even more black than) had been thought and that they radiate in a black-body spectrum. Hawking's demonstration relied on the, by then well-known, non-positivity of energy in QFT. In particular, he was able to show that fluctuations in the energy near the boundary of the black hole were capable of producing radiation modes that propagate to spatial infinity. Consequently the black hole must absorb negative energy, and so it must be decreasing in mass. Hawking did notice the potential contradiction between the non-positivity of energy and the singularity theorems they underwrite. His verdict was that these violations are too small to untrap any surface within the black hole, and that collapse would not be impeded. One might construe this as the first reference to the so-called averaged energy conditions in GR. Of course Hawking was concerned in this case with one particular mode of energy condition violation. His conclusions about the smallness of the effect apply only to the specific process of Black Hole radiation — not to quantum fields in general.

Since 1975, considerable work has been done on averaged energy conditions for quantum fields in general. Because all the classical singularity theorems rely on pointwise energy conditions, and because they are all violated for a large class of quantum fields, the antecedents of none of the classical singularity theorems hold for these fields. To rectify this and other problems, versions of the energy conditions were produced that characterize its average behavior, and the singularity theorems were re-proven for these new conditions. These conditions take forms similar to the following — the averaged strong energy condition (ASEC): $\int (T_{ab} - \frac{T}{2} g_{ab}) t^a t^b d\lambda \geq 0$ along every complete causal geodesic $\gamma(\lambda)$ with affine parameter λ and tangent t^a. As far as I know, the ASEC was the first explicit reference to an averaged energy condition. It was proposed by Tipler[22] who showed that Hawking and Penrose's generalized singularity theorem could be reproven using the ASEC[6]. Somewhat later Roman[18, 19] showed that the AWEC (obtained by replacing the strong energy condtion in the above integral with the weak energy condition) sufficed to reprove Penrose's original singularity theorem. This, and similar, work showed that the AECs are a powerful antidote to some of the weird physics that infects theories of quantized matter fields.

2 Energy conditions and semiclassical GR

The question naturally arises: "But do the AECs hold for quantized fields?" In 1991 Klinkhammer[7][13] undertook to found a research program investigating the conditions under which the averaged energy conditions hold. His initial results were discouraging. He discovered that, although the averaged energy conditions hold in Minkowski spacetime for any free, quantum, scalar test field, there are states even in flat spacetime (with a cylindrical topology) that violate the averaged weak energy condition. Since that time a number of different results have appeared detailing the conditions under which averaged energy conditions of one kind or another hold, and investigating new kinds of energy conditions to replace those that don't hold.

Two main lines of attack have developed. The first is Yurtsever's[28] straightforward generalization of the AECs. He considers inequalities of the form $\beta(k) \equiv \inf_\omega \int_\gamma \langle\omega|T_{ab}|\omega\rangle k^a k^b dv$. He says that $\langle T_{ab}\rangle$ satisfies the generalized ANEC along γ if $\beta(k) > -\infty$ (k^a is a given tangent vector along a complete null geodesic γ). Various bounds can be put on how negative such integrals must be before, e.g., the focusing of null geodesics (the heart of the singularity theorems) fails. So if inequalities of this form obtain then bounds can be put on the magnitude initial focusing can have before a singularity is guaranteed. It is not yet clear whether these inequalities *do* hold or if their bounds are in fact satisfied for the trapped surfaces within black holes.

Ford and Roman[9] have instead investigated what they call quantum inequalities. These are expressions of the form $|F| < (\Delta T)^{-2}$. Here $|F|$ is the magnitude of the negative energy flux and ΔT is its duration. The significance of these inequalities is that the magnitude of EC violation can be bounded, and so a

[6]Tipler also explicitly considered the Parker/Fulling model and dismissed it because it was not clear that their result would hold for any reasonable quantum state. However Rose[20, 21] was able to relax some of the special assumptions of Parker and Fulling and still obtain a bounce-back solution. He still had to make some special assumptions on the quantum state to obtain the same behavior for finite temperatures.

[7]Klinkhammer is also a good reference to the early attention to ECs in quantized theories and the efforts to preserve various results from the classical theory.

characterization of its importance can be assessed. Currently, the conclusions to be drawn from these efforts are not clear. While these inequalities hold on a wide variety of manifolds and for a wide variety of quantum states, no proofs of any singularity theorems have been derived from them. Ford and Roman also show that there is a deep connection between their approach and Yurtsever's. Yurtsever[27] has also noticed this and has used his techniques in order to prove some more extensive difference inequalities involving these quantum inequalities.

The second line of attack generalizes the AECs in a different direction. For example, Flanagan and Wald[7] suggest smearing the integrals used in ANEC with test functions over space-like sections transverse to the geodesics in the ANEC. They show that back reaction terms (derived from imposing the semiclassical EFEs on the quantum fields) can enforce a new class of energy condition in certain cases. They prove positivity of this condition in perturbations about the flat Minkowski metric (except in actual Minkowski space, where it vanishes). They conclude that, for example, macroscopic wormholes are ruled out by this smeared ANEC.

Once again, though intriguing and of great significance for our understanding of the constraints on quantum fields, these results do not establish the focusing of null geodesics required in the singularity theorems. So what is the final story on the singularity theorems in semiclassical GR? That story is still being written. My view of the matter is influenced by an interesting inverse relationship between the simplicity of the topology considered and the strength of the energy conditions that have been shown to hold. I don't have time to consider this here, but such a connection indicates to me that energy conditions strong enough to guarantee singularity theorems may not hold for general 4-d spacetimes satisfying the EFEs. I must emphasize that this is merely an impression — it has not been established.

At present the status of singularities in semiclassical GR is open. What should, however, be clear is that their status depends crucially on the details of QFT and its interaction with GR. In the face of the tremendous difficulties producing acceptable energy conditions satisfied by quantized matter (to say nothing of the explicit models of semiclassical GR without singular behavior), it is necessary to rethink our views on the significance of these theorems. Some suggestions in this direction will be made in the conclusion.

3 Energy conditions and classical GR

Do the results of the preceeding two sections have any bearing on how we understand singularities in classical GR? Did we not already think that quantum mechanics would change dramatically our views on the structure of spacetime? Here, however, we are presented with a great irony. For the investigations into the behavior of quantum fields have shed light as well on the nature of classical matter. It turns out that classical scalar fields can themselves violate all the classical pointwise energy conditions. Indeed, these fields can violate the averaged energy conditions as well. They may violate the energy conditions, and this violation may be arbitrarily great. That classical scalar fields can violate the energy condtions has been known for some time. Both Ellis[5] and Bergmann and Leidnik[3] constructed explicit solutions to the EFEs using classical scalar fields. Both of these solutions avoided singularities, thus they must violate at least some of the classical ECs. Indeed, Ellis' referree complained about his solution precisely because it *did* violate the classical posititivty of energy. So why is the opinion that classical matter necessarily satisfies the energy conditions still so prevalent? In 1995 for example, Yurtsever[27] claims that "the energy conditions (or, more precisely, at least the weak energy condition) are universal in the sense that (i) they are obeyed by the classical stress-energy tensors of all matter fields."(p. 5797) Perhaps the opinion is still prevalent that the fields that violate the energy conditions are "unphysical" in some sense.

Visser and Barceló[8][24] devote considerable attention to the status of scalar fields in modern physics. They argue persuasively that, from the point of view of theoretical physics, scalar fields are indespensible. They discuss four different scalar fields that are reasonably well established theoretically and two more that receive considerable attention from theoretical physicists. Admittedly, of these six, all but one are quantum fields, and the sixth is the "Brans-Dicke scalar," and so involves a modification of GR rather than GR itself. However, the issue is not so much the observational status of classical scalar fields. Rather, the issue concerns the reasonableness of scalar fields as a model for matter. The fact that so much indirect evidence exists for scalar fields and the fact that these fields are considered essential to the majority of physicists indicate that, as a model of matter, scalar fields should now be considered physically "reasonable."

[8] See also Visser[23].

My claim that scalar fields should now seem reasonable receives further justification from the nature of the work that has been done on AECs in semiclassical GR. The first fields known to avoid singularities were scalar. Moreover, it is precisely the scalar fields that have proven so intractable in yielding to new, more general ECs. That scalar fields are taken so seriously, by those for whom a major desiderata is *ruling out their effects*, indicates that these fields cannot simply be ruled unphysical by fiat.

The first violations of the ECs were noticed by considering fluctuations and coherent states of quantum fields. These violations led to considerable activity by physicists to isolate these violations to regimes that posed little danger to the singularity theorems of GR[9]. As I pointed out in §2, the results of this activity are not at all clear. But along the way the tide has apparently turned in favor of scalar fields in physics, and older work that shows how to violate the ECs using classical fields has become relevant to our understanding of the structure of spacetime.

It it thus necessary to repudiate the received wisdom that GR implies the existence of spacetime singularities. It does no such thing. It is constraints on the stress-energy tensor of matter that, in conjunction with EFEs, may imply that a given spacetime is singular. But it must be emphasized that these constraints do not arise from GR. They must be added by hand, so to speak, in accordance with our best view of what fields are possible on spacetime. And our current best view includes scalar fields.

We are now confronted with the somewhat counterintuitive situation (at least from the point of view of those who expected quantum theory to solve our singularity problems) that the status of singularity theorems is up in the air for semiclassical GR, but for classical GR they are clearly unavailable without special pleading or convincing arguments that scalar fields are classically unacceptable.

Conclusion

Whatever the final story that emerges from this flurry of activity, some features of it are already clear. First, the ANECs for classical fields can be violated arbitrarily strongly. Second, the ANECs for quantum fields are

[9]Actually most of this activity seems to be directed toward eliminating the possibility of wormholes, closed timelike curves, negative ADM mass and other weirdnesses. This is not really relevant to my discussion.

also violated. The large scale structure of GR is heavily influenced by the details of the fields on spacetime, and this both classically and quantum mechanically. The singularity theorems of classical and semi-classical GR rely on constraints on these fields. These constraints are not, strictly speaking, part of GR. Nor are they obviously true. There are reasonable looking classical fields that violate every proposed variant of the energy conditions. Quantum mechanically things are murkier, no version of an acceptable energy condition is know to hold for quantized scalar fields. This latter situation may change as further work is done on Flanagan and Wald's "complementary" ANECs.

Some complications for how we understand the relationship between GR and QM follow readily from the considerations of the previous sections:

1. Some standard arguments for quantizing gravitation theory rely on the claim that something goes wrong with our understanding of spacetime at very short length scales/very high curvature. The existence of singularities is taken to be evidence that something goes drastically wrong with GR at such length scales. The claim is that a quantum gravity will smooth this out. If "realistic" spacetimes do not contain singularities, the necessity to smooth out the short scale structure of GR is absent. Thus a strong motivation for quantization is lost. There are, of course, may other arguments for quantizing the gravitational field. However the kind of unitarity violation introduced by evaporating black-holes with central singularities is particularly virulent.

2. Penrose has long held that GR and QM are closely connected at some level. For example in his 1967 Battelle Rencontres lecture he says: "there *is* a deep connection between quantum theory and general relativity, so that it may actually be a mistake to attempt to build the subjects up separately." [17](132) The idea that specific models of quantum fields can play such a decisive role in the singularity structure of GR is perhaps a justification for such a view. Certainly we *can* build up GR separately from QM, but by so doing, we may end up with a skewed picture of the physical world.

3. Because matter fields can so alter the understanding of singularities, a major part of our research into GR, it is best to seek a quantum gravity that is divorced from theories of matter entirely. Only in this

way will we get a true picture of the nature of quantized general relativity. Once we have such a picture we can go back and put in the specific characteristics of matter that are relevant to our universe.

4. Because matter fields can so alter the understanding of singularities, a major part of our research into GR, it is essential that it be quantized using the correct matter fields, else our picture of quantum gravity is likely to be extremely skewed. See item 1.

I cannot here evaluate the strengths and weaknesses of these various positions, nor do I claim to have made much headway in enumerating the possible responses to an awareness of the profound impact matter theories can have on our understanding of GR and GR coupled to a quantum theory of matter. But this list should give some idea of how fruitful this awareness can be for philosophical analysis (as well as physical theorizing).

To end on a provocative note, I now propose a conjecture consistent with what we know about matter fields classically and quantum mechanically[10]. It is in direct opposition to the singularity theorems.

The Unbounded Affine Parameter Conjecture:

- All inextendible geodesics attain arbitrarily large values of their affine parameter.

Now recall the dichotomy of responses to the singularity theorems in §1. The conjecture stakes out a third position. 3. GR is correct and there are no singularities. By drawing whatever implications are necessary for this third option to hold, we can learn something fundamental about the constitution of matter fields on spacetime. In this sense, we can consider GR to be a more powerful theory than we thought. Despite its refusal to comment directly on the nature of matter, GR, via Einstein's equation and the UAP Conjecture, entails a breakdown in the positivity of the energy of matter fields on spacetime. Moreover, it entails that, in regions of very high curvature, inside a region containing closed trapped surfaces, for example, there are scalar fields that prevent the matter from undergoing final, singular collapse.

[10] This remark may be a little too glib. It is possible that any spacetimes immune to the singularity theorems are, by virtue of the fields required for this immunity, plagued with other classes of singularity. Indeed the only stable, ANEC violating solution to the semiclassical EFEs that I know of contains traversable wormholes and naked singularities. These features are, clearly, as disagreeable to our understanding of causality and determinism as singularities from gravitational collapse, if not more so. Again, no general picture of ANEC violating spacetimes is available, so no definite conclusions can be drawn.

I think the conjecture probably fails. And indeed, classically, it seems to serve no purpose other than to pacify those who find singularites aesthetically repugnant[11]. I suggest the conjecture only as a foil to direct attention toward the need to separate our thinking about the global structure of GR from our thinking about the nature of matter fields.

What determines the fields that are present in spacetime? Are there fields with consistent descriptions from the point of view of QFT that have no existence in our universe? What prevents this? Is a fully worked out theory of everything supposed to tell us, in addition to *what* fields there are, also, in some appropriate sense, *why* these fields are? Would it be enough of an answer to say that some properties of the world can hold *only* if fields of a certain type (say scalar fields) exist? This might be a kind of generalized anthropic principle — or perhaps a transcendental deduction. Would such a demonstration shed light on the extent to which the global causal structure of a theory like GR is constrained?

No doubt, in doing GR, I *can* impose various constraints on the fields in spacetime. But when I do so and the spacetime turns out singular, what is the appropriate response? Do we say that GR is incomplete because it is *classical*? Or do we say that it is incomplete because it has improperly constrained the fields it admits on spacetime? We know that GR is incomplete. It is not a theory of matter. The real issue is how we understand the implications of this incompleteness and what we do to augment the theory.

But if we wish to consider GR *simpliciter*, we now have no grounds for asserting that "realistic" spacetimes are singular. GR doesn't speak to fields — "realistic" fields on spacetime are outside its purview.

Finally, a caveat: There is as yet, as far as I'm aware, no demonstration that Flanagan and Wald's transverse ANECs are too weak to rule out non-singular gravitational collapse. This issue is open. On the other hand, the Hadamard condition on quantum states[12] (required for the Flanagan and Wald transverse ANECs) is itself in a position similar to that of the energy conditions in the late 1960s. That is to say, the Hadamard condition is known to capture some features of quantum states that we take to be essential for

[11] It may however solve some problems that arise in semiclassical GR. For example, black hole radiance is not, by itself, responsible for the Hawking information loss paradox. Rather it is the propagation of information into the singularity (the loss of correlation structure) that generates the paradox. No singularities means no unitarity violation.

[12] For definition, discussion and further references, consult Wald[26].

the consistency of semiclassical GR. However, it is not clear that only Hadamard states have this property. Further work, similar to the investigations of the original energy condition assumptions, is necessary to establish the Hadamard condition for reasonable quantum fields.

References

[1] Barceló, C. and M. Visser (1999) "Traversable wormholes from massless conformally coupled scalar fields". Preprint available at xxx.lanl.gov gr-qc/9908029.

[2] G. Belot, J. Earman and L. Ruetsche (1999) "The Hawking Information Loss Paradox: The Anatomy of a Controversy" *The British Journal for the Philosophy of Science* **50**: 189-229.

[3] Bergmann, O. and R. Leipnik (1957) "Space-Time Structure of a Static Spherically Symmetric Scalar Field" *Physical Review* **107**: 1157-1161.

[4] Earman, John (1995) *Bangs, Crunches, Whimpers, and Shrieks*. Oxford: Oxford University Press.

[5] Ellis, Homer (1973) "Ether flow through a drainhole: A particle model in general relativity", *Journal of Mathematical Physics* **14**: 104-118.

[6] H. Epstein, V. Glaser and A. Jaffe (1965) "Nonpositivity of the Energy Density in Quantized Field Theories", *Nuovo Cimento* **36**: 1016-1022.

[7] Flanagan, É. and R. Wald (1996) "Does back reaction enforce the averaged null energy condition in semiclassical gravity?", *Physical Review* **D 54**: 6233-6283.

[8] L. Ford, M. Pfenning and T. Roman (1998) "Quantum inequalities and singular negative energy densities", *Physical Review* **D 57**: 4839-4846.

[9] Ford, L. and T. Roman (1995) "Averaged energy conditions and quantum inequalities", *Physical Review* **D 51**: 4277-4286.

[10] Hawking, Stephen (1975), "Particle Creation by Black Holes", *Communications in Mathematical Physics* **43**:199-220.

[11] Hawking, S. and G. Ellis (1973), *The Large Scale Structure of Space-time*. Cambridge: Cambridge University Press.

[12] Hawking, S. and R. Penrose (1970), "The singularities of gravitational collapse and cosmology", *Proceedings of the Royal Society of London* **A314**: 529-548.

[13] Klinkhammer, Gunnar (1991), "Averaged energy conditions for free scalar fields in flat spacetime", *Physical Review* **D 43**: 2542-2548.

[14] Misner, Charles (1969) "Absolute Zero of Time" *Physical Review* **186**: 1328-1333.

[15] Parker, L. and S. A. Fulling (1973), "Quantized Matter Fields and the Avoidance of Singularities in General Relativity", *Physical Review* **D 7**: 2357-2374.

[16] Penrose, Roger (1965), "Gravitational Collapse and Space-time Singularities", *Physical Review Letters* **14**: 57-59.

[17] Penrose, Roger (1968), "Structure of Spacetime", in DeWitt, C. and J. Wheeler (eds), *Battelle Rencontres: 1967 Lectures in Mathematics and Physics*, New York: W. A. Benjamin, Inc.

[18] Roman, Thomas (1986) "Quantum stress-energy tensors and the weak energy condition", *Physical Review* **D 33**: 3526-3533.

[19] Roman, Thomas (1986) "On the "averaged weak energy condition" and Penrose's singularity theorem", *Physical Review* **D 37**: 546-548.

[20] Rose, B (1986) "Construction of matter models which violate the strong energy condition and may avoid the initial singularity", *Classical and Quantum Gravity* **3**: 975-995.

[21] Rose, B (1987) “A matter model violating the strong energy condition — the influence of temperature”, *Classical and Quantum Gravity* **4**: 1019-1030.

[22] Tipler, Frank (1977) “Energy conditions and spacetime singularities”, *Physical Review* **D 17**: 2521-2528.

[23] Visser, Matt (1995) *Lorentzian Wormholes: From Einstein to Hawking*. Woodbury, NY: AIP Press.

[24] Visser, M. and C. Barceló (2000) “Energy Conditions and Their Cosmological Implications”, plenary talk delivered at Cosmo99, Trieste, Sept/Oct 1999. Preprint available at xxx.lanl.gov gr-qc/0001099.

[25] Wald, Robert (1984), *General Relativity*. Chicago: University of Chicago Press.

[26] Wald, Robert (1994), *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. Chicago: University of Chicago Press.

[27] Yurtsever, Ulvi (1995) “Averaged null energy condition and difference inequalities in quantum field theory”, *Physical Review* **D 51**: 5797-5805.

[28] Yurtsever, Ulvi (1995) “Remarks on the averaged null energy condition in quantum field theory”, *Physical Review* **D 52**: R564-R568.

[29] Zel’dovitch, Ya. (1961) *Zh. Eksp. Teor. Fiz.* **41**:1609 [*Sov. Phys. JETP* **14**:1143 (1962)]. Cited in [13].