WI4680 Applications in Partial Differential Equations

Assignments

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Introduction

This report contains my treatment of the assignments. It will be updated as I progress through the course.

Assignment 1

Variable	Unit	Meaning	Value
θ	-	latitude	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
t	s	time	-
$x = \sin\theta$	-	latitude coordinate	$[-1, 1]$
T	K	temperature	-
R_A	$Js^{-1}m^{-2}$	effective solar radiation	-
Q	$Js^{-1}m^{-2}$	solar radiation	-
Q_0	$Js^{-1}m^{-2}$	solar radiation constant	341.3
α	-	albedo of Earth	-
α_1	-	albedo of ice	0.7
α_2	-	albedo of water	0.289
T^*K	-	temperature at which ice melts	273.15
M	K^{-1}	temperature gradient (?)	-
μ	$Js^{-1}m^{-2}$	greenhouse gas & fine particle parameter	30
R_E	$Js^{-1}m^{-2}$	black body radiation	-
ϵ_0	-	emmisivity of Earth	0.61
σ_0	$Js^{-1}m^{-2}K^{-4}$	Stefan-Boltzmann constant	$5.67 \cdot 10^{-8}$
R_D	$Js^{-1}m^{-2}$	heat dispersion	-
D	$Js^{-1}m^{-2}$	heat dispersion constant	0.3
δ	$Js^{-1}m^{-2}$	heat dispersion at poles	0
C_T	JK^{-1}	heat capacity of Earth	$5 \cdot 10^8$

Table 1: Variables and their meanings

1.1 Introduction

δ cannot be positive (resp. negative) at $x = \pm 1$ (poles), otherwise energy would be artificially entering (resp. leaving) the system. Simply said, the poles cannot be a source or sink of energy. This requires us to set $\delta = 0$ at $x = \pm 1$. Furthermore

$$\left.\frac{dT}{dx}\right|_{x=\pm 1} = 0.$$

However, we run into a problem when we combine the boundary conditions and set $\delta = 0$. The equation for the heat dispersion vanishes at the boundary and we are left with a zeroth order differential equation for which we simply cannot satisfy, one let alone two, boundary conditions.

Hence we must resort to a more basic requirement. Namely, that there exists an equilibrium temperature T_0 at the poles,

$$F(T(x,t))|_{x=\pm} = 0 \forall t > 0, \qquad \implies \left.\frac{dT}{dt}\right|_{x=\pm} = -(R_A - R_E).$$

1.2 Embedding

See the end of section 4.4 in the book on how to construct possible embeddings.

1.3 Conclusion

Assignment 2

2.1 Introduction

2.2 Conclusion

Assignment 3

3.1 Introduction

3.2 Conclusion