Chap. 4: Partitioning Single Stage Networks

partitionability - the ability to divide a
network into independent subnetworks
of different sizes;
each subnetwork size $N' < N$
has all capabilities of complete
network of same type built to be
size N'

necessary for multiple-SIMD and
partitionable SIMD/MIMD

useful for MIMD and SIMD

Homework meeting
with Javier
2:30 Wed Feb 8
my office

no proofs
cube → shuffle
PM2I → shuffle

TEST SCHEDULE

	TEST #1
Covers classes 1-14	Wed 2/15/89
	Room 117
	8:30-9:30 (~10:30) pm
	TEST #2
cover classes 15-27	Thurs 3/30/89
	Room 117
	8:30-9:30 (~10:30) pm
	TEST #3
cover classes 28-42	FINALS

permutation - bijection from a set onto itself

(recall - bijection is one-to-one and onto mapping)

cyclic notation - $(j\ f(j)\ f^2(j)\ \ldots\ f^{k-1}(j))$

j maps to $f(j)$ maps to $f^2(j)\ldots$

where $f^k(j) = j$

$j \rightarrow f(j) \rightarrow f^2(j) \rightarrow \ldots \rightarrow f^k(j) = j$

length of cycle is k

Ex. $p(j) = j+3 \bmod 8$

$p = (\ 0\ \ 3\ \ 6\ \ 1\ \ 4\ \ 7\ \ 2\ \ 5\ )$

Ex. $q(j) = j-2 \bmod 8$

$q = (\ 0\ \ 6\ \ 4\ \ 2\ )(\ 1\ \ 7\ \ 5\ \ 3\ )$

product of permutations –

is the same notation as

$$p*q(j) \equiv \underbrace{pq(j)}_{\text{first apply p then apply q}} \equiv q(p(j))$$

$pq = (0\ \ 3\ \ 6\ \ 1\ \ 4\ \ 7\ \ 2\ \ 5)$

$\quad (0\ \ 6\ \ 4\ \ 2)\ (1\ \ 7\ \ 5\ \ 3)$

$$\begin{aligned} = & \ 0 \rightarrow 3 \rightarrow 1 \\ & \ 1 \rightarrow 4 \rightarrow 2 \\ & \ 2 \rightarrow 5 \rightarrow 3 \\ & \ 3 \rightarrow 6 \rightarrow 4 \\ & \ 4 \rightarrow 7 \rightarrow 5 \\ & \ 5 \rightarrow 0 \rightarrow 6 \\ & \ 6 \rightarrow 1 \rightarrow 7 \\ & \ 7 \rightarrow 2 \rightarrow 0 \end{aligned}$$

$= (0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7)$

$pq(j) = q(p(j)) = (j+3)-2 \bmod 8 = j+1 \bmod 8$

cycle structure of an interconnection function -

unique disjoint cycles representation

Ex. $q(j) = j-2 \bmod 8$

$$= (0\ \ 6\ \ 4\ \ 2)\ (1\ \ 7\ \ 5\ \ 3)$$

Cube Network Cycle Structure

~~permutation~~ product of ordered pairs

$$\text{cube}_i = \prod_{j=0}^{N-1} (j\ \text{cube}_i(j))$$

where i^{th} bit of $j = 0$

Ex. $N = 8$, $i = 1$:

$$\text{cube}_1 = (0\ \ 2)\ (1\ \ 3)\ (4\ \ 6)\ (5\ \ 7)$$

if PEs 0 and 2 inactive:

$$\text{cube}_1' = (1\ \ 3)\ (4\ \ 6)\ (5\ \ 7)$$ still a valid permutation

if just PE 0 inactive, transfer not permutation

- $0 \rightarrow 0$ and $2 \rightarrow 0$ (not one-to-one)
- 2 does not receive data (not onto)

$\therefore$ whole cycle active or whole cycle inactive

to have permutation (or else data destroyed)

Illiac Network Cycle Structure

$$\text{Illiac}_{+1} = (0\ \ 1\ \ 2\ \ldots\ N{-}1)$$

$$\text{Illiac}_{-1} = (N{-}1\ \ldots\ 2\ \ 1\ \ 0)$$

$n = \sqrt{N}$

$$\text{Illiac}_{+n} = \prod_{j=0}^{n-1} (j\ \ j{+}n\ \ j{+}2n\ \ldots\ j{+}N{-}n)$$

$$\text{Illiac}_{-n} = \prod_{j=0}^{n-1} (j{+}N{-}n\ \ldots\ j{+}2n\ \ j{+}n\ \ j)$$

Ex. $N = 16$:

$$\text{Illiac}_{+4} = (0\ \ 4\ \ 8\ \ 12)\ (1\ \ 5\ \ 9\ \ 13)$$
$$(2\ \ 6\ \ 10\ \ 14)\ (3\ \ 7\ \ 11\ \ 15)$$

n ~~networks~~ cycles of size n

PM2I Network Cycle Structure

$$PM2_{+i} = \prod_{j=0}^{2^i-1} (j\ \ j+2^i\ \ j+2*2^i\ \ldots\ j+N-2^i)$$

$$PM2_{-i} = \prod_{j=0}^{2^i-1} (j+N-2^i\ \ldots\ j+2*2^i\ \ j+2^i\ \ j)$$

Ex. $N = 8$, $i = 1$:

$$PM2_{-1} = (0\ \ 6\ \ 4\ \ 2)\ \ (1\ \ 7\ \ 5\ \ 3)$$

Shuffle-Exchange Network Cycle Structure

$$\text{exchange} = \prod_{j=0}^{N-2} (j\ j{+}1)$$

where j is even

$$\text{shuffle} = \prod_{j=0}^{N-1} (j\ \text{shuffle}(j)\ \text{shuffle}^2(j)\ \ldots)$$

where j is not in a previous cycle

Ex. N = 8:

$$\text{shuffle} = (0)\ \ (1\ \ 2\ \ 4)\ \ (3\ \ 6\ \ 5)\ \ (7)$$
$$= (1\ \ 2\ \ 4)\ \ (3\ \ 6\ \ 5)$$

N=16

shuffle = (0) (1 2 4 8) (3 6 12 (9)

(5 10) (7 14 13 11) (15)

maximum cycle length for shuffle is m

because $\text{shuffle}^m(j) = j$, $0 \le j < 2^m$

$\text{shuffle}(p_{m-1/0}) = p_{m-2/0}\, p_{m-1}$

$\text{exchange}(p_{m-1/0}) = p_{m-1/1}\, \bar{p}_0$

Notation

- physical PE addresses $P = \{0,1,...,N-1\}$
- s_i size of partition i (power of 2)
- logical PE addresses for partition i

$$\ell_i = \{\ell_{i/0}, \ell_{i/1}, ..., \ell_{i/(s_i-1)}\}$$

- v number of partitions
- set of all logical PE addresses

$$L = \bigcup_{j=0}^{v-1} \ell_j$$

$$|L| = \sum_{i=0}^{v-1} s_i = N$$

- $t\colon P \rightarrow L$
(translation from physical to logical partitions)
if $t(p_k) = \ell_{i/j}$ then $t^{-1}(\ell_{i/j}) = p_k$

where $p_k \in P$, $\ell_{i/j} \in L$

Partitioning the Cube

Ex. N = 8

Even subnetwork — solid lines

Odd subnetwork — dashed lines

$$\text{t:}\quad 0 \to \ell_{E/0} \quad 2 \to \ell_{E/1} \quad 4 \to \ell_{E/2} \quad 6 \to \ell_{E/3}$$

$$\quad 1 \to \ell_{O/0} \quad 3 \to \ell_{O/1} \quad 5 \to \ell_{O/2} \quad 7 \to \ell_{O/3}$$

physical $\text{cube}_1 = (0\ \ 2)\ \ (4\ \ 6)\ \ (1\ \ 3)\ \ (5\ \ 7)$

$$= \underbrace{(\ell_{E/0}\,\ell_{E/1})\,(\ell_{E/2}\,\ell_{E/3})}_{\text{logical cube}_0}\ \ \underbrace{(\ell_{O/0}\,\ell_{O/1})\,(\ell_{O/2}\,\ell_{O/3})}_{\text{logical cube}_0}$$

$\equiv \text{cube}_{E/0}$ $\qquad \equiv \text{cube}_{O/0}$ odd zero

similarly, physical cube_2 acts as logical cube_1

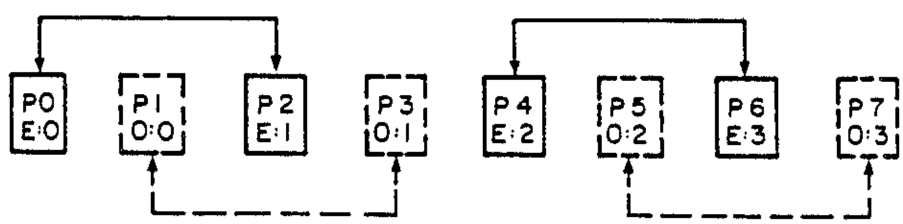
<u>EXAMPLE</u>

$\text{cube}_1(0) = 2 \to$

$\text{cube}_{E/0}(\ell_{E/0})$

$= \ell_{E/1}$

A



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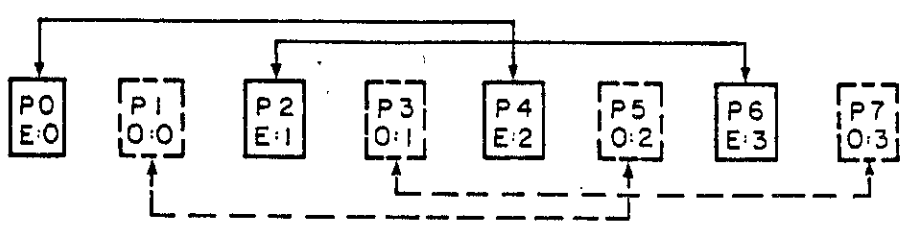


$\text{cube}_i(p_{m-1} \cdots p_i \cdots p_0) = p_{m-1} \cdots \bar{p}_i \cdots p_0, \quad 0 \le i < m$

Partitioning the Cube

in general, constraints on t:

- partition ℓ_i must have $\log_2 s_i$ cube functions, $0 \leq i < v$

  $0 \leq i < v$

  $cube_{i/r}$ means logical $cube_r$

  for partition ℓ_i, $0 \leq r < \log_2 s_i$

- labeling constraints: $\ell_{i/j}$ and $\ell_{i/k} \in \ell_i$

  $\ell_{i/j}$ and $\ell_{i/k}$ connected if and only if $H(j,k) = 1$

  these PEs are only ones $\ell_{i/j}$ can communicate with

- if physical $cube_x \equiv$ logical $cube_{i/r}$ and

  $cube_x(t^{-1}(\ell_{i/j})) = t^{-1}(\ell_{i/k})$ then

  j and k differ only in bit r

$cube_i(p_{n-1} \cdots p_i \cdots p_0) = p_{n-1} \cdots \bar{p}_i \cdots p_0, \quad 0 \leq i < n$

Partitioning the Cube

Intuitive argument, N=8

$\text{cube}_0 = (0\ 1)\ (2\ 3)\ (4\ 5)\ (6\ 7)$

$\text{cube}_1 = (0\ 2)\ (1\ 3)\ (4\ 6)\ (5\ 7)$

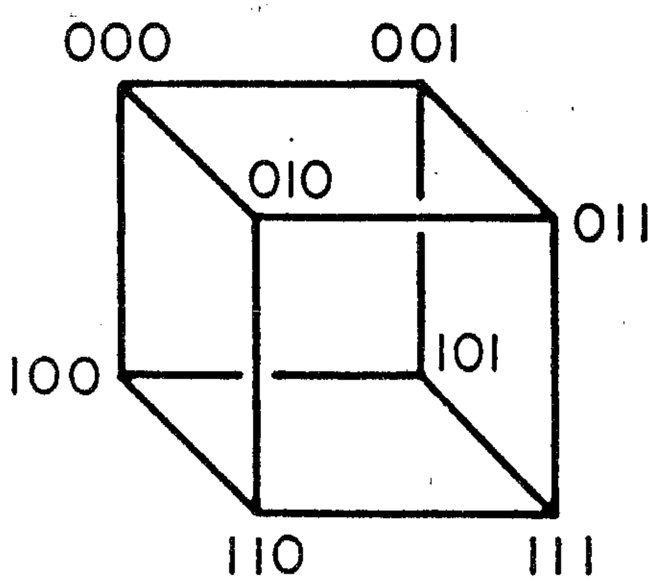
$\text{cube}_2 = (0\ 4)\ (1\ 5)\ (2\ 6)\ (3\ 7)$

For a size $N/2 = 4$ partition you can:

(1) disallow cube_0 [0,2,4,6] [1,3,5,7] p_0 fixed

(2) disallow cube_1 [0,1,4,5] [2,3,6,7] p_1 fixed

(3) disallow cube_2 [0,1,2,3] [4,5,6,7] p_2 fixed



$$\text{cube}_i\ (p_{m-1} \cdots p_i \cdots p_0) = p_{m-1} \cdots \overline{p}_i \cdots p_0$$

Partitioning the Cube

in general, size N Cube into two size N/2 Cubes

pick bit position i, $0 \leq i < m$

N/2 PEs $p_i = 0$, N/2 PEs $p_i = 1$

use $cube_j \quad 0 \leq j < m \quad j \neq i$

for $j \neq i$, N/4 disjoint cycles $cube_j$ $p_i = 0$

N/4 disjoint cycles $cube_j$ $p_i = 1$

repeat process to get smaller partitions

Ex. $N = 16$, $v = 3$, $s_0 = 4$, $s_1 = 4$, $s_2 = 8$

divide on p_3: 0 to 7 $= \ell_2$ 8 to 15 = another group

divide 8 to 15 on p_1: 8,9,12,13 $= \ell_0$ 10,11,14,15 $= \ell_1$

$p_1 = 0$ $\qquad p_1 = 1$

ℓ_2 uses cycles from $cube_0$, $cube_1$, and $cube_2$

ℓ_0 and ℓ_1 use cycles from $cube_0$ and $cube_2$

$cube_i(p_{m-1} \cdots p_i \cdots p_0) = p_{m-1} \cdots \overline{p_i} \cdots p_0, \quad 0 \leq i < m$

Partitioning the Cube

Ex. $N = 16$, $v = 3$, $s_0 = 4$, $s_1 = 4$, $s_2 = 8$

divide on p_3: 0 to 7 $= \ell_2$ 8 to 15 = another group

divide 8 to 15 on p_1: 8,9,12,13 $= \ell_0$ 10,11,14,15 $= \ell_1$

$p_1 = 0$ $p_1 = 1$

once choice for t:

$0 \to \ell_{2/0}$	$1 \to \ell_{2/1}$	$2 \to \ell_{2/2}$	$3 \to \ell_{2/3}$
$4 \to \ell_{2/4}$	$5 \to \ell_{2/5}$	$6 \to \ell_{2/6}$	$7 \to \ell_{2/7}$
$8 \to \ell_{0/0}$	$9 \to \ell_{0/1}$	$10 \to \ell_{1/0}$	$11 \to \ell_{1/1}$
$12 \to \ell_{0/2}$	$13 \to \ell_{0/3}$	$14 \to \ell_{1/2}$	$15 \to \ell_{1/3}$

ℓ_0: (8 9) (12 13) from cube_0 for $\text{cube}_{0/0}$

(8 12) (9 13) from cube_2 for $\text{cube}_{0/1}$

ℓ_1: (10 11) (14 15) from cube_0 for $\text{cube}_{1/0}$

(10 14) (11 15) from cube_2 for $\text{cube}_{1/1}$

ℓ_2: (0 1) (2 3) (4 5) (6 7) from cube_0 for $\text{cube}_{2/0}$

(0 2) (1 3) (4 6) (5 7) from cube_1 for $\text{cube}_{2/1}$

(0 4) (1 5) (2 6) (3 7) from cube_2 for $\text{cube}_{2/2}$

Partitioning the Cube

t for ℓ_0: $8 \rightarrow \ell_{0/0}$ $9 \rightarrow \ell_{0/1}$ $12 \rightarrow \ell_{0/2}$ $13 \rightarrow \ell_{0/3}$

t for ℓ_0: $p_3p_2p_1p_0 \rightarrow p_2p_0$

(8 9) (12 13) from cube_0 for $\text{cube}_{0/0}$ Standard translation of cube

(8 12) (9 13) from cube_2 for $\text{cube}_{0/1}$ "define in terms of p_i" for HW

t' for ℓ_0: $p_3p_2p_1p_0 \rightarrow p_0p_2$ (any permutation)

t' for ℓ_0: $8 \rightarrow \ell_{0/0}$ $9 \rightarrow \ell_{0/2}$ $12 \rightarrow \ell_{0/1}$ $13 \rightarrow \ell_{0/3}$

(8 12) (9 13) from cube_2 for $\text{cube}_{0/0}$

(8 9) (12 13) from cube_0 for $\text{cube}_{0/1}$

t'' for ℓ_0: $p_3p_2p_1p_0 \rightarrow p_2\overline{p}_0$ (complement any bits)

t'' for ℓ_0: $8 \rightarrow \ell_{0/1}$ $9 \rightarrow \ell_{0/0}$ $12 \rightarrow \ell_{0/3}$ $13 \rightarrow \ell_{0/2}$

(9 8) (13 12) from cube_0 for $\text{cube}_{0/0}$

(9 13) (8 12) from cube_2 for $\text{cube}_{0/1}$

$\text{Cube}_i(p_{n-1} \cdots p_i \cdots p_0) = p_{n-1} \cdots \overline{p_i} \cdots p_0, \quad 0 \le i < n$

Partitioning the Cube

Summary of ways to vary t

A. choice of bit positions physical addresses of PEs in partition agree

B. choice of partition to subdivide

C. choice of association of physical bit positions to logical

D. choice of which bit positions in C to complement

A and B: which PEs in partition

C and D: mapping within partition

all PEs in ℓ_i must agree in m-$\log_2 s_i$ bit positions

and use $\log_2 s_i$ cube functions corresponding to the other positions

Partitioning the Illiac

cannot be partitioned into independent subnetworks

need four interconnection functions for each partition

cannot use $\text{Illiac}_{\pm 1}$ since they include all PEs

Ex. to partition into two N/2 subnetworks

need independent logical Illiac_{+1} cycle of size N/2

Partitioning the PM2I

Ex. N = 8

Even subnetwork — solid lines

Odd subnetwork — dashed lines

t: $0 \rightarrow \ell_{E/0}$ $2 \rightarrow \ell_{E/1}$ $4 \rightarrow \ell_{E/2}$ $6 \rightarrow \ell_{E/3}$

$1 \rightarrow \ell_{O/0}$ $3 \rightarrow \ell_{O/1}$ $5 \rightarrow \ell_{O/2}$ $7 \rightarrow \ell_{O/3}$

physical $PM2_{+1} = (0\ 2\ 4\ 6)\ (1\ 3\ 5\ 7)$

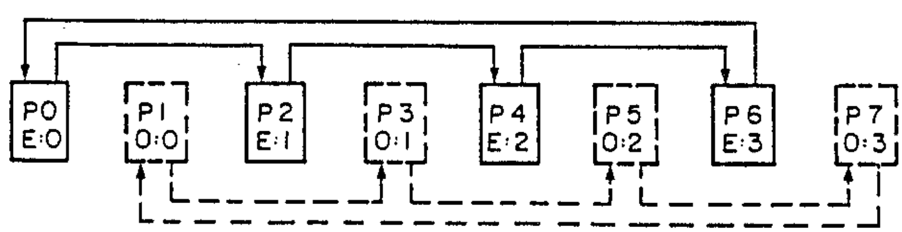
$= (\ell_{E/0}\,\ell_{E/1}\,\ell_{E/2}\,\ell_{E/3})\ (\ell_{O/0}\,\ell_{O/1}\,\ell_{O/2}\,\ell_{O/3})$

logical $PM2_{+0}$ logical $PM2_{+0}$

similarly, physical $PM2_{-1}$ acts as logical $PM2_{-0}$ and

physical $PM2_{\pm 2}$ acts a logical $PM2_{\pm 1}$

A



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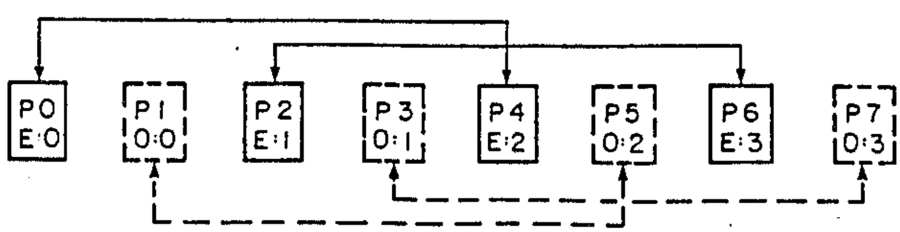


$PM2_{+i}(P) = P + 2^i \bmod N \quad 0 \le i < m$

$PM2_{-i}(P) = P - 2^i \bmod N \quad 0 \le i < m$

Partitioning the PM2I (all arithmetic mod s_i or N)

in general, constraints on t:

- partition ℓ_i must have $2\log_2 s_i$ PM2I functions,

  $0 \leq i < v$

  $PM2_{i/\pm r}$ means logical $PM2_{\pm r}$

  for partition ℓ_i, $0 \leq r < \log_2 s_i$

- labeling constraints: $\ell_{i/j}$ and $\ell_{i/k} \in \ell_i$

  $\ell_{i/j}$ and $\ell_{i/k}$ connected if an only if

  $k = j \pm 2^r \qquad 0 \leq r < \log_2 s_i$

- if physical $PM2_{\pm x} \equiv$ logical $PM2_{i/+r}$ and

  $PM2_{\pm x}(t^{-1}(\ell_{i/j})) = t^{-1}(\ell_{i/k})$ then

  $k = j + 2^r$ for some r, $0 \leq r < \log_2 s_i$

- similar for logical $PM2_{i/-r}$

Partitioning the PM2I

Intuitive argument, N=16

$PM2_{+0} = (\ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ )$

$PM2_{+1} = (\ 0\ \ 2\ \ 4\ \ 6\ \ 8\ \ 10\ \ 12\ \ 14\ )$

$(\ 1\ \ 3\ \ 5\ \ 7\ \ 9\ \ 11\ \ 13\ \ 15\ )$

$PM2_{+2} = (\ 0\ \ 4\ \ 8\ \ 12\ )\ (\ 1\ \ 5\ \ 9\ \ 13\ )$

$(\ 2\ \ 6\ \ 10\ \ 14\ )\ (\ 3\ \ 7\ \ 11\ \ 15\ )$

$PM2_{+3} = (\ 0\ \ 8\ )\ (\ 1\ \ 9\ )\ (\ 2\ \ 10\ )\ (\ 3\ \ 11\ )$

$(\ 4\ \ 12\ )\ (\ 5\ \ 13\ )\ (\ 6\ \ 14\ )\ (\ 7\ \ 15\ )$

For a size $N/2 = 8$ partition: cannot use $PM2_{+0}$
can use even or odd (fix p_0)

For a size $N/4 = 4$ partition:
cannot use $PM2_{+0}$ or $PM2_{+1}$
can use PEs in any one cycle of $PM2_{+2}$ (fix $p_1 p_0$)

Note: $PM2_{+i}$ has 2^i cycles of size $N/2^i$

Partition of size 2^j cannot use $PM2_{+i}$ if $N/2^i > 2^j$
$\rightarrow 2^{m-i} > 2^j \rightarrow m - i > j$

$PM2_{+i}(P) = P+2^i \bmod N \qquad 0 \leq i < m$
$PM2_{-i}(P) = P-2^i \bmod N \qquad 0 \leq i < m$

Partitioning the PM2I

Let $s_i = N/2^a = 2^{m-a}$, $j \in P$, $t(j) \in \ell_i$

- Physical $PM2_{\pm b}$ for $0 \leq b < a$ cannot be used by j
  - — length cycles of $PM2_{\pm b} \geq N/2^{a-1} = 2^{m-a+1} = 2 * s_i$
  - — $PM2_{\pm x}$ has 2^x cycles of length $N/2^x = 2^{m-x}$
  - — $PM2_{\pm 0}$: $2^0 = 1$ cycle of length N

    $PM2_{\pm 1}$: $2^1 = 2$ cycles of length N/2 (odd,even) etc.
  - — since $PM2_{\pm b}$ cycles of length $\geq 2 * s_i$

    must contain some $q \in P$, $t(q) \notin \ell_i$
- ℓ_i needs $2 \log_2 s_i = 2(m-a)$ PM2I functions to use
  - — cannot use a function $PM2_{\pm b}$ for $0 \leq b < a$
  - — must use $2(m-a)$ functions $PM2_{\pm c}$ for $a \leq c < m$

start here next time

Partitioning the PM2I

- $s_i = 2^{m-a}$, $j \in P$, $t(j) \in \ell_i$
- $PM2_{\pm a}$ cycle contains $y + k * 2^a$, for a given y, $0 \leq y < 2^a$, and $0 \leq k < 2^{m-a}$
  - since $t(j) \in \ell_i$, $t(j + k * 2^a) \in \ell_i$ for $0 \leq k < 2^{m-a}$
    $2^{m-a} = s_i = |\ell_i|$, all PEs lo-order a bits = j
- $PM2_{\pm c}$ for $a \leq c < m$
  - must show if cycle of $PM2_{\pm c}$ includes j
    it has no q such that
    $t(q) \notin \ell_i = \{j+k*2^a \mid 0 \leq k < 2^{m-a}\}$

in general, consider $PM2_{+h}$ ($PM2_{-h}$ similar):

$$PM2_{+h} = \prod_{u=0}^{2^h-1} (u \;\; u+2^h \;\; u+2*2^h \;\; \ldots \;\; u+N-2^h)$$

elements in cycle with u: $X^{m-h}u_{h-1/0}$

if $t(j) \in \ell_i$ and $j \in X^{m-a}j_{a-1/0}$ then

if $j \in X^{m-c}u_{c-1/0}$ then $u_{c-1/0} = u_{c-1/a}j_{a-1/0}$

and $X^{m-c}u_{c-1/0} \subseteq X^{m-a}j_{a-1/0}$

$\therefore$ all physical PE numbers in partition size $s_i = 2^{m-a}$ must agree in a lo-order bits (hi-order m-a bits vary)

Partitioning the PM2I

in general, size N PM2I into two size N/2 PM2Is

N/2 PEs $p_0 = 0$, N/2 PEs $p_0 = 1$

use physical $PM2_{\pm i}$ $\quad 0 < i < m$

$2^i/2$ disjoint cycles of $PM2_{\pm i}$ $p_0 = 0$

$2^i/2$ disjoint cycles of $PM2_{\pm i}$ $p_0 = 1$

repeat process to get smaller partitions

Ex. $N = 16$, $v = 3$, $s_0 = 4$, $s_1 = 4$, $s_2 = 8$

must divide on p_0: even $= \ell_2$ $\quad$ odd = another group

must divide on p_1: 1,5,9,13 $= \ell_0$ $\quad$ 3,7,11,15 $= \ell_1$

$p_1 = 0$ $\quad$ $p_1 = 1$

ℓ_2 uses cycles from $PM2_{\pm 1}$, $PM2_{\pm 2}$, and $PM2_{\pm 3}$

ℓ_0 and ℓ_1 uses cycles from $PM2_{\pm 2}$ and $PM2_{\pm 3}$

Partitioning the PM2I

Ex. $N = 16$, $v = 3$, $s_0 = 4$, $s_1 = 4$, $s_2 = 8$

must divide on p_0: even $= \ell_2$ odd $=$ another group

must divide on p_1: $1,5,9,13 = \ell_0$ $3,7,11,15 = \ell_1$

$p_1 = 0$ $p_1 = 1$

one choice for t:

$0 \rightarrow \ell_{2/0}$	$1 \rightarrow \ell_{0/0}$	$2 \rightarrow \ell_{2/1}$	$3 \rightarrow \ell_{1/0}$
$4 \rightarrow \ell_{2/2}$	$5 \rightarrow \ell_{0/1}$	$6 \rightarrow \ell_{2/3}$	$7 \rightarrow \ell_{1/1}$
$8 \rightarrow \ell_{2/4}$	$9 \rightarrow \ell_{0/2}$	$10 \rightarrow \ell_{2/5}$	$11 \rightarrow \ell_{1/2}$
$12 \rightarrow \ell_{2/6}$	$13 \rightarrow \ell_{0/3}$	$14 \rightarrow \ell_{2/7}$	$15 \rightarrow \ell_{1/3}$

ℓ_0: (1 5 9 13) from $PM2_{+2}$ for $PM2_{0/+0}$

(1 9) (5 13) from $PM2_{+3}$ for $PM2_{0/+1}$

ℓ_1: (3 7 11 15) from $PM2_{+2}$ for $PM2_{0/+0}$

(3 11) (7 15) from $PM2_{+3}$ for $PM2_{0/+1}$

ℓ_2: (0 2 4 6 8 10 12 14) from $PM2_{+1}$ for $PM2_{2/+0}$

(0 4 8 12) (2 6 10 14) from $PM2_{+2}$ for $PM2_{2/+1}$

(0 8) (4 12) (2 10) (6 14) from $PM2_{+3}$ for $PM2_{2/+2}$

Partitioning the PM2I

t for ℓ_0: $1 \rightarrow \ell_{0/0}$ $5 \rightarrow \ell_{0/1}$ $9 \rightarrow \ell_{0/2}$ $13 \rightarrow \ell_{0/3}$

t for ℓ_0: $p_3p_2p_1p_0 \rightarrow p_3p_2$

(1 5 9 13) from $PM2_{+2}$ for $PM2_{0/+0}$

(1 9) (5 13) from $PM2_{+3}$ for $PM2_{0/+1}$

t′ for ℓ_0: $p_3p_2p_1p_0 \rightarrow \overline{p}_3\overline{p}_2$

t′ for ℓ_0: $1 \rightarrow \ell_{0/3}$ $5 \rightarrow \ell_{0/2}$ $9 \rightarrow \ell_{0/1}$ $13 \rightarrow \ell_{0/0}$

(13 9 5 1) from $PM2_{-2}$ for $PM2_{0/+0}$ Remember this!

(9 1) (13 5) from $PM2_{-3} = PM2_{+3}$ for $PM2_{0/+1}$

if $y + 2^r = z$ then $\overline{y} - 2^r = \overline{z}$ and

if $y - 2^r = z$ then $\overline{y} + 2^r = \overline{z}$, $0 \leq r < \log_2 s_i$

Partitioning the PM2I

t for ℓ_0: $1 \rightarrow \ell_{0/0}$ $5 \rightarrow \ell_{0/1}$ $9 \rightarrow \ell_{0/2}$ $13 \rightarrow \ell_{0/3}$

t for ℓ_0: $p_3p_2p_1p_0 \rightarrow p_3p_2$

(1 5 9 13) from $PM2_{+2}$ for $PM2_{0/+0}$

(1 9) (5 13) from $PM2_{+3}$ for $PM2_{0/+1}$

t'' for ℓ_0: cyclic relabeling — rotate elements of $PM2_{0/+0}$
$1 \rightarrow \ell_{0/3}$ $5 \rightarrow \ell_{0/0}$ $9 \rightarrow \ell_{0/1}$ $13 \rightarrow \ell_{0/2}$

$$
\begin{aligned}
(1\ \ 5\ \ 9\ \ 13) &= (\ell_{0/0}\,\ell_{0/1}\,\ell_{0/2}\,\ell_{0/3}) \text{ for t} \\
&= (\ell_{0/3}\,\ell_{0/0}\,\ell_{0/1}\,\ell_{0/2}) \text{ for } t''
\end{aligned}
$$

use same cycles of physical PM2I functions for the
logical PM2I functions as t

works for other $PM2_{\pm i}$ cycles since cycle relabeling
does not change distance between
$t''^{-1}(\ell_{0/j})$ and $t''^{-1}(\ell_{0/j+1})$
adjacent physical numbers of PEs in a $PM2_{\pm i}$ cycle
are still $\pm 2^i$ apart

Partitioning the PM2I

Summary of ways to vary t

A. choice of partition to subdivide

B. choice of t: $p_{m-1/0} \rightarrow p_{m-1/m-\log_2 s_i}$ or

$\qquad\qquad$ t: $p_{m-1/0} \rightarrow \bar{p}_{m-1/m-\log_2 s_i}$

C. apply cyclic relabeling

A: which PEs in partition

B and C: mapping within partition

all PEs in ℓ_i must agree in lo-order $m-\overset{\log_2}{s_i}$ bit positions

use $\;2 * \log_2 s_i$ functions $PM2_{\pm j}$ for $(m-\log_2 s_i) \le j < m$

within each partition can use physical $PM2_{+j}$

as logical $PM2_{i/\pm(j-(m-\log_2 s_i))}$

similar for $PM2_{-j}$

Partitioning the Shuffle-Exchange

cannot be partitioned into independent subnetworks

need two interconnection functions for each partition

cannot use shuffle since largest cycle

should be of length $\log_2 s_i$ not m

Ex. $t(1) = \ell_{i/j}$

1 is in cycle with: 1,2,4,8,...,N/2

which is of size m

if $m = \log_2 s_i$ then $s_i = N$

Partitioning for Efficiency in SIMD Mode

Efficiency:

$$\frac{\text{serial time}}{(\#\ \text{PEs}) * (\text{parallel time})}$$

Ex.: factor of N speedup

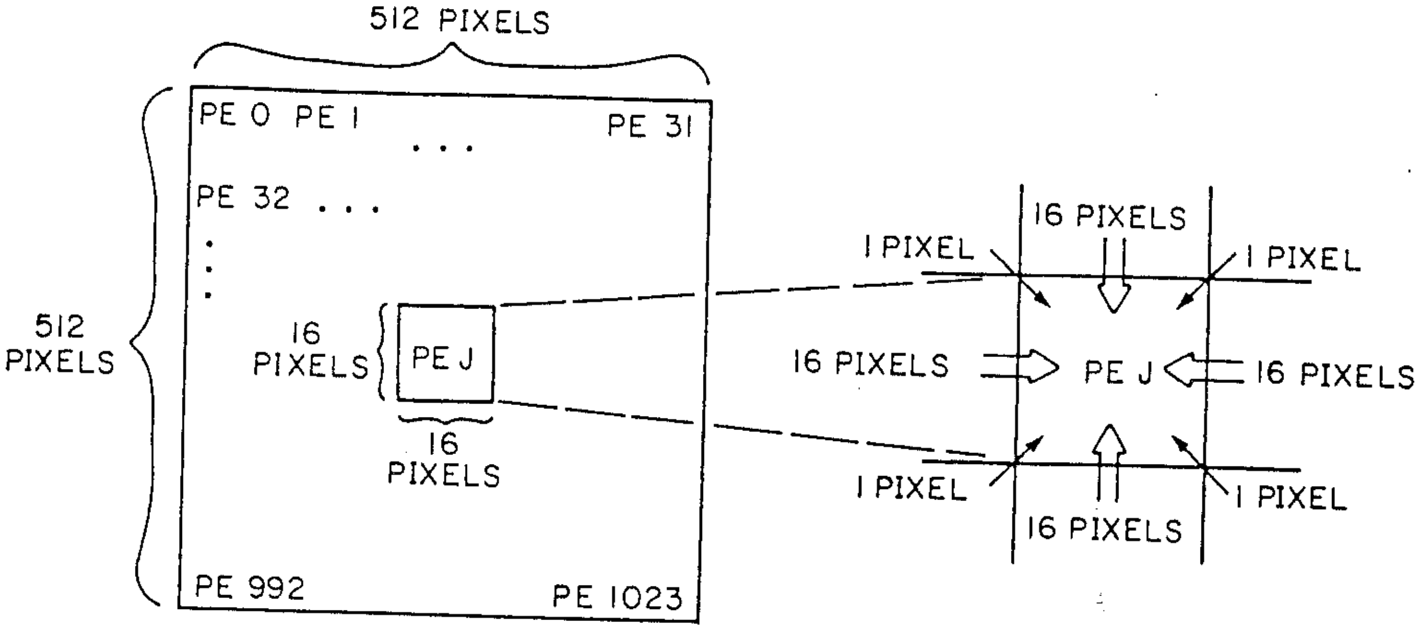
serial time = t

parallel time = t/N

\# PEs = N

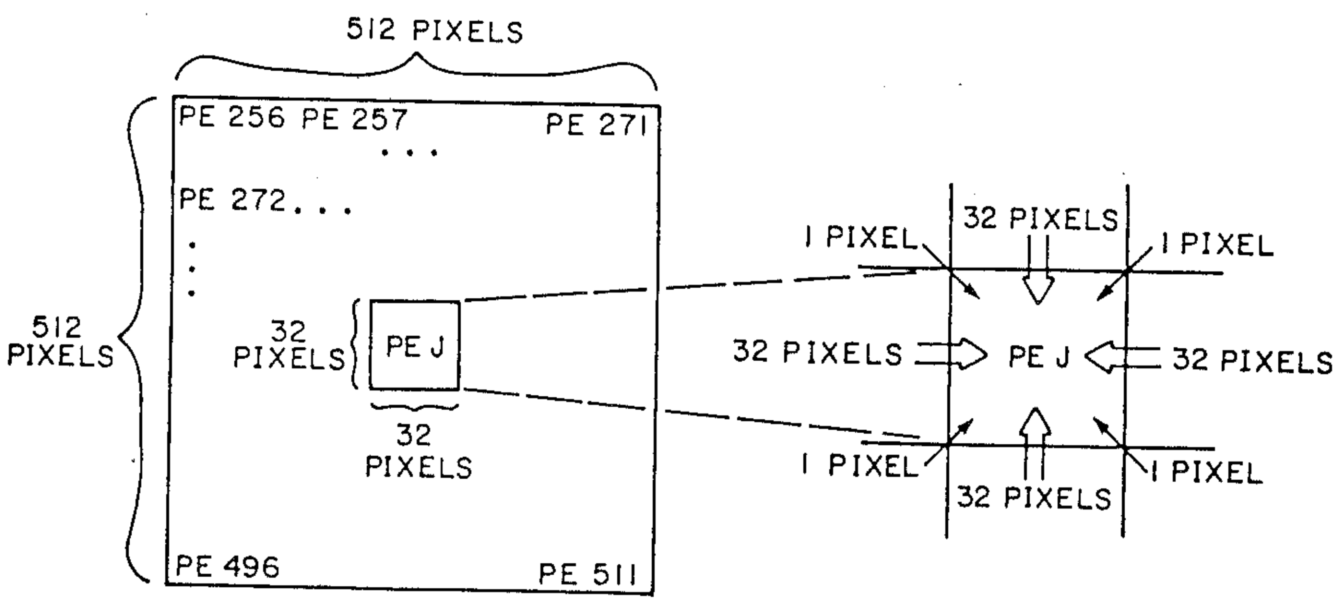
$$\text{Efficiency} = \frac{t}{(N) * (t/N)} = 100\%$$

Partitioning for Efficiency in SIMD Mode



smooth 4 images in sequence: $4*(16^2+68)=1296$ steps

efficiency: $4*512^2/(1024*4*(16^2+68)) = 79\%$



use 4 submachines, smooth 4 images in parallel:

$(32^2+132) = 1156$ steps

efficiency: $4*512^2/(1024*(32^2+132)) = 89\%$

Reasons for Partitioning

- multiple - SIMD machine
  - set of CU's
  - partition PE's into independent SIMD machines
- reconfigurable SIMD/MIMD machines
  - partition system into independent SIMD/MIMD subsystems (PASM)
    - fault tolerance
    - multiple users
    - efficient size
    - program development
    - subtask parallelism
- SIMD machine
  - single CU, same program
  - multiple data sets
  - can improve efficiency
- MIMD machine
  - group PE's which communicate
  - reduce network conflicts