Internet infrastructure and competition in digital markets

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Abstract

Large digital platform companies increasingly integrate vertically by building Internet infrastructure. These proprietary infrastructures confer quality advantages in markets for digital services. I model investment incentives for an infrastructure firm and a vertically integrated firm facing a rival digital services firm without proprietary infrastructure. Small changes in the marginal cost of investment lead to a discontinuous jump in investment incentives both for the infrastructure firm and the vertically integrated firm if the latter has more infrastructure than the former. The infrastructure firm benefits from "commoditization" when it has less infrastructure than the vertically integrated firm. I derive conditions under which the resulting increase in investment is socially efficient. As the market share of the rival firm decreases, a trade-off arises between efficiency and "contestability", a key objective in European competition policy for digital markets.

JEL Codes: L13, L42, L51, L63, L86

Keywords: vertical integration, competition policy, net neutrality, Internet, Internet infrastructure, commoditization

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1 Introduction

Reigning in and regulating "big tech", an umbrella term encompassing diverse companies known by acronyms such as GAFAM or BAT, has emerged as a pressing issue on the political agendas of China, the EU, and the US. Economists have explained the size and power of "big tech" with features that generate "winner-takes-all" markets: (indirect) network effects, platform economics, or the use of big data analytics. However, one aspect of "big tech" market power that remains relatively underexplored is the impact of proprietary Internet infrastructures. This paper seeks to fill this gap by proposing a theory of vertical integration in digital markets to study the effect of proprietary Internet infrastructure on competition.

Over the past decade, the scale and ownership structure of the physical components of the Internet—such as data centers, Internet exchange points, and backbone networks—have undergone significant transformations. Traditionally, large voice carriers connect local networks. These carriers deliver data packages based on principles of net neutrality and best-effort. The largest Internet Service Providers (ISP) interconnect with each other, creating a global network of networks, the Internet.

More recently, consumer-facing firms such as Google, Netflix, or Meta, have increasingly complemented this so-called "public Internet" (operated by private companies nonetheless) with their own investments.[1] Some of the largest digital companies have made joint investments with traditional carriers and also created parallel, proprietary infrastructures.[2] These networks enable higher quality or guaranteed-reliability services. This is essential for quality- or latency-sensitive applications, ranging from entertainment to corporate applications that require nearly 100% uptime. In this paper, I study these companies as firms that integrate vertically by complementing a digital platform business connecting consumers and advertisers (downstream) with proprietary infrastructure (upstream) that enables the digital platform.

Researchers and competition authorities are increasingly aware of proprietary Internet infrastructure but there is little knowledge about its implications. The EU's Digital Markets Act (DMA) recognizes that large platforms, which it calls "gatekeepers", can steer and block access to certain infrastructures. The DMA calls for openness and free choice in its pursuit of "fairness" and "contestability" in digital markets.[3] The German competition authority in its report on "Competition 4.0" singles out content delivery

[1] An article by the Financial Times describes investors' diverging views on the competitive dynamics between data center operators that can be broadly summarized as belonging to the "public Internet" and those operated by "big tech groups". Financial Times, Will the cloud kill the data centre? Jim Chanos thinks so (2022), last accessed 31.10.2022.

[2] Examples include ocean-crossing submarine cables, such as *JUPITER*, connecting the United States, Japan, and the Philippines, owned by a consortium including Amazon Web Services, Meta, NTT, PCCW, PLDT, and Softbank Corp. A transatlantic example, *Havfrue/AEC-2* connects the United States, Ireland, Denmark, and Norway and is owned by Aqua Comms, Bulk, Meta, and Google. Both cables became ready for service in 2020. See also Appendix B.

[3] The DMA discusses network access in recitals 14 and 51 of the preamble. Article 6(1)(e) proposes an unspecified obligation for "gatekeeper" firms not to restrict the choice of Internet access providers. However, it is not clear how the DMA will treat proprietary networks operated by gatekeepers.

networks (CDN) as a piece of Internet infrastructure that has been increasingly used by content firms.[4] However, neither text provides clear guidance on applying economic principles to competition policy for Internet infrastructure.

This paper introduces a theoretical model of vertical integration that characterizes Internet infrastructure as an upstream input to downstream digital services. A pure upstream firm, called the infrastructure firm, undertakes investments in infrastructure and rents out access to it to both a pure downstream firm and a vertically integrated firm. The vertically integrated entity further bolsters its available infrastructure by investing in proprietary infrastructure. As the vertically integrated firm offers its services using more infrastructure, it can offer some unique and superior services (compared to the pure downstream firm) at higher prices, fueling investment incentives for the vertically integrated firm but also influencing the rental fee that the infrastructure firm can charge and its resulting investment decision. Notably, when the industry's cost dynamics result in the vertically integrated firm's infrastructure surpassing that of the infrastructure firm, both firms have higher investment incentives.

The model sheds light on the incentives driving the "commoditization" of the infrastructure industry, a concern often addressed in industrial policy discussions, extending beyond telecommunications to sectors like automotive. As a second main result, the model demonstrates a trade-off between contestability (the market share of the pure downstream firm) and efficiency (social welfare). The model suggests accounting for the role of infrastructure in the review of vertical mergers in digital markets. It explains side payments that have occurred between vertically integrated firms and infrastructure firms such as Netflix and Comcast. Finally, the model suggests that expanding net neutrality to certain kinds of infrastructure, such as CDN, would likely harm consumers. Notably, the model's robustness is demonstrated through sensitivity analyses conducted across various assumptions concerning congestion, bargaining mechanisms, and product differentiation.

[4] Bundeskartellamt (2016) Working Paper: Market power and platforms [in German]. The authors mention that on-demand server and network services allow small-scale entry, while many large firms invest additionally in CDN to reduce response times. The report does not contain conclusions for the competitive assessment of these CDN.

2 Literature

This paper relates to the literature on the economics of Internet infrastructure (Greenstein, 2020). Wilson, Xiao, and Orazem (2021) analyze the investment decisions of ISPs and find the long-term effects of investment delays on infrastructure quality. Greenstein and Fang (2020) find that data centers are being built primarily where customers are located, rather than in locations with favorable (land- and energy-)cost structures.

The topic of this paper is related to Buehler, Schmutzler, and Benz (2004) and Avenali, Matteucci, and Reverberi (2014). The former studies investment incentives by an upstream industry when network quality is not verifiable and the downstream (retail) industry is one-sided. They argue that vertical separation enhances incentives to invest in network quality most of the time. One main channel here is the quality sensitivity of retail demand. However, Buehler, Schmutzler, and Benz (2004) study a chain of monopolies and do not consider vertical integration.

Chaturvedi, Dutta, and Kanjilal (2021) investigate ISP pricing, in the presence of complementarity between broadband and content. Avenali, Matteucci, and Reverberi (2014) analyze functional and ownership separation for broadband networks and find ambiguous effects of vertical integration. By contrast, this setup focuses on competing investments by a pure infrastructure firm (for example, Akamai, a CDN operator) and a vertically integrated firm (for example, Google). I do not analyze the last-mile connections in which broadband providers have monopoly access[5] but focus on the digital services for which large content firms integrate vertically. These papers focus on the monopoly standing of last-mile ISPs for residential connections. Infrastructure in this article refers to the IT infrastructure used by businesses to provide digital services, such as Content Delivery Networks. Section 3 explains the model alongside a comprehensive example.

Investment in this kind of infrastructure improves data management over what is provided on the public Internet in the presence of net neutrality. Net neutrality is the imposition of zero-termination fees[6] and nondiscrimination of data by carriers. Even though net neutrality is not uniformly enforced (for an early overview of the literature, see Schuett, 2010), it poses economic questions and trade-offs as described by Economides and Tåg (2012) or Greenstein, Peitz, and Valletti (2016). Current net neutrality regulation is uneven, focusing on ISPs while leaving open bypass opportunities and loopholes for cloud services and content providers (Stocker, Smaragdakis, and W. Lehr, 2020). This paper studies the investments into infrastructure that enable net-

[5]Some attempts at competing in this area by content providers, such as Google Fibre, have failed in the past and do not play a large role in the marketplace.

[6]For example, an ISP such as AT&T cannot charge Netflix for traffic that terminates in an AT&T network such as a residential building where Netflix customers live. The price paid by the final consumer to the ISP is understood to compensate the terminating network, no matter where data packages originate.

work management practices that arise to cope with the limitations of net neutrality.[7] It contributes to the debate on net neutrality by modeling investment incentives into the network management practices that already exist to adapt to net neutrality restrictions.

This setting where advertisement is served alongside digital services is closely related to Armstrong (2006)'s competitive bottleneck in two-sided markets. My paper introduces a true vertical structure with an intermediate input, in contrast with other papers on two-sided markets that call agreements with one market side "vertical" (Lee, 2013; D'Annunzio, 2017; Carroni, Madio, and Shekhar, 2018). The infrastructure firm is not a platform member, but an input supplier. The infrastructure firm cannot pick a downstream monopolist (in the style of "exclusive dealing" papers on platforms) because it cannot make a binding commitment not to negotiate with both firms. Its inability to commit against opportunistic renegotiation rules out exclusivity. Instead, I show how the infrastructure firm benefits when its network is smaller than the vertically integrated firm's (corresponding to a platform with market power in a two-sided market model). Even though the infrastructure firm loses its strategic role and essentially sells a "commodity", it ends up extracting a greater surplus from the vertically integrated firm.

The phenomena studied in this paper are well-documented in a growing descriptive literature that has been largely ignored by economists. Some large digital services firms have pursued vertical integration strategies through the construction of private backbone networks, edge computing facilities, and owned CDNs that improve their ability to expand and change their digital infrastructure to improve the performance and quality of their services (Arnold et al., 2020; Arnold, 2020; Sermpezis, Nomikos, and Dimitropoulos, 2017; Motamedi et al., 2019). Depending on the business model, private infrastructure can result in cost decreases because of hardware that is fit for purpose or increasing connection quality from faster delivery of data packages at the router of the last-mile ISP.

This paper is the first approach to analyze the effects of this shift in Internet ownership structure. Stocker, Knieps, and Dietzel (2021) document the geographic and virtual dimensions of private networks and their implications for firm costs, service quality, and innovation. Lehr et al. (2019) and Balakrishnan et al. (2021) describe the functional disparities between services that rely on the public Internet versus services that are supported by proprietary networks and clouds. Using publicly available data, we add to the description of proprietary networks by showing that increasing numbers and shares of submarine cables have firms including Amazon, Meta, Microsoft, and Google among their owners (see Appendix B). By analyzing the previously overlooked competitive effect of a feature of digital markets that mainly concerns the largest digital firms, this paper contributes to the academic debate on regulation and antitrust towards large technology companies (see also Petit, 2020).

[7] For example, infrastructure that ensures that one content provider's data packages arrive earlier and in a specific order at a router from where they are treated on a first-come-first-served basis with the data packages of other content providers. Thus this infrastructure improves speed at the stage preceding the point where the net neutrality principle comes into play. See Easley, Guo, and Krämer (2018).

3 Model

This model has two key components:

1. Infrastructure, which is a capital investment either of an infrastructure firm or of a vertically integrated firm. The size of a firm's infrastructure is denoted k. An example are CDN, which are decentralized networks of servers that are used to distribute files, such as large media files, to consumers.

2. Digital services, which are provided to consumers by a pure digital services firm or by a vertically integrated firm using infrastructure as an input. Quantities of digital services are denoted q. An example of digital services is video streaming services which are offered both by vertically integrated firms and pure digital services firms.

Firms sell digital services (in the "downstream" market) to consumers, generating revenue through these sales (independent of whether sales happen through subscriptions, tokens, or in other ways) as well as through advertising. To sell digital services, a firm needs infrastructure as an input, either owned or rented (the "upstream" market). Firms can rent infrastructure from an infrastructure firm. In addition, we assume one firm to be vertically integrated and able to build some infrastructure for its own use.

There are three players:

1. A vertically integrated firm (V) which makes a costly investment in infrastructure, purchases access to additional infrastructure from an infrastructure firm, sells digital services to consumers, and shows advertisement alongside these digital services. An example would be Google's YouTube, a video-sharing platform.

2. A digital services firm (Q) which purchases infrastructure access from an infrastructure firm, sells digital services to consumers, and shows advertisement alongside these digital services. Think of this as a smaller company offering a video sharing platform, such as Dailymotion.

3. An infrastructure firm (I) which makes a costly investment in infrastructure and sells access to its infrastructure to V and Q. Think of this as a CDN provider, such as Akamai or Limelight.

The vertical structure is represented in Figure 1.

Downstream there is a large pool of potential consumers, but firms are limited in their ability to sell services by their available infrastructure (either their own or rented infrastructure). For example, as a video-streaming service adds additional infrastructure, its average latency decreases so far that consumers watch one additional unit of video content. Renting (or owning, for firm V) one additional unit of infrastructure allows firms V and Q to serve exactly one more unit of services to consumers at zero marginal cost and to show one more unit of advertisement alongside these services.[8] In other

[8] Zero marginal cost is a common assumption to focus on pricing in two-sided markets (Hagiu and Lee, 2011; D'Annunzio, 2017).

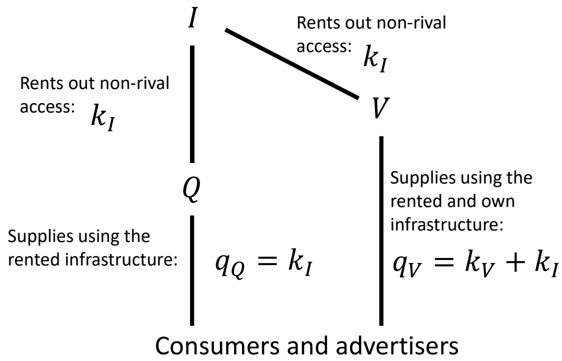

Figure 1: Infrastructure rental and downstream service supply

words, firms' ability to sell to consumers is constrained by their available infrastructure. The marginal unit of infrastructure is defined implicitly through consumers taste: it is the required service improvement that will attract one additional consumer into the market.

Firms compete for consumers in services that they both sell, while we assume that the firm with more infrastructure is the only one that can offer services at a superior level. The idea is that the additional infrastructure allows the firm to offer a tiered service that cannot be offered by the firm with less infrastructure, and which is offered to consumers as a separate product. In the example of video-streaming platforms, this could represent videos at a higher resolution which can only be delivered at acceptable speeds by the firm with the greater infrastructure (a premium version of YouTube, which would be sold alongside free access to a basic version of YouTube and a free access to the rival firm's video sharing service). Therefore, the firm with more infrastructure can charge consumers for this premium service while offering a free, advertisement-funded service that is equivalent to the free, advertisement-funded service of its rival. This premium service forms what we call the "monopolistic segment" on which the vertically integrated firm can set separate prices and face separate demand.

Firm Q can only choose whether to rent firm I's infrastructure k_I, so its downstream capacity is $q_Q = \{k_I, \emptyset\}$. Firm V can choose to rent access to k_I in addition to its own infrastructure k_V, so it offers $q_V = \{k_I + k_V, k_V\}$. Under this capacity structure, the size of the competitive segment is $min(q_V, q_Q)$ and the size of the monopolistic segment is $max(q_V, q_Q) - min(q_V, q_Q)$.[9] The relationship between available infrastructure and competition for consumers is visualized in Figure 2. In this example, $\bar{q}_Q = k_I$ and $\bar{q}_V = k_I + k_V$. The Figure illustrate that the available infrastructure determines cumulatively the size of the competitive and monopolistic segment. The size of the lat-

[9]Firms never have an incentive to hold back capacity, so we do not allow for this possibility. The reason is that in contrast with a standard Cournot model, prices are not decreasing in quantity.

ter is endogenously determined by the amount of infrastructure that is available to the downstream firm that has access to less infrastructure (here, and as we will see later, in equilibrium, Q). Therefore, Q and V compete Bertrand-style over the first k_I units of demand, while over the remaining k_V, V acts as a monopolist.

In the base model, access to infrastructure is non-rival, so I can rent its infrastructure to both V and Q simultaneously. While infrastructure in reality is capacity-limited (for example, storage space in a data center, capacity on a fiber-optic cable), in the base model we focus on the type of infrastructure that enables new kinds of services (for example, enabling latency-sensitive applications by building additional servers closer to final users) rather than congestion.

In our example, I offers a CDN for storing large media files to video-streaming services. In this example, infrastructure investment would correspond to building additional servers in local networks closer to consumers. The model emphasizes the quality advantage of being able to reach consumers locally (and thus decreasing latency) which enables higher quality services (such as live-streaming, and higher video resolutions) for which platforms can charge through premium services (such as YouTube Premium). We consider congestion in a robustness check in Section 6.1 and find that it is not important to the results of the model.

Firms compete in prices and their products (on the competitive segment) are perfect substitutes to consumers. Hence, consumers buy services from the cheapest firm if prices do not exceed their willingness-to-pay a. When two firms set identical prices for their services, they split consumer demand equally. The digital services firm and the vertically integrated firm sell advertisement space alongside services to advertisers at a constant price of r (irrespective of the segment on which the advertisement is displayed).[10]

I's objective is to maximize profits which are the sum of rental transfers t_V, t_Q minus the investment cost $c_I(k_I)$. V's objective is to maximize profits which are the revenue from selling digital services and advertisement downstream, minus the rental transfer t_V and investment cost $c_V(k_V)$. Q maximizes profits by selling digital services and advertisement downstream, minus its rental transfer t_Q. Firms' outside options are normalized to zero, and they accept transfers that allow them to at least break even. Note that firms V and Q can always choose not to rent from I but they will be (weakly) better off doing so. We indicate the monopolistic and competitive segments by subscript $h = c$ (for competition), m (for monopoly).[11] Firm j's demand on h is $d_{j,h}$ and its price to consumers is $p_{j,h}$. Hence the payoffs:

[10] Exogeneity of r is a strong assumption but not important to derive the results. It is equivalent to the "competitive bottleneck" configuration (Armstrong and Wright, 2007) in two-sided markets. The pricing is similar to the "per-reader advertising charges" in Armstrong (2006) or advertisers' willingness-to-pay in Gans (2022). The downstream market of our model is a simplified version of this two-sided market. A secondary source of revenues enables Q to pay a positive transfer to the infrastructure firm, which is required for competition in equilibrium.

[11] From the demand structure above it follows that only one firm, either V or Q, can offer services on the monopolistic segment and serve consumers thereon.

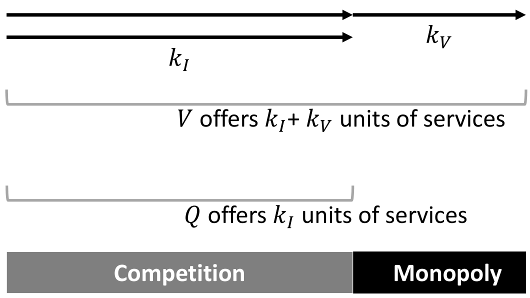

Figure 2: Market segmentation for differently-sized infrastructure

$$\Pi_I = t_Q + t_V - c_I(k_I) \tag{1}$$

$$\Pi_V = \sum_{h=c,m} d_{V,h}(p_{V,h} + r) - t_V - c_V(k_V) \tag{2}$$

$$\Pi_Q = \sum_{h=c,m} d_{Q,h}(p_{Q,h} + r) - t_Q \tag{3}$$

The cost function for the creation of infrastructure is firm-specific and entails diminishing returns. We consider a convex cost function of the form $c_i(k) = k^\alpha + \beta_i(k - k_0), \alpha > 1, \beta_i > 0$ for some exogenous values $\beta_i, k_0 \in \mathbb{R}_+$, $k \geq 0$, $i = V, Q$. This form will simplify results later, by confining the firm-specific cost parameter to a linear term while including a strictly convex term that guarantees a unique interior solution.

The timing is as follows: First, V and I simultaneously choose non-negative levels of investment. Second, I simultaneously posts take-it-or-leave-it offers to buy access to k_I at transfers t_V, t_Q to V and Q, respectively. V and Q choose whether to accept or reject these offers. Player I cannot commit not to renegotiate prices opportunistically, so V and Q are accepting prices expecting competition by the other firm.[12] Third, firms set prices for consumers and advertisers. Fourth, consumers and advertisers decide simultaneously from which firm to buy and payoffs are realized.

[12]More precisely, after V and Q have announced whether to accept the take-it-or-leave-it offer, if any of them rejected the offer, I can make one additional, revised take-it-or-leave-it offer. This prevents I from initially announcing a high transfer that can only be recouped under monopoly, but not under competition, and thereby attempting to create a downstream monopoly. Any firm facing such a transfer would anticipate an opportunistic offer to its rival and reject it. In equilibrium V and Q correctly anticipate that the rival will enter. The idea is similar to franchising where the franchiser would like to commit to limit downstream competition but has an incentive to opportunistically award more franchises after a contract has been made (McAfee and Schwartz, 1994). With a similar logic, a durable goods monopolist is disciplined by competition from its future self (Bulow, 1982). In our model, upstream competition could alternatively be a second infrastructure firm or a competitive fringe of infrastructure firms but given our demand structure, this is a convenient way of limiting upstream market power.

4 Analysis

We start by defining a subgame-perfect Nash equilibrium (SPNE) of this game.

Definition SPNE: A subgame-perfect Nash equilibrium of this game is a profile of prices $p_{j,h}$, transfers t_j $j = V, Q$, rental decisions, and investment levels k_i, $i = I, V$, such that these are optimal in every sub-game.

Now, we state the first main result regarding the types of equilibria of this game. This result states that firm I will attain the greater investment level only if it has a certain level of cost advantage (Theorem 1). We state the main propositions regarding the property of equilibria, namely that investment by both V and I is higher when V has the greater investment level (Proposition 1) and conditions for the existence of unique equilibria (Proposition 2). Then, we state the second main result regarding social welfare in this model (Theorem 2). We then analyze the game, proving propositions 1 and 2, and the resulting Theorem 1, followed by an analysis of social welfare. The proof of Theorem 2 is not interesting in itself, however, so it appears in the Appendix.

Theorem 1: There are two types of equilibrium in which either $k_I > k_V$ or $k_V \geq k_I$. $k_V \geq k_I$ in equilibrium if $\beta_V - \beta_I \leq r/2$. The converse is true otherwise.

First, we show:

Proposition 1: Both I's and V's optimal level of investment are higher in the equilibrium when $k_V \geq k_I$ than in the equilibrium where $k_I > k_V$.

and

Proposition 2: When $\beta_V > \beta_I - \frac{r}{2}$ ($\beta_V < \beta_I - \frac{r}{2}$) there is a unique equilibrium where $k_I > k_V$ ($k_I < k_V$).

Theorem 2: Social welfare is decreasing in marginal costs β_I, β_V but has a discontinuity when $\beta_V - \beta_I = r/2$. Social welfare increases at this point if

$$(a+r)^{\alpha-1}\left[\frac{2a+r}{\alpha}\right] > \left[\frac{2a+r}{\alpha}\right]^{\alpha} + (\beta_I+\beta_V)^{\alpha-1}\left(\frac{a+\frac{r}{2}}{\alpha}\right) \tag{4}$$

and decreases otherwise.

We analyze the game and prove Theorem 1 and Proposition 1 in Appendix A.1.

One interpretation of the case $k_I > k_V$ is that I plays a strategic role by "pushing" V into the region where it has market power - without it, V faces tough competition and only reaps low profits. In the second case, k_I does not have such a strategic role: Q's ability to constrain V's pricing power is limited and additional infrastructure at the margin allows V to increase its revenue by $a + r$, independent of whether it makes this investment privately or whether I makes this investment (as long as $k_I < k_V$).

I call the intuition behind the case $k_V > k_I$ "commoditization": I's infrastructure becomes interchangeable with marginal investment for V because it only adds to the monopolistic segment. For this reason, V's willingness-to-pay is higher in this scenario.[13]

In the first-order-conditions for the investment of I and V (see equations 37 and 41 in Appendix A.1) we assume implicitly that investment is as in the case of $k_V > k_I$ if the condition holds with equality. This is purely for completeness, there is nothing in the model that tells us that this particular equilibrium will be chosen when the condition holds with equality. The following section shows that there is a set of parameters for which either outcome (the upper or lower branch of these equations) is possible.

4.1 Existence of equilibria

So far, we have computed the transfers and prices that maximize firm profits for given levels of investment as well as the optimal levels of investment as a function of model parameters and whether k_I or k_V is larger. Without further restrictions, there is no guarantee that the optimal levels of investment are indeed consistent with the size ordering of the two infrastructures.

From the first-order-conditions for the investment of I and V we see that I (V) will never choose a level of investment greater than 0 if $\beta_I > a + (3r/2)$ ($\beta_V > a + r$). In general, strictly positive levels of investment only arise in equilibrium if $\beta_V < r/2$ and $\beta_I < r$. To investigate only interesting cases, we are going to restrict the parameter space accordingly. Furthermore, for any set of parameters to result in an equilibrium, the parameters also need to result in the correct size ordering of the two infrastructures, i.e., $k_I > k_V$ or the converse must be true when computing equilibrium investment levels. We now show that such equilibria exist. As the proof is short, we leave it in the main text.

Proof of proposition 2: Take the first-order conditions from the upper cases of equations 37 and 41, impose $k_I > k_V$, and some algebra yields

$$\beta_V > \beta_I - \frac{r}{2}. \tag{5}$$

Therefore, given marginal costs that fulfill the above inequality, the infrastructure firm has indeed the larger network. Similarly, comparing the lower cases of equations 37 and

[13] The idea of commoditization, that is, converting a product into a standardized and interchangeable input, is common in the discussion of industrial policy and vertical integration of some big tech firms. Beyond Internet infrastructure firms, it also appears in discussions of other industries such as the automotive sector. One concern in that sector is whether automotive firms will become mere suppliers of hardware as the value generation shifts to digital platform companies, using the data generated by cars and drivers. A major German daily compares the fear of Internet infrastructure firms like Deutsche Telekom to become "dumb tubes", or commoditized suppliers to data-driven consumer-facing platform companies. The corresponding fear of car companies and other manufacturing companies is to become "dumb wheels" or "extended workbenches" as profit shifts to digital platform companies. FAZ, Autobranche im Spagat (2021) [in German], last accessed 31.10.2022.

41 and solving for $k_V \geq k_I$ yields

$$\beta_V \leq \beta_I - \frac{r}{2}. \tag{6}$$

Now, setting the upper and lower cases of equations 37 and 41 equal yields:

$$\left(\frac{r/2 - \beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} = \left(\frac{r - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} \qquad \leftrightarrow \quad \beta_V = \beta_I - \frac{r}{2} \tag{7}$$

$$\left(\frac{a + r - \beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} = \left(\frac{a + (3r/2) - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} \qquad \leftrightarrow \quad \beta_V = \beta_I - \frac{r}{2}. \tag{8}$$

The profit functions are strictly concave ($c''_I(k_I) < 0$, $c''_V(k_V) < 0$), so the first-order conditions are sufficient to describe the unique values of k_I, k_V where Π_I, Π_V attain their global maxima. Also, the equilibrium value of k_I does not depend on k_V and vice versa except through the ordering of size of these two variables. Therefore, when either $\beta_V > \beta_I - \frac{r}{2}$ or $\beta_V < \beta_I - \frac{r}{2}$, the equilibrium is unique. □

It is not surprising that the firm with lower marginal costs will generally have the larger infrastructure. Also the "wedge" of $r/2$ that separates the marginal cost values which equalize investment is not interesting in itself - it is a consequence of the no-congestion-assumption which allows I to charge Q an additional $r/2$ for every unit of k_I added in addition to charging its contribution to V's profit.[14]

There is one combination of parameters where both orderings of the two infrastructure variables are an equilibrium. When $\beta_V = \beta_I - \frac{r}{2}$, both $k_V > k_I$ and $k_I > k_V$ can be optimal. For completeness, I have defined the conditional functions above such that that $k_V > k_I$ will be chosen. However, there is nothing inherent in the model which tells us that this equilibrium will be chosen. This is interesting because it gives us a configuration of parameters for which we can directly compare the two types of equilibrium. We will use this below to compare social welfare for the two types of equilibrium, holding parameters constant.

Figure 3 illustrates equilibrium investment values as a function of β_I for some set of parameters. The vertically integrated firm's infrastructure investment k_V does not depend on β_I, and it can only take two values for a given set of parameters, represented by horizontal black lines: one corresponding to the case $k_V > k_I$ and one where $k_I > k_V$. We denote V's optimal level of investment k_V^* in the former case and k_V^{**} in the latter case. k_I depends on I's cost parameter β_I as well as other parameters and on which firm has the larger infrastructure, so it is represented by two decreasing functions. Given all other parameter values, a value of β_I admits an equilibrium in which infrastructure levels are k_V^*, k_I^* (k_V^{**}, k_I^{**}) whenever it is true that $k_V^* > k_I^*$ ($k_I^{**} > k_V^{**}$). These regions are marked with solid black lines. When $\beta_I = \beta_V + r/2$, both equilibria are possible.

The model is silent on which equilibrium to expect in case of indifference. However, we are going to compare the two equilibria and show that $k_V > k_I$ dominates the

[14] Accordingly, this wedge decreases in robustness checks (in Section 6) that afford less bargaining power to I or that consider congestion.

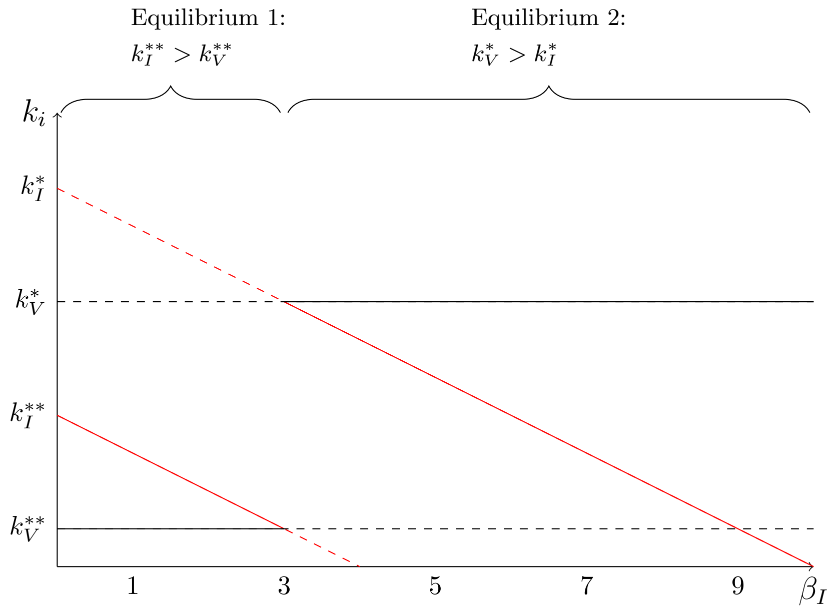

Figure 3: Equilibrium investment decisions as a function of β_I for $\beta_V = 1, a = 4, r = 4, \alpha = 2$.

other in terms of firm profits and sometimes on social welfare. The presence of the two different kinds of equilibria is consistent with the increasingly large role played by vertically integrated firms in infrastructure which is documented in the literature (see Section 2) and Appendix B.

4.2 Social welfare

We consider whether the equilibria we just found maximize social welfare. We define social welfare as the total sum of surplus minus costs. This implies that both parts of the network are used to supply services. In terms of the game we presented in the previous section, this is the case whenever $q_V = k_I + k_V$. We define social welfare S as follows:

$$S = (k_I + k_V)(a + r) - c_I(k_I) - c_V(k_V) \tag{9}$$

It is easy to show that social welfare is maximized when

$$c_I'(k_I) = c_V'(k_V) = a + r \tag{10}$$

Comparing this optimum to the outcome of the decentralized model, we see that social welfare is generally not maximized. Furthermore, social welfare is a function both of the parameters (higher costs clearly imply lower social welfare) as well as the two different types of equilibrium described in Theorem 1. As mentioned before, we want to compare social welfare when the parameters admit both types of equilibria. This allows us to ignore the levels of marginal cost parameters β_I, β_V to analyze the role of the two different cases where either V or I owns more infrastructure.

Proof of Theorem 2: In the Appendix.

It is unsurprising that social welfare decreases when marginal costs increase. The interesting question is what happens with social welfare when we jump between the two equilibria of the decentralized game that we have identified earlier, either because we have marginal cost parameters that admit both types of equilibria as shown above in Propositions 1 and 2, or if we consider a small change of β_V, β_I such that we move between these two equilibria (while having a negligible direct effect of the change). This proposition shows that the benefit from the increased investment incentives in the $k_V > k_I$-equilibrium dominates additional investment cost if β_I, β_V are low or if α, a and r jointly fulfill the condition described in equation 4.

Now compare the socially optimal investment with the Nash-equilibrium investment levels which we have computed in equations 37 and 41. In the case where $k_I > k_V$, we have underinvestment, as $c'_I(k_I) = r$, $c'_V(k_V) = r/2$. In the case where $k_V > k_I$, we have overinvestment as $c'_I(k_I) = a + (3r/2)$, $c'_V(k_V) = a + r$. The reason for overinvestment is that absent congestion, I's private return exceeds the public return from each additional unit of investment, as it gets to charge Q $r/2$ for the additional investment (which correspondingly lowers V's profit by $r/2$ due to competition). However, the net effect on social welfare is positive as long as the condition in equation 4 is fulfilled.

The problem could also be viewed as a social planner's problem who chooses k_V and k_I to maximize social surplus. Here, it is necessary to explicitly state a side-condition that the total sum of demand equals the maximum capacity from the combined networks of V and I. This planner problem can be written as

$$\max_{k_I,k_V} S = (k_I + k_V)(a + r) - c_I(k_I) - c_V(k_V) \tag{11}$$

$$\text{subject to} \sum_{i=V,Q} \sum_{h=c,m} d_{i,h} = k_I + k_V. \tag{12}$$

and it is clearly also maximized where $c'_I(k_I) = c'_V(k_V) = a + r$. The side-condition in equation 12 is also fulfilled in the Nash equilibrium as both Q and V find it optimal to sign the contract with I. This side-condition rules out inefficient configurations for the social planner's problem in which V would not have access to k_I and the total quantity of digital services served downstream would be lower than in the Nash-equilibrium.

The intuition for the result on social welfare comes from the comparison of two competing forces: when comparing the equilibrium where $k_I > k_V$ to the equilibrium where $k_V \geq k_I$, we are moving from a situation in which both k_I and k_V are below the social optimum to a situation where k_I is above and k_V is exactly at the social optimum. Whether social welfare increases then simply depends on whether the increase in k_I is "too much" relative to the increase in k_V.

5 Applications

5.1 Contestability

How does vertical integration into Internet infrastructure by V impact the "contestability" of the downstream market? Contestability is a policy objective in the regulation of digital markets. The European Commission implicitly defines contestability as the absence of "very high barriers to entry or exit, including high investment costs, which cannot, or not easily, be recuperated in case of exit, and the absence of, or reduced access to, some key inputs in the digital economy, such as data", also referring to the network effects implied by access to more data.[15]

The justification for pursuing contestability is that in its absence, platform markets in particular are prone to "tipping", resulting in entrenched market power, leading to higher costs for consumers and less innovation.[16] A straightforward way to approach this question in the context of this model is to look at how the market shares of V and Q (both on the advertising side and the consumer side) behave between the two equilibria.

Market shares in an antitrust context are typically measured in terms of revenue (rather than profit). Revenue on the advertising side is proportional to the number of connected users for both V and Q. Revenue on the consumer side is nil for Q due to Bertrand competition and $k_V a$ for V due to market power for digital services on the monopolistic segment.

We denote the market shares of firm j as MS_j. It is measured in terms of revenue in the digital services market across both sides. It can be simplified by dividing by r and substituting infrastructure levels for demand. Now, market share is a function of infrastructure levels k_V, k_I as well as the ratio of the value generated by digital services and advertisement (a/r).

$$MS_Q = \frac{r(k_I/2)}{r(k_I + k_V) + ak_V} = \frac{(k_I/2)}{(k_I + k_V) + (a/r)k_V} \tag{13}$$

$$MS_V = 1 - MS_Q \tag{14}$$

We analyze the change in MS_Q in the case where two equilibria are possible. Denoting the equilibrium where $k_V \geq k_I$ ($k_V < k_I$) with superscript ** (*) we find:

Proposition 3: $MS_Q^{**} - MS_Q^{*}$ is negative and decreasing in r, β_I.

[15] The DMA discusses contestability repeatedly without giving a simple definition. Its meaning is implicitly explained most clearly in recitals 3 and 13 of the preamble.

[16] For example, the preamble of the EU's DMA asserts that "specific features of core platform services make them prone to *tipping*: once a service provider has obtained a certain advantage over rivals or potential challengers in terms of scale or intermediation power, its position may become unassailable and the situation may evolve to the point that it is likely to become durable and entrenched in the near future." (emphasis added), DMA, preamble, paragraph 25.

Proof: In the Appendix.

For the cases when we find that social welfare increases at the higher levels of vertical integration that come with the $k_V > k_I$-case, the model illustrates a trade-off: the dynamics of Internet infrastructure and vertical integration that can lead to increased efficiency and greater social welfare come at the cost of lower market share for non-integrated rival platforms. Pursuing the EU's policy goal of contestability in digital markets comes at a cost in terms of efficiency. It is at odds with the equilibrium that arises when V's infrastructure dominates.

High market shares are not harmful to consumers per se but often serve as shortcuts to market power analysis by competition authorities. Under some conditions, such as incomplete capital markets or network effects, high market shares can constitute and further enhance market power, however, which may justify concerns about the market structure of the downstream market.

5.2 Merger: I buys Q

What happens when I is not a pure upstream player but can integrate downstream as well and provide digital services? While there is no notable example of a CDN operator offering consumer-facing services, such as media entertainment, there are traditional Internet infrastructure firms active in downstream consumer markets. For example, Deutsche Telekom operates on multiple levels of Internet infrastructure for business customers (including data centers and cloud computing) but also has media offerings including sports and streaming for final consumers.[17]

To remain as close as possible to the base model, we consider a merger between I and Q, forming a new merged entity, M. Now, the two firms V and M are symmetric except for the potentially different marginal cost variables β_M, β_V. In the first stage, M and V invest in infrastructure k_M, k_V. At the second stage M makes a take-it-or-leave-it-offer to V for access to k_M. Both firms anticipate downstream competition at the third stage. We analyze M's decision to rent its infrastructure to V as well as M's and V's infrastructure investment decisions.

In the base model, a firm never has an incentive to provide less digital services than its available infrastructure allows (as infrastructure costs are sunk at the phase of downstream competition and digital services are provided at zero marginal cost). Now, however, if M offers more digital services downstream while it has a smaller network, it provides additional competition to its rival V which may decrease V's ability to pay high transfers for network access. The equilibrium depends on whether M can commit to a level of downstream services q_M alongside the transfer t_V at the second stage.[18]

[17] See Deutsche Telekom website, for business customers, for media and entertainment [both in German], last accessed 31.10.2022.

[18] While the base model allowed no commitment of I towards V not to renegotiate transfers with Q, it seems more plausible that an integrated entity could credibly commit not to compete with its business customers which is why we check both cases.

This is described in Proposition 4 which comes in two parts, 4a) describes the case without commitment by M for the quantity q_M, while 4b) describes the case with commitment:

Proposition 4a: If M cannot commit to a level of q_M at the second stage, $q_M = k_M$ in equilibrium. Furthermore,

$$\left.\begin{aligned} c'_M(k_M) &= r \\ c'_V(k_V) &= r/2 \\ t_V &= (k_M - k_V)(r/2) + k_V(a+r) \end{aligned}\right\} \text{ if } \beta_V > \beta_M + r/2$$

and

$$\left.\begin{aligned} c'_M(k_M) &= (3r/2) + a \\ c'_V(k_V) &= a + r \\ t_V &= k_M(a+r) \end{aligned}\right\} \text{ if } \beta_V < \beta_M + r/2$$

Proposition 4b: If M can commit to a level of q_M at the second stage, M offers

$$\left.\begin{aligned} t_V &= k_M(a+r) \\ q_M &= k_M \end{aligned}\right\} \text{ if } \beta_V < \beta_M + r/2$$

at the second stage and first-stage investment is given by $c'(k_M) = a + (3r/2)$ and $c'(k_V) = a + r$.

$$\left.\begin{aligned} t_V &= k_M(a+r) \\ q_M &= 0 \end{aligned}\right\} \text{ if } \beta_V > \beta_M + r/2$$

at the second stage and first-stage investment is given by $c'(k_M) = a + r$ and $c'(k_V) = a + r$. In both cases, if $\beta_V = \beta_M + r/2$, either one of these sets of investment levels and transfers can arise in equilibrium.

Proof: In the Appendix.

The first part of the proposition tells us that without commitment to a quantity level, we obtain the same result as in the base model. As M will always find it attractive at the third stage to offer digital services on its infrastructure and V anticipates this at the second stage, we have similar equilibrium profits and transfers and the condition on the difference in marginal cost that determines the equilibrium outcome is identical.

The second part of the proposition tells us that when the newly merged entity can commit in advance to a downstream strategy, it may find it useful to set $q_M = 0$, or shut down its digital services operations and become a pure infrastructure provider. This is the case when its infrastructure is larger than that of the vertically integrated firm. The intuition is that in this case, it can create a downstream monopoly and participate in its profits by charging monopoly prices for access to k_M. As now the size of the competitive segment is $\emptyset$ and the size of the monopolistic segment is $k_M + k_V$, consumer welfare is 0 and lower than in the base model when $\beta_V > \beta_I + r/2$.

This result does not have novel implications per se, competition authorities already investigate vertical mergers for the ability and incentive of a merged entity to divert business either upstream or downstream. While harmful to consumers, M shutting down its downstream business maximizes social welfare and increases investment (as marginal investment fulfills the conditions of equation 10). The proof also involves checking that the foregone downstream revenue is smaller than the additional transfer fee but this is implied by the condition $\beta_V > \beta_M + r/2$.

5.3 Efficient side-payments

One question of current interest is the contribution of digital content firms to telecommunication networks.[19] We observe side payments between some large content platforms, such as Netflix, and traffic carriers such as Comcast.[20] This model can explain why such side payments can occur when I's network is only used to a limited extent by Q. There are several reasons why smaller competitors may be limited in their ability to compete with a large digital platform even in the presence of sufficient infrastructure: intellectual property (patents, technology or media content), network effects, brand attraction, or internal technical ability. For example, the lack of exclusive content may limit a video-streaming platform's attractiveness to consumers.

We think of these various limitations as some exogenous limit to the demand served by Q. If Q cannot serve more than some amount $\bar{q}_Q$ of services and V and I can agree, before $t = 1$ of the base game, that V will pay a transfer t'_V (in addition to t_V) to I conditional on choosing a certain value k_I. To make the model interesting, we only consider cases where $\bar{q}_Q$ is binding in equilibrium. As the proof is short, we do not move it to the Appendix.

Proposition 5: If Q is capacity-constrained and conditional side-payments are possible and $\beta_V > \beta_I + r/2$, I's and Q's incentives to invest align. Investment levels are given by $c'_V(k_V) = c'_I(k_I) = a + r$.

Proof: When Q is capacity-constrained so that it can serve only a portion of the demand that is smaller than the size of I's network, every unit $k_I > \bar{q}_Q$ can be rented out only to V. V generates a profit of $(a+r)(k_V + k_I - \bar{q}_Q) + r/2\bar{q}_Q - k_V(k_V) - t'_V - t_V$. In equilibrium, I and V can jointly maximize their surplus by setting a value of k_I such that $c'_I(k_I) = a + r$. Call this value k_I^{**} and k_I^* the equilibrium level of I's investment. Then this can be implemented and is incentive compatible when V offers any value of t'_V such that $(k_I^{**} - k_I^*)(a+r) > t'_V > c_I(k_I^{**}) - c_I(k_I^*)$ if and only if $k_I = k_I^{**}$. This value

[19]See Reuters, EU's Vestager assessing if tech giants should share telecoms network costs (2022), last accessed 01.05.2023.

[20]See Wall Street Journal, Netflix to Pay Comcast for Smoother Streaming (2014), last accessed 01.05.2023.

exists and is positive from the fact that $c_I(k_I)$ is increasing and convex and its marginal value is $a+r$ only at $c_I(k_I^{**})$ and lower at any lower level. □

This result illustrates that in the base model, competition from Q serves as a friction that prevents I and V from maximizing their joint surplus. Absent this constraint, the value k_I that maximizes total surplus is simply the one that equates marginal cost and revenue. An enforceable contract on the choice of k_I is a simple mechanism to implement the efficient investment levels k_V, k_I.

5.4 Net neutrality

Net neutrality is the absence of termination charges, i.e., content providers do not pay those Internet service providers (ISP) into whose network their traffic is delivered. According to other definitions, net neutrality does not allow ISP to offer a "fast lane" in exchange for payment.

The model only translates to this setup approximately as I does not represent an ISP operating last-mile-networks (we generally do not observe competition at this stage due to fixed costs that imply a natural monopoly). However, there is some discussion as to whether certain infrastructure operators should follow the same rules as ISP. For example, in the US CDN are exempt from net neutrality rules, following lobbying activity.[21]

Consider a net neutrality scenario in which I is constrained to charge a single price $t_Q = t_V = t$. Then, access to this infrastructure can be described as a price posted by the upstream industry instead of the bilateral bargaining of the base model. More precisely, after investment decisions in k_V, k_I have been made, I posts a price under which platforms can purchase non-rival access to k_I. V and Q decide simultaneously whether to pay the price, and finally downstream competition takes place. Here, we do not consider renegotiation of the transfer t (as there is only a single profit-maximizing price, renegotiation makes no difference in this case).

Proposition 6: Under net neutrality, I charges $t = k_I(a+r)$ and chooses $c'(k_I) = a+r$. In equilibrium, Q chooses not to pay this price and $q_Q = \{\emptyset\}$, $q_V = k_V + k_Q$. $p_{V,m} = a$, $d_{V,m} = q_Q$.

Proof: In the Appendix.

Net neutrality for infrastructure providers thus harms downstream competition as Q is essentially excluded from the market and consumers pay higher prices due to the absence of a competitive segment. The intuition behind this result is that I has no

[21] See the FCC's NPRM in 2014 which introduces their thinking on CDN. Protecting and Promoting the Open Internet NPRM (2014). Akamai and other CDNs lobbied to FCC to amend the Open Internet Order which would exempt them from regulation See filings received by Akamai between 02.02.2014 and 31.03.2015. This was reflected in the final order. FCC Releases Open Internet Order (2015). All links last accessed 17.01.2023.

interest in downstream competition as it reduces industry profit. There is no equilibrium where Q accepts the offer and V rejects because in this case, V's best response would be to invest at least up to the point where $c_V'(k_V) = r/2$. However, due to the presence of competition, Q cannot operate profitably when $t = k_I(a + r)$. The resulting outcome maximizes social welfare as the resulting equilibrium levels of investment fulfill the conditions of equation 10.

Net neutrality in this model unravels the assumption that I cannot commit not to renegotiate and sell to a second firm, limiting its power to create a downstream monopoly. Other papers show similarly how the effects of net neutrality hinge on an infrastructure provider's ability to commit to its pricing (see Footnote 7 of Schuett, 2010) or alternatively on pricing commitment by the downstream industry (Greenstein, Peitz, and Valletti, 2016).

6 Robustness

6.1 Congestion

So far, we have abstracted from the possibility of congestion. Each unit of infrastructure could be used as a non-rival input in the provision of digital services by both V and I. This most accurately describes investments that enhance capabilities rather than capacity. For example, in a network that does not suffer from congestion, additional investment may represent new network nodes that reduce the average (physical) distance that data packages travel between digital services firms and consumers. In such a case, it may be reasonable to abstract away from congestion and to focus on the role of the network in increasing demand through enhanced quality.

In reality, there are few examples of infrastructure that are truly non-rival, and the aspect of congestion may just be more or less important. To reflect this, we consider a robustness check in which returns on the competitive segment of the market are decreased. We introduce a *congestion factor* $1/2 \leq \phi < 1$. We consider adjusted demand functions on the competitive segment:

$$d'_{V,c} = \phi d_{V,c} \quad \text{and} \quad d'_{Q,c} = \phi d_{Q,c}. \tag{15}$$

The closer ϕ is to 1, the smaller is the role played by congestion and for $\underset{\phi \to 1}{lim}$, we are back in the base model. As ϕ moves closer to $1/2$, congestion reduces the returns on the competitive segment.

Simultaneous use of infrastructure (i.e., on the competitive but not on the monopolistic segment), decreases the quantity of digital services the lower is ϕ. For example, the time that videos buffer before playing increases or the time that websites take to load and consumers switch away.[22] As firms compete Bertrand-style on the competitive segment, this results in lower advertisement revenues.

Proposition 7: Under congestion, investment is given by

$$\begin{cases} k_V = \left(\frac{(r\phi/2)-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_V = \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \quad \text{and} \quad \begin{cases} k_I = \left(\frac{r\phi-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_I = \left(\frac{a+(1+\phi/2)r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{16}$$

Proof: In the Appendix.

[22] A leaked e-mail by Mark Zuckerberg about Facebook from February 14, 2008 is instructive about the relationship between transmission quality and the quantity of services consumed: "We have a lot of stats that show that usage of the site is basically tied to how fast the site is. The faster we make the site, the more activity we see. I believe the latest data I saw was that if we made the site 100ms faster we'd have about 3% more activity and if we made the site a second faster we'd have about 20% more activity. That's a really big deal. What it means is that even if users don't consciously notice the speed, it's subconsciously making them do fewer pager views and less activity." The Zuckerberg Files, "Six4Three v. Facebook sealed exhibits", last accessed 27.02.2023.

The overall impact of congestion on the model is low. Investment in the case $k_I > k_V$ is decreased overall for both V and I. The part of the equations for k_I, k_V related to revenues is multiplied by ϕ. When I has the larger network, the transfer it gets from V reflects that marginal investment by V allows V not only to sell more services but also to escape the "congestion tax" (the term $k_V(1-\phi/2)r$ in equation 63 is greater than the corresponding term $k_V(r/2)$ in the base model, equation 32). Interestingly, in the case where $k_V \geq k_I$, investment by V is unaffected and investment by I is reduced to a lesser degree: only the part of the expression that pertains to the transfer paid by Q is multiplied by ϕ in equation 61. The intuition is that the transfer t_V is solely determined by the returns of V on the monopolistic segment which are unaffected by congestion.

6.2 Nash-in-Nash bargaining

In the preceding analysis, we have not focused on surplus division between I on the one hand and V and Q on the other hand. In the base model, I proposes a take-it-or-leave-it offer that allows it to extract the entire surplus from the rental of k_I, only being disciplined by future competition from itself, due to the lack of commitment. Focusing on revenues instead of profits, for example in the analysis of market shares in Section 5.2, has allowed us to side-step this issue.

This assumption is less credible the more we think of the upstream industry as competitive. Nevertheless, intuitively it is not clear that limiting I's ability to extract surplus from the transaction should fundamentally change the model, rather than modifying the marginal-revenue conditions that determine equilibrium investment. Having introduced a source of interdependence of bargaining outcomes in the form of congestion, it seems desirable, however, to study the effect of an alternative surplus-division rule on the outcome of the model.

The Nash-in-Nash bargaining framework seems to be the most reasonable alternative surplus division rule for our model. The model satisfies the conditions of weak conditional decreasing marginal contributions, feasibility, and gains from trade posed by Collard-Wexler, Gowrisankaran, and Lee (2019). Alternative frameworks to intensify competition upstream, such as entry from a competitive fringe or adding one or several additional upstream firms, would require additional assumptions on the timing of investment and combinatorics for the now exponentially increased number of constellations in which V and Q can have access to different subsets of networks. These assumptions may drive results in addition to any potential effect from alternative modes of surplus division, making the model less tractable.

Again we consider the case with congestion, in which demand on the competitive segment is modified by a congestion parameter $1/2 \leq \phi < 1$. In addition, we define bargaining weights $0 < \delta_j < 1$, $j = V, Q$ which represent the share of the surplus that firm I can extract in the negotiation with V and Q, respectively. The impact of these changes on the model is that we now assume agreements in period 2 to occur immediately and transfers t_Q, t_V to be given by the following expressions:

$$t_Q = \delta_Q \phi k_I (r/2) \tag{17}$$

$$t_V = \begin{cases} \delta_V (k_I \frac{r\phi}{2} + k_V(a + r(1 - \frac{\phi}{2}))) & \text{if } k_I > k_V \\ \delta_V k_I (a + r) & \text{if } k_V \geq k_I \end{cases} \tag{18}$$

Proposition 8: In the case with congestion and Nash-in-Nash bargaining, equilibrium investments are given by

$$\begin{cases} k_V = \left(\frac{(1-\delta_V)a + r(1-\delta_V(1+(\phi/2))) - \beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} \\ k_V = \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} \end{cases} \qquad \begin{cases} k_I = \left(\frac{(\delta_V+\delta_Q)\frac{r\phi}{2} - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_I = \left(\frac{\delta_V(a+r) + (\delta_Q \phi r/2) - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{19}$$

Proof: In the Appendix.

V's marginal investment when $k_V \geq k_I$ does not depend on δ_V. When $k_I > k_V$, however, V's investment now also depends on a as I no longer extracts the whole surplus on this margin. The new expressions reinforce the intuition about the cases in which V's profit drives investment by V and I, respectively. Unsurprisingly, I is now left with lower investment incentives although cases in which I over-invests are still possible (but ruled out in cases where $\delta_V + \frac{\delta_Q \phi}{2} \leq 1$.

Overall, modeling surplus division via Nash-in-Nash bargaining makes the model richer as it highlights the cases in which individual parameters do (a for V in the case $k_I > k_V$) or don't (δ_V for V's investment in the case $k_V \geq k_I$) matter. It does not fundamentally change the model however, reassuring us that the upstream monopoly is a relatively innocent assumption.

6.3 Product differentiation

Finally, we are interested how the introduction of product differentiation impacts the model. Until now, V and Q have offered undifferentiated services in the downstream market. On the competitive segment, consumers view their offers as perfect substitutes. Now, we consider that consumers have an innate preference for the products offered by one firm or the other. This may be due to personal preference, for example, as a result of branding, or differentiated product offering, such as exclusive content in the case of video-streaming platforms.

We follow the well-known framework of Shubik and Levitan (1980) for a differentiated goods model. This framework has the advantage that the total market size is unaffected by the number of products or their degree of substitutability. We have already considered a modification of total market size through congestion which shrinks the total amount of services sold on the competitive segment of the downstream market. We maintain both congestion and Nash-in-Nash bargaining as in the previous two subsections.

Denoting the degree of substitutability $\mu \in [0, \infty]$ for $n = 2$ products (digital services by firms V and Q, respectively), following the notation of Motta (2004) with a as the demand scaling parameter, we write indirect demand for $i = V, Q$ as

$$p_{i,c} = a - \frac{1}{1+\mu}\left(nq_i + \mu\sum_{j=1}^{n} q_j\right), \tag{20}$$

and direct demand as

$$d_{i,c} = \frac{1}{n}\left[a - p_i(1+\mu) + \frac{\mu}{n}\sum_{j=1}^{n} p_j\right]. \tag{21}$$

We leave demand on the monopolistic segment unaffected. On the competitive segment, price competition will now not generally result in marginal-cost pricing because a firm that increases its price above marginal cost will still face positive demand. However, due to the presence of advertisement revenues r, it is not clear that this price increase will be profitable. It could be that the marginal loss of advertisement revenue, which is linear in $q_{i,c}$, outweighs the marginal gain from a price increase for any price above marginal cost.

Under price competition for two firms and interpreting r as a negative marginal cost (as in Gans, 2022), equilibrium prices are

$$p_{i,c} = \frac{2a - r(2+\mu)}{4+\mu} \tag{22}$$

First, note that if advertisement revenues are too large, firms will indeed lower their price below marginal cost (0). As the model only allows non-negative prices, to rule out this case, we impose

$$r < \frac{2a}{2+\mu}. \tag{23}$$

Proposition 9: Equilibrium investments are now given by

$$\begin{cases} k_I &= \left(\frac{(\delta_Q+\delta_V)\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right)-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} \\ k_I &= \left(\frac{\delta_Q\frac{\phi(a+r)}{4+\mu}+\delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right)}{\alpha}\right) \end{cases} \begin{cases} k_V &= \left(\frac{a+r-\delta_V\left(\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right)-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} \text{ if } k_I > k_V \\ k_V &= \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} \text{ if } k_V \geq k_I \end{cases} \tag{24}$$

The $k_I > k_V$-equilibrium arises if

$$\beta_I \geq \beta_V + \delta_Q\left(\frac{\phi(a+r)}{4+\mu}\right) + \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) - (a+r) \tag{25}$$

and the $k_V \geq k_I$-equilibrium arises otherwise.

Proof: In the Appendix.

The resulting expressions for equilibrium investment levels and the parameter thresholds that give rise to the different equilibria are now more complicated but have some interesting properties. The new parameter μ does not merely shift all outcomes. Product differentiation always increases the share of the surplus that I can capture when $k_I > k_V$. In the case $k_V \geq k_I$, it also increases t_Q but decreases (increases) the share captured by I from V if $a + r > (<)\ 1$. Notably, investment by V is still at the efficient level if $k_V \geq k_I$. When $k_I > k_V$, the deviation from the efficient level of k_V is governed by the share of surplus captured by V and only this part is affected by congestion (the intuition for which was given in the previous section), and product differentiation.

6.4 Discussion of the model

The key prediction of the model is a change in the relationship between the traditional carriers of data on the Internet and vertically integrated platform companies. Depending on the role that the upstream firm plays in the vertically integrated firm's value creation, we observe a drastic, non-continuous change in investment incentives. I argue that this mechanism points to issues beyond digital services. Collaborations between digital platform companies and other industries, such as the automotive industry, have led to fears about the future focus of value creation. This model shows that the shift in outside options through vertical integration can have a disruptive impact on an industry.

This model is intended as a first attempt to analyze the questions posed by the vertical integration of digital platforms through proprietary infrastructures. Both competition in downstream markets as well as Internet infrastructure are complex and technical issues, and sector-specific regulation differs between Europe, North America, and other regions of the world. As such, it is not the purpose of this model to predict exactly the behavior and contracts that will arise in any specific geographic or product market. Instead, the model illustrates key features of proprietary Internet infrastructure: the potential efficiency and new goods and services provided by big tech investment, but also the interaction in the marketplace with smaller players which can be harmed by well-intended regulation or marginalized through increased efficiency.

The setup is standard except for the way that upstream investment determines the size of the downstream market. Intuitively, this one-dimensional measure of consumer demand can be read either as the intensive margin of demand (existing customers demanding additional services as bandwidth increases) or the extensive margin of demand (new customers are won as networks improve and bandwidth increases). For a more stringent exposition, I have focused only on the first interpretation in this paper but note that investment can also affect the extensive margin, especially for new services.

I make use of a stylized environment to approach this complex environment. This model illustrates the incentives for a large digital services company to invest in proprietary Internet infrastructure. Even when applying this model to an example as specific as video-on-demand streaming, we are still folding several kinds of differentiated products downstream and different kinds of infrastructure upstream into a simple framework that relates the amount of the available infrastructure to the effective quantity of services that can be provided.

The issue of downstream foreclosure (I refusing to deal with a downstream firm) is sidestepped by limiting I's ability to commit to a deal for monopoly. While this is partially motivated by the existence of upstream competition (and absent upstream competition, the essential facilities doctrine in competition law can limit anticompetitive foreclosure), a more economic formulation could point out that two forces in the model drive the incentive for foreclosure: congestion makes downstream foreclosure more attractive, as letting a second firm compete downstream limits the revenue I can extract from the first firm, while product differentiation points in the exact opposite direction: allowing a second firm in expands the downstream market. For the sake of exposition, I chose to move both of these effects into robustness checks but it seems worth pointing out that foreclosure can also be controlled through appropriate assumptions on the relative strength of these forces.

What is k? We are looking at proprietary networks that consist of many different parts. For the purpose of this model, we are not interested in, for example, what share of investment goes towards data centers vs. submarine cables. Instead, we are interested in the service improvement that can be purchased at a given price. Therefore, infrastructure k can be thought of as a measure of real quality gain from a given level of investment. This only requires downstream demand to be at least somewhat elastic to the quality improvement induced by investment which seems a reasonable assumption (see also Footnote 22).

The 1:1 ratio between investment and downstream capacity is a free variable, as even relatively inelastic consumer demand can be expressed in the slope of $c_i(k_i)$. A steep investment cost function means that it is very costly to expand the market through further investment, and a low elasticity of demand with respect to quality improvements implies that large investments are needed to expand downstream demand. The main assumptions on investment in the model are therefore that investment costs are convex (increasing demand through additional infrastructure becomes more expensive as efficient and low-cost investment opportunities are realized) and potentially different for I and V.

The model is balanced on a "knife's edge" as for given β_I there is only one value of β_V that gives rise to the two types of equilibria that we compare in our analysis. The key mechanism, also throughout the robustness checks, is the change in the outside option. A more elegant but much more complicated approach would assume that the own-price elasticity of demand differs along the dimension of q_i independent of whether a particular level of infrastructure is reached by one firm or many, and potentially decreases at higher levels of q_i. This would also generate higher markups in the area where the vertically integrated firm does not face competition. This kind of demand could correspond to markets where infrastructure supports innovative and novel services that have fewer alternatives than basic ones.

This alternative formulation might be useful to model the entire ecosystem of a large digital services company, where the range of services offered through increasingly sophisticated infrastructure could range from e-mail, online search, and real-time virtual realities (a "Metaverse") to future technologies. Such a demand structure would, however, assume the existence of a wider market with substitutes of varying qualities for all kinds of services offered by such a firm. For a tighter exposition and a more specific example, I choose to concentrate on a market in which the consumers' choice of options is always well-defined, as each segment of demand represents a product that is offered either by a monopolist or a Bertrand oligopolist and an outside option of 0.

While it seems natural to think about vertical separation, a counterpart to the forward integration considered in Section 5.2, it is not a case that I analyze. The technical reason is that with two independent upstream networks that both downstream players could access, the number of possible permutations of network configurations increases from 4 to 16 (as each downstream firm can have access to either no network, network one, network two, or both networks). Not only does this make the model a lot less tractable but it also requires additional assumptions on the timing and commitment regarding the access of downstream firms to particular networks. More practically, vertical separation in this context seems a less credible policy than in the natural monopoly applications where it normally arises (railroads, electricity networks, and other utilities). It seems harder to justify punishing a firm for improving its IT. Moreover, since the defining characteristic of proprietary networks is their specialization to the business and demand of a particular firm, the value of V's infrastructure would likely be diminished or completely lost upon separation.

The most promising avenue for future research appears to be innovation in proprietary networks: While the motivation to study proprietary networks is partly also the ability of platforms to steer innovation in their ecosystems, innovation is not an explicit feature of the model. In part, the increased demand as a result of increased investment can be understood as a demand for innovative services that only become feasible with increased infrastructure. However, a practical concern around proprietary networks owned by big tech firms (e.g., deployment of software products through the cloud services of firms such as Microsoft, Amazon, and Google) is that innovation might shift into their "walled gardens", decreasing overall benefits from innovation by limiting spillover and network effects.

7 Conclusion

The Internet has affected the global economy on many levels, underscoring the critical importance of understanding the economics governing its infrastructure. It has enabled some platform businesses to grow to spectacular size in a short amount of time. Understanding the economics underpinning its infrastructure is key to successful economic policy and regulation. In particular, an effects-based assessment of regulation, potential anti-competitive conduct, and merger review needs economic guidance. This paper illustrates the economic effects of the increasing vertical integration into Internet infrastructure by digital platform companies.

The model presented in this paper outlines the investment incentives driving competition downstream in digital markets. It demonstrates that investment incentives surge for both pure upstream players and vertically integrated firms as the latter increases its ownership of infrastructure. This phenomenon arises from the vertically integrated firm's market power over additional demand, even in the absence of additional infrastructure from the infrastructure firm.

Marginal investment by the infrastructure firm increases the total surplus to be shared between it and the vertically integrated firm. As each unit of upstream investment results in a constant increase in revenue for the vertically integrated firm, the infrastructure of the infrastructure firm becomes fully commoditized. An important exception arises when the infrastructure firm has lower investment costs and fringe players cannot compete with the vertically integrated firm over the whole range of services, for example, because of a lack of technical ability, patents, or exclusive content. In this case, the vertically integrated firm may prefer to subsidize investment by the infrastructure firm directly. At least in this model, there is no market failure and no need for intervention for this to happen.

As a consequence, this model explains different aspects of the rise of private, proprietary infrastructure. I predict that this additional investment is socially desirable but might increase the market shares of the largest companies. High market shares are not problematic per se but often serve as shortcuts to market power analysis by competition authorities. Under specific circumstances, such as incomplete capital markets, present strength can beget future strength, however, justifying a concern about the market structure of the downstream market.

One policy conclusion that follows directly from the model and which is robust to different settings and applications is that expanding net neutrality, a pet policy of some Internet activists, to the infrastructure firms currently not covered, may lead to the exclusion of rivals and harm to consumers. Entry downstream by the upstream firm may also have the surprising effect of reducing downstream services whenever the newly vertically integrated firm prefers infrastructure separation, yielding an inefficient outcome that does not occur in the equilibrium of the base model.

Caution is advised before drawing further policy conclusions from a literal reading of the model. It is understood that this model is not a full simulation of any particular downstream market with its generic features such as paid-for premium services and advertisement revenues. Nor does it necessarily describe the market structure for the upstream industry. Instead, I purposefully aggregate infrastructure investment into a black box variable to study the effect of quality-improving infrastructure investment. Intervention in any particular market would need to carefully evaluate the sources of revenue and business models of firms in the downstream market and identify the most important components of Internet infrastructure related to an industry.

Nevertheless, this paper can help policymakers and enforcers ask the right questions both for a competitive analysis of a digital market and for a forward-looking market investigation: First, it describes how even efficient and increasing investment in proprietary and public networks can enhance the unequal footing on which vertically integrated firms and smaller rivals compete. Second, it illustrates the kingmaker role of third-party infrastructure providers, especially when they become active downstream themselves.

Finally, the model points to questions beyond digital services. Large technology firms have begun vertical integration in other fields, including automotive, where questions about the future focus of value creation have also been asked. The model allows for many rich expansions as discussed above. In addition, the analysis can be expanded by appropriate data to test model predictions empirically.

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A Proofs

A.1 Proof of Theorem 1

Proof of proposition 1: At the third and final stage, firm $j = V, Q$ choose consumer prices $p_{j,h}$ for $h = c, m$. On the monopoly segment, firm j maximizes profits by setting $p^*_{j,m} = a$. On the competitive segment $p^*_{i,c} = 0$ is the only equilibrium: firms have zero marginal cost and products are perfect substitutes, so Bertrand pricing prevails.[23]

If both firms decide to rent infrastructure access from I (we'll verify next that they do so in equilibrium), the resulting available infrastructure is as in Figure 2 and resulting demand is:

$$d_{Q,c} = d_{V,c} = k_I/2 \tag{26}$$
$$d_{V,m} = k_V \tag{27}$$
$$d_{Q,m} = 0. \tag{28}$$

Given these prices and demand, profits are

$$\Pi_Q = k_I(r/2) - t_Q \tag{29}$$
$$\Pi_V = k_I(r/2) + (r + a)k_V - t_V - c_V(k_V). \tag{30}$$

Recall that when setting transfers, I cannot commit not to renegotiate. For example, if Q rejects the initial offer, I can always offer $t'_Q = k_I(r/2)$ which allows Q to break even (anticipating $d_{Q,c} = k_I/2$ and revenues $rd_{Q,c}$) and is accepted. Anticipating that it will face competition for the first k_I units of services, the highest price that V is willing to pay to I is

$$t_V = \frac{r}{2}k_I + k_V(a + r) - k_V\frac{r}{2} - max\{0, (k_V - k_I)(r/2 + a)\} \tag{31}$$

which can be rewritten as

$$t_V = \begin{cases} (k_I + k_V)\frac{r}{2} + k_V a & \text{if } k_I > k_V \\ k_I(a + r) & \text{if } k_V \geq k_I. \end{cases} \tag{32}$$

Importantly, V's willingness-to-pay for access to k_I depends on whether its own infrastructure is greater than I's. Levels of infrastructure are selected in the first period and are fixed and observed when I makes its offers. When k_I is larger than k_V and V does not rent access to k_I, V can only serve k_V units of digital services and it faces competition from Q on this range. V's revenue in this case is $k_V\frac{r}{2}$, the third term in equation 31. Its willingness-to-pay is therefore the difference between the total revenue it makes with k_I minus this outside option.

[23] In spite of the presence of capacities, this is not an example of Bertrand-Edgeworth pricing because in this model, prices are not declining in quantity. Every unit of infrastructure enables provision of an additional unit of services, valued at a constant value a.

If k_V exceeds k_I, V will be able to act as a monopolist for some part of demand $(k_V - k_I)$ even without access to I's infrastructure. At the margin, one additional unit of infrastructure allows V to serve one more unit of digital services at the monopolist's markup $a + r$.

At the first stage, V and I make their investment decisions. We now set up the profit functions and solve for the profit-maximizing level of investment. V's first period problem is

$$\underset{k_V}{max}\ \Pi_V = d_{V,c}(p_{V,c} + r) + d_{V,m}(p_{V,m} + r) - t_V - c_V(k_V). \tag{33}$$

Now we can write

$$\frac{\partial \Pi_V}{\partial k_V} : a + r - c_V'(k_V) - \frac{\partial t_V}{\partial k_V} = 0 \tag{34}$$

$$\frac{\partial t_V}{\partial k_V} = \begin{cases} \left(\frac{r}{2} + a\right) & \text{if } k_I > k_V \\ 0 & \text{if } k_V \geq k_I \end{cases} \tag{35}$$

$$c_V'(k_V) = \alpha k_V^{\alpha-1} + \beta_V \tag{36}$$

which yields the first order conditions for k_V:

$$\begin{cases} \frac{r}{2} = c_V'(k_V) & \text{if } k_I > k_V \\ a + r = c_V'(k_V) & \text{if } k_V \geq k_I \end{cases} \leftrightarrow \begin{cases} k_V = (\frac{r/2 - \beta_V}{\alpha})^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_V = (\frac{a+r-\beta_V}{\alpha})^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{37}$$

The second order conditions are fulfilled from the convexity of $c_V(k_V)$.

I's first stage problem is

$$\underset{k_I}{max}\ \Pi_I = t_Q + t_V - c_I(k_I) \tag{38}$$

with

$$\frac{\partial \Pi_I}{\partial k_I} : \quad \frac{r}{2} + \frac{\partial t_V}{\partial k_I} - c_I'(k_I) = 0 \tag{39}$$

$$\frac{\partial t_V}{\partial k_I} = \quad \begin{cases} \frac{r}{2} & \text{if } k_I > k_V \\ a + r & \text{if } k_V \geq k_I. \end{cases} \tag{40}$$

which yields the first order conditions for k_I:

$$\begin{cases} r = c_I'(k_I) & \text{if } k_I > k_V \\ a + (3r/2) = c_I'(k_I) & \text{if } k_V \geq k_I \end{cases} \leftrightarrow \begin{cases} k_I = \left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_I = \left(\frac{a+(3r/2)-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{41}$$

Comparing the upper and lower branches of equations 37, 41, we see that there are two types of equilibrium and that these are characterized by either $k_I > k_V$ or $k_V \geq k_I$. In the former case, investment is lower than in the latter case, as stated in Proposition 1. $\square$

A.2 Proof of Theorem 2

For given β_V, β_I, we can write social welfare when $k_V \geq k_I$ as

$$\begin{aligned} S|_{k_V \geq k_I} = (a+r)&\left[\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} + \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \\ &- \left[\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_I\left(\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} - k_0\right)\right] \\ &- \left[\left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_V\left(\left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} - k_0\right)\right] \end{aligned} \tag{42}$$

while if the inequality is reversed, we have

$$\begin{aligned} S|_{k_I > k_V} = (a+r)&\left[\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} + \left(\frac{r/2-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \\ &- \left[\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_I\left(\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} - k_0\right)\right] \\ &- \left[\left(\frac{r/2-\beta_V}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_V\left(\left(\frac{r/2-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} - k_0\right)\right] \end{aligned} \tag{43}$$

Now, for $S|_{k_V \geq k_I} - S|_{k_I > k_V} > 0$, it must be that

$$\begin{aligned} &(a+r)\left(\left[\frac{2a+\frac{5}{2}r-\beta_I-\beta_V}{\alpha}\right]^{\frac{1}{\alpha-1}} - \left[\frac{\frac{3}{2}r-\beta_I-\beta_V}{\alpha}\right]^{\frac{1}{\alpha-1}}\right) \\ &\quad - \left[\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_I\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \\ &\quad - \left[\left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_V\left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \\ &+ \left(\frac{r-\beta_I}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_I\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \left(\frac{\frac{r}{2}-\beta_V}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \beta_V\left(\frac{\frac{r}{2}-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} > 0 \end{aligned} \tag{44}$$

$$\leftrightarrow (a+r)\left[\frac{2a+r}{\alpha}\right]^{\frac{1}{\alpha-1}} - \left(\frac{2a+r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \beta_I\left(\frac{a+\frac{r}{2}}{\alpha}\right)^{\frac{1}{\alpha-1}} - \beta_V\left(\frac{a+\frac{r}{2}}{\alpha}\right)^{\frac{1}{\alpha-1}} > 0 \tag{45}$$

$$\leftrightarrow (a+r)^{\alpha-1}\left[\frac{2a+r}{\alpha}\right] > \left[\frac{2a+r}{\alpha}\right]^{\alpha} + (\beta_I+\beta_V)^{\alpha-1}\left(\frac{a+\frac{r}{2}}{\alpha}\right) \tag{46}$$

which is the expression in the proposition. □

A.3 Proof of proposition 3

Substitute the expressions from equations 37, 41 into

$$
\begin{aligned}
MS_Q^{**} - MS_Q^{*} &= \frac{k_I^{**}/2}{k_I^{**} + (1+a/r)k_V^{**}} - \frac{k_I^{*}/2}{k_I^{*} + (1+a/r)k_V^{*}} \qquad (47)\\
&= \frac{\frac{1}{2}\left(\frac{a+\frac{3r}{2}-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{a+\frac{3r}{2}-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} + \left(1+\frac{a}{r}\right)\left(\frac{a+r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}} - \frac{\frac{1}{2}\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} + \left(1+\frac{a}{r}\right)\left(\frac{\frac{r}{2}-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}}}\\
&= \frac{1}{2}\left(\frac{a+\frac{3}{2}r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{r-\beta_I}{\alpha}^{\frac{1}{\alpha-1}} + \left(1+\frac{a}{r}\right)\left(\frac{\frac{r}{2}-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}}\right)\right]\\
&\quad - \frac{1}{2}\left(\frac{r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{a+\frac{3r}{2}-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} + \left(1+\frac{a}{r}\right)\left(\frac{a+r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\\
&= \left(a+\frac{3}{2}r-\beta_I\right)\left(r-\beta_I+\left(1+\frac{a}{r}\right)^{\alpha-1}\left(\frac{r}{2}-\beta_V\right)\right)\\
&\quad - (r-\beta_I)\left[a+\frac{3}{2}r-\beta_I+\left(1+\frac{a}{r}\right)^{\alpha-1}(a+r-\beta_I)\right]
\end{aligned}
$$

which, after multiplying out the brackets and some simplifications, becomes

$$
\begin{aligned}
&\left(a+\frac{3r}{a}-\beta_I\right)\left(\frac{r}{2}-\beta_V\right) - (r-\beta_I)(a+r-\beta_I) \qquad (48)\\
&= -\frac{ra}{2} - \frac{r^2}{4} + \beta_I\frac{r}{2} + (\beta_I-\beta_V)a + \beta_I - \beta_V\frac{3}{2}r + \beta_I\beta_V - \beta_I^2.
\end{aligned}
$$

As we consider the case where both equilibria can exist, we substitute $\beta_V = \beta_I - \frac{r}{2}$.

$$
\begin{aligned}
-\frac{ra}{2} - \frac{r^2}{4} + \beta_I\frac{r}{2} + (\beta_I-\beta_I+\frac{r}{2})a + \beta_I r - \frac{3}{2}\beta_I r - \frac{3}{4}r + \beta_I^2 - \beta_I\frac{r}{2} - \beta_I^2 \qquad (49)\\
= -\frac{r^2}{4} - \frac{3}{4}r - \beta_I\frac{r}{2}
\end{aligned}
$$

which is clearly negative, given that $r > 0$, $\beta_I > 0$, proving the proposition.

A.4 Proof of proposition 4

First, note that in the case without commitment, at the third stage, M will always compete downstream, so $q_M = k_M$ because doing so strictly increases M's profit, at least by $k_M(r/2)$ if $k_M \leq k_V$ and $k_M(r/2) + max\{0, (k_M - k_V)(a + r/2)\}$ otherwise, while withholding its capacity at the third stage yields no profit. Anticipating this, the highest transfer t_V that V is willing to pay is the difference between the profit with or without k_M given that the competitive segment downstream will be k_M if V agrees to rent access to M's infrastructure or k_V if it does not and $k_V \leq k_M$. But then the analysis is equivalent to the base model with k_I instead of k_M.

In the case with commitment, if $k_M > k_V$, offering $q_M = k_M$ limits M to charge $t_V \leq (k_M - k_V)\frac{r}{2} + k_V(a+r)$, or the additional profit V would make competing when M's infrastructure is larger. If M commits to $q_M = 0$, the highest transfer t_V that V would accept is $k_M(a+r)$. There are only these two candidate values of q_M as profits and the highest transfer t_V are linear in the value of q_M that M offers. Comparing profit under these two candidate commitments, we find

$$k_M(a+r) > k_M\frac{r}{2} + k_V(a+\frac{r}{2}) \tag{50}$$

which is true only if $k_M > k_V$. The proposition follows from this. □

A.5 Proof of proposition 6

There are two candidate prices for t, either the highest price that both Q and V are willing to pay or the highest price that a single firm is willing to pay. The highest price that both Q and V are willing to pay must be $k_I(r/2)$ that is, Q's revenue under competition.[24] The highest revenue that V could make renting k_I is the monopoly profit $k_I(a+r)$ which is higher. The highest revenue that Q could make, given k_V, is $k_I(r/2) + max(0,(k_I - k_V)(a+r/2))$. This is lower than $k_I(a+r)$ for all $k_V > 0$. Whenever V sets $k_V > 0$, only V will accept. By setting $t = k_I(a+r)$, I ensures that V agrees to rent at the proposed price. Off-path, V never has the incentive to choose $k_V = 0$ as $c'_V(0) = \beta_V$ which is smaller than the marginal revenue of $r/2$ by assumption. □

A.6 Proof of proposition 7

We now analyze the model and its applications under congestion. Third stage prices conditional on available infrastructure remain unchanged: as products are still perfect substitutes and marginal costs are zero, the only prices that can emerge in equilibrium are $p_{V,c} = p_{Q,c} = 0$. However, revenues on this segment are now lower. Given that both firms rent access to I's infrastructure, V's price on the monopolistic segment remains unchanged at $p_{V,m} = a$.

The resulting profits are

$$\Pi_Q = \phi k_I(r/2) - t_Q \tag{51}$$

$$\Pi_V = \phi k_I(r/2) + (a+r)k_V - t_V - c_V(k_V) \tag{52}$$

[24] V's profit always exceeds Q's profit. The low candidate price for t is therefore firm Q's profit. Formally, the low candidate price for t is the lowest possible profit of either firm, which would be $min\{k_I(r/2),(k_I + k_V)(r/2) + max\{0,(k_V - k_I)(a+r/2)\}\}$ for which $k_I(r/2)$ is the only consistent solution.

Following the analysis from the base model, resulting transfers are

$$t_Q = \phi k_I(r/2) \tag{53}$$

$$t_V = \begin{cases} k_I \frac{r\phi}{2} + k_V\left(a + r(1-\frac{\phi}{2})\right) & \text{if } k_I > k_V \\ k_I(a+r) & \text{if } k_V \geq k_I \end{cases} \tag{54}$$

Note that the expression for t_V if $k_V \geq k_I$ is identical to the base model but if $k_I > k_V$, the expression for the transfer is different and lower.[25] In particular, the contribution of k_I to the transfer that I can demand is now lower, as each additional unit of k_I that increases the competitive segment has a lower return due to congestion. At the same time, the return to investment by V is higher because an increase in k_V and therefore an expansion of V's monopolistic segment allows V not only to escape competition with Q but also to escape the congestion "tax".

The resulting first-period investments follow:

$$\max_{k_V} \Pi_V = d'_{V,c}(p_{V,c}+r) + d'_{V,m}(p_{V,m}+r) - t_V - c_V(k_V) \tag{55}$$

$$\frac{\partial \Pi_V}{\partial k_V} = a + r - c'_V(k_V) - \frac{\partial t_V}{\partial k_V} = 0 \tag{56}$$

$$\frac{\partial t_V}{\partial k_V} = \begin{cases} a + r(1-\frac{\phi}{2}) & \text{if } k_I > k_V \\ 0 & \text{if } k_V \geq k_I \end{cases} \tag{57}$$

$$\frac{\partial \Pi_V}{\partial k_V} = \begin{cases} c'_V(k_V) = \frac{r\phi}{2} \\ c'_V(k_V) = a + r \end{cases} \leftrightarrow \begin{cases} k_V = \left(\frac{r\phi-\beta_V}{2\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_V = \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{58}$$

$$\max_{k_I} \Pi_I = t_Q + t_V - c_I(k_I) \tag{59}$$

$$\frac{\partial \Pi_I}{\partial k_I} = \frac{\phi r}{2} + \frac{\partial t_V}{\partial k_I} - c'_I(k_I) \tag{60}$$

$$= \begin{cases} c'_I(k_I) = r\phi \\ c'_I(k_I) = a + (1+\phi/2)r \end{cases} \leftrightarrow \begin{cases} k_I = \left(\frac{r\phi-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_I = \left(\frac{a+(1+\phi/2)r-\beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{61}$$

□

[25] Which can be easily seen from solving $(k_V + k_I)\frac{1}{2} > k_I\frac{\phi}{2} + k_V(1-\frac{\phi}{2}) \leftrightarrow k_I\frac{1-\phi}{2} > k_V\frac{1-\phi}{2}$ which is true when $k_I > k_V$.

A.7 Proof of proposition 8

Working from the expressions for t_V, t_Q,

$$t_Q = \delta_Q \phi k_I (r/2) \tag{62}$$

$$t_V = \begin{cases} \delta_V \left(k_I \frac{r\phi}{2} + k_V \left(a + r\left(1 - \frac{\phi}{2}\right)\right)\right) & \text{if } k_I > k_V \\ \delta_V k_I (a+r) & \text{if } k_V \geq k_I \end{cases} \tag{63}$$

we work out the resulting first-period investments:

$$\frac{\partial t_V}{\partial k_V} = \begin{cases} \delta_V \left(a + r\left(1 - \frac{\phi}{2}\right)\right) & \text{if } k_I > k_V \\ 0 & \text{if } k_V \geq k_I \end{cases} \tag{64}$$

$$\frac{\partial \Pi_V}{\partial k_V} : \begin{cases} c_V'(k_V) = (1-\delta_V)a + r\left(1 - \delta_V\left(1 + \frac{\phi}{2}\right)\right) & \text{if } k_I > k_V \\ c_V'(k_V) = a + r & \text{if } k_V \geq k_I \end{cases} \tag{65}$$

$$\tag{66}$$

Using the cost function, we obtain the following levels of investment:

$$\leftrightarrow \begin{cases} k_V = \left(\frac{(1-\delta_V)a + r(1-\delta_V(1+(\phi/2))) - \beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_V = \left(\frac{a+r-\beta_V}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{67}$$

Similarly for firm I:

$$\frac{\partial \Pi_I}{\partial k_I} = \frac{\partial t_Q}{\partial k_I} + \frac{\partial t_V}{\partial k_I} - c_I'(k_I) \tag{68}$$

$$\frac{\partial t_Q}{\partial k_I} = \frac{\delta_Q \phi r}{2} \tag{69}$$

$$\frac{\partial \Pi_I}{\partial k_I} : \begin{cases} c_I'(k_I) = (\delta_V + \delta_Q)\frac{r\phi}{2} & \text{if } k_I > k_V \\ c_I'(k_I) = \delta_V(a+r) + \frac{\delta_Q \phi r}{2} & \text{if } k_V \geq k_I \end{cases} \tag{70}$$

$$\tag{71}$$

Using the cost function, we obtain the following levels of investment:

$$\leftrightarrow \begin{cases} k_I = \left(\frac{(\delta_V+\delta_Q)\frac{r\phi}{2} - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_I > k_V \\ k_I = \left(\frac{\delta_V(a+r) + (\delta_Q \phi r/2) - \beta_I}{\alpha}\right)^{\frac{1}{\alpha-1}} & \text{if } k_V \geq k_I \end{cases} \tag{72}$$

□

A.8 Proof of proposition 9

V and Q charge identical prices on the competitive segment and face demand $d'_{V,c} = d'_{Q,c} = \phi k_I/2$. From this, we can compute third-period profits for given transfers and infrastructure size.

$$\Pi_Q = \phi\frac{k_I}{2}(p_{Q,c}+r) - t_Q \tag{73}$$
$$= \phi\frac{k_I}{2}\left(\frac{2a+2r}{4+\mu}\right) - t_Q$$
$$\Pi_V = \phi\frac{k_I}{2}(p_{V,c}+r) + (r+a)k_V - t_V - c_V(k_V) \tag{74}$$
$$= \phi\frac{k_I}{2}\left(\frac{2a+2r}{4+\mu}\right) + (r+a)k_V - t_V - c_V(k_V) \tag{75}$$

Given these profits, we can now write transfers

$$t_Q = \delta_Q\phi\frac{k_I}{2}\left(\frac{2a+2r}{4+\mu}\right) \tag{76}$$
$$t_V = \begin{cases} \delta_V\left(\frac{2a+2r}{4+\mu}\phi\frac{k_I}{2} + k_V\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right) & \text{if } k_I > k_V \\ \delta_V k_I\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) & \text{if } k_V \geq k_I \end{cases} \tag{77}$$

Finally, first period investment follows from the FOC:

$$\frac{\partial t_V}{\partial k_V} = \begin{cases} \delta_V\frac{4+(a+r)(\mu-2\phi)}{4+\mu} & \text{if } k_I > k_V \\ 0 & \text{if } k_V \geq k_I \end{cases} \tag{78}$$
$$\frac{\partial \Pi_V}{\partial k_V} : \begin{cases} c'_V(k_V) = r + a - \delta_V\left(\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right) & \text{if } k_I > k_V \\ c'_V(k_V) = a + r & \text{if } k_V \geq k_I \end{cases} \tag{79}$$

Similarly for firm I:

$$\frac{\partial \Pi_I}{\partial k_I} = \frac{\partial t_Q}{\partial k_I} + \frac{\partial t_V}{\partial k_I} - c'_I(k_I) \tag{80}$$
$$\frac{\partial t_Q}{\partial k_I} = \frac{\delta_Q\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) \tag{81}$$
$$\frac{\partial \Pi_I}{\partial k_I} : \begin{cases} c'_I(k_I) = (\delta_Q+\delta_V)\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) & \text{if } k_I > k_V \\ c'_I(k_I) = \delta_Q\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) + \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) & \text{if } k_V \geq k_I \end{cases} \tag{82}$$

The parameter values underpinning the $k_I > k_V$ equilibrium are:

$$k_I = \left(\frac{(\delta_Q + \delta_V)\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) - \beta_I}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{83}$$

$$k_V = \left(\frac{r + a - \delta_V\left(\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right) - \beta_V}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{84}$$

$$k_I > k_V \leftrightarrow \tag{85}$$

$$(\delta_Q + \delta_V)\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) - \beta_I > r + a - \delta_V\left(\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right) - \beta_V \leftrightarrow \tag{86}$$

$$\beta_V > \beta_I + (\delta_Q + \delta_V)\frac{\phi}{2}\left(\frac{2a+2r}{4+\mu}\right) + r + a - \delta_V\left(\frac{4+(a+r)(\mu-2\phi)}{4+\mu}\right) \leftrightarrow \tag{87}$$

$$\beta_V > \beta_I + r + a + \delta_Q\left(\frac{\phi(a+r)}{4+\mu}\right) - \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) \tag{88}$$

And for the $k_V \geq k_I$ equilibrium:

$$k_I = \left(\frac{\delta_Q \frac{\phi(a+r)}{4+\mu} + \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right)}{\alpha} \right) \tag{89}$$

$$k_V = \left(\frac{a + r - \beta_V}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{90}$$

$$k_V \geq k_I \leftrightarrow a + r - \beta_V \geq \delta_Q\left(\frac{\phi(a+r)}{4+\mu}\right) + \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) - \beta_I \tag{91}$$

$$\beta_I \geq \beta_V + \delta_Q\left(\frac{\phi(a+r)}{4+\mu}\right) + \delta_V\left(\frac{4+(a+r)(\mu-\phi)}{4+\mu}\right) - (a+r) \tag{92}$$

$$\tag{93}$$

□

B Submarine cable ownership

In this Section I present data on submarine cable ownership by some firms that are traditionally labeled "big tech" firms. The purpose of this exercise is firstly, to motivate the model describing vertical integration by firms offering digital services, and secondly, to justify the assumption that only either very large or specialized infrastructure firms have the scale to make these investments.

Bischof, Fontugne, and Bustamante (2018) describe the increasing role of these firms in the submarine cable infrastructure: "The latest construction boom, however, seems to be driven by content providers, such [as] Google, Facebook, Microsoft, and Amazon. According to Telegeography's Research Director Alan Mauldin, the amount of capacity deployed by content providers has risen 10-fold between 2013 and 2017, outpacing all other customers of international bandwidth."

I analyze data from Telegeography on submarine cables underlying the Submarine Cable Map.[26] The data is publicly available and as of September 2022 contains data on 516 submarine cables. The data set also includes location data on the cables which I do not use. Each observation of the data set includes the name of the cable, its length in kilometers, a list of its owners, a list of suppliers, and the year (and sometimes month) when the cable became or will become ready for service, ranging from 1989 to 2026.

Description

I search among the list of owners in our data set for Google, Amazon, Meta, Apple, Microsoft, Baidu, Alibaba, and Tencent. These firms are sometimes referred to with catch-all abbreviations such as GAMAM (GAFAM, before Facebook changed its name to Meta in 2021) or BAT. Neither Apple nor any of the BAT firms appear on the list of owners, but only Meta, Microsoft, Google, and Amazon Web Services. However, Alibaba does own terrestrial backbone within Asia (Corneo et al., 2021) Thus I identify cables that have one of the above-mentioned firms among its owners. This does not indicate sole ownership. Indeed, except for 7 purely Google-owned cables, all cables listed here as having "GAMAM owners" have co-owners. This is unsurprising, given that submarine cables typically include several fibre-optic cables and firms can own individual fibres.

I also search the list of owners for other firms and as a general observation, I remark that the list of owners includes mostly telecommunications firms and governments, as well as a few electricity companies, but no other firms that mainly sell digital services. While this is just one example of Internet infrastructure, it is consistent with the data previously collected in the literature (see references in this Section and Section 2) that describes the emergence of proprietary networks as a phenomenon driven by just a few of the largest technology firms.

Extending the range of the data of the previous paper by 7 years, I find that among submarine cables getting ready for service 2022-2024, the share of GAMAM climbs to

[26] https://www.submarinecablemap.com/

between 20 and 27% (see Figure 4).[27] This is higher than the share of new cables owned by these first in the previous decade, which only exceeded 20% in one year (2018).

At the same time, the absolute number of new GAMAM-owned cables has quadrupled, from 1.1 new cables per year between 2010 - 2019, to 4.4 new cables for 2020 - 2024 (Figure 5). The overall increase in cables going ready for service has increased by a third during this period, from 15.7 new cables per year to 21 new cables for year. In other words, the cables added with the large technology firms as co-owners contribute more than half of the increase in the number of added cables between the the period before and after 2020. The phenomenon of these firms owning submarine cables is not a recent one, however, with the first such cable registered in 2010. Overall, the data confirms the increasing role of content firms among investors of infrastructure.

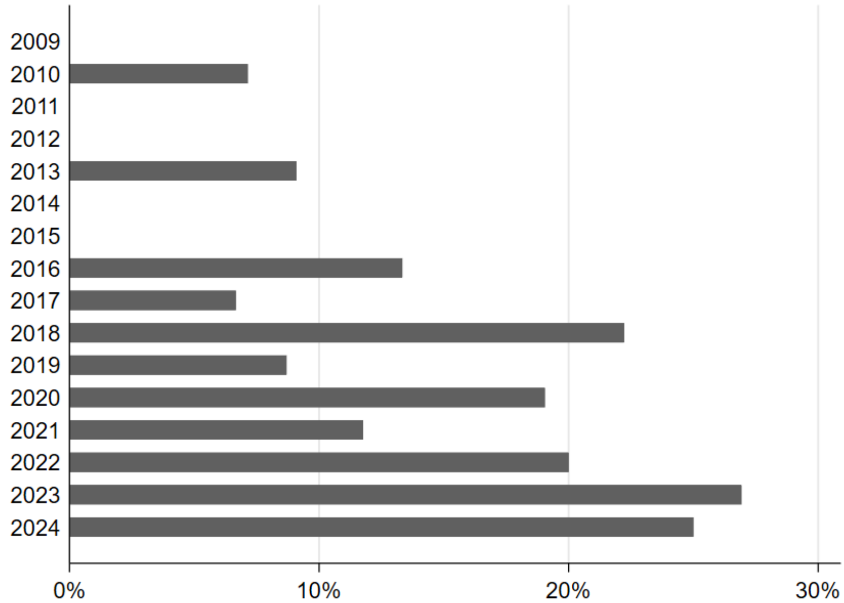

Figure 4: Share of submarine cables with GAMAM owners by ready-for-service date

[27] Only 7 announcements have been made regarding cables that are ready for service in 2025 and only 1 for 2026, none of them involving any of the above-mentioned firms.

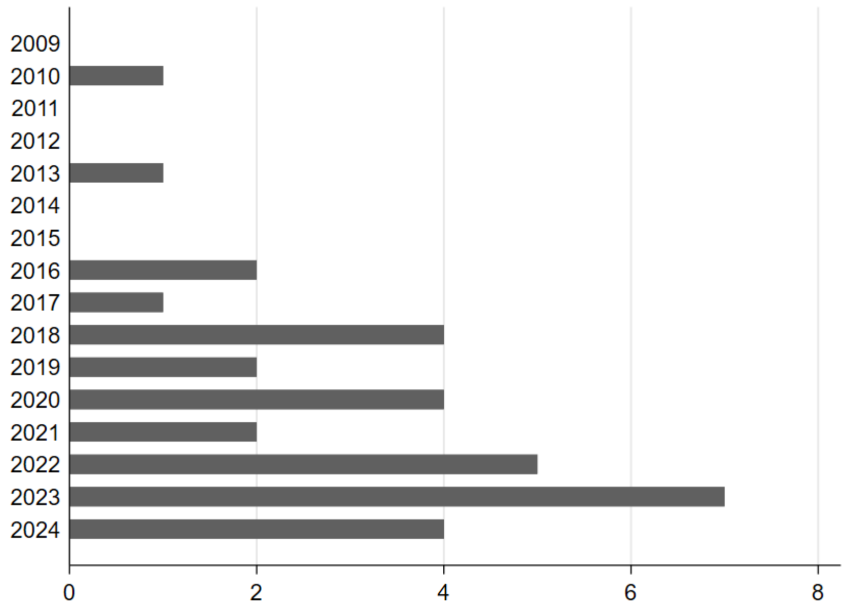

Figure 5: Sum of submarine cables with GAMAM owners by ready-for-service date