Deep Neural Network

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Abstract

The purpose of this report is to develop a neural net that can identify handwritten digits in the MNIST database at near human levels of accuracy. The neural net will be developed without the assistance of libraries such as Python's tensor flow or MATLAB's Deep Learning.

The author would like to express his gratitude to Professor Hicken for the suggestion of this project. The author would also like to thank Theodore Ross and Varun Rao for their assistance with artificial neural networks.

1 Introduction

In recent years the Have it solve the MNIST with a simple simple thing then try diffrent layers and stuff

Go over tan h vs sigmoid Explain batch testing

1.1 The MNIST database

The Modified National Institute of Standards and Technology database or MNIST database [5] is a database of handwritten numbers used to train image processing systems. It contains 60,000 training images and 10,000 testing images. The database is comprised of images that are made up of a grid of 28x28 pixels. Some of these are seen in figure 1.

A number of attempts have been made to get the lowest possible error rate on this dataset. As of August 2018 the the lowest achieved so far is a error rate of 0.21% or an accuracy of 99.79%. For comparison human can accurately recognize digits at a rate of 98.5% [6].

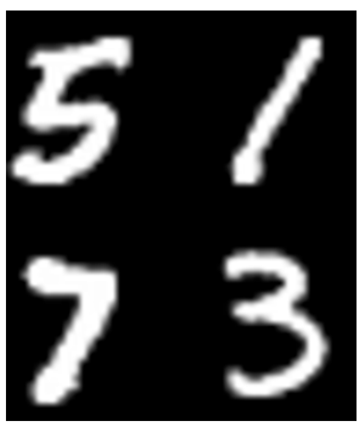


Figure 1: Sample numbers from MNIST [1].

1.2 Artificial neural network

An artificial neural network (referred to as a neural network in this paper) is a computation system that mimics the biological neural networks found in animal brains. A nerual network is not an explict A Neural networks may be trained for tasks, such as the number recognition in this report.

1.3 Neural Network Walkthrough

Forward and backward propogation are visulized in figure 3

The four steps

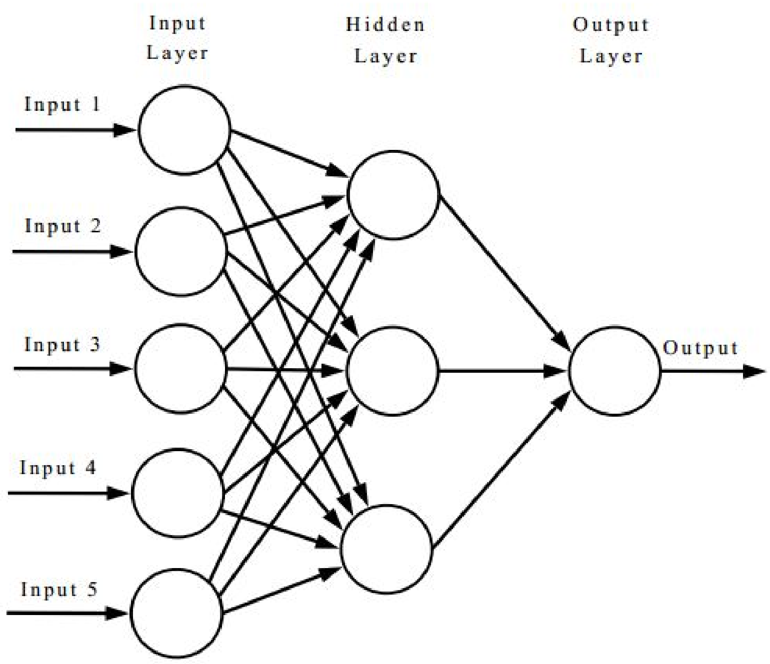


Figure 2: Visualization of a neural network [2].

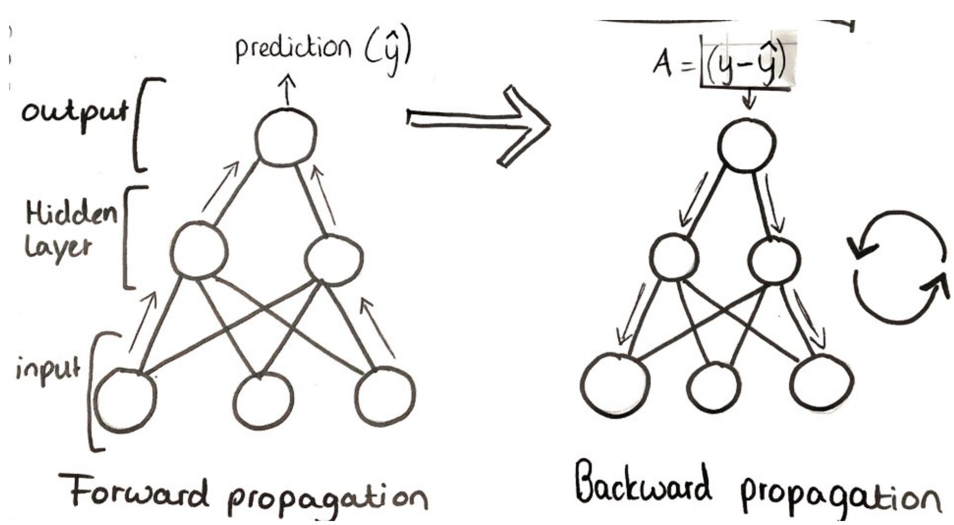


Figure 3: A visulization of forward and backward propogation [3].

1. Initialize weights and biases.
2. Forward propagation
3. compute loss
4. back prop

1.3.1 Parameter Initialization

The first step in training a neural net is to initialized the bias vectors and weight matrices. They are initialized with random numbers between 0 and 1, then multiplied by a small scalar around the order of 10^{-2} so that the units are not on the region where the derivative of the activation function are close to zero. The initial parameters should be different values (to keep the gradients from being the same).

There are various forms of initialization such as Xavier initialization or He-et-al Initialization, but a discussion on methods of initialization outside the scope of this paper. In this paper we will stick with random parameter initialization.

1.3.2 Forward Propagation

The next step is the forward propagation. The network takes the inputs from a previous layer, computes their transformation, and applies an activation function. Mathematically this is represented by equation 1.

$$\begin{aligned} z_i = A_{i-1} * W_i + b \\ A_i = \phi(z_i) \end{aligned} \tag{1}$$

Where z is the input vector, A is the layer, W is the weights going into the layer, b the bias, and ϕ the activation function. This process then repeats for the next layer until it reaches the end of the neural net.

1.3.3 Cost

The cost or the loss

1.3.4 Backward propagation

After going forward through the neural net in the forward propagation step, the next step is backwards propagation. Backwards propagation is the updating of the weight parameters via the derivative of the error function with respect to the weights of the neural net. For the output layer this is seen in equation 2 and equation 3 for all other layers.

$$dW_{i=\text{end}} = \phi'(z_{i=\text{end}}) * (A_{i=\text{end}} - y) \tag{2}$$

$$dW_i = \phi'(z_i) * \left(W^T_{(i+1)} * dW_{(i+1)}\right) \tag{3}$$

Once these derivatives have been computed, the weights are updated by equation 4

$$W_i = W_i = \alpha * dW_i * A^T_{(i-1)} \tag{4}$$

Where for the first layer $z^T_{(i-1)}$ will be the input vector and for all the following layers it will be the vector from the previous layer.

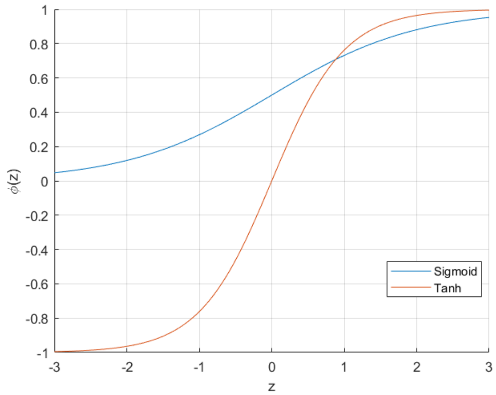


Figure 4: Visualization of sigmoid and Tanh function

1.4 Gradient Decent

Also known as steepest decent, gradient decent is a first order optimization algorithm. It is used to find the minimum of a function. Equation 5 shows gradient decent implemented in a neural net.

$$\Delta W(t) = -\alpha \frac{\partial E}{\partial W(t)} \tag{5}$$

Where

1.5 Activation Function

The activation function was previously mentioned as a function used to convert the input signal to the output signal. Activation functions introduce non-linear properties to the neural net's functions, allowing the neural net to represent complex functions [3].

The two most common activation functions used in neural nets for the gradient decent are sigmoid and hyperbolic tangent (Tanh). The formula for Tanh is seen in equation 6, and the derivative of Tanh is seen in equation 7

$$\phi_{\text{Tanh}}(z) = \frac{1 - e^{-2z}}{1 + e^{-2z}} \tag{6}$$

$$\phi'_{\text{Tanh}}(z) = \frac{4}{(e^{-z} + e^{z})^2} \tag{7}$$

The formula for the sigmoid function is seen in equation 8, the formula for it's derivative is seen in equation 9.

$$\phi_{\text{Sigmoid}}(z) = \frac{1}{1 + e^{-z}} \tag{8}$$

$$\phi'_{\text{Sigmoid}}(z) = \frac{e^{-z}}{(e^{-z} + 1)^2} \tag{9}$$

The sigmoid and Tanh function are visualized in figure 4.

Both functions have relativly simple mathematical formulas and are differentiable. In this paper the sigmoid function is used over the Tanh function. WHY?. Sigmoid and Tanh are not the only activation functions. Other functions that should be noted are the Rectified Linear Unit (ReLU) and the Leaky Rectified Linear Unit function. While these functions can perform better than Tanh and Sigmoid, they are more complex and a proper discussion of them is outside the scope of this paper. Expalain importance of activation function

ID	architecture (number of neurons in each layer)	test error for best validation [%]	best test error [%]	simulation time [h]	weights [milions]
1	1000, 500, 10	**0.49**	0.44	23.4	1.34
2	1500, 1000, 500, 10	**0.46**	0.40	44.2	3.26
3	2000, 1500, 1000, 500, 10	**0.41**	0.39	66.7	6.69
4	2500, 2000, 1500, 1000, 500, 10	**0.35**	0.32	114.5	12.11

Figure 5: Run times for various neural network architectures [4].

1.6 Pitfalls

The most important thing to stear clear of is over traning. Overtraning occurs when the neural net trains too much to the traning data. While it will have a high accuracy for the traning data, it's performance for the test data will decay, as it has become too well attuned to the training data.

The other problem is the time it takes to train. A three layer neural net can be trained to 97% accuracy within 10 minutes, however it will not improve far beyond that. Larger nets will take longer to train, but will take far longer to train.

2 Implementation

2.1 Object Oriented Programming in MATLAB

This neural net had to be made without the use of any built in libraries [7] and the code had to be modular [8]. To create code for neural network subject to these constraints the author decided to create their own neural net class in MATLAB.

The MATLAB class philipNeuralNet.m was written for this neural net project. It has the parameters learningRate and Level. The learningRate paramter is obviously the learning rate. The Level parameter has four parameters attached to it: W (weight), dW (weight derivative), z (input), and A (the vector for the layer). By having this class we have avoided hard coding the propagation of the neural net and it is possible to test different neural net architectures on this code.

The MATLAB class also has an activation function and a derivative of the activation function. The actFunSwitch variable allows either the Sigmoid or the Tanh function to be selected. Additionally the enableBias variable allows for biases to be used or not used in the code's execution.

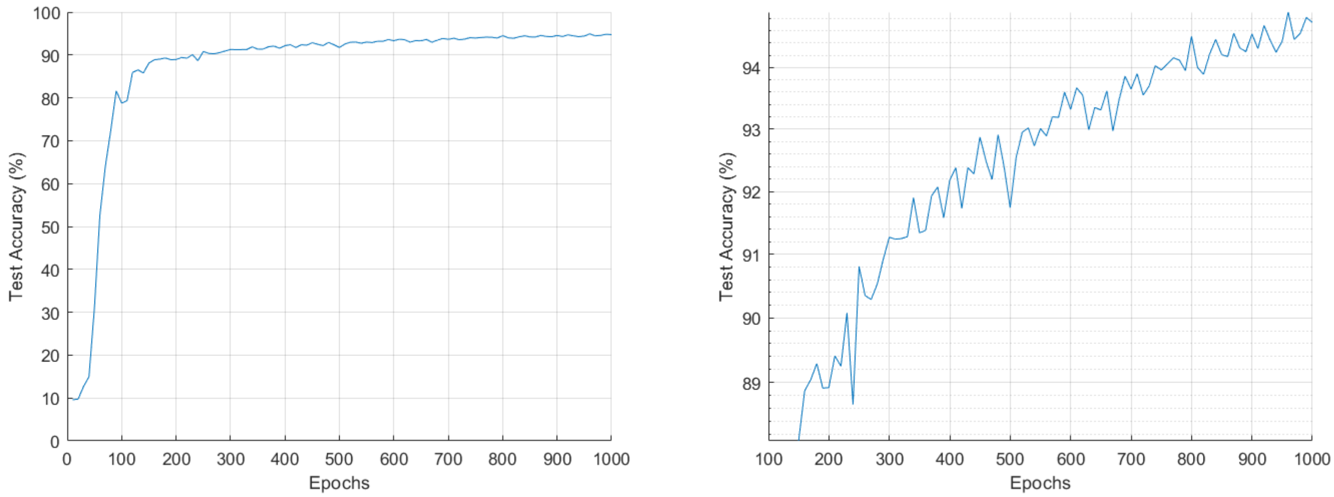
2.2 MATLAB Code

The MALAB code first initializes a neural net from given parameters. It then uses the handleTrainNet function to train the net. This function implements batch training, using the backward propagation function. It then computes and displays the training error and the testing accuracy after a specified number of runs.

3 Results

3.1 Simple Neural Net

The best results were found for simplex neural net examined; a one hidden layer with 250 nodes a learning rate of 0.1, and no biases. For this simple a test accuracy of 98% was achieved. The first 1000 epochs of this net are visualized in figure 6.



(a) The results for the first 1000 epochs. (b) The results for the first epochs scaled logarithmically

Figure 6: The results from the simple network for the first 1000 epochs.

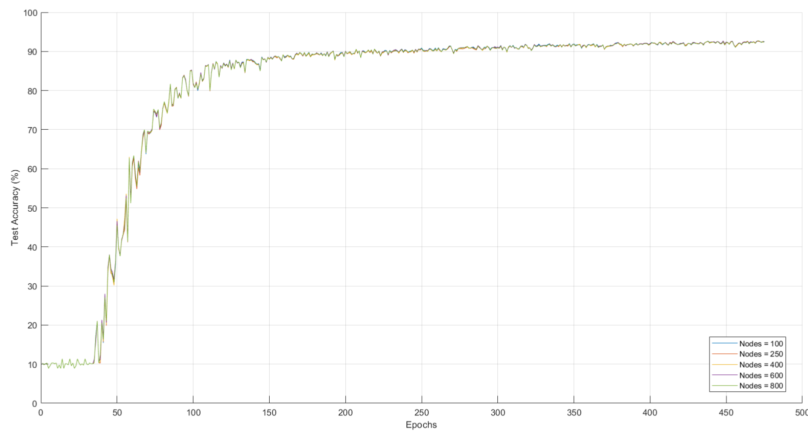


Figure 7: Net accuracy for different layer sizes.

Looking at the results, It was discovered that

3.2 Comparison of different hidden layer sizes

From Shure [9], the optimal size for the hidden layer in a three layer neural network is 250 nodes. Comparing the results for hidden layers show in figure 7, different sizes of hidden layer do not have a large effect on the accuracy of these results. However what is different is the time it takes to run each net. The more nodes in a net, the longer it takes for the net to train, as there are more operations to perform. Thus if a neural net with 250 nodes will have the same accuracy as a net with 800 nodes, the first net is preferable, as it will be trained faster.

3.3 Multiple hidden layers

4 Conclusion

The simple neural net achieved an accuracy of 98.37%. This is on par with human recognition, and is about as accurate as a simple neural net can achieve. Higher accuracy rates are achieved via the use of constitutional neural networks, such as the LeNet-5 [**?**]. Due to the long run time of large multi layered neural networks they were not studied, but could provide a more accurate identification with out convolution.

References

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Appendix 1 - MATLAB code

NN_Master.m

```
% This will throw an error so you will look here and look at your notes
% below
%abc =valNothere
% see
% ...
    C:\Users\Philip\Documents\MATLAB\Fall2018\DesignOpt\checkNN\matlab-mnist-two-layer-percep
%% Philip Hoddinott NN
% Neural Net for MNIST numbers
%% Setup

clear all; close all;

inputValues = loadMNISTImages('train-images.idx3-ubyte');
labels = loadMNISTLabels('train-labels.idx1-ubyte');

% Transform the labels to correct target values.
targetValues = 0.*ones(10, size(labels, 1));
for n = 1: size(labels, 1)
    targetValues(labels(n) + 1, n) = 1;
end

nn_input_dim=784;
nn_hdim=250;
nn_output_dim=10;

numberOfHiddenUnits=nn_hdim;

sizeArr=[250;10];
learningRate=.1;
chunk=1; % creat esub arrays

%pNet=philipNetSixLayer(inputValues,targetValues,numberOfHiddenUnits,sizeArr,learningRate)
enableBias=0; %  0 for off, 1 for on
actFunSwitch =0;  % 0 for sigmoid, 1 for Tanh
pNet=philipNeuralNet(inputValues,sizeArr,learningRate,enableBias,actFunSwitch)
% http://ufldl.stanford.edu/wiki/index.php/Using_the_MNIST_Dataset

inputValuesTest = loadMNISTImages('t10k-images.idx3-ubyte');
labelsTest = loadMNISTLabels('t10k-labels.idx1-ubyte');

batchSize = 100;
epochs = 500;
numEpoch=2;

fprintf('Train twolayer perceptron with %d hidden units.\n', nn_hdim);
fprintf('Learning rate: %d.\n', learningRate);

%[pNet,pNetBest,errorM,testAccM]=handleTrainNet(pNet, inputValues, targetValues, ...
    epochs, batchSize, inputValuesTest, labelsTest,numEpoch,sizeArr);
load('wksp_plots')
figure; hold on
plot(10.*[(1:1:length(testAccM))],testAccM)
xlabel('Epochs')
```

```
ylabel('Test Accuracy (%)')
grid on
%set(gca, 'YScale', 'log')

figure; hold on
plot(10.*[(15:1:length(testAccM))],testAccM(15:end))
xlabel('Epochs')
ylabel('Test Accuracy (%)')
grid on
set(gca, 'YScale', 'log')
function [pNet,pNetBest,errorM,testAccM] = handleTrainNet(pNet,  inputValues, ...
    targetValues, epochs, batchSize, inputValuesTest, labelsTest,numEpoch,sizeArr)

    trainingSetSize = size(inputValues, 2);
    errorM=[];
    testAccM=[];

    n = zeros(batchSize);
    errorBest=100;
    accBest=-1;

    figure; hold on;
    ylabel('Training Error')
    %xlabel('Neural Net Evaluations')
    xlabel('Epochs')
    tic
    for t = 1: numEpoch*epochs

        for k = 1: batchSize

            % Select which input vector to train on.
            n(k) = floor(rand(1)*trainingSetSize + 1);
            % get inputs and targets
            inputVector = inputValues(:, n(k));
            targetVector = targetValues(:, n(k));
            % Propagate the input vector through the network.
            for i=1:length(sizeArr)
                if i==1
                    pNet.Level(i).z=pNet.Level(i).W*inputVector;
                else % output
                    pNet.Level(i).z=pNet.Level(i).W* pNet.Level(i-1).A;
                end
                pNet.Level(i).A=pNet.actFunc(pNet.Level(i).z+pNet.Level(i).b);%%%% ...
                    NEW
            end
            iArr=linspace(length(sizeArr),1,length(sizeArr));
            pNet=backprop(pNet,sizeArr,inputVector,targetVector);
        end % end foor loop

        % Calculate the error for plotting.
        error = 0;
        for k = 1: batchSize
            inputVector = inputValues(:, n(k));
            targetVector = targetValues(:, n(k));
            outputVector = pNet.netOutput(inputVector,sizeArr);
            error=error+norm(outputVector- targetVector, 2);

        end
        error = error/batchSize; errorM=[errorM,error];
```

```
        plot(t,error,'*')

        if error<errorBest
            pNetBest=pNet; errorBest=error;
        end

        if mod(t,10)==0 %100
            [correctlyClassified, classificationErrors] = testAcc(pNet, ...
                inputValuesTest, labelsTest,sizeArr);
            acc=100*(correctlyClassified) / ...
                (correctlyClassified+classificationErrors);
            if acc>accBest
                accBest=acc;
            end
            testAccM=[testAccM,acc];
            fprintf('Epoch = %d,error = %.4f, best acc = %.4f\n',t, error,accBest)
            grid on;
            toc
        end
        drawnow
    end

end

function pNet=backprop(pNet,sizeArr,inputVector,targetVector) % function to ...
    perform backpropgation
    learningRate=pNet.learningRate;
    iArr=linspace(length(sizeArr),1,length(sizeArr));
    for i=iArr
        if i==length(sizeArr) % cost at output
            pNet.Level(i).dW=pNet.dactFunc( pNet.Level(i).z).* ...
                (pNet.Level(i).A-targetVector);
        else % hidden
            pNet.Level(i).dW = pNet.dactFunc( ...
                pNet.Level(i).z).*(pNet.Level(i+1).W'* pNet.Level(i+1).dW);
        end
        dz=pNet.dactFunc(pNet.Level(i).z);
        pNet.Level(i).db=(1/length(dz))* sum(dz,2); %%% NEW
        %pNet.Level(i).dW=pNet.Level(i).dW*(1/length(pNet.Level(i).dW)); %% NEW
    end

    for i=iArr
        if i≠1 % output
            pNet.Level(i).W= ...
                pNet.Level(i).W-learningRate*pNet.Level(i).dW*pNet.Level(i-1).A';
        else % hidden
            pNet.Level(i).W= ...
                pNet.Level(i).W-learningRate*pNet.Level(i).dW*inputVector';
        end
        if pNet.enableBias==1 % switch for enable bias
            pNet.Level(i).b=pNet.Level(i).b-learningRate*pNet.Level(i).db; %%% NEW
        end
    end
end
```

philipNeuralNet.m

```
classdef philipNeuralNet
    %philipNeuralNet Summary of this class goes here
    %   Detailed explanation goes here

    properties
        learningRate;
        Level;
        enableBias;
        actFunSwitch;
    end

    methods
        function obj = ... philipNeuralNet(inputValues,sizeArr,learningRate,enableBias,actFunSwitch)
            % initalized values
            inputDim = size(inputValues, 1);

            for i =1:length(sizeArr)
                if i==1
                    obj.Level(i).W=rand(sizeArr(i),inputDim);
                    obj.Level(i).W=( obj.Level(i).W)./size( obj.Level(i).W,2);
                else
                    obj.Level(i).W=rand(sizeArr(i),sizeArr(i-1));
                    obj.Level(i).W=( obj.Level(i).W)./size( obj.Level(i).W,2);
                end
                obj.Level(i).z=learningRate*rand(sizeArr(i),1);
                obj.Level(i).A=learningRate*rand(sizeArr(i),1);
                obj.Level(i).dW=learningRate*rand(sizeArr(i),1);
                obj.Level(i).b=learningRate*zeros(sizeArr(i),1);
                obj.Level(i).db=learningRate*obj.Level(i).b;

            end
            obj.learningRate=learningRate;
            obj.enableBias=enableBias;
            obj.actFunSwitch=actFunSwitch;
        end
         function funcVal = actFunc(obj,x)
            %actFunc activation function
            %   depending on the activation funciton switch, this performs
            %   sigmoid or TanH
            if obj.actFunSwitch==0 % for sigmoid
                funcVal = 1./(1 + exp(-x));
            elseif obj.actFunSwitch==1 % for tanh
                funcVal=tanh(x);
            end
         end

        function funcD = dactFunc(obj,x)
            %dactFunc activation func derivative
            %   depending on the activation funciton switch, this performs
            %   derivative of sigmoid or Tanh
            if obj.actFunSwitch==0
                funcD = obj.actFunc(x).*(1 - obj.actFunc(x));
            elseif obj.actFunSwitch==1
                funcD=(1-tanh(x).^2);
            end
        end

```

```
        function outputVector = netOutput(pNet, inputVector,sizeArr)
            % netOutput get the output of the neural net given an input and
            %
            % INPUT:
            % pNet : net
            % inputVector : input to net
            % sizeArr : net architecture
            %
            % OUTPUT:
            % outputVector : output of net
            for i=1:length(sizeArr)
                if i==1
                    pNet.Level(i).z=pNet.Level(i).W*inputVector;
                else
                    pNet.Level(i).z=pNet.Level(i).W*pNet.Level(i-1).A;
                end
                pNet.Level(i).A=pNet.actFunc(pNet.Level(i).z);
            end
            outputVector=pNet.Level(i).A;
        end

    end
end
```