Computer Lab 3 Computational Statistics

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Contents

Question 1: Cluster sampling

An opinion pool is assumed to be performed in several locations of Sweden by sending inter- viewers to this location. Of course, it is unreasonable from the financial point of view to visit each city. Instead, a decision was done to use random sampling without replacement with the probabilities proportional to the number of inhabitants of the city to select 20 cities. Explore the file population.xls. Note that names in bold are counties, not cities.

1. Import necessary information to R.

```r
# set working directory
# load the data
data = read.csv2("population.csv")
data[,1] <- as.character(data[,1])
```

2. Use a uniform random number generator

to create a function that selects 1 city from the whole list by the probability scheme offered above (do not use standard sampling functions present in R).

```r
# create the select city function - based on max
selectCity = function(data){
  # proportional to the number of inhabitants of the city
  data$proportinalPopulation = data$Population/sum(data$Population)
  # generate random numbers - include to dataset
  data$rn_uniform = runif(n=nrow(data),0,1)
  # probabilities proportional to the number of inhabitants
  data$prob_proportinalPopulation = data$proportinalPopulation * data$rn_uniform
  # take one city - max prob_proportinalPopulation
  city_index = which.max(data$prob_proportinalPopulation)
  # print(data[city_index,][1]) # make the print nicer
```

```
    return(data[city_index,1])
}
```

3. Use the function you have created in step 2

as follows: (a) Apply it to the list of all cities and select one city (b) Remove this city from the list (c) Apply this function again to the updated list of the cities (d) Remove this city from the list (e) . . . and so on until you get exactly 20 cities.

```
# create a function to select 20 cities
select20cities = function(data){
  cities_remain <- data
  selected_cities <- c()
  for (i in 1:20) {
  selected_city <- selectCity(cities_remain)
  selected_cities <- c(selected_cities,selected_city)
  cities_remain <- cities_remain[-which(cities_remain[,1]==selected_city),]
  }
  return(data.frame(Municipality=selected_cities))
}
```

4. Run the program.

Which cities were selected? What can you say about the size of the selected cities?

```
# list of all selected cities
selected_cities <- select20cities(data)
idx <- match(selected_cities$Municipality, data[,1])
selected_cities$Population <- data[idx,2]
selected_cities <- selected_cities[order(selected_cities[,2],decreasing = TRUE),]
rownames(selected_cities) <- c()
selected_cities
```

```
##     Municipality Population
## 1      Stockholm     829417
## 2       Göteborg     507330
## 3         Malmö     293909
## 4       Uppsala     194751
## 5     Linköping     144690
## 6      Västerås     135936
## 7        Örebro     134006
## 8     Norrköping     129254
## 9   Helsingborg     128359
## 10    Jönköping     126331
## 11         Umeå     114075
## 12         Lund     109147
## 13        Borås     102458
## 14    Eskilstuna      95577
## 15    Sundsvall      95533
## 16        Gävle      94352
## 17     Halmstad      91087
## 18    Södertälje      85270
## 19      Botkyrka      81195
## 20       Haninge      76237
```

5. Plot one histogram showing the size of all cities of the country.

Plot another histogram showing the size of the 20 selected cities. Conclusions?

```
# histogram - 2 selected cities
histplot_20cities = ggplot(selected_cities)+
  geom_histogram(binwidth=10000,aes(x=Population)) +
  ggtitle("Size of the 20 selected cities") +
  scale_x_continuous(labels = scales::comma)

histplot_cities = ggplot(data)+
  geom_histogram(binwidth=10000,aes(x=Population)) +
  ggtitle("Size of all cities in Sweden")+
  scale_x_continuous(labels = scales::comma)

# put the plots together
grid.arrange(histplot_20cities, histplot_cities, ncol = 2)
```
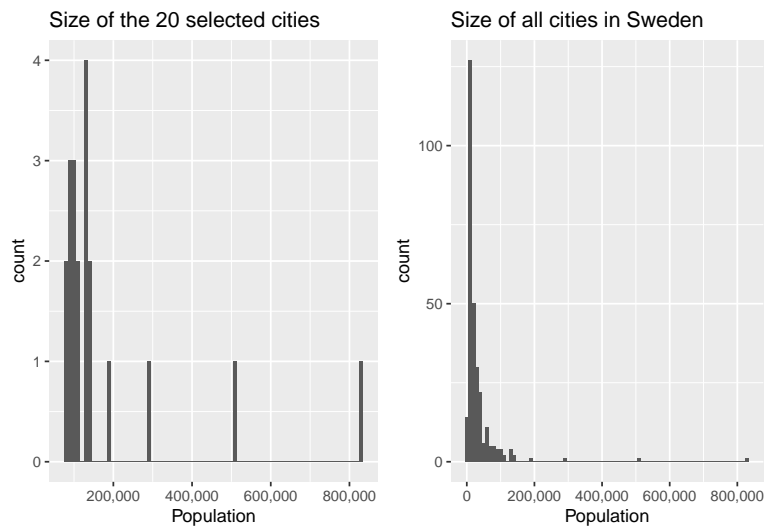
Question 2: Different distributions

The double exponential (Laplace) distribution is given by formula:

$$DE(\mu, \alpha) = \frac{\alpha}{2} \exp(-\alpha \mid x - \mu \mid)$$

1. Write a code generating double exponential distribution DE(0, 1)

from Unif(0, 1) by using the inverse CDF method. Explain how you obtained that code step by step. Generate 10000 random numbers from this distribution, plot the histogram and comment whether the result looks reasonable.

Firstly, we create a sequence of random numbers which are uniform distributed,

$$u \sim \text{Unif}(0, 1).$$

Since the cumulative distribution function (CDF) of Laplace distribution is

$$F(u) = \frac{1}{2} + \frac{1}{2}\text{sgn}(u - \mu)[1 - \exp(-\alpha|u - \mu|)], \quad u > 0$$

and the deduction

$$P(X < y) = P(F^{-1}(u) < y) = p(u < F(y)) = F(y),$$

we can assume that X statisfies

$$X = F^{-1}(u) = \mu - \frac{\text{sgn}(u - 0.5)}{\alpha}\ln(1 - 2|u - 0.5|).$$

Thus, we conclude that $X \sim DE(0, 1)$ and thereby get a sequence x following Laplace distribution by the inverse CDF function of Laplace distribution.

```
# generate random numbers
rm(list=ls())
n=10000
set.seed(12345)
x_rand <- runif(n=n, min=0, max=1)
data1 <- data.frame(unif=x_rand)
ggplot(data1,aes(x=unif))+
   geom_histogram(aes(y=..density..),
                  colour="black",
                  fill="white",
                  bins=30)+
  geom_density(alpha=.2, fill="#FF6666")
```

4
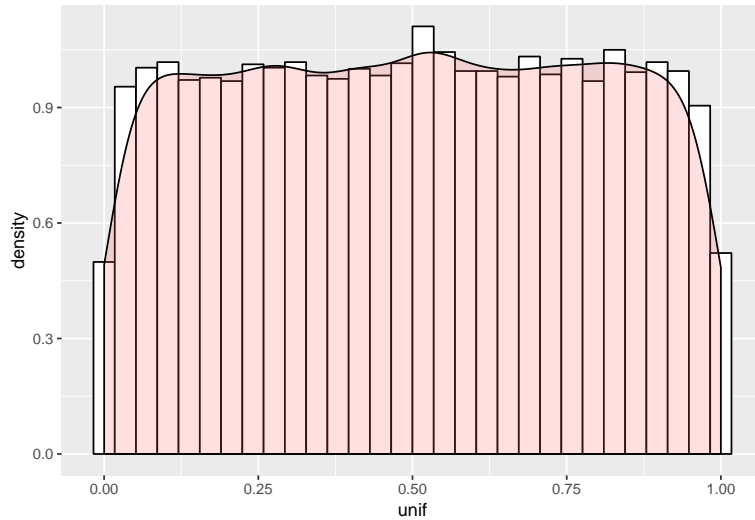
```r
laplace_distribution = function(mu, alpha, p){
  result <- mu-(1/alpha)*sign(p-0.5)*log(1-2*abs(p-0.5))
  # result = alpha/2 * exp(-alpha * (abs(x_rand -  mu)))
  return(result)
}


data1$lap <- laplace_distribution(0,1, x_rand)

ggplot(data = data1, aes(x=lap)) +
 geom_histogram(aes(y=..density..),
                colour="black",
                fill="white",
                bins=30)+
  geom_density(alpha=.2, fill="#FF6666")
```
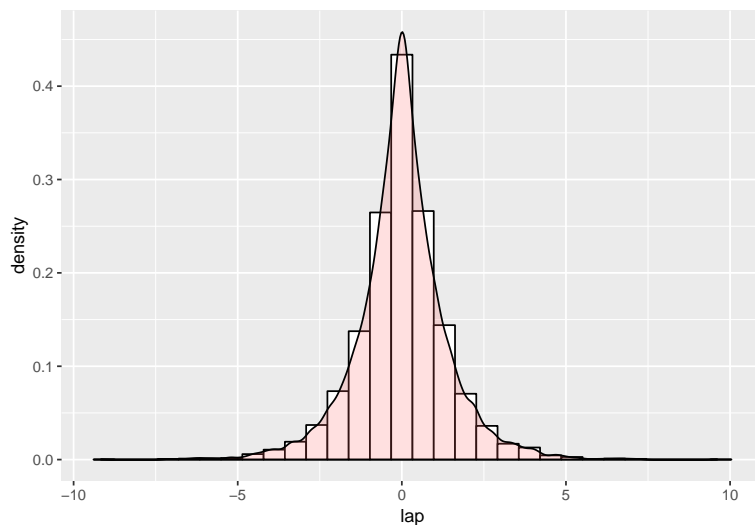

2. Use the Acceptance/rejection method

with DE(0,1) as a majorizing density to generate N(0,1) variables. Explain step by step how this was done. How did you choose constant c in this method? Generate 2000 random numbers N(0,1) using your code and

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plot the histogram. Compute the average rejection rate R in the acceptance/rejection procedure. What is the expected rejection rate ER and how close is it to R? Generate 2000 num- bers from N(0,1) using standard rnorm() procedure, plot the histogram and compare the obtained two histograms.

Firstly, we treat the sequene `data1$lap` as $Y \sim f_Y$. Then we find the probability density function (PDF) of Laplace distribution DE(0,1), which is

$$f_Y(x) = \frac{1}{2}\exp(-\mid x \mid).$$

Meanwhile, we find the PDF of normal distribution N(0,1),

$$f_X(x) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2}).$$

We want a c which can let f_Y cover f_X. Since we have

$$\frac{f_X}{f_Y} = \sqrt{\frac{2}{\pi}}\exp(-\frac{x^2}{2}+|x|),$$

it will get its maximum when $x = \pm 1$. So

$$c = \max\frac{f_X}{f_Y} = \sqrt{\frac{2}{\pi}}\exp(\frac{1}{2}) \approx 1.315$$

By treating `data1$unif` as u, if it statisfies

$$u_i \le \frac{f_X(Y_i)}{cf_Y(Y_i)},$$

we would accept that Y_i follows N(0,1).

```r
pdf_norm = function(x){
  exp(-(x**2)/2)/(sqrt(2*pi))
}
pdf_lap = function(x){
  (1/2)*exp(-abs(x))
}

c <- sqrt(2/pi)*exp(0.5)
n2 <- 2000
accept <- c()
for (i in 1:n) {
  if(length(accept)>=n2){
    break()
  }
  u <- data1[i,1]
  Y <- data1[i,2]
  q <- pdf_lap(Y)
  p <- pdf_norm(Y)
  if(u<=p/(c*q)){
    accept <- c(accept,Y)
  }
}
data2 <- data.frame(val=accept, id="esti")
# ggplot(data = data2, aes(x=val)) +
#   geom_histogram(aes(y=..density..),
#                  colour="black",
```
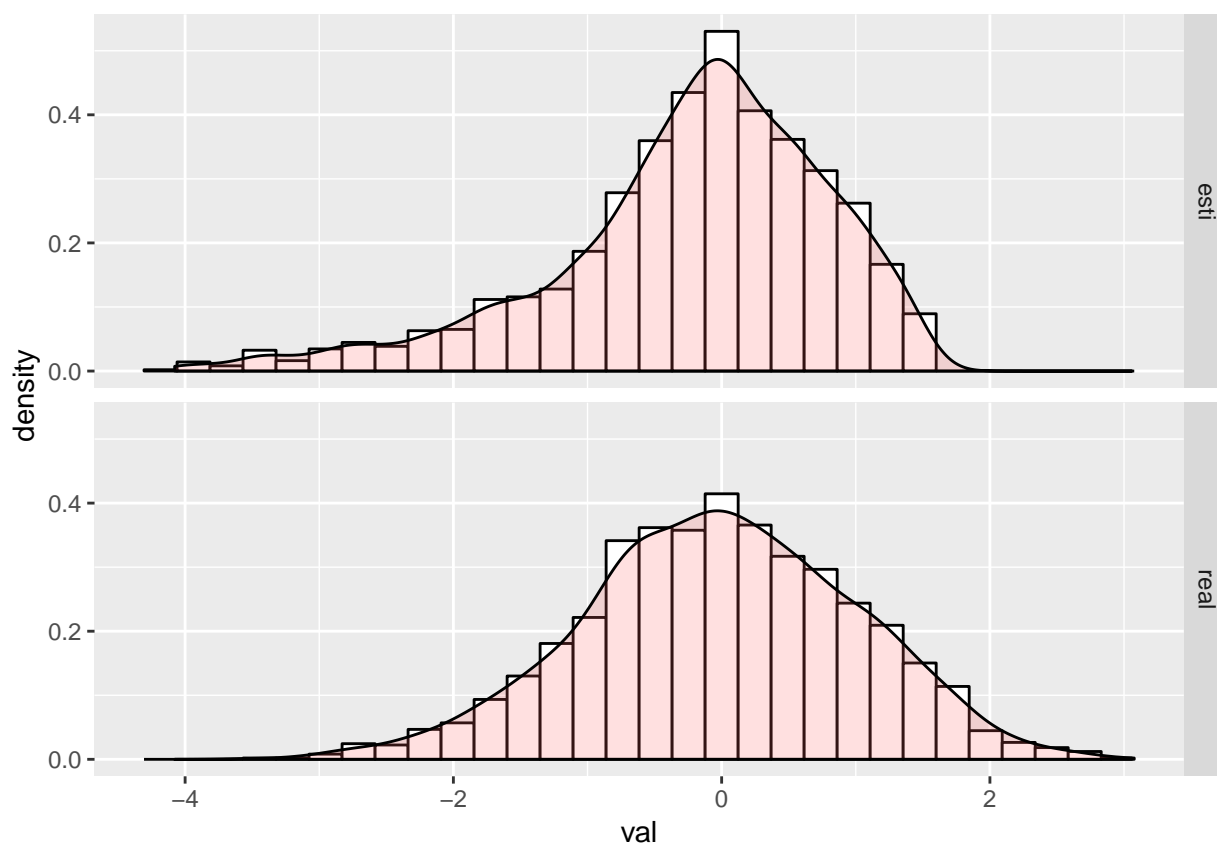
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```
#                   fill="white",
#                   bins=30)+
#    geom_density(alpha=.2, fill="#FF6666")
```

```
data3 <- data.frame(val=rnorm(n=n2,0,1),id ="real")
data3 <- rbind(data2,data3)

ggplot(data = data3, aes(x=val)) +
 geom_histogram(aes(y=..density..),
                colour="black",
                fill="white",
                bins=30)+
  geom_density(alpha=.2, fill="#FF6666")+
  facet_grid(id~.)
```



```
knitr::opts_chunk$set(echo = TRUE, out.height = "200px")
Sys.setlocale(locale="english")
# libraries used
rm(list=ls())
library(ggplot2)
#install.packages("gridExtra") # to put the plots togeter
library(gridExtra)

# set working directory
# load the data
data = read.csv2("population.csv")
```

```r
data[,1] <- as.character(data[,1])
# create the select city function - based on max
selectCity = function(data){
  # proportional to the number of inhabitants of the city
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  return(data.frame(Municipality=selected_cities))
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idx <- match(selected_cities$Municipality, data[,1])
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```

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laplace_distribution = function(mu, alpha, p){
  result <- mu-(1/alpha)*sign(p-0.5)*log(1-2*abs(p-0.5))
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data1$lap <- laplace_distribution(0,1, x_rand)

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  u <- data1[i,1]
  Y <- data1[i,2]
  q <- pdf_lap(Y)
  p <- pdf_norm(Y)
  if(u<=p/(c*q)){
    accept <- c(accept,Y)
  }
}
data2 <- data.frame(val=accept, id="esti")
# ggplot(data = data2, aes(x=val)) +
#  geom_histogram(aes(y=..density..),
#                 colour="black",
#                 fill="white",
#                 bins=30)+
#   geom_density(alpha=.2, fill="#FF6666")


data3 <- data.frame(val=rnorm(n=n2,0,1),id ="real")
data3 <- rbind(data2,data3)
```

```
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 geom_histogram(aes(y=..density..),
                colour="black",
                fill="white",
                bins=30)+
  geom_density(alpha=.2, fill="#FF6666")+
  facet_grid(id~.)
```