1. $h_{hp}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{hp}(e^{j\omega}) e^{j\omega n} d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{j\omega n} d\omega$

$= \frac{1}{2\pi jn}\left(e^{-j\omega_c n} - e^{-j\pi n}\right) + \frac{1}{2\pi jn}\left(e^{j\pi n} - e^{j\omega_c n}\right)$

$= \frac{1}{\pi n}\left(\frac{e^{j\pi n} - e^{-j\pi n}}{2j}\right) - \frac{1}{\pi n}\left(\frac{e^{j\omega_c n} - e^{-j\omega_c n}}{2j}\right)$

$= \frac{\sin(\pi n)}{\pi n} - \frac{\sin(\omega_c n)}{\pi n}$

$= \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$

2. (a) $\sum_{n=0}^{N-1} e^{-j2\pi n/N}$

$= (1-(e^{-j2\pi/N})^N) / (1-e^{-j2\pi/N})$

$= (1-e^{-j2\pi}) / (1-e^{-j2\pi/N})$

$= 0$

(b) If $k \neq 0$:

$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$

$= \sum_{n=0}^{N-1} e^{-j2\pi kn/N}$

$= \left(1-(e^{-j2\pi k/N})^N\right) / (1-e^{-j2\pi k/N})$

$= (1-e^{-j2\pi k}) / (1-e^{-j2\pi k/N})$

$= 0$

If $k = 0$:

$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$

$= \sum_{n=0}^{N-1} 1 = N$

3. (a) $x[n] = [2, 0, 1, 0]$ $\quad y[n] = [1, -1, 0, 0]$

$$s[n] = x[n] \otimes y[n]$$

$$= \sum_{m=0}^{3} x[m]\, y[(n-m)_4]$$

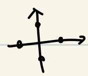
$$s[0] = 2 \cdot 1 + 0 + 1 \cdot 0 + 0 = 2$$

$$s[1] = 2 \cdot -1 + 0 + 1 \cdot 0 + 0 = -2$$

$$s[2] = 2 \cdot 0 + 0 + 1 \cdot 1 + 0 = 1$$

$$s[3] = 2 \cdot 0 + 0 + 1 \cdot -1 + 0 = -1$$

$\therefore\ x[n] \otimes y[n] = [2, -2, 1, -1]$, $n = 0, 1, \cdots, 3$



(b) $$X[k] = \sum_{n=0}^{3} x[n]\, e^{-j2\pi nk/4}$$

$$= x[0] + x[1]\, e^{-j\frac{\pi k}{2}} + x[2](-1)^k + x[3]\, e^{-j\frac{3\pi k}{4}}$$

$$= 2 + (-1)^k$$

$X(k) = [3, 1, 3, 1]$, $k = 0, 1, \cdots 3$

$$Y[k] = \sum_{n=0}^{3} y[n]\, e^{-j2\pi nk/4}$$

$$= y[0] + y[1]\, e^{-j\frac{\pi k}{2}} + y[2](-1)^k + y[3]\, e^{-j\frac{3\pi k}{4}}$$

$$= 1 - e^{-j\frac{\pi k}{2}}$$

$Y(k) = [0, 1+j, 2, 1-j]$, $k = 0, 1, \cdots 3$

(c) $Z(k) = [0, 1+j, 6, 1-j]$, $k = 0, 1, \cdots, 3$

(d) $$z[n] = \frac{1}{4}\sum_{k=0}^{3} Z[k] e^{j2\pi kn/4}$$
$$= \frac{1}{4}\left((1+j)e^{j\frac{\pi n}{2}} + 6\, e^{j\pi n} + (1-j)\, e^{-j\frac{\pi n}{2}} \right)$$

$$z[n] = [\, 2 , -2 , 1, -1 \,] , \quad n = 0, 1, \cdots, 3$$

4.

$$V[k] = \sum_{n=0}^{2N-1} v[n]\, W_{2N}^{kn}$$
$$G[k] = \sum_{n=0}^{N-1} g[n]\, W_N^{kn} = \sum_{n=0}^{N-1} v[2n]\, W_N^{kn} = \sum_{n=0}^{N-1} v[2n]\, W_{2N}^{2nk} \qquad \left(\because \left(W_{2N}^{kn}\right)^2 = W_N^{kn} \right)$$
$$H[k] = \sum_{n=0}^{N-1} h[n]\, W_N^{kn} = \sum_{n=0}^{N-1} v[2n+1]\, W_N^{kn} = \sum_{n=0}^{N-1} v[2n+1]\, W_{2N}^{2nk}$$

$$V[k] = v[0]\, W_{2N}^{0k} + v[1]\, W_{2N}^{k} + v[2]\, W_{2N}^{2k} + \cdots + v[2N-1]\, W_{2N}^{(2N-1)k}$$
$$= \left(v[0]\, W_{2N}^{0k} + v[2]\, W_{2N}^{2k} + \cdots + v[2N-2]\, W_{2N}^{(2N-2)k} \right) + \left(v[1]\, W_{2N}^{k} + \cdots + v[2N-1]\, W_{2N}^{(2N-1)k} \right)$$
$$= G[k \bmod N] + W_{2N}^{k} \left(v[1]\, W_{2N}^{0k} + v[3]\, W_{2N}^{2k} + \cdots + v[2N-1]\, W_{2N}^{(2N-2)k} \right)$$
$$= G[k \bmod N] + W_{2N}^{k} H[k \bmod N] \, , \quad 0 \le k \le 2N-1$$

$$\therefore \; f[k] = W_{2N}^{k} = e^{-j\frac{2\pi}{2N}k}$$