Empirical Finance: Methods & Applications
Individual Coursework 1

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1 Optimization Model

At each time step t, we solve the following optimization problem to find the optimal hedge weights while ensuring sparsity, integer constraints, and turnover stability.

1.1 Variables

- $R_t \in \mathbb{R}^T$: Vector of synthetic inventory returns over the past T days (constructed using today's inventory weights).
- $H_t \in \mathbb{R}^{T\times N}$: Matrix of hedging instrument returns over the past T days (each column corresponds to an asset).
- $w_t \in \mathbb{Z}^N$: Vector of integer hedge positions (number of whole shares held in each hedging instrument).

1.2 Optimization Problem

$$\min_{w_t} \quad \|R_t - H_t w_t\|_2^2 + \lambda_1 \|w_t\|_1 + \lambda_2 \|w_t - w_{t-1}\|_2^2 \tag{1}$$

1.3 Constraints

- **Integer Constraint (Whole Shares Only)**:

$$w_t \in \mathbb{Z}^N \tag{2}$$

- **Sparsity Constraint (Limit the Number of Active Hedge Instruments)**:

$$\|w_t\|_0 \leq k \tag{3}$$

  This constraint ensures that at most k instruments have nonzero positions.

- **Turnover Constraint (Reduce Excessive Trading)**:

$$\|w_t - w_{t-1}\|_2^2 \leq \tau \tag{4}$$

  This constraint limits excessive changes in hedge positions over time.

1.4 Explanation of Each Term

- $\|R_t - H_t w_t\|_2^2$ minimizes the tracking error between inventory returns and hedged returns.
- $\lambda_1 \|w_t\|_1$ encourages sparsity by reducing the number of active hedge instruments.
- $\lambda_2 \|w_t - w_{t-1}\|_2^2$ stabilizes trading by penalizing excessive changes in hedge positions.
- The ℓ_1-norm sparsity constraint directly limits the number of hedge instruments used.

Appendix

References