Target-Based Adaptation Using the Discontinuous Petrov-Galerkin Method

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Don't forget NSERC+McGill Funding

ABSTRACT

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ABRÉGÉ

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CHAPTER 1
Introduction

1.1 Motivation

The use of computational fluid dynamics (CFD) tools for the numerical analysis of fluid flows has significantly reduced costs associated with aerodynamic design over the past several decades. As computing systems have become increasingly powerful, there has been a corresponding advance in the equations employed for flow simulation (initially beginning with potential equations and now using the full Navier-Stokes equations) as well as in the resolution of complex flow phenomena.

Finite volume methods currently represent the industry and, to a great extent, academic standard for the solution of partial differential equations (PDEs) in the aerospace community. This is in large part due to their robustness in the presence of steep gradients in the flow as well as their ability to model geometrically complex objects as a result of the possibly unstructured nature of the volumes. However, finite volume schemes are inherently second order accurate, making their usage inefficient when flow solutions are smooth. In these cases, it is advantageous to use higher-order functions for the representation of the solution, resulting in solution convergence at greater rates than second order as well as necessitating fewer degrees of freedom (DOF) to obtain similar levels of solution resolution. One would thus like to employ spectral methods because of their exponential convergence properties, however these methods are unsuitable in the presence of complex geometry. Pseudo-spectral methods (which can be thought of as a spectral method being used within each control volume of a finite volume scheme) have become the most popular choice when attempting to address the concern of employing high-order accurate (greater than second order) solution representation in the presence of complex geometry.

The discontinuous Galerkin (DG) method, initially proposed by Reed et al. [1] and subsequently analyzed for the solution of systems of conservation laws [2, 3, 4, 5, 6], has become the most popular choice of high-order scheme in the CFD community. Despite its widespread usage, there are still several major issues related to the standard DG method:

- High computational complexity with increasing order of accuracy;
- Restrictive Courant-Friedrichs-Lewy (CFL) conditions for explicit time stepping and suboptimal dissipation and dispersion characteristics for wave propagation;
- Difficulty in generating meshes for complex geometrical objects as well as in converting low-order meshes provided by standard mesh generators to high-order meshes;
- Improper test space for best approximation in the energy norm when applied to hyperbolic PDEs.

The last of these issues provides the motivation for the thesis, however, many difficulties arising from the other listed problems have already been encountered simply while setting up the framework to begin the novel research. With particular regard to the representation of complex curved geometry, it has been observed that improper geometry treatment can contribute significant error to a flow simulation, potentially eliminating any advantages obtained from the use of better test spaces, for example.

1.1.1 Computational Complexity

Provide links to sections where related results are provided if available

Significant progress has been made with regard to the DG method's computational complexity, notably through the exploitation of sum factorization techniques, originally proposed by Orszag [7] and now employed in the tensor-product Spectral element method (SEM) of Kopriva [8] and for general elements using collapsed tensor-product spaces by Karniadakis et al. [9]. When explicit methods are used, use of the sum factorization technique reduces the growth in computational complexity from $O(N^{2d})$ to $O(N^{d+1})$ where d is the dimension of the problem and $N = P + 1$ where P is the order of the method. Savings for implicit schemes are even greater and further, the appropriate decomposition of element bases into corner, edge, facet, and volume modes allows for the ability to statically condense out the volume modes, significantly reducing the growth rate of globally coupled DOF as the order of the solution is increased in global matrix inversion stages.

Another technique pursued for computational complexity minimization is the use of collocated schemes. Collocated schemes, notably the nodal DG scheme [10, 11], minimize interpolation and integration costs by collocating solution and flux interpolation nodes with cubature nodes. While this culminates in satisfactory results for linear PDEs solved on straight-sided elements, the use of higher-order accurate cubature rules is advocated in curved elements or when solving nonlinear systems of equations, destroying the collocation property of the scheme. It has also been demonstrated that simplex element "alpha-optimized" nodes [12] associated with the nodal DG scheme, were unsuitable when solving nonlinear PDEs due to significant aliasing errors [13]. This in turn spurred research into the derivation of triangular and tetrahedral element nodes which possess good interpolation properties and have an associated cubature rule, recovering at least volume collocation; this has been pursued by several authors [13, 14, 15]. However, cubature rules associated with these nodes are weak in the sense that they can not even exactly integrate linear PDEs on linear elements when cast in the variational weak DG framework which might degrade their accuracy and high-order convergence properties; the conclusions of Bassi et al. [16] suggest that this occurs for tensor-product elements.

A final avenue which has been pursued for the minimization of computational cost associated with high-order methods is the use of pre-integrated DG schemes [17] or DG-type schemes cast in differential form, notably the Energy Stable Flux Reconstruction (ESFR) schemes introduced in §1.1.2. In these versions of the DG scheme, integral operations associated with the weak formulation of the DG scheme are performed in a preprocessing stage, eliminating many of the cubature operations associated with the original scheme. It is shown in link to scheme derivation section that the majority of operators can be pre-computed in this fashion for both weak and strong forms of DG-type schemes and that it would in fact be naive not to do so.

1.1.2 Restrictive CFL Condition and Suboptimal Dissipation and Dispersion Properties

As there is some level of subcell resolution in high-order schemes, the maximum CFL number decreases with the order of approximation of the solution on a fixed mesh. This results in restrictive time step sizes when solving problems explicitly. The recently introduced Flux Reconstruction (FR) schemes, initially proposed for tensor-product elements [18] and subsequently generalized to simplex elements [19], were originally presented as a unifying framework for DG-type schemes where stable time step limits greatly in excess of the standard DG scheme were achieved. However, restrictions on the type of flux reconstruction allowable while maintaining linear stability of the various schemes proposed by Huynh [18] were initially unclear. Appropriate choices for energy stability resulted in the derivation of the ESFR schemes, energy stable for linear advection on tensor-product elements [20] and for advection-diffusion on tensor-product elements [21]; the extension to simplex elements has also been made [22, 23, 24].

Within the ESFR framework, with different schemes parametrized simply by two parameters c and k (which can be associated with a scaling of the highest mode of the numerical flux correction), investigations were performed to find the values of c and k achieving the maximum stable time steps for linear advection-diffusion problems. These values were denoted as c_+ and k_+, allowing for time steps approximately twice as large as in the original DG formulation while maintaining the optimal order of convergence [21, 22, 23, 24]. A similar analysis relating to filtering of the numerical flux in the standard DG formulation also demonstrated that the stable time step could be increased while maintaining the optimal order of convergence [25]. More recent work has focused on optimizing within an ESFR framework to find optimal flux reconstruction operators for the minimization of dissipation and dispersion [26]. Interestingly, it was found that there existed an FR operator, not fitting within the ESFR framework, which had significantly improved performance as compared to the optimal ESFR operator for the minimization of dissipation and dispersion. While the authors were unable to conclude that this method was energy stable, the extension of the energy stability envelope may make this possible [27].

As the formulation of the ESFR schemes is extremely similar to that of the original DG scheme, it was natural that comparisons between the two would be made and it was eventually determined that ESFR schemes could be interpreted as modally filtered DG schemes. This was initially demonstrated in 1D and for straight-sided tensor product extensions [28] and subsequently generalized to straight-sided simplex elements [24]. A related investigation proving the equivalence of certain ESFR schemes with DG variants on both straight-sided and curved tensor-product elements was also performed [29, 30]. More recently, equivalence between all ESFR schemes and modally filtered DG schemes on curved tensor-product and simplex elements was demonstrated, with the important conclusion that ESFR schemes can be formulated as a weak DG scheme where discontinuous edge flux is substituted for numerical edge flux correction [31]. Given the theoretical justifications for the numerical flux and the significant computational effort expended to compute it, this interpretation puts the modification of its contribution to the residual into question, despite increasing the stable time step size. It was further demonstrated that ESFR schemes are inherently less efficient than DG schemes in weak form when solving problems implicitly.

1.1.3 Treatment of Complex Geometry

1.2 Thesis Overview

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1.3 Testing

$$
\begin{aligned}
&\frac{\partial}{\partial t}u(\boldsymbol{x}^c,t)+\nabla\cdot\boldsymbol{f}(u(\boldsymbol{x}^c,t))=0, t\geq 0, \boldsymbol{x}^c \coloneqq \begin{bmatrix} x & y & z \end{bmatrix} \in \boldsymbol{\Omega},\\
&u(\boldsymbol{x}^c,0)=u_0(\boldsymbol{x}^c), \boldsymbol{x}^c\in\boldsymbol{\Omega},
\end{aligned}
$$

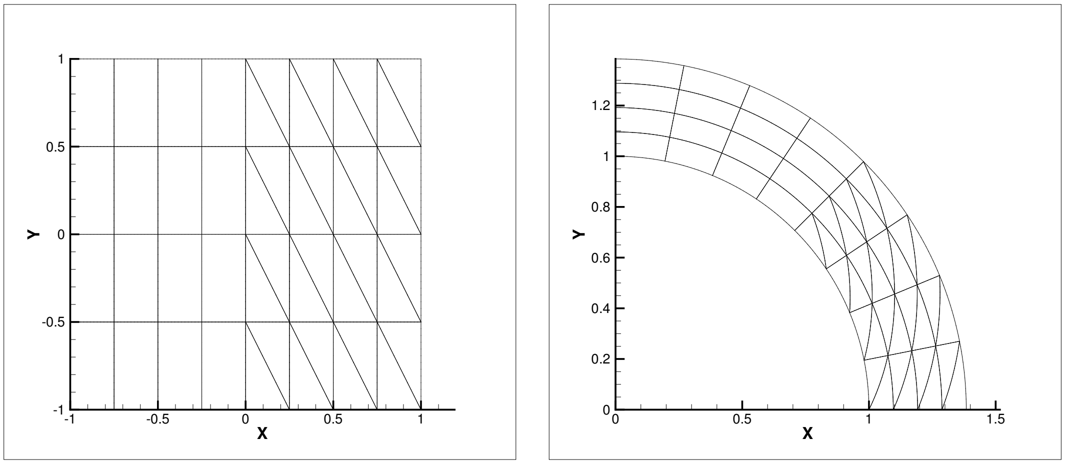
$$
\begin{aligned}
\int_{\boldsymbol{\Omega}_r}\boldsymbol{\chi}(\boldsymbol{\xi}^r)^T J_m^{\Omega}\boldsymbol{\chi}(\boldsymbol{\xi}^r)\frac{d}{dt}\hat{\boldsymbol{u}}_m(t)^T d\boldsymbol{\Omega}_r + \int_{\boldsymbol{\Omega}_r}\boldsymbol{\chi}(\boldsymbol{\xi}^r)^T\frac{\partial}{\partial\xi}\boldsymbol{\chi}(\boldsymbol{\xi}^r)\hat{\boldsymbol{f}}^r_{1m}(t)^T d\boldsymbol{\Omega}_r\\
+\int_{\boldsymbol{\Omega}_r}\boldsymbol{\chi}(\boldsymbol{\xi}^r)^T\frac{\partial}{\partial\eta}\boldsymbol{\chi}(\boldsymbol{\xi}^r)\hat{\boldsymbol{f}}^r_{2m}(t)^T d\boldsymbol{\Omega}_r + \int_{\boldsymbol{\Omega}_r}\boldsymbol{\chi}(\boldsymbol{\xi}^r)^T\frac{\partial}{\partial\zeta}\boldsymbol{\chi}(\boldsymbol{\xi}^r)\hat{\boldsymbol{f}}^r_{3m}(t)^T d\boldsymbol{\Omega}_r\\
+\int_{\boldsymbol{\Gamma}_r}\boldsymbol{\chi}(\boldsymbol{\xi}^r)^T J_m^{\Gamma}\hat{\boldsymbol{n}}_m\cdot\boldsymbol{f}_m^C d\boldsymbol{\Gamma}_r = \boldsymbol{0}^T,
\end{aligned}
\tag{1.3.1}
$$

Selecting suitable sets of surface and volume cubature nodes and using (1.3.1) is given discretely by [22, eq. (4.24)], [24, eq. (23)],

presented in §1.4 to numerically verify

		L_2 Error			Conv. Order		
Polynomial	Mesh Size	ρ	p	s	ρ	p	s
P1	7.91e-02	1.66e-01	2.42e-01	4.52e-02	-	-	-
	3.95e-02	4.93e-02	6.58e-02	1.58e-02	1.75	1.88	1.52
	1.98e-02	1.63e-02	2.08e-02	5.63e-03	1.60	1.66	1.49
	9.88e-03	5.56e-03	6.90e-03	2.00e-03	1.55	1.59	1.49
P2	1.09e-01	1.75e-03	1.78e-03	1.55e-03	-	-	-
	5.46e-02	2.20e-04	2.06e-04	2.38e-04	2.99	3.11	2.70
	2.73e-02	2.75e-05	2.47e-05	3.22e-05	3.00	3.06	2.89
	1.36e-02	3.43e-06	2.99e-06	4.15e-06	3.01	3.05	2.96
	6.82e-03	4.29e-07	3.69e-07	5.25e-07	3.00	3.02	2.98
P3	8.33e-02	6.68e-04	8.98e-04	6.25e-05	-	-	-
	4.17e-02	7.09e-05	9.54e-05	4.82e-06	3.24	3.24	3.70
	2.08e-02	7.25e-06	9.89e-06	4.05e-07	3.29	3.27	3.57
	1.04e-02	7.64e-07	1.06e-06	2.76e-08	3.24	3.22	3.87
	5.21e-03	7.23e-08	1.03e-07	1.76e-09	3.40	3.36	3.97
P4	6.74e-02	6.33e-06	7.24e-06	4.53e-06	-	-	-
	3.37e-02	2.58e-07	2.68e-07	2.39e-07	4.62	4.76	4.24
	1.69e-02	8.27e-09	7.60e-09	8.92e-09	4.96	5.14	4.74
	8.43e-03	2.56e-10	2.22e-10	2.91e-10	5.01	5.10	4.94

Table 1–1: Errors and Convergence Orders - $PI_c = 2P$ (GL/Cools), $PI_{cf} = 2P$ (GL), $PG = P$, EFE = 1



(a) Mesh before Mapping (b) Mapped Mesh

Figure 1–2: Supersonic Vortex Structured Mixed Mesh

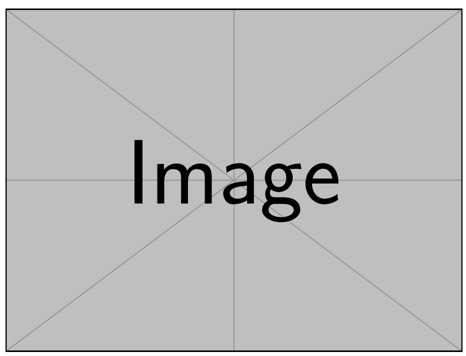


Figure 1–1: figure caption

- setspace (2000/12/01)
- ulem (2000/05/26)

  You can use `\uline{text}` to underline text. The documentation of the ulem package provides more information.
- sectsty (1999/04/12)

  Since underlining in section headings is rather nontrivial, the sectsty package was used to manipulate the formatting of sectional headings.

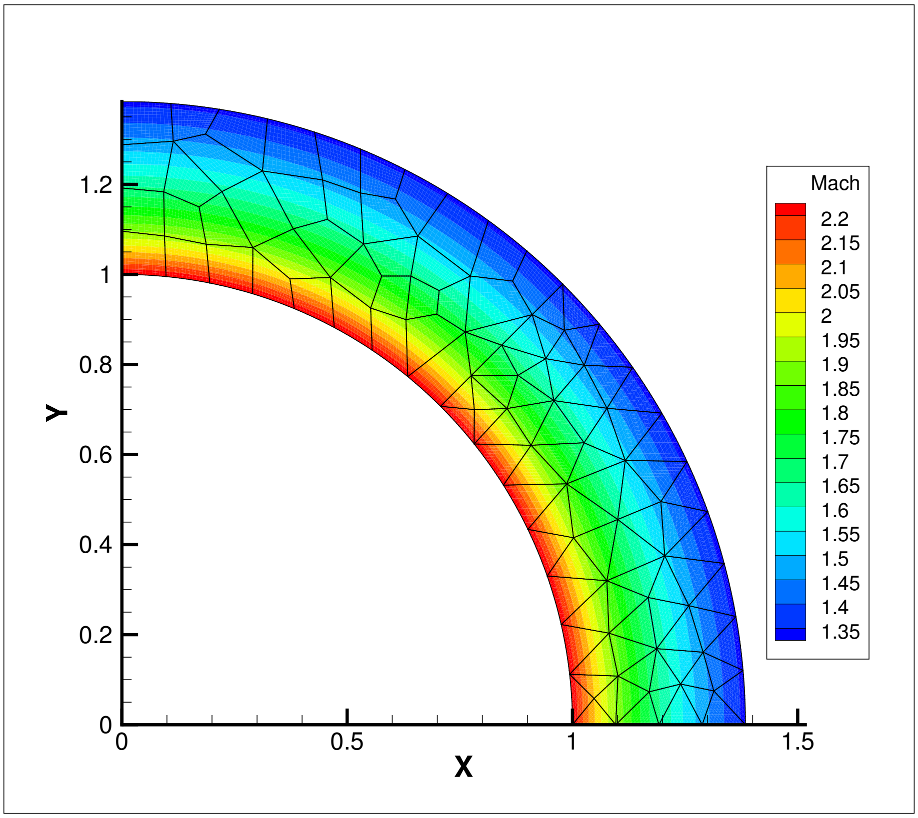


Figure 1–3: Mach Number Distribution: $P = 4$, $PG = 4$, $PF = 5$, $PFr = 8$, $PIs = 8$, $PIc = 11$, $PIsf = 8$, $PIcf = 12$, $EFE = 1$

- ragged2e (1999/06/08)

  This package adds support to handle ragged right justification appropriately.

- everysel (1999/06/08)

  Required by the ragged2e package.

1.4 General Guidelines

- Please use commands provided by mcgilletdclass, otherwise your thesis may not be able to pass the validation check.

- If you run into problems using this stylesheet, please contact us as soon as possible.
- Files you must submit: *.tex, *.dvi and *.bib files.

CHAPTER 2
Usage

An example of how to use the mcgilletdclass document class is provided in mcgilletd-sample.tex. Details of how to compile the example are provided in a later section. Use this class file the same way as the report class, by putting `\documentclass{mcgilletdclass}` at the beginning of the $\LaTeX\,2_\varepsilon$ file. There are only a few options available with this document class, which will be described in the following sections. Due to the requirements of the McGill University, this document class does not support **two-sided printing or two-column page layout** .

CHAPTER 3
The document class options

3.1 Font Size Options: 10pt, 11pt or 12pt

These are the font sizes that are available in the standard LaTeX 2_εreport document class. The 12pt font size is the preferred option for the final typeset version of the document. Obviously, only one of these options should be used in the `\documentclass` command. The 12pt option is selected by default if none of these options are specified in the `\documentclass` command.

CHAPTER 4
Title Page

The document preamble is the part that occurs before the `\begin{document}` statement. Throughout the document, information about the author, the title, etc., is **required** . The following commands are used in the document preamble to define all of the required text strings that are used to personalize the document.

- `\SetTitle{text}` :This is the title as it appears on the title page. The title must be entered using all upper-case letters. Line-breaks can be entered in the title by using the `\\` command.
- `\SetAuthor{text}` :The author's name, using capital letters where appropriate.
- `\SetDegreeType{text}` :Something like "Master of Science" or "Doctor of Philosophy".
- `\SetDepartment{text}` :The department or faculty
- `\SetUniversity{text}` :McGill University
- `\SetUniversityAddr{text}` :Montreal, Quebec
- `\SetThesisDate{text}` :Date of your thesis
- `\SetRequirements{text}` :Text of requirements statements
- `\SetCopyright{text}` :Copyright Statements

CHAPTER 5
The Thesis FrontSection

In this section, we present the commands used to add all pages prior to the first chapter of the thesis. Some of these commands are required while some of them are optional.

5.1 Title page(required)

The command `\maketitle` formats the title page.

5.2 Dedication

The optional dedication can be inserted by using the command `\SetDedicationName{text},\SetDedic` and `\Dedication`, where text is the contents of the dedication.

5.3 Acknowledgments

In order to format the optional acknowledgments, use the command `\SetAcknowledgeName{text},\Set` and `\Acknowledge{text}`. The title "ACKNOWLEDGMENTS" is automatically added to the table of contents.

5.4 Table of Contents, List of Tables and Figures(required)

These lists are generated with the commands `\tableofcontents`, `\listoftables and \listoffigure` The titles of these lists may be changed by renewing the `\TOCHeading{text}, \LOTHeading{text}`, and the `\LOFHeadling{text}` macros, respectively.

5.5 Abstract(required)

This command, which actually is an environment, sets up the required text for the abstract. The proper use is : `\SetAbstractEnName{text}` for the heading of the abstract and `\SetAbstractEnText{text}` for the text of the abstract. The command `\AbstractEn` is used to incorporate the heading of the abstract and the abstract text into your document. `\SetAbstractFrName{text},\SetAbstracFrText{text} and \AbstractFr` are the equivalent commands for the French abstract, and `\SetAbstractOtName{text}`, `\SetAbstracOtText{text}` and `\AbstractOt` are for an abstract in another language. And abstract in English and French are required by the University.

CHAPTER 6
The Thesis "Mainmatter"

In this section, we describe the commands used to format the main chapters of the thesis. Please refer to the sample thesis file mcgilletdsample.tex for examples of how these commands are used.

6.1 Chapter Titles

Use the `\chapter` command to start a new chapter in the document. If the boolean SetDSpace is true, then the chapter will be typeset using double-spacing. In general, the class file sets the SetDSpace boolean appropriately so that the selection of singlespacing or double-spacing is transparent to the user. Note that the chapter title may extend over more than one line by using the following form: `\chapter`[Title Line 1 `\protect\newline` Title Line 2]Title Line 1 \\ Title Line 2

6.2 Section Headings

Use the `\section` command for first-level subheadings. Use the `\subsection` command for second-level subheadings. Since it may be difficult to distinguish third-level subheadings from second-level subheadings, it is suggested that the `\paragraph` command be used for all third-level subheadings. Note that the text of the heading may extend over more than one line. In this case, use the same command as shown for the `\chapter` command above.

6.3 Tables and Figures

Use the `\begin{table} and \end{table}` to contain a new table.

Use the `\begin{figure} and \end{figure}` to include a new figure. Captions must be placed above tables and below figures. `\TableCaption{text}` is for tables and `\FigureCaption{text}` is for figures. Please note that captions are required for both figures and tables, and must be supplied on a strictly one-to-one basis.

`\TableCaptionOpt{text1}{text2}` is another command for the similar use. Here, text1 will appear in the list of tables; text2 is the actual caption of a table. Similarly, `\FigureCaptionOpt{text1}{text2}` is for figures.

CHAPTER 7
The Thesis ”Backmatter”

The following commands are used to control the appearance and content of appendices and other thesis end-matter. You might have to tailor some of these commands to suit your requirements. Use the `\ETDAppendix{Appendix}{text}` and `\ETDAppendix{Appendix}{text}` commands to add an appendix; you may have more than one appendix if necessary.

7.1 List of References

We only support the BibTex tool. Use the `\bibHeading{text}` command to set the heading of your references. Use the `\bibliography{bibtexdatabase}` to incorporate your BibTex list. Use the `\bibliographystyle{text}` to set the citation style of your bibliography.

Here is a simple example to demonstrate the use of BibTex.

The following text and latex commands (in conjunction with the mgilletd.bib file included in the mcgilletd package):

produce:

The references themselves are inserted in the document at the point of the `\bibliography{mcgilletd}` command (see the end of this document).

CHAPTER 8
Special Commands

The commands described in this section are used to aid in the commenting of the source file, and also describe support for special formatting. These commands are optional, in that they are not required in order to format the document.

8.1 Automatically Generated Indices

Several commands have been included in this document class file to facilitate the creation of automatically generated lists with the MakeIndex program. Use this command in the document preamble to enable the creation of an index file. If only one index is to be created, then just use the

command. However, if there is to be more than one index, or it is desirable to name the index file something besides the default, use the following command

A unique filename for each index must be assigned, so a command should exist for each index, in the document preamble. Issue this command at the point in the document where the index is to be typeset. Typically, this will be after the lists of figures and tables (if included). The command creates a new chapter and sets up the initial page formatting for the list to be generated. The form of the command directly corresponds to the form of the command used in the document. This is the command that is used to actually define what is going to be added to the index list. As with the other indexing commands there are two forms that can be used, depending on how the command was issued. For generating only one index file, use the command shown below If there are more than one index files, or if a custom filename was used for the index, then the following form of the command must be used: Of course, for each index file that is created, a run of the MakeIndex program will be required.

8.2 Changing the Code . . .

It might be necessary to modify some of the commands defined in the documentclass. However, you do NOT have to (and SHOULD NOT!) edit the mcgilletdclass.cls file directly. Instead, place any commands you wish to create or modify in a file called mcgilletdclass.cfg and put it in the same directory as your main thesis file. The mcgilletdclass documentclass will automatically load this file. Please note that required commands must not be changed if your thesis is to be correctly converted to XML for the McGill EThesis Project.

8.3 An Example Thesis File

In order to typeset the example file, the following files should exist: mcgilletdclass.cls, mcgilletdsample.tex, mcgilletd.bib, chick.eps, and the additional packages described at the beginning of this document. (See the documentation for your local installation of LaTeX 2ε for information on how to install local customizations.)

Use the following commands to typeset the mcgilletdsample file:

```
latex mcgilletdsample

bibtex mcgilletdsample

latex mcgilletdsample

latex mcgilletdsample
```

8.4 Special Cases

An objective of the McGill Electronic Thesis Project is to create a means to ensure the long-term preservation and long-term accessibility of theses created at McGill University. The submitted theses will be archived as XML, which is a subset of SGML, a markup language for the interchange of structured data. The basis for SGML/XML is the description of the contents, not the layout. Therefore, the mcgilletdclass documentclass adds sematic support to text layout commands. Some of them are described as below:

- Emphasis ProperName
  - i `\BProperName` is to set text to boldface; the text will be treated as a proper name.
  - ii `\BIProperName` is to set text to boldface and italic; the text will be treated as a proper name.
  - iii `\BIUProperName` is to set text to boldface, italic and underline; the text will be treated as a proper name.
  - iv `\BUProperName` is to set text to boldface and underline; the text will be treated as a proper name.
  - v `\IUProperName` is to set text to italic and underline; the text will be treated as a proper name.
  - vi `\UProperName` is to set text to underline; the text will be treated as a proper name.
  - vii `\IProperName` is to set text to italic; the text will be treated as a proper name.

- Emphasis Technical Term

  i `\BTechnicalTerm` is to set text to boldface; the text will be treated as a technical term.

  ii `\BITechnicalTerm` is to set text to boldface and italic; the text will be treated as a technical term.

  iii `\BIUTechnicalTerm` is to set text to boldface, italic and underline; the text will be a treated as technical term.

  iv `\BUTechnicalTerm` is to set text to boldface and underline; the text will be treated as a technical term.

  v `\IUTechnicalTerm` is to set text to italic and underline; the text will be treated as a technical term.

  vi `\UTechnicalTerm` is to set text to underline; the text will be treated as a technical term.

  vii `\ITechnicalTerm` is to set text to italic; the text will be treated as a technical term.

- Emphasis Title

  i `\BTitle` is to set text to boldface; the text will be treated as a title.

  ii `\BITitle` is to set text to boldface and italic; the text will be treated as a title.

  iii `\BIUTitle` is to set text to boldface, italic and underline; the text will be a treated as title.

  iv `\BUTitle` is to set text to boldface and underline; the text will be treated as a title.

v `\IUTitle` is to set text to italic and underline; the text will be treated as a title.

vi `\UTitle` is to set text to underline; the text will be treated as a title.

vii `\ITitle` is to set text to italic; the text will be treated as a title.

- Emphasis ForeignWord

  i `\BForeignWord` is to set text to boldface; the text will be treated as a Foreign Word.

  ii `\BIForeignWord` is to set text to boldface and italic; the text will be treated as a Foreign Word.

  iii `\BIUForeignWord` is to set text to boldface, italic and underline; the text will be a treated as Foreign Word.

  iv `\BUForeignWord` is to set text to boldface and underline; the text will be treated as a Foreign Word.

  v `\IUForeignWord` is to set text to italic and underline; the text will be treated as a Foreign Word.

  vi `\UForeignWord` is to set text to underline; the text will be treated as a Foreign Word.

  vii `\IForeignWord` is to set text to italic; the text will be treated as a Foreign Word.

- Visual Emphasis

  i `\BVisualEmphasis` is to set text to boldface; the text will be treated as Visual Emphasis.

ii `\BIVisualEmphasis` is to set text to boldface and italic; the text will be treated as Visual Emphasis.

iii `\BIUVisualEmphasis` is to set text to boldface, italic and underline; the text will be treated as Visual Emphasis.

iv `\BUVisualEmphasis` is to set text to boldface and underline; the text will be treated as Visual Emphasis.

v `\IUVisualEmphasis` is to set text to italic and underline; the text will be treated as Visual Emphasis.

vi `\UVisualEmphasis` is to set text to underline; the text will be treated as Visual Emphasis.

vii `\IVisualEmphasis` is to set text to italic; the text will be treated as Visual Emphasis.

- Quotation

  i Use `\SingleInlineQuote` command to do ‘inline quote’.

  ii Use `\DoubleInlineQuote` command to do “inline quote”.

  iii Use `\begin{quote}` and `\end{quote}` for block quote

- Verse

  i Use `\verse` command for verse.

- Lists

  i Use `\begin{BulletList}` and `\end{BulletList}` commands to create bulleted lists.

  ii Use `\begin{RomanList}` and `\end{RomanList}` commands to create upper case Roman numbered lists.

iii Use `\begin{romanList}` and `\end{romanList}` commands to create lower case roman numbered lists.

iv Use `\begin{AlphList}` and `\end{AlphList}` commands to create upper case Alphabetical lists.

v Use `\begin{alphList}` and `\end{alphList}` commands to create lower case alphabetical lists.

vi Use `\begin{arabicList}` and `\end{arabicList}` commands to create arabic numbered lists.

- Formula

i Use `\begin{MathFormula}` and `\end{MathFormula}` commands to embed normal math formula as per the following example:

```
\begin{MathFormula}
\begin{math}
...
\end{math}
\end{MathFormula}
```

Example:

$$\bar{u} = \int_I^E g(x)dx \tag{8.4.1}$$

ii Use `\begin{ChemFormula}` and `\end{ChemFormula}` commands for chemical formulae, similar to the above example.

```
\begin{ChemFormula}
...
\end{ChemFormula}
```

8.5 Testing figures and tables.

Table 8–1: The First Table

First	Second	Third	Fourth
12	26	12	33
17	93	88	3
12	26	12	33
12	26	12	33
12	26	12	33
17	93	88	3
12	26	12	33
12	26	12	33
12	26	12	33
17	93	88	3
12	26	12	33
12	26	12	33

CHAPTER 9
TESTING FOOTNOTES

We will now work on testing footnotes.[1] And now let's add some spurious text so that we can have a second[2] footnote.

[1] This is the first footnote.

[2] This is the second footnote. To test if the footnote is typeset single-spaced, we add meaningless text until we can see a second line.

Appendix A

Here is the text of an Appendix.

Appendix B

Here is the text of a second, additional Appendix

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