CS 5110/6110 Program 4: Vickrey Clark Grove Auctions

You want to sell weekly placements of ads on your website. You have three slots, call them S1, S2, and S3, where you will display the ads. From past experience, you know that the ad in S1 will get 500 clicks per week, S2 will get 300 clicks per week, and S3 will get 100 clicks per week. (For simplicity, we'll assume you don't want to run the same ad in two different spots, that the number of clicks per week is constant and independent of the ad, and that you're running the same ad in each spot for the entire week.)

Suppose you have found five advertisers, A1, A2, ... A5, who want to advertise on your site. You decide to use VCG to determine whose ads are placed in each slot.

You ask each advertiser to make a bid of how much they are willing to pay per click. We will assume that they are willing to pay for any of the slots. The bids you get back are:

A1: $.50
A2: $.40
A3: $.30
A4: $.20
A5: $.10

For convenience, let's call Pn the price that advertiser An is willing to pay". For example, P2 = .40. We'll also call C1 = 500, C2 = 300, and C3 = 100, the number of clicks for each ad slot (S1,S2, S3)t.

Under VCG, the rule for assigning the winners is what you would expect: you give the better slots to the better bids. So you would assign slots S1, S2, and S3 to advertisers A1, A2, and A3 respectively. A4 and A5's ads would not appear at all.

The rules for determining what each winner pays are more complicated. Think of each winner as "displacing" the bidders below it. For example, if A3 hadn't bid, A4 would have taken the slot S3 instead of getting nothing. Thus A3's bid represents a loss of C3 clicks to A4, the displacement deprives A4 of clicks worth C3 * P4 = 100 * $.20 = $20. A3 pays $20, the "cost" of its displacement.

Likewise, if A2 hadn't bid, A4 would have taken S3 as before, but additionally A3 would have taken S2. Since being in S2 would result in (C2 - C3) = 200 more clicks than slot S3, A3 is deprived of clicks worth 200 * $.3 = $60. We have already calculated the cost of depriving A4 of S3 to be $20 above, so the amount A2 pays is $60 + $20 = $80.

Algebraically, you start to see a pattern:

A3 pays P4 * (C3 - 0)
= .2 * (100 - 0) + 1 * (0 - 0)
= .2 * 100
= 20

A2 pays P3 * (C2 - C3) + P4 * (C3 - 0)
= .3 * (300 - 100) + 2 * (100 - 0) + 1 * (0 - 0)
= .3 * 200 + .2 * 100
= 60 + 20
= 80

A1 pays P2 * (C1 - C2) + P3 * (C2 - C3) + P4 * (C3 - 0)
= .4 * (500 - 300) + .3 * (300 - 100) + .2 * (100 - 0)
= .4 * 200 + .3 * 200 + .2 * 100
= 80 + 60 + 20
= 160

A4 and A5's ads don't run, so they pay nothing.

Consider controlling the following parameters:

a. Number of bidders and the value per click for each bidder.
b. Closeness of bids (or pattern of bids). For example, (.5,.4,.3, .2, .1) or (.5, .45, .2, .1, .1)
c. Number of clicks expected for each slot (in decreasing order)
d. Number of advertising slots

Come up with five interesting parameter combinations.

Assume one person controls several of the bids. Can the person manipulate the system to get a better payoff? Explain. Allow the users to try various strategies for each parameter combination.

In the video, demonstrate what you learned.