WI4680 Applications in Partial Differential Equations

# Assignments

Philip Soliman (4945255)

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**Supervisor:**
Dr. H.M. Schuttelaars
Delft University of Technology
Faculty of Electrical Engineering, Mathematics and
Computer Science

# Contents

## Introduction

This report contains my treatment of the assignments. It will be updated as I progress through the course.

# Assignment 1

| Variable | Unit | Meaning | Value |
|---|---|---|---|
| $\theta$ | - | latitude | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ |
| $t$ | $s$ | time | - |
| $x = \sin\theta$ | - | latitude coordinate | $[-1, 1]$ |
| $T$ | $K$ | temperature | - |
| $R_A$ | $Js^{-1}m^{-2}$ | effective solar radiation | - |
| $Q$ | $Js^{-1}m^{-2}$ | solar radiation | - |
| $Q_0$ | $Js^{-1}m^{-2}$ | solar radiation constant | 341.3 |
| $\alpha$ | - | albedo of Earth | - |
| $\alpha_1$ | - | albedo of ice | 0.7 |
| $\alpha_2$ | - | albedo of water | 0.278 |
| $T^*K$ | - | temperature at which ice melts | 273.15 |
| $M$ | $K^{-1}$ | temperature gradient (?) | - |
| $\mu$ | $Js^{-1}m^{-2}$ | greenhouse gas & fine particle parameter | 30 |
| $R_E$ | $Js^{-1}m^{-2}$ | black body radiation | - |
| $\epsilon_0$ | - | emmisivity of Earth | 0.61 |
| $\sigma_0$ | $Js^{-1}m^{-2}K^{-4}$ | Stefan-Boltzmann constant | $5.67 \cdot 10^{-8}$ |
| $R_D$ | $Js^{-1}m^{-2}$ | heat dispersion | - |
| $D$ | $Js^{-1}m^{-2}$ | heat dispersion constant | 0.3 |
| $\delta$ | $Js^{-1}m^{-2}$ | heat dispersion at poles | 0 |
| $C_T$ | $JK^{-1}$ | heat capacity of Earth | $5 \cdot 10^8$ |

**Table 1:** Variables and their meanings

## 1.1 Introduction

$\delta$ cannot be positive (resp. negative) at $x = \pm 1$ (poles), otherwise energy would be artificially entering (resp. leaving) the system. Simply said, the poles cannot be a source or sink of energy. This requires us to set $\delta = 0$ at $x = \pm 1$. Furthemore

$$\left.\frac{dT}{dx}\right|_{x=\pm 1} = 0.$$

However, we run into a problem when we combine the boundary conditions and set $\delta = 0$. The equation for the heat dispersion vanishes at the boundary and we are left with a zeroth order differential equation for which we simply cannot satisfy, one let alone two, boundary conditions.

The remedies for this are to either give $delta$ a small positive value, or to require that there is no dispersion of energy to and from the poles

$$R_D|_{x=\pm 1} = 0.$$

## 1.2 Conclusion

# Assignment 2

## 2.1 Introduction

## 2.2 Conclusion

# Assignment 3

## 3.1 Introduction

## 3.2 Conclusion