Optimal Test Functions for Linear Advection

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1 Face Test Functions

The linear test functions on either side of the 1D face (point) take the exact form:

$$v^l_{\hat{\phi}_i} = a^l_0 + a^l_1 r,$$
$$v^r_{\hat{\phi}_i} = a^r_0 + a^r_1 r.$$

The coefficients can be computed by solving the following linear system

$$\mathbf{A} = \begin{pmatrix} a-c & a-c & c & -c \\ a-c & a-c+\frac{4}{h} & c & -c \\ c & c & a-c & a-c \\ -c & -c & a-c & a-c+\frac{4}{h} \end{pmatrix}.$$

$$\mathbf{coef} = \begin{pmatrix} \frac{\left(ac-c^2\right)h+a}{a^2-2\,ac-(ac^2-c^3)h} \\ 0 \\ -\frac{\left(a^2-ac\right)h+2\,a}{2\,(a^2-2\,ac-(ac^2-c^3)h)} \\ \frac{\left(a^2-ac\right)h}{2\,(a^2-2\,ac-(ac^2-c^3)h)} \end{pmatrix}.$$