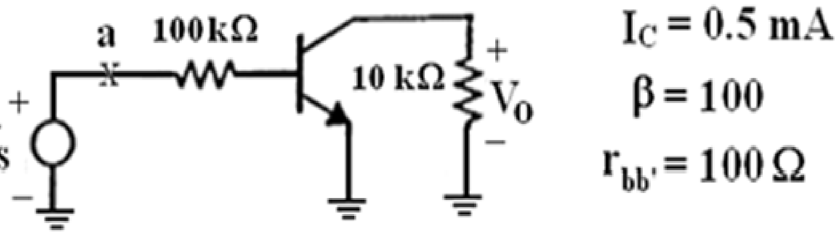
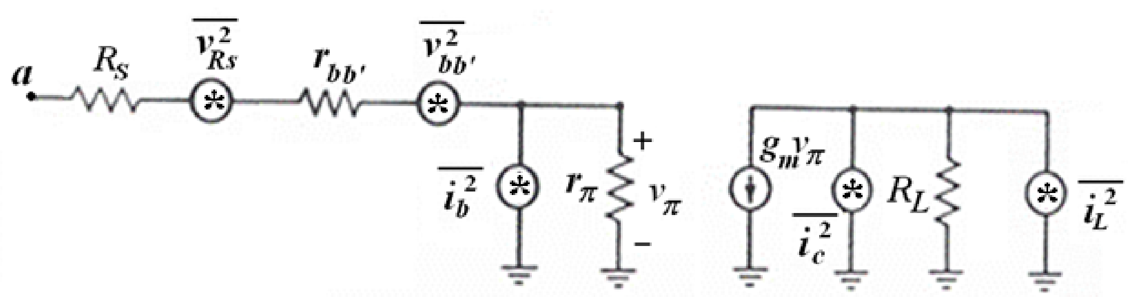
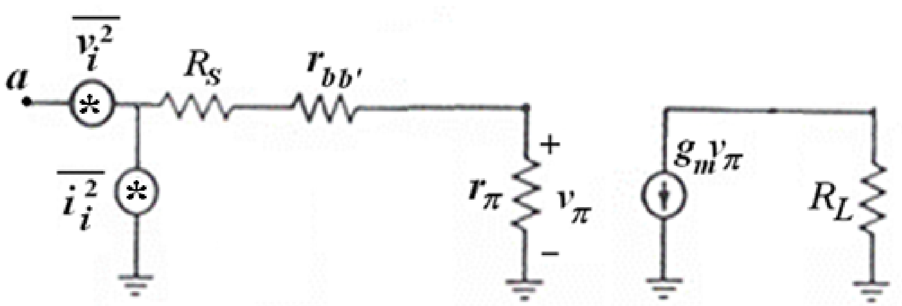
EE4341 TUTORIAL 3 SOLUTION

1. (i)







$$r_\pi = \frac{V_T}{I_B} = \frac{26\text{ mV}}{5\ \mu\text{A}} = 5.2\text{ k}\Omega$$

Short-circuit input at point "a":

$$g_m v_i\left(\frac{r_\pi}{R_s + r_\pi + r_{bb'}}\right) = g_m\left(v_{Rs} + v_{bb'}\right)\left(\frac{r_\pi}{R_s + r_\pi + r_{bb'}}\right) + g_m i_b\left(\frac{\left(R_s + r_{bb'}\right)r_\pi}{R_s + r_\pi + r_{bb'}}\right) + i_c + i_L$$

$$\because R_s \ \ \& \, r_\pi >> r_{bb'}$$

$$\therefore g_m v_i\left(\frac{r_\pi}{R_s + r_\pi}\right) = g_m\left(v_{Rs} + v_{bb'}\right)\left(\frac{r_\pi}{R_s + r_\pi}\right) + g_m i_b\left(\frac{R_s r_\pi}{R_s + r_\pi}\right) + i_c + i_L$$

$$v_i = v_{Rs} + v_{bb'} + i_b R_s + \left(i_c + i_L\right)\left(\frac{R_s + r_\pi}{g_m r_\pi}\right)$$

$$\overline{v_i^{\,2}} = \overline{v_{Rs}^{\,2}} + \overline{v_{bb'}^{\,2}} + \overline{i_b^{\,2}} R_s^{\,2} + \left(\overline{i_c^{\,2}} + \overline{i_L^{\,2}}\right)\left(\frac{R_s + r_\pi}{g_m r_\pi}\right)^2$$

$$= 4kT\left(R_s + r_{bb'}\right) + 2qI_B R_s^{\,2} + \left(2qI_C + \frac{4kT}{R_L}\right)\left(\frac{R_s + r_\pi}{g_m r_\pi}\right)^2$$

$$= 1.66\times10^{-15} + 1.60\times10^{-14} + 1.79\times10^{-16}$$

$$= 1.78\times10^{-14} \ \ \mathrm{V}^2/\mathrm{Hz}$$

Open-circuit input at point “a”:

$$\beta i_i = \beta i_b + i_c + i_L$$

$$i_i = i_b + \frac{i_c + i_L}{\beta}$$

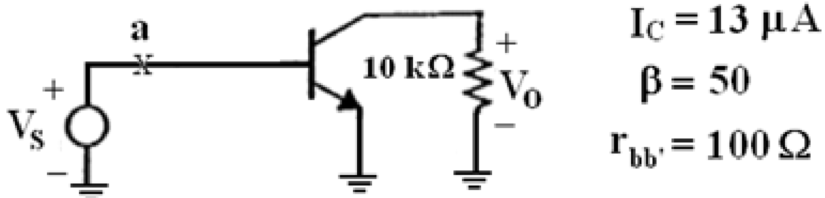
$$\overline{i_i^{\,2}} = \overline{i_b^{\,2}} + \frac{1}{\beta^2}\left(\overline{i_c^{\,2}} + \overline{i_L^{\,2}}\right)$$

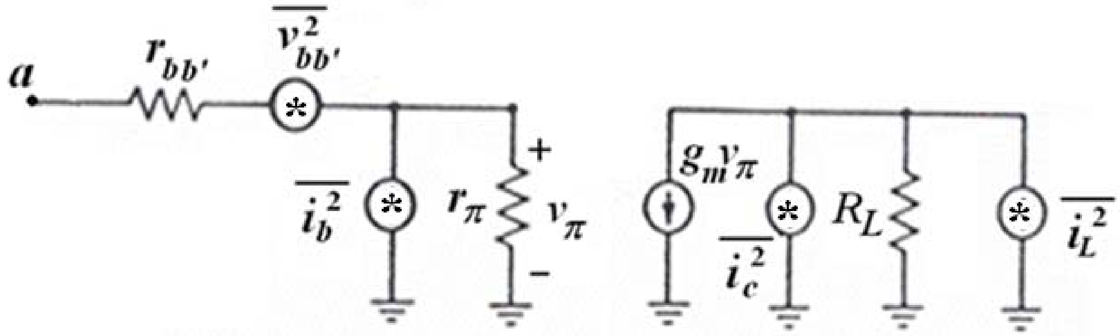
$$= 2qI_B + \frac{1}{\beta^2}\left(2qI_C + \frac{4kT}{R_L}\right)$$

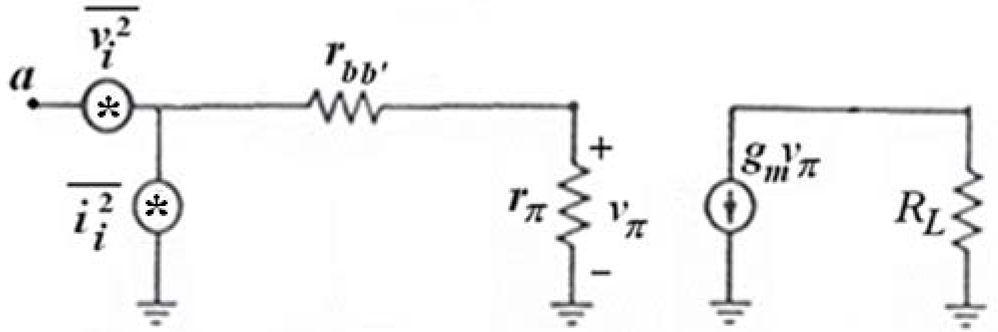
$$= 1.60\times10^{-24} + 1.62\times10^{-26}$$

$$= 1.62\times10^{-24} \ \ \mathrm{A}^2/\mathrm{Hz}$$

1. (ii)







$$r_\pi = \frac{V_T}{I_B} = \frac{26\ \text{mV}}{0.26\ \mu\text{A}} = 100\ \text{k}\Omega$$

Short-circuit input at point "a":

$$g_m v_i \left(\frac{r_\pi}{r_\pi + r_{bb'}} \right) = g_m v_{bb'} \left(\frac{r_\pi}{r_\pi + r_{bb'}} \right) + g_m i_b \left(\frac{r_{bb'} r_\pi}{r_\pi + r_{bb'}} \right) + i_c + i_L$$

$$\because r_\pi >> r_{bb'}$$

$$\therefore g_m v_i = g_m v_{bb'} + g_m i_b r_{bb'} + i_c + i_L$$

$$v_i = v_{bb'} + i_b r_{bb'} + \frac{i_c + i_L}{g_m}$$

$$\overline{v_i^{\,2}} = \overline{v_{bb'}^{\,2}} + \overline{i_b^{\,2}}\, r_{bb'}^{\,2} + \left(\overline{i_c^{\,2}} + \overline{i_L^{\,2}}\right)\left(\frac{1}{g_m^{\,2}}\right)$$

$$= 4kTr_{bb'} + 2qI_B r_{bb'}^{\,2} + \left(2qI_C + \frac{4kT}{R_L}\right)\left(\frac{1}{g_m^{\,2}}\right)$$

$$= 1.656\times10^{-18} + 8.32\times10^{-22} + 2.33\times10^{-17}$$

$$= 2.50\times10^{-17}\ \ \mathrm{V}^2/\mathrm{Hz}$$

Open-circuit input at point “a”:

$$\beta i_i = \beta i_b + i_c + i_L$$

$$i_i = i_b + \frac{i_c + i_L}{\beta}$$

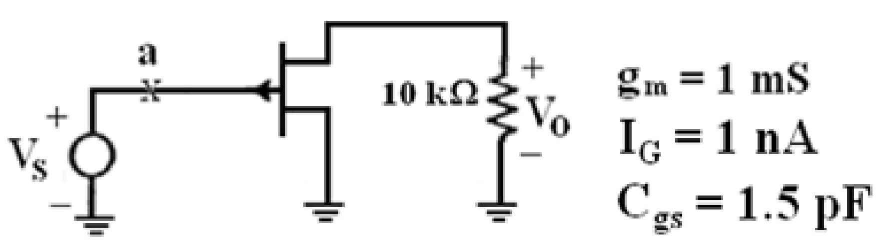
$$\overline{i_i^{\,2}} = \overline{i_b^{\,2}} + \frac{1}{\beta^2}\left(\overline{i_c^{\,2}} + \overline{i_L^{\,2}}\right)$$

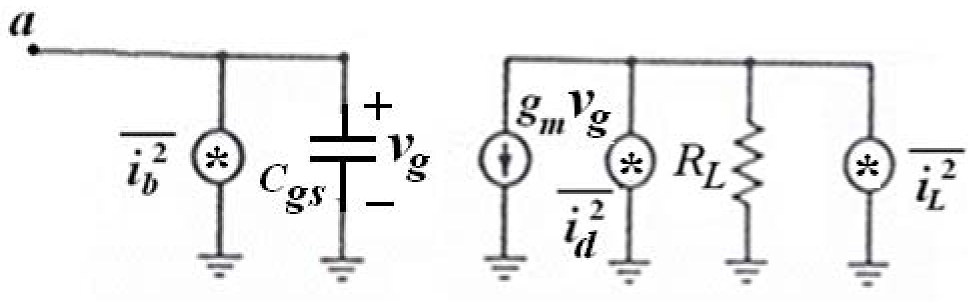
$$= 2qI_B + \frac{1}{\beta^2}\left(2qI_C + \frac{4kT}{R_L}\right)$$

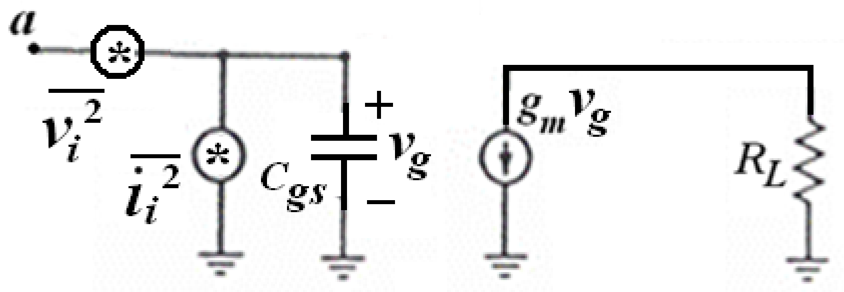
$$= 8.32\times10^{-26} + 2.33\times10^{-27}$$

$$= 8.55\times10^{-26}\ \ \mathrm{A}^2/\mathrm{Hz}$$

1. (iii)







Short-circuit input at point “a”:

$$g_m v_i = i_d + i_L$$

$$v_i = \frac{1}{g_m}(i_d + i_L)$$

$$\begin{aligned}
\overline{v_i^2} &= \left(\frac{1}{g_m^2}\right)\left(\overline{i_d^2} + \overline{i_L^2}\right) \\
&= \left(\frac{1}{g_m^2}\right)\left(4kT\left(\frac{2g_m}{3}\right) + \frac{4kT}{R_L}\right) \\
&= 4kT\left(\frac{2}{3g_m}\right) + \frac{4kT}{g_m^2 R_L} \\
&= 1.1\times10^{-17} + 1.656\times10^{-18} \\
&= 1.27\times10^{-17} \ \ \mathrm{V^2/Hz}
\end{aligned}$$

Open-circuit input at point "a":

$$g_m i_i\left(\frac{1}{j\omega C_{gs}}\right)=g_m i_g\left(\frac{1}{j\omega C_{gs}}\right)+i_d+i_L$$

$$i_i=i_g+\left(i_d+i_L\right)\left(\frac{j\omega C_{gs}}{g_m}\right)$$

$$\overline{i_i^{\,2}}=\overline{i_g^{\,2}}+\left(\overline{i_d^{\,2}}+\overline{i_L^{\,2}}\right)\left|\frac{j\omega C_{gs}}{g_m}\right|^2$$

$$=\overline{i_g^{\,2}}+\left(\overline{i_d^{\,2}}+\overline{i_L^{\,2}}\right)\left(\frac{\omega C_{gs}}{g_m}\right)^2$$

$$=2qI_G+\left(4kT\left(\frac{2g_m}{3}\right)+\frac{4kT}{R_L}\right)\left(\frac{\omega C_{gs}}{g_m}\right)^2$$

$$=3.2\times10^{-28}+1.13\times10^{-39}f^2\ \ \mathrm{A}^2/\mathrm{Hz}$$

Note: the term associated with f^2 equal to the first term when f = 532 kHz.

Case	$\overline{v_i^{\,2}}$	$\overline{i_i^{\,2}}$
(i)	$1.78\times10^{-14}\ \ \mathrm{V}^2/\mathrm{Hz}$	$1.62\times10^{-24}\ \ \mathrm{A}^2/\mathrm{Hz}$
(ii)	$2.50\times10^{-17}\ \ \mathrm{V}^2/\mathrm{Hz}$	$8.55\times10^{-26}\ \ \mathrm{A}^2/\mathrm{Hz}$
(iii)	$1.27\times10^{-17}\ \ \mathrm{V}^2/\mathrm{Hz}$	$3.2\times10^{-28}+1.13\times10^{-39}f^2\ \ \mathrm{A}^2/\mathrm{Hz}$

For a signal source with very low source resistance, the effect of $\overline{v_i^{\,2}}$ dominates that of $\overline{i_i^{\,2}}$.

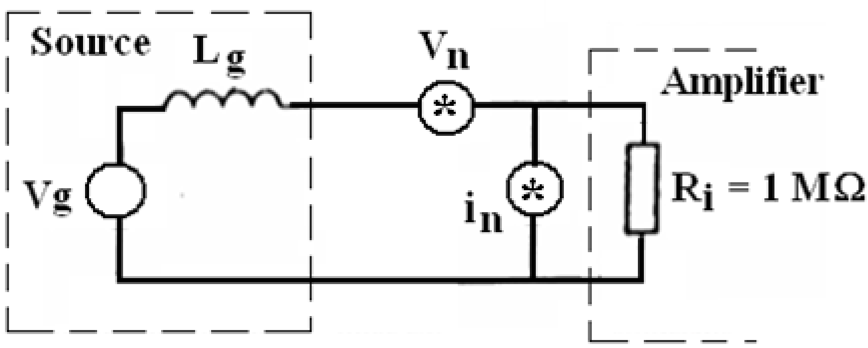
(i) $V_i=\sqrt{\overline{v_i^{\,2}}\Delta f}=\sqrt{1.78\times10^{-14}\times20k}=19\ \ \mu\mathrm{V}$

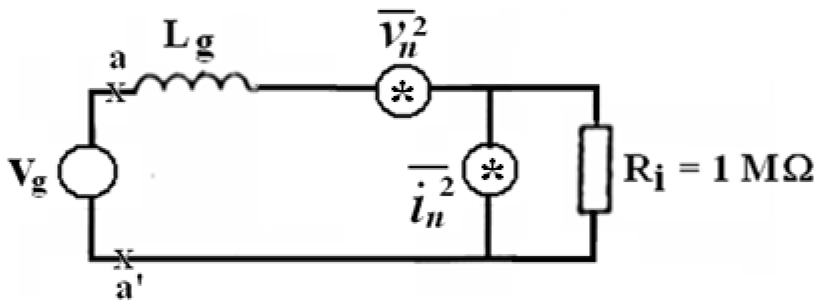
(ii) $V_i=\sqrt{\overline{v_i^{\,2}}\Delta f}=\sqrt{2.50\times10^{-17}\times20k}=0.7\ \ \mu\mathrm{V}$

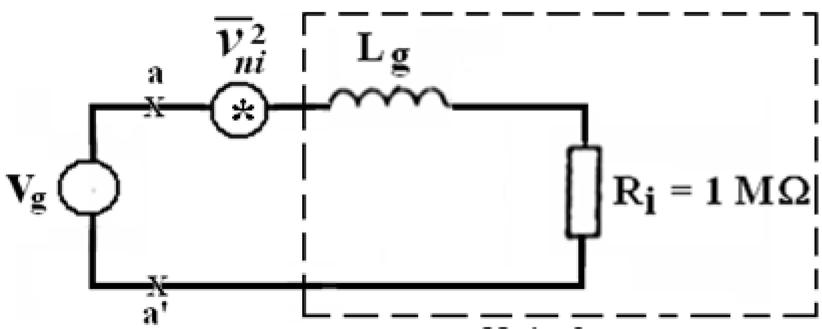
(iii) $V_i = \sqrt{\overline{v_i^2}\Delta f} = \sqrt{1.27\times10^{-17}\times 20k} = 0.5\ \mu\text{V}$

$\therefore$ For a high input impedance amplifier to be interfaced with a low impedance signal source, circuit (iii) is the best configuration to achieve excellent low-noise performance.

2.





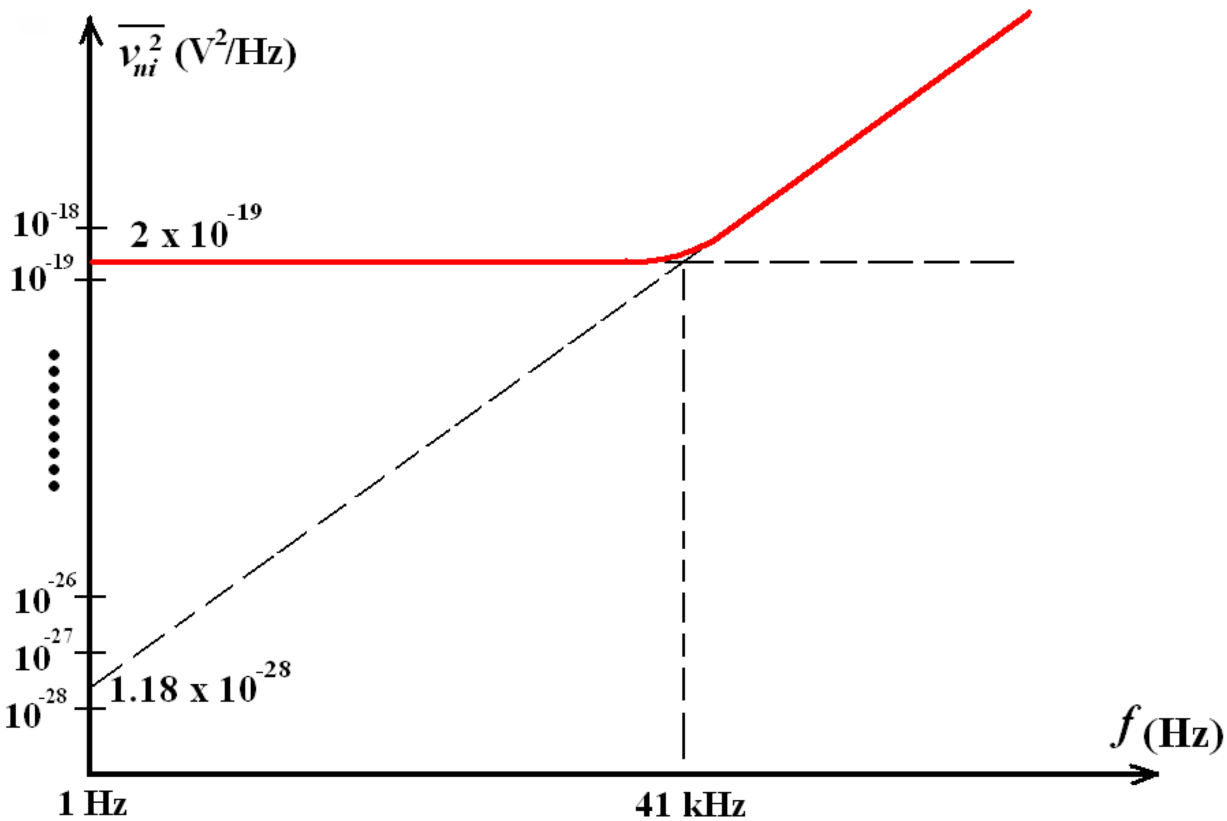


Short-circuit a-a':

$$v_{ni}\left(\frac{R_i}{R_i + j\omega L_g}\right) = v_n\left(\frac{R_i}{R_i + j\omega L_g}\right) + i_n\left(\frac{j\omega L_g R_i}{R_i + j\omega L_g}\right)$$

$$v_{ni} = v_n + i_n\left(j\omega L_g\right)$$

$$
\begin{aligned}
\overline{v_{ni}^{\ 2}} &= \overline{v_n^{\ 2}} + \overline{i_n^{\ 2}}\left|j\omega L_g\right|^2 \\
&= \overline{v_n^{\ 2}} + \overline{i_n^{\ 2}}\left(\omega L_g\right)^2 \\
&= 2\times 10^{-19} + 3\times 10^{-24}\left(2\pi f \times 1m\right)^2 \\
&= 2\times 10^{-19} + 1.18\times 10^{-28} f^2 \quad \mathrm{V}^2/\mathrm{Hz}
\end{aligned}
$$



$$
\begin{aligned}
V_{ni} &= \sqrt{\int_{f_1}^{f_2} \overline{v_{ni}^{\ 2}}\, df} = \sqrt{\int_{f_1}^{f_2} \left(2\times 10^{-19} + 1.18\times 10^{-28} f^2\right) df} \\
&= \sqrt{\left[2\times 10^{-19} f\right]_{f_1}^{f_2} + 1.18\times 10^{-28}\left[\frac{f^3}{3}\right]_{f_1}^{f_2}} \\
&= \sqrt{2\times 10^{-19}\left(100k - 0\right) + \frac{1.18\times 10^{-28}}{3}\left[\left(100k\right)^3 - 0\right]} \\
&= 0.244 \quad \mu\mathrm{V}
\end{aligned}
$$

$$SNR(dB) = 20\,log\,\frac{V_g}{V_{n1}} = 20\,log\,\frac{1mV}{0.244\mu V} = 72.3\ \text{dB}$$