Design of a MPC to Simulate Heat Transfer Effectiveness of a Thermoelectric Cooling Device

Philip Nakashita (3039643141), Liam Parrish (3039648679), William Xu (3034675411)
December 2023

ABSTRACT

This paper provides a method to find a finite-time, optimal temperature modeling system utilizing a thermo-electric coolant between a fluid reservoir and a heat exchanger plate. It will also demonstrate how it performs significantly better than a traditional PID controller attempting to model the same situation. GitHub Repository: (http://tinyurl.com/3mdk7r4n)

1. INTRODUCTION

Temperature control and cooling remain a significant challenge in modern engineering practices. Due to the development of more advanced modeling methods, it is beneficial to simulate the temperature and heat transfer behavior of a thermal model in order to better inform design decisions. In this paper, we explore the use of a thermo-electric (TEC) device to control the temperature of fluid inside a reservoir. This system is especially challenging to control since TECs power output is non-linear with respect to temperature. The system dynamics of the fluid, reservoir, and accompanying heat exchanger plate are derived from a first principles thermodynamics and heat transfer based approach, which can then be used to simulate the system. We also describe how a Model Predictive Controller (MPC) could be designed for a temperature vs time reference tracking control task. Finally, the performance of a typical PID controller and the designed MPC controller can be compared in the context of a reference tracking problem.

2. THEORY

2.1 Thermo-Electric Device

A thermo-electric device is a type of temperature control device that makes use of the "Peltier Effect". A temperature differential is created through the introduction of voltage to the device . Often, thermo-electric coolers feature a collection of semiconductors that are sandwiched between ceramic plates [1].

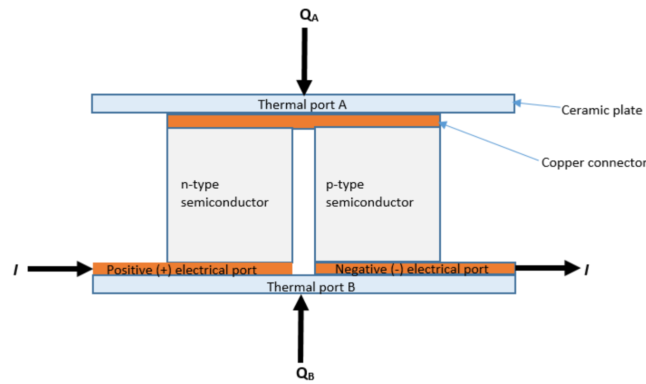

Figure 1: TEC Diagram [2]

These devices are often used in cooling applications, with voltage being the driving factor.
The defining equations are as follows [2]:

$$Q_A = \alpha T_A I - \frac{1}{2} I^2 R + K(T_A - T_B) \qquad \text{(Eq 1)}$$

$$Q_B = -\alpha T_B I - \frac{1}{2} I^2 R + K(T_B - T_A) \qquad \text{(Eq 2)}$$

Qa, Qb and Ta, Tb describe heat flow and temperatures into their respective ports. Alpha is the seebeck coefficient, K is the thermal conductance, V is the voltage potential, I is the current, and R is the total electrical resistance

$$W = VI \qquad \text{(Eq 3)}$$

$$W + Q_A + Q_B = 0 \qquad \text{(Eq 4)}$$

2.2 Heat Transfer

Modeling complex systems such as this often require heavy finite element computation to determine temperature and heat transfer across surfaces. For the sake of simplifying the current model, several valid assumptions will be made.

First and foremost, thermal entities will be modeled under the lumped capacitance assumption. This allows for a simplified representation of a physical system as exhibiting equivalent temperature throughout its entirety.

$$Bi = \frac{h}{k} L \qquad \text{(Eq 5)}$$

Utilizing the Biot number, which compares the convective heat transfer to the conductive, this can be applied to the problem model. In this case, temperature distribution across the thickness of the reservoir and heat exchanger plate will be neglected, and only the effects of convective heat transfer from its surface will be considered.

Secondly, all convective heat transfers will be modeled assuming a natural convection from the ambient environment following Newton's law of cooling where the heat transfer rate is proportional to the temperature difference divided by a thermal resistance. A calculation for more dynamic convection could create complications in the analysis that isn't necessary.

3. MODEL

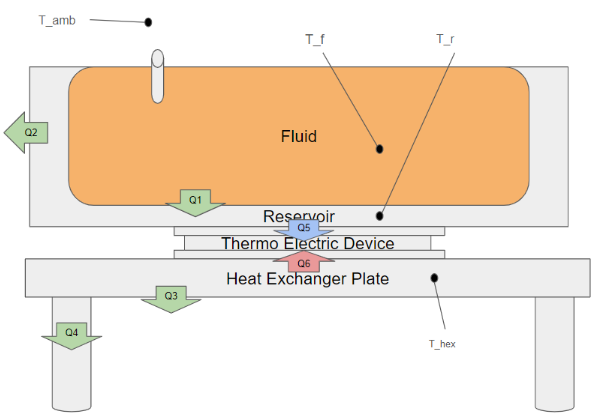

Figure 2: Thermal System Model

3.1 Overview

Our problem will model how effectively a TEC can cool a thermal reservoir filled with fluid. The TEC will also be connected to a heat exchanger plate on its bottom face to cycle its absorbed heat away.

Our system's state variables will be the three temperatures that we would like to track: Fluid temperature, Reservoir temperature, and Heat Exchanger temperature. Our input variables are the voltage applied through the TEC, and the heat exchanger's duty cycle. The most important state to keep track of is the temperature of the fluid, while the others are less so. Thus, we model our stage cost Q and input cost R as:

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3.2 System Dynamics

The system was given by the following equations that describe the heat transfer across different surfaces:

$$q_{Fluid-Ambient} = \frac{T_{Fluid} - T_{Ambient}}{R_{Fluid-Ambient}} \quad \text{(Eq 6)}$$

$$q_{Reservoir-Ambient} = \frac{T_{Reservoir} - T_{Ambient}}{R_{Reservoir-Ambient}} \quad \text{(Eq 7)}$$

$$q_{HEXplate-Ambient} = \frac{T_{HEXplate} - T_{Ambient}}{R_{HEXplate-Ambient}} \quad \text{(Eq 8)}$$

$$q_{Fluid-Reservoir} = \frac{T_{Fluid} - T_{Reservoir}}{R_{Fluid-Reservoir}} \quad \text{(Eq 9)}$$

$$q_{TEC-Reservoir} = \frac{\alpha_{TEC} T_{Reservoir} V_{TEC}}{R_{TEC}} - \frac{V_{TEC}^2}{2R_{TEC}} + K_{TEC}(T_{Reservoir} - T_{HEXplate})$$

(Eq 10)

$$q_{TEC-HEXplate} = \frac{-\alpha_{TEC} T_{HEXplate} V_{TEC}}{R_{TEC}} - \frac{V_{TEC}^2}{2R_{TEC}} + K_{TEC}(T_{HEXplate} - T_{Reservoir})$$

(Eq 11)

In these equations, C represents the heat capacity, R representing thermal resistances, and T being the temperatures. These values were calculated and used to determine the rate of change for each of the state variables given by the following equations.

$$\frac{dT_{Fluid}}{dt} = \frac{-q_{Reservoir}}{C_{Fluid}} \quad \text{(Eq 12)}$$

$$\frac{dT_{Reservoir}}{dt} = \frac{(q_{Fluid-Reservoir} - q_{Reservoir-Ambient} - q_{TEC-Reservoir})}{C_{Reservoir}} \quad \text{(Eq 13)}$$

$$\frac{dT_{HEXplate}}{dt} = \frac{(q_{HEXplate-Ambient} - q_{HEX} - q_{TEC-Hexplate})}{C_{HEXplate}} \quad \text{(Eq 14)}$$

Using a time step of five seconds, an euler discretization was then performed across the time horizon of the simulation.

$$T_{k+1} = T_k + \frac{dT_k}{dt}\Delta t \quad \text{(Eq 15)}$$

3.3 System Constraints

State and input constraints will be implemented into the model. First and foremost, each of the state temperatures will share the same lower and upper boundaries, being $5^{\circ}C$ and $150^{\circ}C$ respectively. Our voltage will be constrained between -12V and 12V, while our heat exchanger duty cycle will be bound between 0 and 1, much like a PWM signal.

3.4 Constrained Finite Time Optimal Control (CFTOC) Formulation

$$\min_{x_0,\ldots,x_N,u_0,\ldots,u_{N-1}} \sum_{k=0}^{N} (x_k - \bar{x})^T Q (x_k - \bar{x}) + u_k^T R u_k$$
$$\begin{aligned} & x_{k+1} = x_k + f(x_k, u_k)\Delta t & \\ & 278K \leq x_k \leq 423K & \forall k = \{0,\ldots,N-1\} \\ & u_{\min} \leq u_k \leq u_{\max} & \forall k = \{0,\ldots,N\} \\ & x_0 = \bar{x}_0 & \forall k = \{0,\ldots,N-1\} \end{aligned}$$

Figure 4: Cost Function

Using the constraints and system dynamics as described in the previous sections, the cost function can be compiled into the optimization problem as observed in Figure 4. Where $\bar{x}$ is the reference temperature in which we want our controller to track. An example temperature vs time profile is shown below in Figure 5.

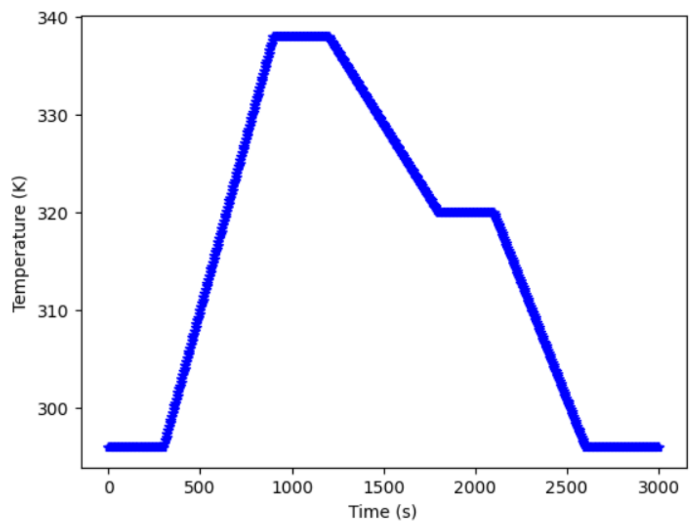

Figure 5: Reference Temperature

This is merely the reference temperature for our fluid temperature however. For the other two states, their reference temperature is just a constant value which is at room temperature, or 298K.

3.5 PID Controller

To compare against the MPC controller described earlier, we simulated a PID controller to attempt to control the same system. The PID controller computes error (e_k) between the fluid temperature and reference temperature to compute the actuation signal for the thermoelectric device using the following equation:

$$u_k = K_P * e_k + K_I * \sum_{k=0}^{k} e_k * T_s + K_D \frac{e_k - e_{k-1}}{T_s} \quad \text{(Eq 17)}$$

The PID controller gains were tuned heuristically in simulation to minimize reference tracking error and to minimize oscillations. One particular challenge with tuning a PID controller for this system is that the system has significant 'thermal inertia' and the fluid temperature responds very slowly to an actuation signal, this results in a system that is very prone to overshooting and oscillating. For the heat exchanger pump duty cycle the PID controller will turn on the pump to match the TEC voltage duty cycle if the system is trying to decrease the temperature. Additionally, the PID controller actuation signal is subject to the same input constraints

4. SIMULATION RESULT

To analyze the effectiveness of the controllers, we first directly compared the MPC vs. PID controller's performance for the reference tracking problem, then we analyzed the performance of both controllers where the state feedback has gaussian random noise added into the signal, and finally we analyzed the computation time and CFTOC horizon length trade-off for the MPC controller.

4.1 MPC vs PID Simulation Results

Simulation was run using the CFTOC and constraints listed in the previous section. This was then compared to a PID model for the exact same reference temperature.

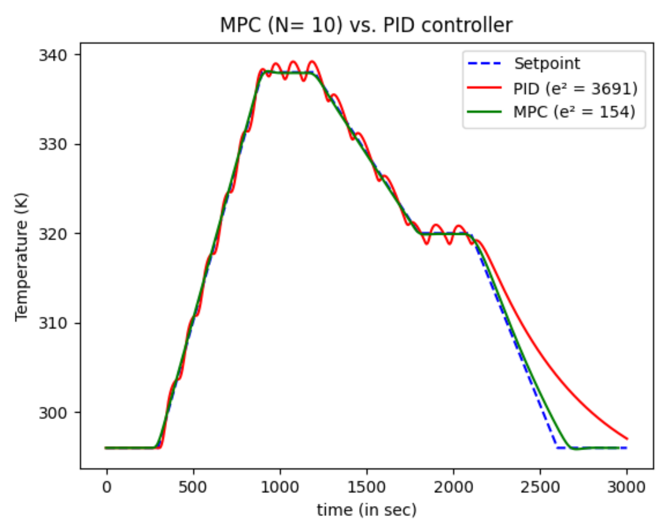

Figure 6: MPC vs PID performance with RSS error

To compare how well the MPC and PID trajectories follow the reference temperature, residual sum squared error was calculated to show how much the MPC and PID trajectory deviated. As seen in figure 6, the MPC least squares sum was around 154, a much lower value than that of the PID simulation, which yielded a value of 3691.

4.2 Simulation with Gaussian Noise

Since all real systems have some form of noise, we wanted to explore the effect of noise on our control system. To simulate noise on the system, noise sampled from a gaussian distribution with standard deviation of 1 deg Kelvin was added to the state feedback used by the controllers to compute the next input.

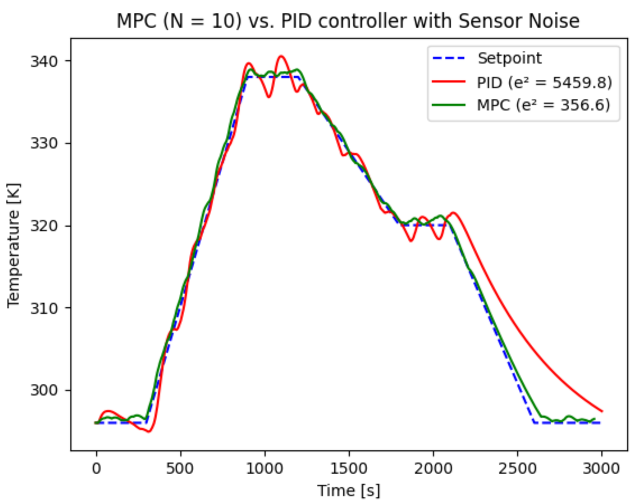

Figure 7: MPC vs. PID controller performance with added gaussian random noise

From Figure 7 we can see that the performance of both controllers gets worse in the presence of noise, which is expected, however the MPC controller performs better than the PID controller by an even larger margin with residual sum squared values of 5459.8 and 356.6 respectively.

4.3 MPC Controller With Different Time Horizons

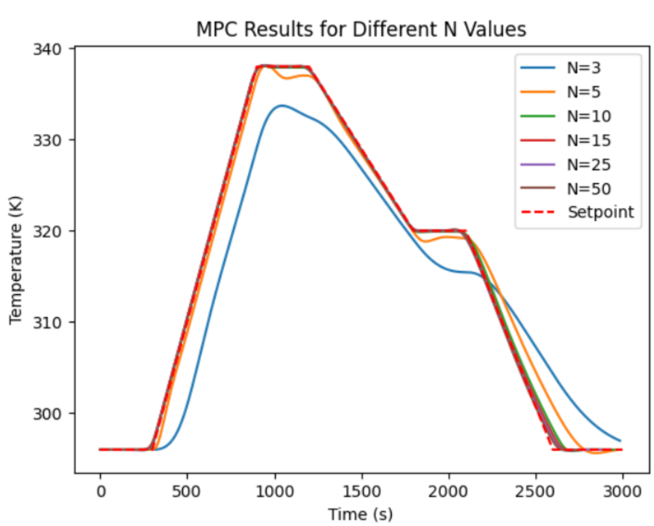

Figure 8: MPC Results for Various N Values

We also wanted to explore the effects that different time horizons have on the overall tracking of the MPC compared to the reference temperature. This was mainly for the purpose of determining at what time horizon does the MPC controller reach diminishing returns. It can be seen that in Figure 8, a time horizon of below 5 can yield results that are fairly off base. It can also be observed that for a time horizon of N = 10 and onwards, our MPC simulation almost matches our reference signal almost exactly.

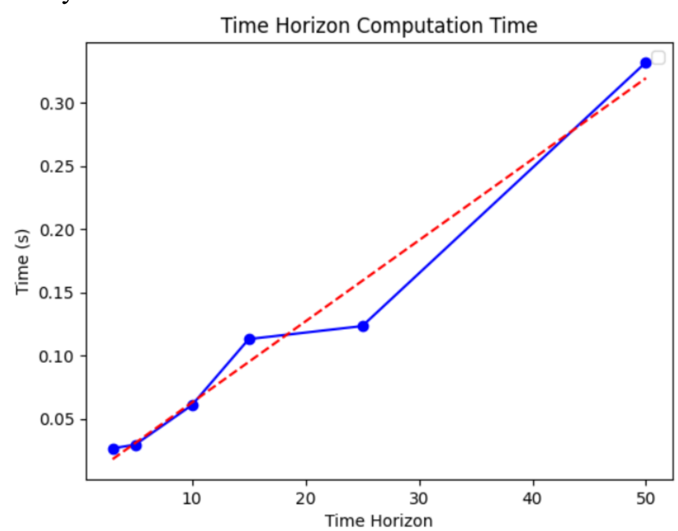

Figure 9: Time Horizon Computation Time per CFTOC

As we also see in figure 9, the computation time it takes to simulate a single optimization problem scales as such for an increasing time horizon. But since the computation time required to solve the CFTOC problem at one time step is much less than our sampling time of 5 seconds, which indicates that running an MPC controller in real time is a scalable solution to this specific task.

5. CONCLUSION

In this paper we formulated and solved an MPC problem for a system with non-linear dynamics and actuation and compared it against a PID controller. We found that the MPC controller when given a sufficient time horizon outperformed the PID controller by almost an order of magnitude even in the presence of noise added to the system.

6. REFERENCES

[1] Woodward, W. S. (2019) *Simple Design Equations for Thermoelectric Coolers*

[2] Mathworks (2018) *Peltier Device: Electrothermal Converter*