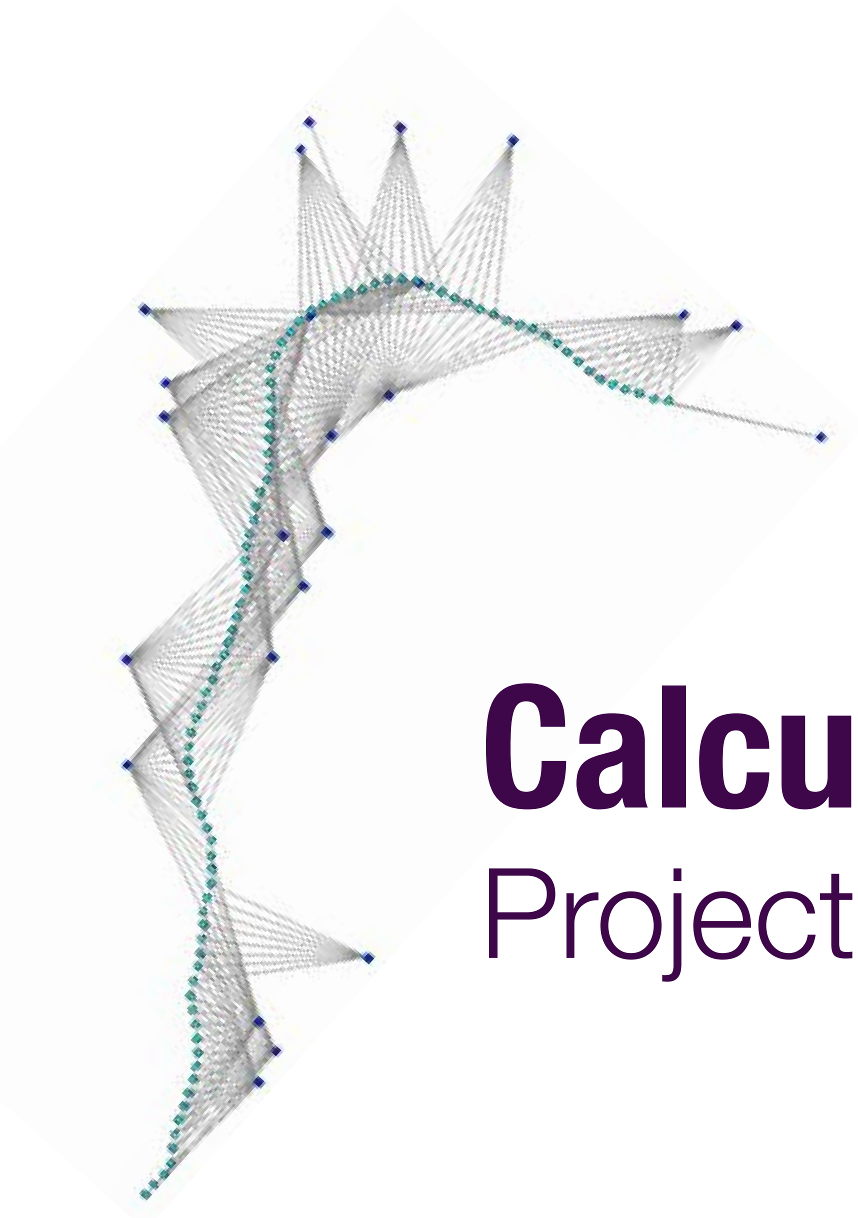

Calculus III

Project Discussion

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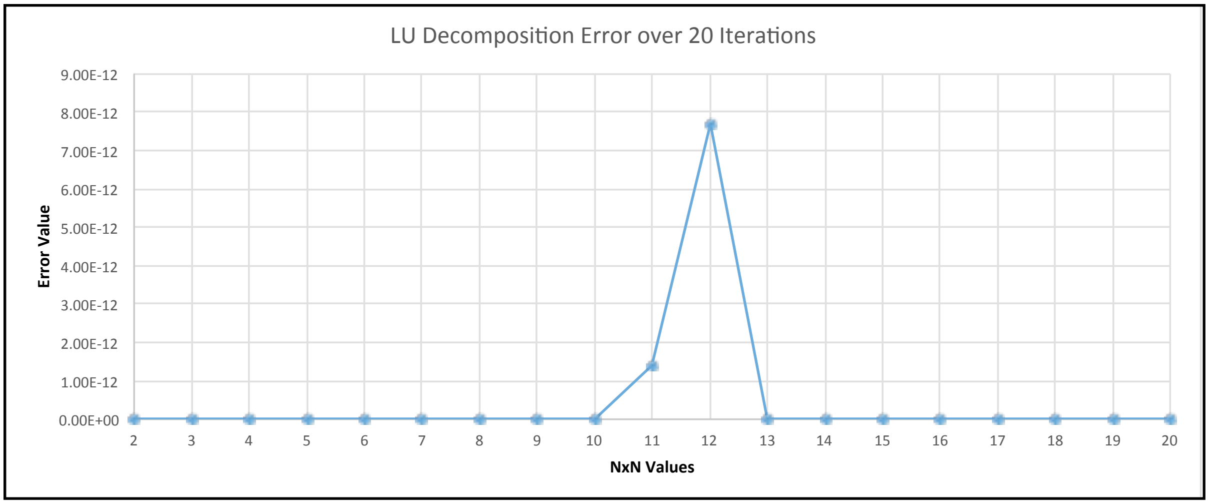
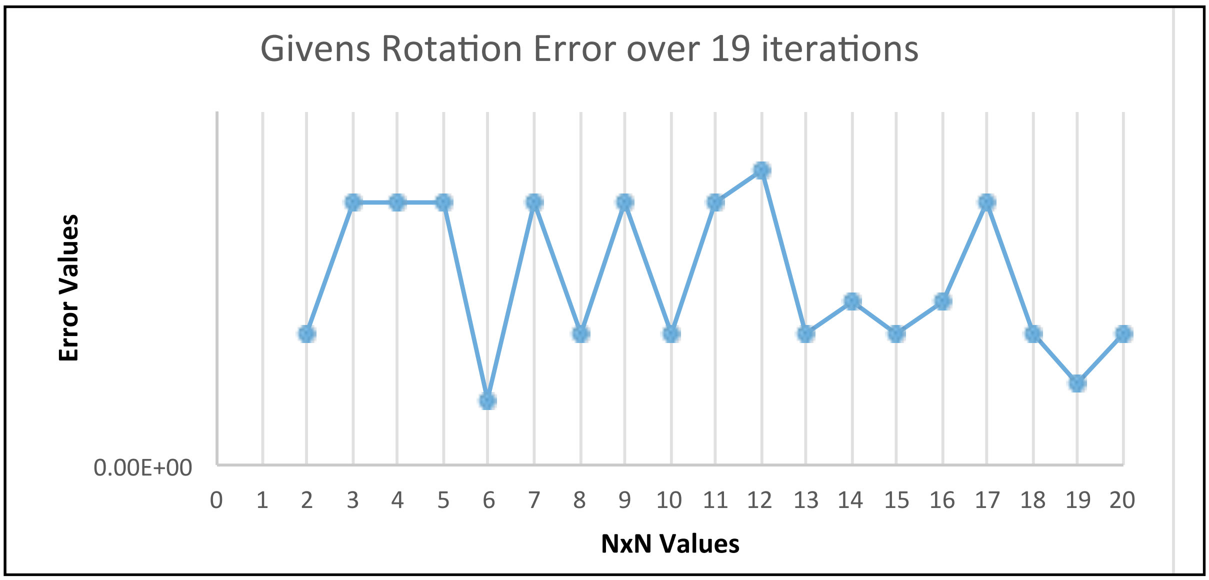
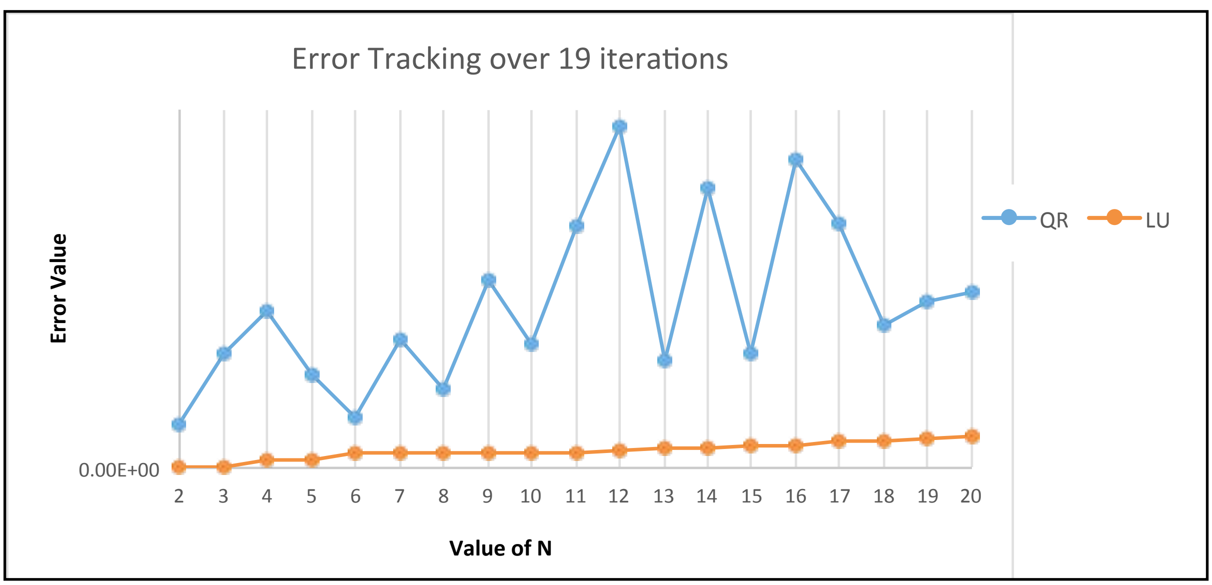
Part 1 Graphs







Part 1 Discussion

After plotting the errors obtained as a function of n, our team was able to conclude that it is justified to use the LU or QR-factorizations as opposed of calculating an inverse matrix for a couple of reasons. First and foremost, as is clearly shown on the graphs, when observing the errors of both LU or QR-factorizations and the errors of calculation by inverse matrix, there is a marginally lower error rate for LU or QR-factorization use that supports its use as a superiorly accurate process.

In addition, the increased speed and efficiency of the LU or QR-factorization process far exceeds the less effective task of finding an inverse matrix, which in comparison to using an LU or QR-factorization is more arduous and complicated. Thus, the benefit of using LU or QR-factorizations in this way is very clear. In terms of conditioning error, the output value changes very little in response to a small argument, meaning the condition number is lower and therefore considered more efficient. By inferring that increased stability exists for a well-conditioned value, it is clear that an LU or QR-factorization process has increased benefit in addition to simply being justified.

Part 2 Discussion

In the Jacobi Method, the role of n plays an important part when decoding algorithms, because n represents the number of iterations that must occur to get to the closest approximation. Thus, a higher n value tends to lead to more iterations before the method converges. The results for this method usually ends up being larger in size. It took roughly double the number of iterations to obtain the desired precision while using the Jacobi method that it took to solve Gauss-Seidel.

Meanwhile, with Gauss-Seidel Method, the role of n when decoding algorithms tells a very different story. Just like the higher n value leads to more iterations, less iterations show an increase in efficiency by using less iterations. The results for the Gauss-Seidel Method, therefore, usually ends up having a lower n and better efficiency rate. It took half the number of iterations to obtain the desired precision while using the Gauss-Seidel method than were necessary for the Jacobi method. Considering these factors, the length of the initial stream n is absolutely important, because in order to understand the benefits of the Gauss-Seidel method in computer programming, one must understand that n is vital to the efficiency factor of the method. Indeed, n definitely has an effect on the number of iterations required to achieve the error tolerance.

Part 3 Discussion

1. Interpret the data in the matrix, and discuss the social factors that influence those numbers.

There are several social factors that can influence the size of a population. The population in question has a dramatic decrease in population before bouncing back 20%. This could be due in part to the population's food source not being able to keep up with the increasing population. Or there could have been a war that killed off many of the population. The dramatic increase could be associated with peacetime and an influx of returning soldiers. This could be similar to the baby boom that America experienced after WWII.

2. What will the population distribution be in 2010? 2020? 2030? 2040? 2050? Calculate also the total population in those years, and by what fraction the total population changed each year.

Below are the population distributions, total population, and fraction of change calculations for the given years.

2010- {{ 3.023809524 },{ 0.7 },{ 0.769047619 },{ 0.771428571 },{ 0.9 },{ 0.838095238 },{ 0.647619048 },{ 0.44 },{ 0.171428571 }}; 8.261428571 * (10^15) people in the population; 41.821% decrease

2020 -{{ 0.816929134 },{ 0.7 },{ 0.196771654 },{ 0.228897638 },{ 0.229606299 },{ 0.26192126 },{ 0.221732283 },{ 0.164913386 },{ 0.058204724 }}}; 2.878976 * (10^15) people in the population, 65.152 % decrease

2030 - { 0.7 },{ 0.728337349 },{ 0.216780723 },{ 0.252173494 },{ 0.247333012 },{ 0.256493494 },{ 0.208994699 },{ 0.080747952 }}; 2.690860723 * (10 ^15) people in the population; 6.534 % decrease

2040 - {{ 1.183044815 },{ 0.7 },{ 0.37814399 },{ 0.416596222 },{ 0.123994781 },{ 0.14103352 },{ 0.12575125 },{ 0.125518377 },{ 0.053129472 }}; 3.247212427 * (10^15) people in the population; 20.676 % increase

2050 - {{ 1.389038221 },{ 0.7 },{ 0.502939527 },{ 0.287672611 },{ 0.316925103 },{ 0.092232691 },{ 0.095369858 },{ 0.081846825 },{ 0.042439095 }}; 3.508463931 * (10^15) people in the population; 8.045 % increase

Part 3 (Cont'D)

3. Use the power method to calculate the largest eigenvalue of the Leslie matrix A. The iteration of the power method should stop when you get 8 digits of accuracy. What does this tell you? Will the population go to zero, become stable, or be unstable in the long run? Discuss carefully and provide the mathematical arguments for your conclusion. You might want to investigate the convergence of ||Ak||.

After implementing the Power method to find eigenvalues and eigenvectors, the iteration of the power method stopped within 8 digits of accuracy and provided us with the following number: 1.28865623. Based on our mathematical calculations, we predict that the population will become unstable in the long run. We have ample mathematical arguments to support this conclusion. When we look at the convergence of ||Ak||, it is clear that the approximated eigenvalue lambda and approximated eigenvector v output lead to a largest eigenvalue that mathematically proves that the population is growing or shrinking faster than manageable. A lower eigenvalue (less than one) leads to slower growth and a more stable rate, but our rate of convergence is clearly the mark of an unstable population. Also, looking at the population distributions by year, we also see an exponential growth or dramatic decay in population in 2020 and 2030 and in 2040 and 2050, respectively. Using the rules of the Power Method and the results of the iterations, we can clearly see that the population is unstable.

4. Suppose we are able to decrease the birth rate of the second age group by half in 2020. What are the predictions for 2030, 2040 and 2050? Calculate again the largest eigenvalue of A (to 8 digits of accuracy) with your program and discuss its meaning regarding the population in the long run.

If we were to cut in half the birthrate for the second age group in 2020, we expect to see stability in the following decades of 2020-2050. We found the largest eigenvalue to be 1.16790271 with up to eight digits of accuracy. In the long run, this means the population population that we are studying would be more stable, as it is closer to 1.

References

Weir, Maurice D., Joel Hass, and George B. Thomas. Thomas' Calculus: Early Transcendentals. Boston: Addison-Wesley, 2010. Print.

Picture from http://www.ananth.in/Projects/Entries/2010/12/7_Real-time_Smoothing_and_Mapping.html