Target-Based Adaptation using the Discontinuous Petrov-Galerkin Method

by

Philip Zwanenburg
Bachelor of Engineering, McGill University (2014)

A thesis submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering

at

MCGILL UNIVERSITY

September 2018

Author . . . .
Department of Mechanical Engineering
September 1, 2018

Certified by . . . .
Siva Nadarajah
Associate Professor
Thesis Supervisor

Certified by . . . .
Mathias Legrand
Associate Professor
Thesis Supervisor

Certified by . . . .
Jean-Christophe Nave
Associate Professor
Thesis Supervisor

Accepted by . . . .
NAME
CHAIRMAN, DEPARTMENT COMMITTEE ON GRADUATE THESES

Dedication

This thesis is dedicated to those who have fuelled my interest in numerical analysis through their genious, creativity and passion. Include best graphic or logo

Acknowledgments

ToBeDone

Don't forget NSERC+McGill Funding

Abstract

ToBeDone

Abrégé

ToBeDone

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 ToBeModified

Chapter 2

Methodology

2.1 Governing Equations

Following the notation of Pletcher et al. [1, Chapter 5], the continuity, Navier-Stokes and energy equations with source terms neglected are given by

$$\frac{\partial \boldsymbol{W}}{\partial t} + \boldsymbol{\nabla} \cdot \left(\boldsymbol{F^i}(\boldsymbol{W}) - \boldsymbol{F^v}(\boldsymbol{W}, \boldsymbol{Q}) \right) = \boldsymbol{0}, \tag{2.1}$$

where the vector of conservative variables is given by

$$\boldsymbol{W} \coloneqq \begin{bmatrix} \rho & \rho \boldsymbol{v} & E \end{bmatrix},$$

and where the inviscid and viscous fluxes are defined as

$$\boldsymbol{F^i}(\boldsymbol{W}) \coloneqq \begin{bmatrix} \rho \boldsymbol{v}^T & \rho \boldsymbol{v}^T \boldsymbol{v} + p \underline{\underline{\boldsymbol{I}}} & (E+p)\boldsymbol{v}^T \end{bmatrix}, \tag{2.2}$$

$$\boldsymbol{F^v}(\boldsymbol{W}, \boldsymbol{Q}) \coloneqq \begin{bmatrix} \boldsymbol{0}^T & \underline{\underline{\boldsymbol{\Pi}}} & \underline{\underline{\boldsymbol{\Pi}}} \boldsymbol{v}^T - \boldsymbol{q}^T \end{bmatrix}. \tag{2.3}$$

The various symbols represent the density, ρ, the velocity vector, $\boldsymbol{v}$, the total energy per unit volume, E, the pressure, p, defined according to the equation of state for a calorically ideal gas,

$$p = (\gamma - 1)\left(E - \frac{1}{2}\rho \boldsymbol{v}\boldsymbol{v}^T \right), \ \gamma = \frac{c_p}{c_v},$$

where the specific heats at constant volume, c_v, and at constant pressure, c_p, are constant, the stress tensor, $\underline{\underline{\mathbf{\Pi}}}$, given by

$$\underline{\underline{\mathbf{\Pi}}} = 2\mu \left(\underline{\underline{\boldsymbol{D}}} - \frac{1}{3} \boldsymbol{\nabla} \cdot \boldsymbol{v} \underline{\underline{\boldsymbol{I}}} \right), \; \underline{\underline{\boldsymbol{D}}} \coloneqq \frac{1}{2} \left(\boldsymbol{\nabla}^T \boldsymbol{v} + \left(\boldsymbol{\nabla}^T \boldsymbol{v} \right)^T \right),$$

where μ is the coefficient of shear viscosity and where the coefficient of bulk viscosity was assumed to be zero, and the energy flux vector, $\boldsymbol{q}$, defined by

$$\boldsymbol{q} = \kappa \boldsymbol{\nabla} T,$$

where T represents the temperature and

$$\kappa = \frac{c_p \mu}{Pr},$$

with Pr representing the Prandtl number. In the case of the Euler equations, the contribution of the viscous flux is neglected.

2.2 Boundary Conditions

Bibliography

[1] R. Pletcher, J. Tannehill, D. Anderson, Computational Fluid Mechanics and Heat Transfer, Second Edition, Series in Computational and Physical Processes in Mechanics and Thermal Sciences, Taylor & Francis, 1997.
URL `https://books.google.ca/books?id=ZJPbtHeilCgC`