Helmholtz’s Naturalised Conception of Geometry

and his Spatial Theory of Signs

In 1917, *Die Naturwissenschaften* published a short review of two lectures of Hermann von Helmholtz on Goethe, in which the reviewer called particular attention to Helmholtz’s views on epistemology and their relation to Kant, concluding with the appeal: „Dear reader! To summarise would be profane. Read them yourself!“ The editors of the 1921 edition of Helmholtz’s writings on epistemology were similarly enthusiastic in recommending his epistemological work, which, they emphasised, examined the „problems of knowledge in general ... from the aspect which concerns the physiology of the senses:“ „May both scholars and students always continue to be able to find a rich source of intellectual profit and pleasure in their dealings with Helmholtz the epistemologist!“ The author of the review[1] was Albert Einstein, who had read Helmholtz since his youth, and the editors of the *Epistemological Writings*[2] were Paul Hertz and Moritz Schlick. All three were products of an intellectual milieu suffused with Helmholtz’s views.

Helmholtz himself described his epistemology as a „*physiologische Erkenntnistheorie*.“ But what was that theory, and what was its particular attraction to these authors? In this paper, I will try to answer these questions by looking at his „The Facts in Perception,“ a *Rektoratsrede* held at the Friedrich-Wilhelms-Universität in Berlin on 3 August 1878, when Helmholtz took up the chair in Physics. As this lecture weaves together a number of themes, I will focus on only two: his manifold theory of perception, which construes the human perceptual field as a data-space; and

[1] (Einstein 1917)

[2] (Helmholtz 1977)

Helmholtz's use of this theory, particularly in the second appendix to the lecture, to rebut neo-Kantian criticisms of his earlier papers on non-Euclidean geometry.

The argument that (some species of) geometry is itself a physical science is probably the best known aspect of Helmholtz's philosophical work. What is not so well known, however, is the role of the perceptual theory that he advanced simultaneously, and which influenced both Wittgenstein's and the Vienna Circle's notion that our experience plays out in a phenomenological manifold, or „space." Helmholtz's use of the term manifold was not metaphorical:

> Riemann calls a system of differences in which the individual element (*das Einzelne*) can be determined by *n* measurements, an n-*fold manifold*, or a *manifold of* n *dimensions*. Thus the space that we know and in which we live is a three-fold extended manifold, a plane a two-fold, and a line a one-fold one, as is indeed time. The system of colours also constitutes a three-fold manifold, in that each colour can be represented ... as the mixture of three elementary colours, of each of which a definite quantum is to be chosen. ... We could just as well describe the domain of simple tones as a manifold of two dimensions, if we take them to be differentiated only by pitch and volume ... [3]

On this theory of perception, all data of our experience can be resolved into primitive concatenations of spatial properties with sensible properties, both of which are organised into manifolds. Spatial and sensible properties are indivorcible aspects of primitive, unanalysable experiences, and it is this fusion of the manifolds of sensibilia with the spatial manifold which underwrote Helmholtz's claim that geometry was not an *a priori* science: on this theory we have no experience or acquaintance with space that is not connected to sensible properties, and we have no concept of number without observing how the field of perception changes. This deliberate conflation of spatial- and sense-qualities is essential to Helmholtz's argument concerning geometry, and represents, as we shall see, a fundamental break with Kant.

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[3] (Helmholtz 1870), pp. 16-17.

Helmholtz called these primitive elements in the perceptual manifold *signs*, to distinguish them from representations of large-scale objects—*pictures*—which consist in temporally stable aggregates of the elements, such that every large-scale object (including, of course, a measuring-rod) is already the reflection of a temporal regularity. Propositions which assert such regularities among the occurrences in the field of elementary perceptions have objective content, whereas those describing the structure of that field itself are (at least for the subject) both atemporal and necessary. As Helmholtz put it in his second lecture on Goethe: „Only with respect to temporal sequence can sensations be pictures of objects... . Among the determinations of temporal sequence is number. In these relations they afford more than mere signs would do."[4] In calling the elements of the perceptual manifold signs, Helmholtz was only one step away from the Wittgenstein/Vienna Circle conception of scientific language. For if the elements of our experience are signs of external physical systems, and all possible experience is bounded by such a space of signs, it follows that—if our language and thought is to connect to the external world at all—then it can only be by means of a mapping (an *Abbildung*) onto that primitive sign-system. Helmholtz's theory postulated a closed data-space in which all perceptions occur as, we might say, phenomenological event-points. It allowed him a new definition of „conceivable experience" which, to use Reichenbach's words, „pioneered the solution to the problem of intuition in geometry."[5] Something is intuitively imaginable, Helmholtz claimed, when we can imagine completely the sense-impressions which „the object would cause in us ... under all thinkable conditions of observation."[6] In what follows, we shall see how this definition was applied.

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[4] Helmholtz, „Goethes Vorahnung kommender naturwissenschaftlicher Ideen," in (Helmholtz 1896), p. 358.

[5] (Reichenbach 1928), p. 78.

[6] (Helmholtz 1883), p. 644.

In 1878 Helmholtz published his fourth and last paper on geometry, entitled „The Origin and Meaning of the Geometrical Axioms," in *Mind*. This paper, which was subsequently abridged and reprinted as the third appendix to „The Facts in Perception," is philosophically the most sophisticated of the four papers, since it was written in direct response to a neo-Kantian critic of Helmholtz's earlier work. Here, Helmholtz argues that both idealists and realists must concede—if they are at all prepared to admit that our representations of the world have causes—that our simplest perceptions, since they always have both spatial and material (that is, sense-qualitative) aspects, must in turn be viewed as the effects of two kinds of unknown causes. These causes are whatever gives rise to the two aspects—the spatial and the qualitative—of our primitive experiences. As he is careful to emphasise, it is irrelevant to his argument whether they are considered to be located in the subject or in the external world. This indifference is essential to his approach, for it immunises his argument against the idealist objection that, even should the world „in itself" be non-Euclidean, the transcendental subject will always experience it through a Euclidean screen of pure intuition.

> Now we find it to be a fact of consciousness, that we believe ourselves to perceive objects which are at specific places in space. That an object should appear at a particular specific place, and not at another one, will have to depend on the kind of real conditions evoking the representation. We must infer that in order to effect the occurrence of the perception of another place for the same object, other real conditions would have had to be present. Thus in the real there must exist some or other relationships, or complexes of relationships, which specify at what place in space an object appears to us. I will call these, to use a brief term for them, *topogeneous moments*. Of their nature we know nothing; we know only that the coming about of spatially different perceptions presupposes a difference in the topogeneous moments.
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> Besides this there must, in the domain of the real, be other causes which effect our believing that at the same place we perceive at different times different material things having different properties. I will allow myself to give them the name of *hylogeneous moments*.[7]

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[7] Helmholtz, „The Applicability of the Axioms to the Physical World," in (Helmholtz 1977), pp. 159-160. I have retained Helmholtz's terms „topogeneous moments" and „hylogeneous

Topogeneous moments are responsible for an object's appearing at a given location, and not at another. The definition does not say how we identify an object, nor does it make any mention of time. In fact, it expresses a necessary connection between identity and location: if two things are identical with one another then they are, at any given time, at the same location. Hylogeneous moments account for the presence of qualitatively different things at the same place at different times. Here we assume that we do have such a criterion for identity—identical qualities, excluding spatial ones—and that two things not identical with each other must, if they are at the same place, be there at different times.

From the example that Helmholtz gives us of "something that must have a ground"—the perception: Red at this location now—and the lengthier, less abstract account in "The Facts in Perception," we know that he thinks of primitive perceptual judgments as points in a manifold of possible sensations. Space has a special role to play here, since spatial determinations show up in conjunction with all external quality-fields (*Qualitätenkreise*): I can correlate a spatial location with tactile qualities as well as with visual ones—space is the common sense. Thus Kant, at the opening of the Transcendental Aesthetic, states that space "is not an empirical concept that can be derived from external experience," because only by means of space can we represent objects to ourselves as being outside of us, individuated and alongside one another. Space grounds all of these last three aspects of experience for Helmholtz as well; however, he denies the conclusion that all spatial properties are consequently prior to experience. In essence he rejects the second part of Kant's contention that, "one can never make oneself a representation that there is no space, while one can very well think to oneself that no objects are to be encountered in it." (*Critique of Pure Reason*, B38) To think of a space empty of *qualia* is, on Helmholtz's account, possible *only if*

moments."

we prescind from the qualitative part of the internal manifold: if we do so, we prescind also from the possibility of measurement. From the fact that every *Qualitätenkreis* is associated with the spatial manifold, but not conversely, it does not follow that the latter can be thought without reference to the former: I cannot imagine a meter without imagining a meter-stick. Qualia are as much an essential part of the perceptual manifold as spatial determinations are, for no point in space is ever experienced in the absence of a sensible quality.

The real correlates of such qualia, the hylogeneous moments, are unknown, and in actual perceptions, the effects of the hylogeneous moments are always conjoined with spatial determinations. So the perceptions "white at [3, 10]", "hard at [3, 10]" may combine to form the perception that something white and hard is situated at location [3, 10]. Helmholtz's definitions serve to divide the attributes attached to a given complex of perceptions into two groups: the spatial data, on the one hand, and the other qualia on the other. Since the species of sensation are finite, we may assume that we are able to associate with each point in space a finite vector of qualities, say $[x, y, z][A, B, ..., Z]$, where the first three coordinates are spatial, the following ones indexes of various *Qualitätenkreise*. The causes of the first three coordinates in a given experience are its topogeneous moments, and those of the rest its hylogeneous moments. The properties he gives these moments follow directly from this characterisation, if we adopt the simple definition that two entities are identical if they consist of the same vector (or vectors) of qualia.

Consider first the hylogeneous moments. They account for the various material things with diverse qualities appearing at the same place at different times, that is, for the fact that we, when confining our attention to the location $[x, y, z]$ observe changes in some of the qualia $[A, \ldots, Z]$. Of course "to see another material thing" need mean nothing more than to see another set of qualities at that location: we never assumed a thing which was the bearer of these predicates except in so far as we considered the spatial location itself to be such a bearer. The matter is slightly more complicated when we come to the topogeneous moments. They account for the fact that a given *object* appears where it does: “in order to effect the occurrence of the perception of another place for the same object, other real conditions would have had to be present.”[8] In the first definition (of hylogeneous moments) Helmholtz does not speak of objects, but of material things, *stoffliche Dinge*, where the pairing of *hyle* and *Stoff* is deliberate. Changes in the hylogeneous moments associated with a topogeneous moment—changes in the qualities appearing at a given spatial location—are the changes that mark the passage of time. It is the unifying and individualising role of the topogeneous moments that makes the hylogeneous moments appear to us as *things*. If we associate with the concept “object” the entity responsible for a *set* of elementary perceptions, thus treating it as the correlate of a *completed picture* (which notion, Helmholtz argues, does not fit the elements of our simplest representations) then it is clear that “object” contains “spatially situated” as part of its definition, that a determinate object is always spatially situated. The object corresponding to $[x, y, z][A, B, \ldots, Z]$ is the entity responsible for all the elementary perceptions making up this picture. However, that there is one such entity is an unjustified inference, a consequence only of the perceptions being unified by their simultaneous situation at a single location in space. The topogeneous moments are indeed responsible for our perception of *this* object’s being at *this* location, for anything located somewhere else must need be a different object.

But if the unique spatial location is part of the definition of the object, then is the definition not trivial? It says that the topogeneous moments are responsible for an object’s appearing where it does, and I have just said that the concept “object”

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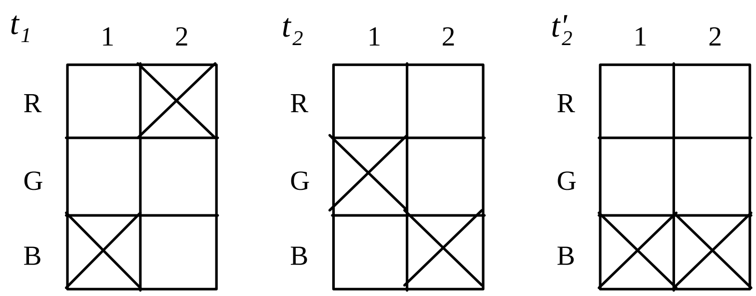
[8] Helmholtz, „The Applicability of the Axioms to the Physical World,“ p. 160.

includes spatial situation in its definition. And surely the fact that we see one and the same object in motion overturns it on the first stroke? If an object's definition includes its location, then surely by changing the location we change the definition of the thing. In answer to the first objection we should recall that the relation of the spatial coordinates $[x, y, z]$ to the coordinates of the *Qualitätenkreise* is asymmetric: the other coordinates are unified in their all being associated with the single spatial location—according to Helmholtz, "white and hard" is a concept, not an elementary perception like "white at [2, 3,4]" or "hard at [2, 3,4]". So to say that the spatial location is an essential constituent of the object, that real conditions would have to have been at hand for the same object to appear at a different place is strained, but not trivial. Helmholtz's definition refers to a single moment in time—the instant in which the observer directs her attention to a group of elementary representations—and it expresses the following thought: representations which obtain simultaneously at different places are not, that is are not experienced, as being identical. And to say that the *same* object could have appeared at a different place at this moment means: an object qualitatively identical, indistinguishable with respect to the consequences of its hylogeneous moments, but at another location. The value of the definition becomes clear when we answer the second question, and turn our attention to descriptions of moving objects, for the perception of a single object moving through space can be arrived at only by means of a regular connection between groups of the topogeneous and hylogeneous moments.

I will illustrate this connection by means of the following diagrams:

	1	2
t_1	B	R
t_2	G	B
t_2'	B	B

Each row represents two locations 1 and 2 in a finite one-dimensional space. B, R, and G are three qualities from a single *Qualitätenkreis*, which we will take, for simplicity's sake, to consist only of these three qualities. Each pair of cells in a row is associated with a time t_n—I consider initially only the first two—t_1 and t_2. I refer to the sum of these various elements as the *perceptual manifold*, the appropriateness of the name being clearer if we represent these states as:



We consider the diagrams for t_1 and t_2 to represent the movement of an object from location 1 to location 2. How are we to describe this in terms of topogeneous and hylogeneous moments? A physical process, according to Helmholtz, is the coming about of certain hylogeneous moments and their consequences (that is, the perceptions they cause) and the procession (*Ablauf*) of these moments in conjunction with groups of distinct topogeneous moments. In my greatly simplified case, we will consider the *one* consequence of some hylogeneous moments (the sensation B) conjoined in succession to the group of two distinct topogeneous moments (the locations 1 and 2). At t_1 the topogeneous moments associated with 1 are conjoined to some hylogeneous moments giving rise to the sensation of B, and together they give us the perception "B at 1” or B(1), for short. At t_2 the hylogeneous moments associated with B are conjoined to the topogeneous moments of 2, and yield the perception B(2). But these conjunctions alone need not give rise to the perception that a single object has moved from 1 to 2. That perception will only result if the perceptions at 2 and 1, at times t_1 and t_2 respectively, are correlated with this process in such a way that a single quality appears to be *displaced*, *i.e.* that B *was not* perceptible at 2 at t_1 , and *is no longer* perceptible at 1 at time t_2. If we had t_2' instead of t_2, then there would be a perception of something stretching perhaps, but not of the displacement of a single thing. And if the perceptual manifold were to change randomly, like “snow” on a television screen, we would not be able to form coherent images of any stable objects, let alone see them move in a regular manner.

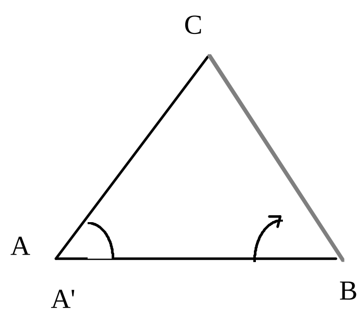
The perception of a displacement of a B[lue] object from one location to another is the consequence of an intricate interaction of topogeneous and hylogeneous moments. “What we perceive directly” are only *regularities* in ever-changing associations of such. The movement of a B object appears as the sequence: t_1: B(1), R(2); and t_2: G(1), B(2); so the hylogeneous moments associated with the sensation B *proceed* from t_1 to t_2 in conjunction with the group of topogeneous moments (1, 2), and, simultaneously other hylogeneous moments come to be in conjunction, from t_1 to t_2, with the group (2, 1). Without such regularities in the effects of the two species of moments, there will be no stable objects to observe, and it is indeed only these stable, large-scale objects that make up the world that we consciously perceive.

Now that we have an understanding of Helmholtz’s perceptual theory, we can perhaps make better sense of his account of physically equivalent processes and their relation to on the one hand *physical*, and on the other hand *pure intuitive* geometry:

> When we observe that diverse physical processes can proceed in congruent spaces in equal periods of time, this means that, in the domain of the Real, equal aggregates and consequences of specific hylogeneous moments

> can come to be and unfold in conjunction with specific definite groups of different topogeneous moments—namely those which give us the perceptions of physically equivalent parts of space. And if experience then teaches us that *every* conjunction or *every* consequence of hylogeneous moments that can exist or unfold with that one group of topogeneous moments is also possible in conjunction with every physically equivalent group of *other* topogeneous moments—well this is in any case a proposition that has a real content, and the topogeneous moments thus doubtless influence the unfolding (*Ablauf*) of real processes.[9]

"Physically equivalent" spatial magnitudes are ones in which the same processes may "exist and unfold" (*bestehen und ablaufen*) under the same conditions and in equal periods of time. The „most commonly used process for determining physically equivalent magnitudes," he goes on, „is the transport of rigid bodies, such as compasses and rulers, from one location to another." But on Helmholtz's account, such processes are represented as complexes of primitive sensations, implying changing connections between groups of hylogeneous and topogeneous moments. A statement about a rotation to the effect that, say,



"The rod A'B may be rotated about B in such a way that, without the rod AC moving, A' is made to coincide with C" will be a statement with real content, since, as we have seen, the very identity of the bodies AC and AB as well as, of course, their rigidity, depends on regularities in our perceptions—regularities in no way implied in the thin fabric of Helmholtz's definitions. But this proposition would appear, from a pure

[9] Helmholtz, „Die Anwendbarkeit der Axiomen auf die Physische Welt," in (Helmholtz 1896), p. 403 (my translation), cf. (Helmholtz 1977), pp. 160-161.

intuitive perspective, as necessarily true, and, furthermore, as being quite independent of any facts concerning the actual physical behaviour of a rotating rod.

Helmholtz now proposes a shift in perspective that allows us to see how a particular kind of consciousness might come to experience Euclidean geometry as *a priori*. He suggests that our consciousness might be related to the topogeneous moments in a two-fold manner:

> One could, *e.g.*, assume that an intuition of the alikeness [equivalence] of two spatial magnitudes might be produced immediately, without physical measurement, by the influence of topogeneous moments on our consciousness, thus that certain aggregates of topogeneous moments might even be equivalent in respect of a psychic, immediately perceivable effect.[10]

Physical measurement, as we have seen, always assumes regularities in our perceptions which manifest themselves as the appearance of stable, subsisting objects, which in turn may be moved, *i.e.* put in association with groups of spatial points (the effects of the topogeneous moments). But the spatial points never appear divested of sensible qualities, just as the sensible qualities are always experienced as being located somewhere in space. If on the other hand the topogeneous moments could act on us directly, without our needing to measure their relationships, and, furthermore, they did so in such a way that their effects on our consciousness mirrored that of the metric of Euclidean space, then we would intuit a pure geometry:

> Let us assume that the intensity of that psychic effect , whose alikeness [equality] appears in our representation as the alikeness of distance between two points, depends in the same way on some or another three functions of the topogeneous moments of any point, as the distance in *Euclidean* space depends on the three coordinates of any point. Then the system of pure geometry of such a consciousness would have to satisfy the axioms of Euclid, however else the topogeneous moments of the real world and their physical equivalence behaved.[11]

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[10] Helmholtz, „The Applicability of the Axioms to the Physical World," p. 161.

[11] Helmholtz, „The Applicability of the Axioms to the Physical World," p. 161.

The physical equivalencies referred to in the last sentence are the metric of space as determined by physical geometry, by measurement with rigid bodies in time. If both that metric *and* the metric given by the direct effects of the topogeneous moments on our consciousness were to agree with the Helmholtz/Riemann analytic description of the metric of Euclidean space, then we would have a pre-established harmony between physical and psychological space, a psycho-physical parallelism of the sort Helmholtz denied.

The kind of intuitive geometry which is here being described is a static, atemporal one. Geometric statements are always statements about "regular (*gesetz-mäßige*) connections between topogeneous moments,"[12] and in the scenario Helmholtz envisages, we have direct access to such *connections*, although of course *never to the topogeneous moments themselves*. It is as if I considered the space of my perceptions at an instant, abstracting from the other properties distributed within it, and thus also from the possibility of utilising them as a means of comparison. This form of imagination cannot peel this space off from that of my basic experiences entirely: I must be able to identify the direct and indirect effects of the same topogeneous moments with each other, otherwise the geometry based on pure intuitions of space would have no empirical referent, and I would literally be unable to apply it physically. There is here no question of the one or the other metric being "correct," that is given by the *actual* relations of the topogeneous moments to one another outside of perception, for we have no direct knowledge of them. So the two-fold relation Helmholtz describes, though it lends itself to a straightforward realistic interpretation, is, as he maintained at the outset, neutral on the question of realism or idealism. The agreement or disagreement between the two geometries could play out

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[12] Helmholtz, „Die Anwendbarkeit der Axiomen auf die Physische Welt," in (Helmholtz 1896), p. 403, cf. (Helmholtz 1977), p. 160.

entirely within an idealistic and solipsistic consciousness, without any assumption of an independent reality. The one geometry would be an atemporal one, abstracted from the elementary perceptions that make up our experiences, the other a temporal one, derived from observed regularities among these elementary perceptions.

Conclusion

Although Helmholtz stripped space of its Euclidean properties, he did so, on his own account, by stretching Kantian intuition to include „topological“ properties of the colour-space, of the auditory manifold, and of the various other *Qualitätenkreise*. This epistemological view proved most influential—certainly readers familiar with the *Tractatus* will notice similarities between the theory I have adumbrated and Wittgenstein’s notion of a logical space, as Schlick himself cannot fail to have done.[13] For both Helmholtz and Wittgenstein, the distinction between *a priori* logico-mathematical truths and empirical, natural scientific ones, is reflected in a distinction between atemporal, structural properties of a space, and temporal, empirical appearances *within* that space.

Indeed, the language of „spaces“ of experience (or indeed of „reasons“) has become common coin within analytic philosophy. The attraction of this term lies in its providing an easy way of reconciling two species of truth with one another, of giving them a shared semantic basis. Thus it is worth noting that, in Helmholtz’s epistemology, the widening of the space of experience was accompanied by an increasing emphasis on the role of time—in particular, as he himself explained in *Zählen und Messen,* on the notions of measurement and prediction, both of which he conceived of as natural, indeed physiological functions of the scientist. Helmholtz, as I mentioned at the outset, described his theory as a „*physiological* epistemology.“ This

[13] cf. (Carruthers 1990) for a phenomenological interpretation of the *Tractatus’* logic.

label expresses on the one hand the origin of the theory in his research in sense-physiology, research which was specifically concerned with the manner in which the human organism records and processes information concerning systems in the external world by means of its sense-organs. Realistically interpreted, the theory considers the organism itself as a physical system, thus as a system whose thoughts and actions are themselves *motions*. It is this temporal aspect of perceptions and measurement that determines their ‘objectivity’ for Helmholtz, and it is this fact which explains (cf. the quotation from the lecture on Goethe on pp. 3-4) why physical geometry is of use in describing the world. Measurements map numbers onto our perceptual manifold, yielding us a language for describing precisely the regular behaviour of appearances in the internal field of perception. On this view, the language of mathematical science gains its validity not because it derives from some platonic source prior to or above experience. It results instead from the physical constraints on the human organism’s construction, and from that organism’s interaction with its physical environment. For Helmholtz, to use Philip Kitcher’s phrase, geometry emerges „from what the world will let us do to it.“[14]

David Jalal Hyder
Max-Planck-Institut für Wissenschaftsgeschichte
Wilhelmstr. 44
10117 Berlin
Germany

jalal@mpiwg-berlin.mpg.de

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[14](Kitcher 1983) p. 108.

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