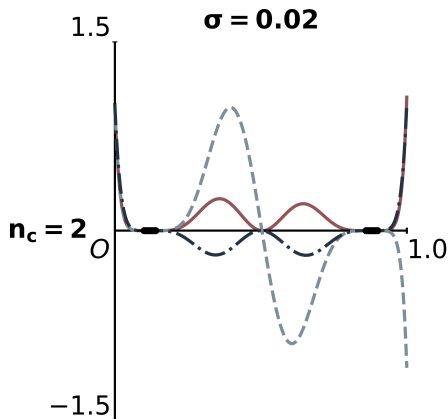# Sharpened CG Iteration Bound for High-contrast Heterogeneous Scalar Elliptic PDEs

Going Beyond Condition Number

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- Main R.Q.: "How can improve existing estimates on the total number of necessary CG iterations?"

Structure

- CG method: under the hood
- Classical iteration bound
- Influence of eigenvalue distribution
- High-contrast coefficients: split eigenspectrum
- Two-cluster bound
- Multi-cluster bound
- Partitioning
- Results on sharpness
- Practical estimation of new bounds