Machine Learning – Initial GitHub Commit

Philip Hartout

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x	**y**
3	2
1	2
0	1
4	3

Table 1: Training sample

Gradient descent algorithm application

We set our learning rate $\alpha = 0.1$. Recall that $J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$
Initialisation
$\theta_0 = 0$, $\theta_1 = 1$. So, $h_\theta(x) = 0 + 1 \cdot x$
Variable update #1
Now we update θ_0 and θ_1

$$\begin{aligned}
&\theta_0 := \theta_0 - \alpha \cdot \frac{\partial}{\partial \theta_0} J(\theta_0) \\
&\Leftrightarrow \theta_0 := \theta_0 - \alpha \frac{1}{4} \sum_{i=1}^{4} h_\theta(x^{(i)}) - y^{(i)} \\
&\Leftrightarrow \theta_0 := 0 - 0.1 \frac{1}{4} \left[(3-2) + (1-2) + (0-1) + (4-3)\right] \\
&\Leftrightarrow \theta_0 := 0
\end{aligned}$$

Now, we update θ_1 simultaneously, i.e. we still use our hypothesis function we had at the beginning: $h(x) = 0 + 1 \cdot x$. This yields:

$$\begin{aligned}
&\theta_1 := \theta_1 - \alpha \cdot \frac{\partial}{\partial \theta_1} J(\theta_1) \cdot x^{(i)} \\
&\Leftrightarrow \theta_1 := \theta_1 - \alpha \frac{1}{4} \sum_{i=1}^{4} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \\
&\Leftrightarrow \theta_1 := 1 - 0.1 \frac{1}{4} \left[(3-2) \cdot 3 + (1-2) \cdot 1 + (0-1) \cdot 0 + (4-3) \cdot 4\right] \\
&\Leftrightarrow \theta_1 := 0.85
\end{aligned}$$

Variable update #2
Now we update θ_0 and θ_1 with $h_\theta(x) = 0 + 0.85 \cdot x$

$$\begin{aligned}
&\theta_0 := \theta_0 - \alpha \cdot \frac{\partial}{\partial \theta_0} J(\theta_0) \\
&\Leftrightarrow \theta_0 := \theta_0 - \alpha \frac{1}{4} \sum_{i=1}^{4} h_\theta(x^{(i)}) - y^{(i)} \\
&\Leftrightarrow \theta_0 := 0 - 0.1 \frac{1}{4} \left[(2.55-2) + (0.85-2) + (0-1) + (3.4-3)\right] \\
&\Leftrightarrow \theta_0 := 0.03
\end{aligned}$$

Now, we update θ_1 simultaneously, i.e. we still use our hypothesis function we had at the beginning: $h(x) = 0 + 1 \cdot x$. This yields:

$$
\begin{aligned}
&\theta_1 := \theta_1 - \alpha \cdot \frac{\partial}{\partial \theta_1} J(\theta_1) \cdot x^{(i)} \\
&\Leftrightarrow \theta_1 := \theta_1 - \alpha \frac{1}{4} \sum_{i=1}^{4} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \\
&\Leftrightarrow \theta_1 := 1 - 0.1 \frac{1}{4} \left[(2.55 - 2) \cdot 3 + (0.85 - 2) \cdot 1 + (0 - 1) \cdot 0 + (3.4 - 3) \cdot 4 \right] \\
&\Leftrightarrow \theta_1 := 0.7975
\end{aligned}
$$