1. $S_1: y_1[n] = x_1[n] + x_1[n-1]$

$S_2: y_2[n] = x_2[n] + 2x_2[n-1] - x_2[n-2]$

$S_3: y_3[n] = x_3[n-1] + x_3[n-2]$

Impulse response of S_1 : $h_1[n] = \delta[n] + \delta[n-1]$

" S_2 : $h_2[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$

" S_3 : $h_3[n] = \delta[n-1] + \delta[n-2]$

$$
\begin{aligned}
y[n] &= x[n] * h_1[n] * h_2[n] * h_3[n] \\
&= x[n] * (\delta[n] + \delta[n-1]) * (\delta[n] + 2\delta[n-1] - \delta[n-2]) * (\delta[n-1] + \delta[n-2]) \\
&= x[n] * \big(\delta[n-1] + \delta[n-2] + 2\delta[n-2] + 2\delta[n-3] - \delta[n-3] - \delta[n-4] \\
&\qquad + \delta[n-2] + \delta[n-3] + 2\delta[n-3] + 2\delta[n-4] - \delta[n-4] - \delta[n-5] \big) \\
&= x[n] * (\delta[n-1] + 4\delta[n-2] + 4\delta[n-3] - \delta[n-5]) \\
&= x[n] * h[n]
\end{aligned}
$$

(a) The impulse response $h[n] = \delta[n-1] + 4\delta[n-2] + 4\delta[n-3] - \delta[n-5]$

(b) It's a FIR system. This system is a linear combination of finite previous inputs.

$y[n] = x[n] * h[n] = x[n-1] + 4x[n-2] + 4x[n-3] - x[n-5]$

2. $h(n) = \delta(n-1) + 4\delta(n-2) + 4\delta(n-3) - \delta(n-5)$

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \delta(n-1) e^{-j\omega n} + 4\sum_{n=0}^{\infty} \delta(n-2) e^{-j\omega n} + 4\sum_{n=0}^{\infty} \delta(n-3) e^{-j\omega n} - \sum_{n=0}^{\infty} \delta(n-5) e^{-j\omega n}$$

$$= e^{-j\omega} + 4e^{-j2\omega} + 4e^{-j3\omega} - e^{-j5\omega}$$

(a) The frepnency response $H(e^{j\omega}) = e^{-j\omega} + 4e^{-j2\omega} + 4e^{-j3\omega} - e^{-j5\omega}$

(b) It's a causal system.

$\because h(n) = \delta(n-1) + 4\delta(n-2) + 4\delta(n-3) - \delta(n-5)$

$\Rightarrow h(n) = 0 \quad \forall n < 0$