# MATH5210 Analysis
# Exam Rework
# Philip Nelson

## 3

Use the definition of convergence to prove: If $a_n$ is a bounded sequence and $\lim_{n\to\infty} b_n = 0$, then $\lim_{n\to\infty} a_n b_n = 0$.

**Proof:** Assume $a_n$ is a bounded sequence, let $M$ be the upper bound of $a_n$ s.t. $M > a_n$, and let $b_n$ be a sequence s.t. $\lim_{n\to\infty} b_n = 0$.
Then for all $\epsilon > 0$ there exists $N \in \mathbf{N}$ s.t. $n \geq N$ and $|b_n - 0| = |b_n| < \frac{\epsilon}{M}$
Using the definition of convergence

$$\begin{aligned}
|a_n - A| &< \epsilon \\
\Rightarrow |a_n b_n - 0| &< \epsilon \\
\Rightarrow |a_n b_n| &< \epsilon \\
\Rightarrow |M b_n| &< \epsilon \\
< |M \frac{\epsilon}{M}| &= \epsilon
\end{aligned}$$

Therefore if $a_n$ is a bounded sequence and $\lim_{n\to\infty} b_n = 0$, then $\lim_{n\to\infty} a_n b_n = 0$.

∎

## 6

Prove that the sequence defined recursively by $a_1 = 2, a_{n+1} = \sqrt{6 + a_n}$ converges to 3.

**Proof:**