Machine Learning – Written Assignment 4

Philip Hartout

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1. (a) The decision boundaries can be found below.

   (b) It appears that only decision trees, 1-nearest neighbour and logistic regression with quadratic terms classify all data points correctly all data points, but it also appears that they are prone to overfitting[1]; therefore, some form of trade-off is required to make optimal decisions. An idea to highly increase accuracy is to run all algorithms on the same dataset, and, with a given data point belonging to a test set, counting to which class it belongs to according to all 4 algorithms. If, say, all 4 algorithms indicate that the point should be a positive example, then it is highly likely that it is, on the other hand, being aware of the fact that some classifiers do not categorise a given point of a test set as a positive example should require a more careful consideration.

2. First, to do one iteration of the k-means clustering algorithm, we assign each value to the closest cluster centroid, which have means $\mu_{c^{(1)}} = 1$, $\mu_{c^{(2)}} = 3$, $\mu_{c^{(3)}} = 8$, by calculating the squared distance of each point to each cluster centroid. Hence, we end up with the following points assigned to the following cluster centroids:

$$
\begin{aligned}
c^1 &= 1 \\
c^2 = \ldots = c^7 &= 2 \\
c^8 = \ldots = c^{16} &= 3
\end{aligned}
$$

From there, we can estimate the cost before updating the coordinates the cluster centroids. The formula for the cost function is given as follows.

$$J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K) = \frac{1}{m} \sum_{i=1}^{m} \left\| x^i - \mu_{c^i} \right\|^2$$

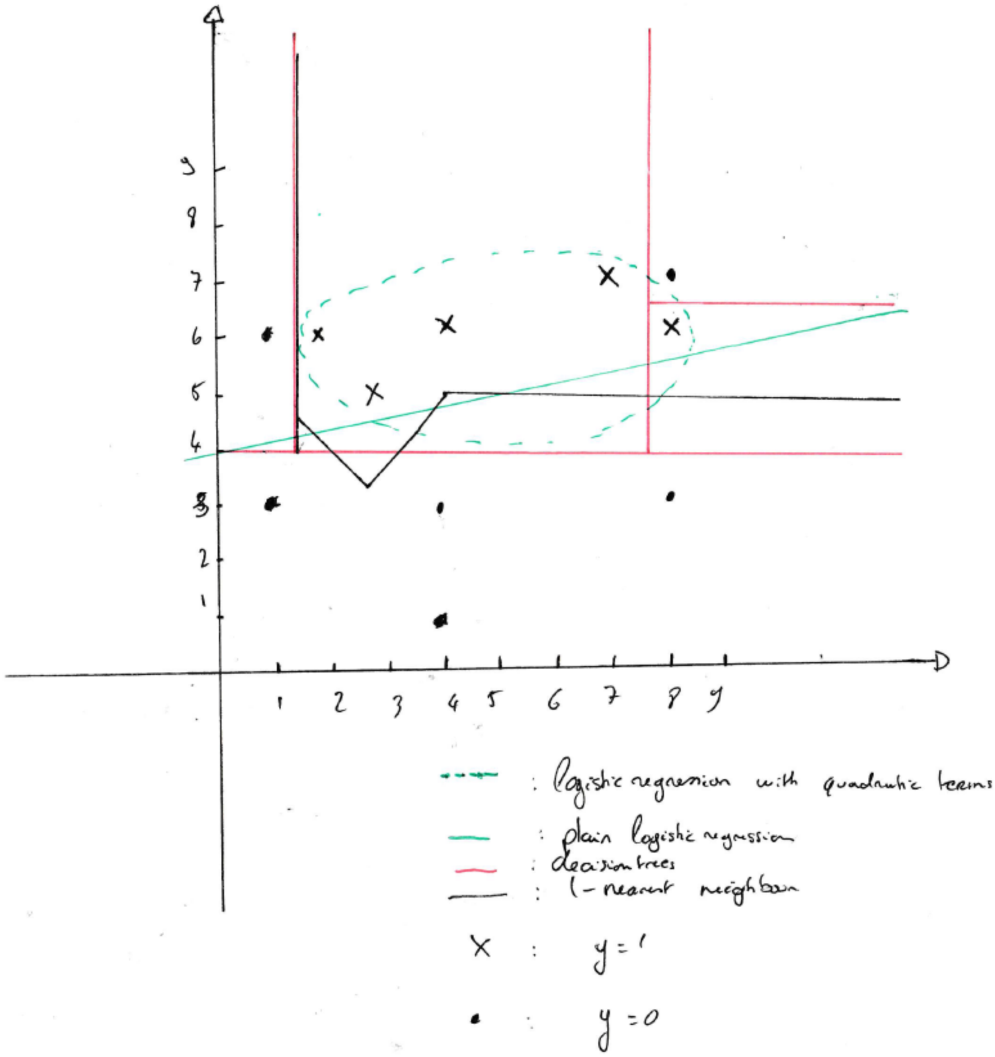
Here, in particular:

$$J(c^{(1)}, \ldots, c^{(16)}, \mu_1, \mu_2, \mu_3) = \frac{1}{16} [\ (1-1)^2 + (2-3)^2 + (3-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2 + (5-3)^2$$
$$+ (7-8)^2 + (10-8)^2 + (11-8)^2 + (13-8)^2 + (14-8)^2 + (15-8)^2 + (17-8)^2 + (20-8)^2 + (21-8)^2\ ] = 33$$

Now that we have (1) assigned each data point to a cluster and (2) calculated the cost function, we can now calculate the new position of each cluster centroid, which we achieve by calculating the mean of each cluster, as follows:

$$
\begin{aligned}
&\mu_1 = \frac{1}{1} \cdot 1 = 1 \\
&\mu_2 = \frac{1}{6} \cdot (2 + 3 + 3 + 4 + 5 + 5) = \frac{11}{3} = 3.667 \\
&\mu_3 = \frac{1}{9} \cdot (1 + 2 + 3 + 3 + 4 + 5 + 5 + 7 + 10 + 11 + 13 + 14 + 15 + 17 + 20 + 21) = \frac{151}{9} \approx 16.778.
\end{aligned}
$$

[1] Except, perhaps, logistic regression with quadratic terms.


: Logistic regression with quadratic terms
: plain logistic regression
: decision trees
: 1-nearest neighbour
X : y = 1
• : y = 0

Which leads us to re-assign the data points to new clusters:

$$
\begin{aligned}
c^1 = c^2 &= 1 \\
c^3 = \ldots = c^9 &= 2 \\
c^{10} = \ldots = c^{16} &= 3
\end{aligned}
$$

Finally, we estimate the cost function:

$$
J(c^{(1)}, \ldots, c^{(16)}, \mu_1, \mu_2, \mu_3) = \frac{1}{16}\,[\,(1-1)^2 + (2-1)^2 + \left(3 - \frac{11}{3}\right)^2 + \left(3 - \frac{11}{3}\right)^2 + \left(4 - \frac{11}{3}\right)^2 + \left(5 - \frac{11}{3}\right)^2
$$

$$
+ \left(5 - \frac{11}{3}\right)^2 + \left(7 - \frac{11}{3}\right)^2 + \left(10 - \frac{11}{3}\right)^2 + \left(11 - \frac{151}{9}\right)^2 + \left(13 - \frac{151}{9}\right)^2 + \left(14 - \frac{151}{9}\right)^2 + \left(15 - \frac{151}{9}\right)^2
$$

$$
+ \left(17 - \frac{151}{9}\right)^2 + \left(20 - \frac{151}{9}\right)^2 \left(21 - \frac{151}{9}\right)^2\,] = \frac{12493}{1296} \approx 9.639
$$