EE 6225 – Multivariable Control System

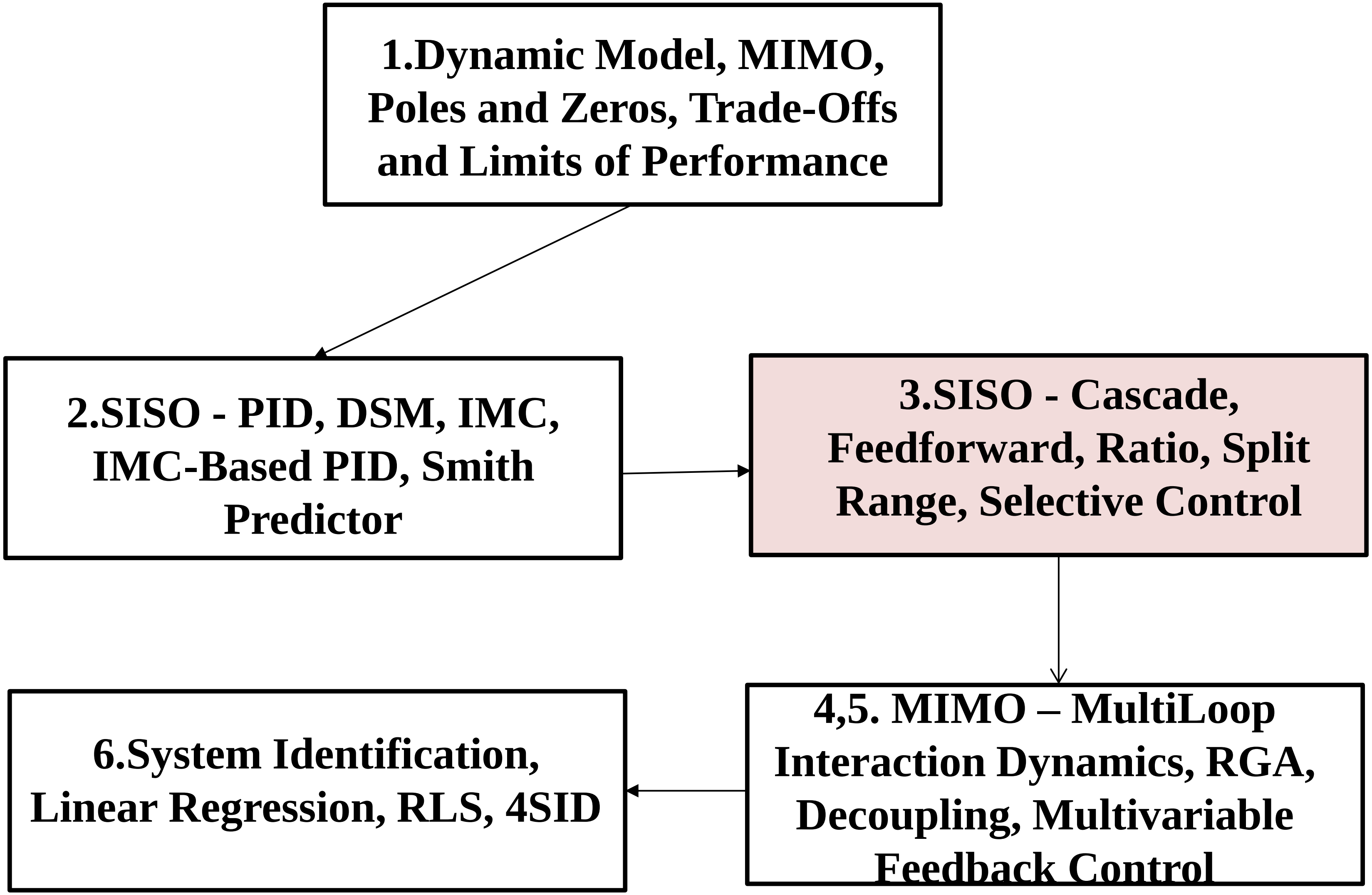
Part I – Advanced Process Control

Dr Poh Eng Kee

Professor (Adjunct)

e-mail: eekpoh@ntu.edu.sg

Course Outline



3. Beyond Feedback Control (Example: Furnace)

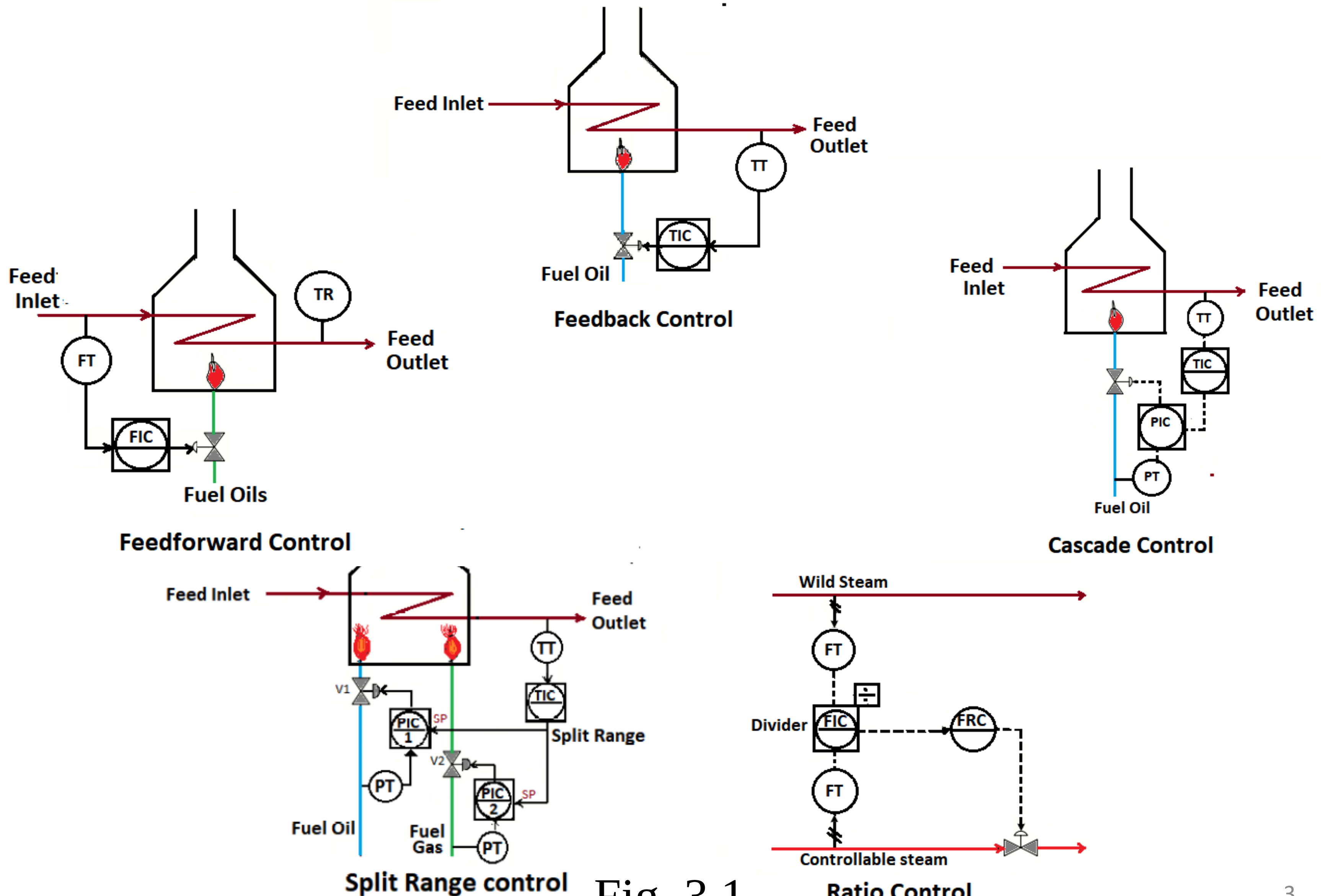


Fig. 3.1

Outline of Lecture 3

3.1 Cascade Control

3.1.1 Inner (Secondary) Closed Loop Transfer Function
3.1.2 Outer (Primary) Closed Loop Transfer Function
3.1.3 IMC-Based PID Cascade Control

3.2 Feedforward Control

3.2.1 Feedforward/Feedback Control Structure
3.2.2 Feedforward Design with Perfect Compensation
3.2.3 Feedforward Design without Perfect Compensation

3.3 Other Control Strategies

3.3.1 Ratio Control
3.3.2 Split Range Control
3.3.3 Select Control

3. Learning Objectives

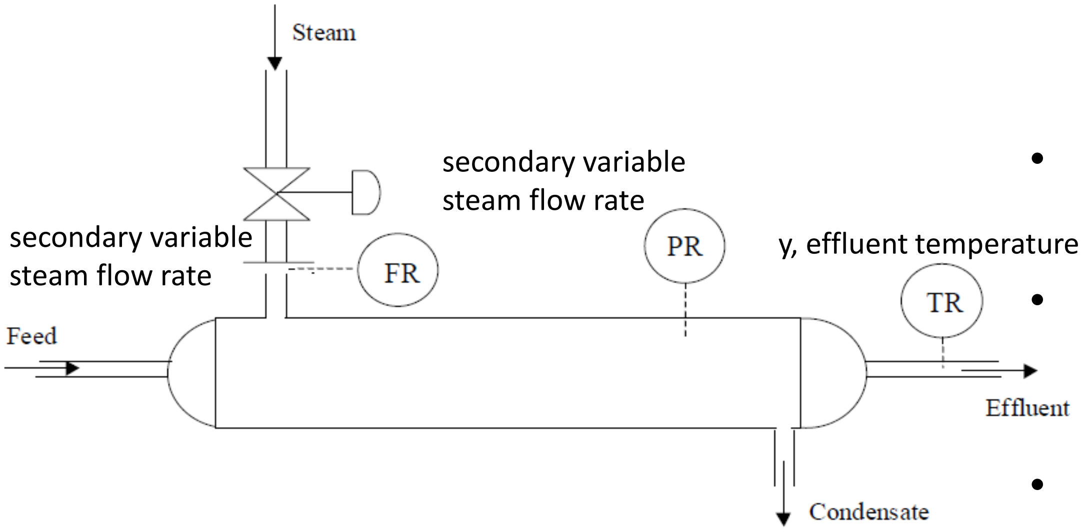
- Feedback control system will only react after the effects of disturbances are perceived in the process output
- Use of cascade control system to a process with a primary controlled variable (process output) that is much slower than its secondary controlled output
- Apply feedforward control to compensate for measured disturbances before the disturbances affect the system
- Other control (heuristics) strategies for process control, such as ratio, split range and selective control, which are able to switch between manipulated variables (controller inputs) or select from several controlled outputs

3.1 Cascade Control

- Cascade control can improve (faster) control system performance over single-loop control whenever either:
  - Disturbances affect a measurable intermediate or secondary process output that directly affects the primary process output that we wish to control; or
  - Gain of the secondary process, including the actuator, is nonlinear.

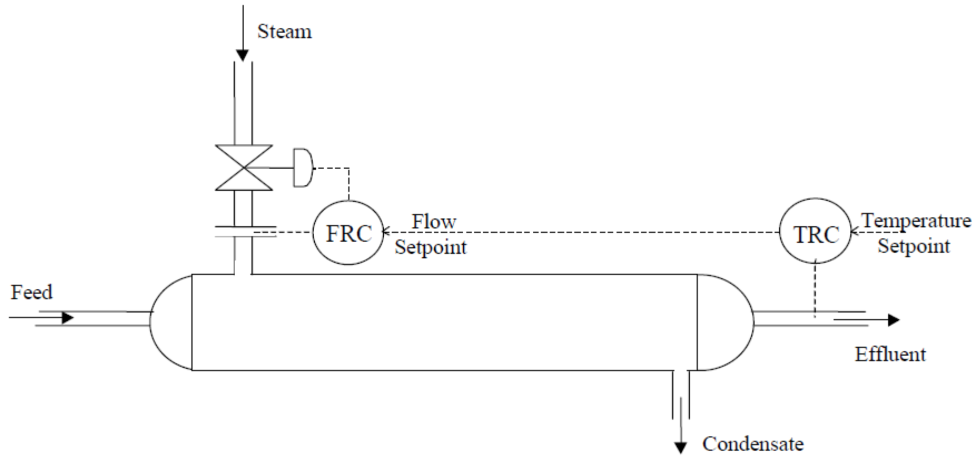
- In the first case, a cascade control system can limit the effect of the disturbances entering the secondary variable on the primary output. May not be able to model the disturbance transfer function exactly for feedforward control.
- In the second case, a cascade control system can limit the effect of actuator or secondary process gain variations on the control system performance. Such gain variations usually arise from changes in operating point due to setpoint changes or sustained disturbances.

3.1 Example Shell and Tube Exchanger

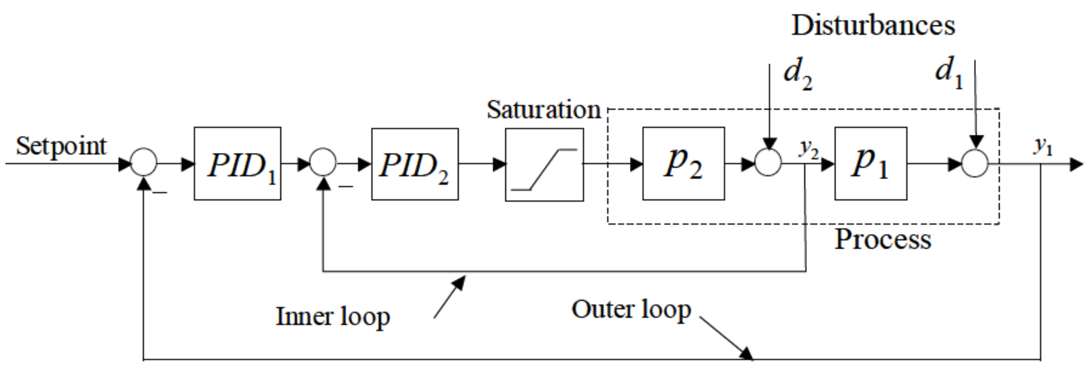


- The primary <u>controlled variable (process output *y*)</u> is the temperature of the tube side effluent stream.
- There are two possible secondary <u>manipulated variables (controller inputs</u>), the flow rate of steam into the exchanger and the steam pressure in the exchanger.
- The steam flow rate affects the effluent temperature through its effect on the steam pressure in the exchanger.
- The steam pressure in the exchanger affects the effluent temperature by its effect on the condensation temperature of the steam.
- Therefore, either the steam flow rate or the steam pressure in the exchanger can be used as the secondary output in a cascade control system. The choice of which to use depends on the disturbances that affect the effluent temperature

3.1 Example Shell and Tube Exchanger



Effluent temperature controller adjusts flow setpoint



Cascade Control Block Diagram

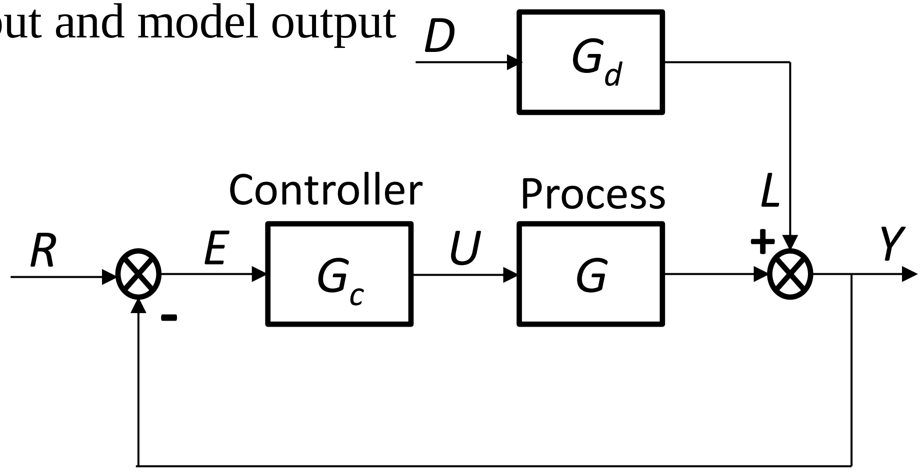
- If the main disturbance is variations in the steam supply pressure, then controlling the steam flow with the control valve is most likely to be the best choice.
- However, it is still necessary to have control of the effluent temperature to be able to track effluent temperature setpoint changes and to reject changes in effluent temperature due to feed temperature and flow variation.
- Since there is only one control effort, the steam valve stem position, traditional cascade control uses the effluent temperature controller to adjust the setpoint of the steam flow controller, as shown in left top figure
- The PID cascade control system block diagram (Seborg et al.,1989) is shown with the secondary process variable y_2 is the steam flow rate, while the primary variable y_1 is the effluent temperature

RECAP: 2.3 IMC

Feedback the error between the process output and model output

Equivalent conventional controller:

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$



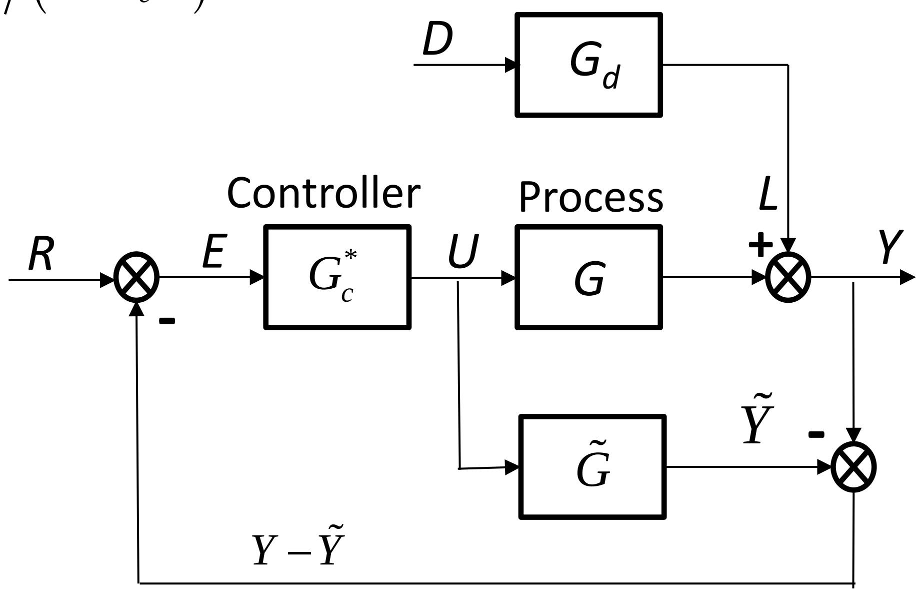
Use block diagram algebra:

$$\tilde{Y} = \tilde{G}U, \quad E = R - \left(Y - \tilde{Y}\right) = R - Y + \tilde{G}U$$

$$U = G_c^* E = G_c^* \left(R - Y + \tilde{G}U\right) \Rightarrow U = G_c^* \left(R - Y\right) \Big/ \left(1 - G_c^* \tilde{G}\right)$$

$$Y = GU + L = G\, G_c^* \left(R - Y\right) \Big/ \left(1 - G_c^* \tilde{G}\right) + L$$

$$\Rightarrow \left(1 + GG_c^* - G_c^* \tilde{G}\right) Y = GG_c^* R + \left(1 - G_c^* \tilde{G}\right) L$$



Hence,

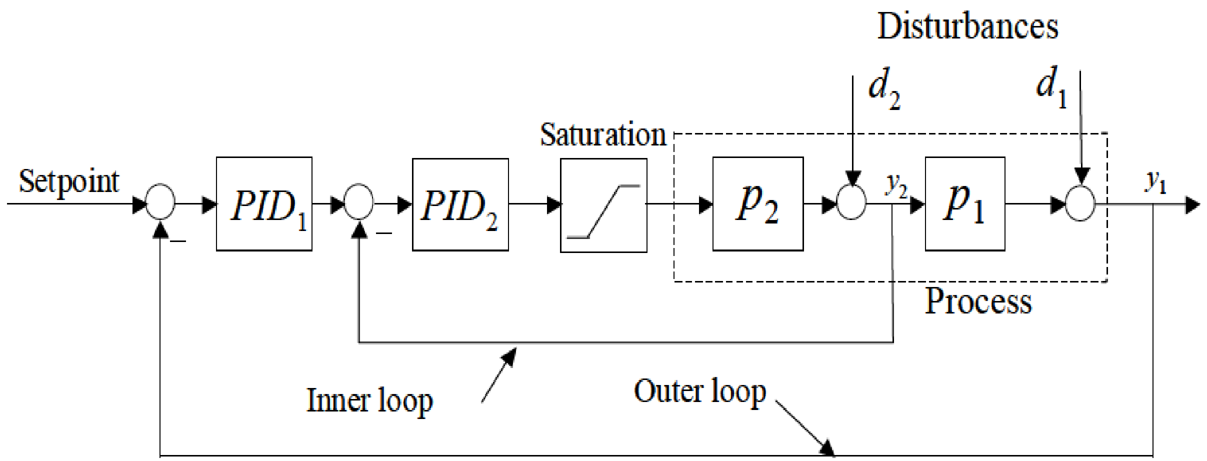
$$Y = \frac{G_c^* G}{1 + G_c^* \left(G - \tilde{G}\right)} R + \frac{\left(1 - G_c^* \tilde{G}\right)}{1 + G_c^* \left(G - \tilde{G}\right)} L$$

If $\tilde{G} = G$,

$$Y = G_c^* G R + \left(1 - G_c^* \tilde{G}\right) L$$

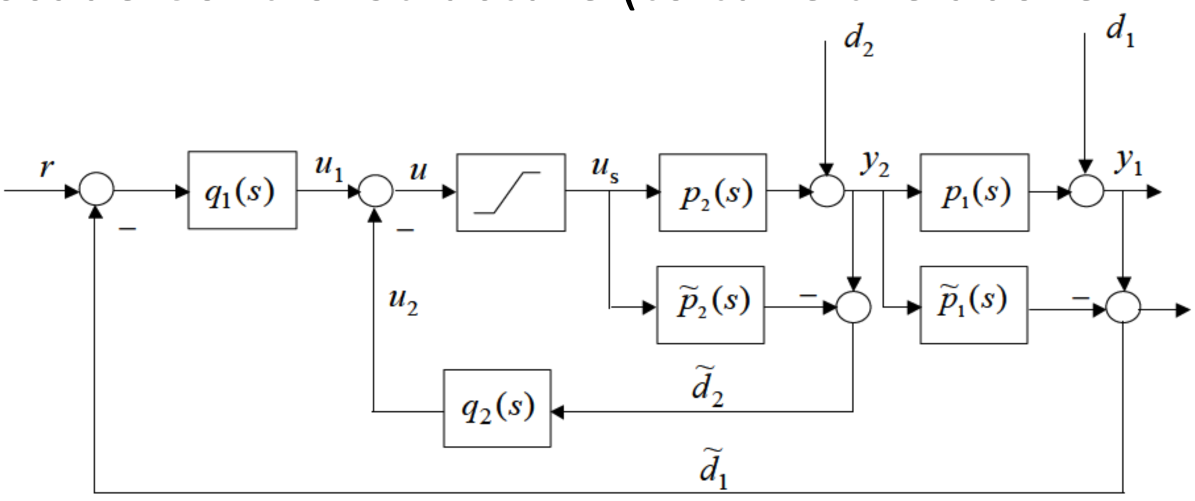
3.1 Cascade Structure and Controller Design

- Traditional PID Cascade Control Structure



(A)

- IMC Cascade Control Structure (to tune the above PID controller)



(B)

3.1.1 Secondary (Inner) Closed Loop Transfer Function

- Ignoring the saturation block, consider the secondary (inner) loop transfer function from u_1 and d_2 to the secondary process output y_2

$$u_S(s) = u_1(s) - u_2(s)$$

$$u_2(s) = q_2(s)\left[y_2(s) - \tilde{p}_2(s)u_S(s)\right]$$

Therefore,

$$u_S(s) = u_1(s) - q_2(s)\left[y_2(s) - \tilde{p}_2(s)u_S(s)\right]$$

$$\Rightarrow u_S(s) = \frac{u_1(s) - q_2(s)y_2(s)}{1 - \tilde{p}_2(s)q_2(s)}$$

Now,

$$y_2(s) = p_2(s)u_S(s) + d_2(s)$$

$$= p_2(s)\left[\frac{u_1(s) - q_2(s)y_2(s)}{1 - \tilde{p}_2(s)q_2(s)}\right] + d_2(s)$$

$$\Rightarrow y_2(s) = \frac{p_2(s)u_1(s) + \left(1 - \tilde{p}_2(s)q_2(s)\right)d_2(s)}{\left(1 + \left(p_2(s) - \tilde{p}_2(s)\right)q_2(s)\right)}$$

3.1.2 Primary (Outer) Closed Loop Transfer Function

- The transfer function between the setpoint *r* and the disturbances to the primary process output y_1 is given by

$$y_1(s) = p_1(s)\,y_2(s) + d_1(s)$$

Hence,

$$y_2(s) = \frac{y_1(s) - d_1(s)}{p_1(s)} \quad \text{and}$$

$$u_1(s) = q_1(s)\left[r(s) - \left(y_1(s) - \tilde{p}_1(s)\,y_2(s)\right)\right]$$

$$\Rightarrow u_1(s) = q_1(s)\left[r(s) - \left(y_1(s) - \tilde{p}_1(s)\frac{\left(y_1(s) - d_1(s)\right)}{p_1(s)}\right)\right]$$

Therefore, suppressing the dependence on s in the expression for brevity,

$$y_1(s) = p_1 y_2(s) + d_1(s) = p_1\left[\frac{p_2 u_1(s) + (1 - \tilde{p}_2 q_2)\,d_2(s)}{\left(1 + (p_2 - \tilde{p}_2)\,q_2\right)}\right] + d_1(s)$$

q_2 can be chosen (i) so that the zeros of $(1 - \tilde{p}_2 q_2)$ cancels the slow poles of p_1 or (ii) to invert part of the process $\tilde{p}_2$ (standard IMC design procedure)

q_1 should approximately invert $\tilde{p}_1 \tilde{p}_2$

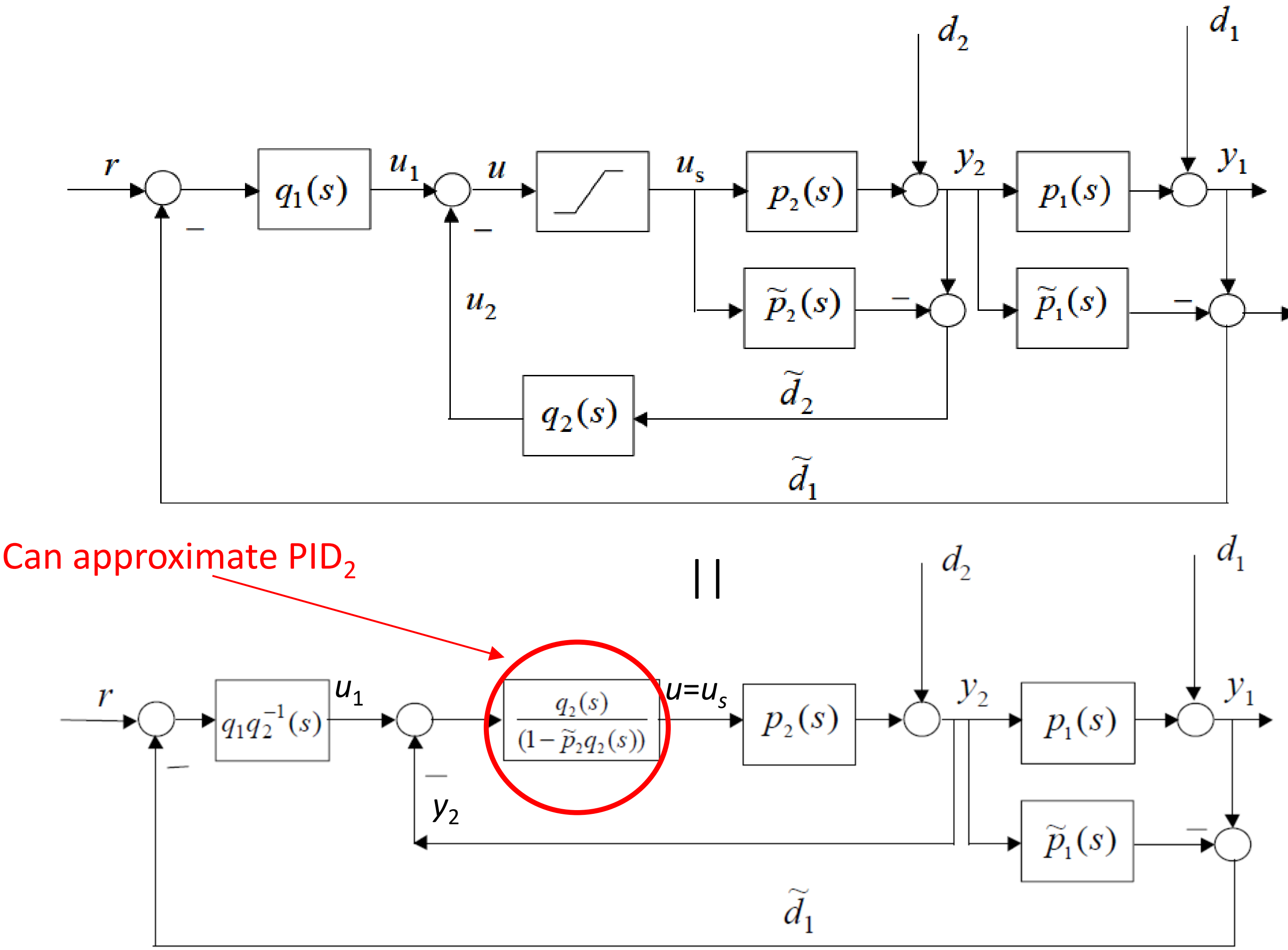
$$= p_1\left[\frac{p_2 q_1\left[r(s) - \left(y_1(s) - \tilde{p}_1\frac{\left(y_1(s) - d_1(s)\right)}{p_1}\right)\right] + (1 - \tilde{p}_2 q_2)\,d_2(s)}{\left(1 - (p_2 - \tilde{p}_2)\,q_2\right)}\right] + d_1(s)$$

$$\Rightarrow y_1(s)\left(1 + (p_1 - \tilde{p}_1)\,p_2 q_1 + (p_2 - \tilde{p}_2)\,q_2\right) = p_1 p_2 q_1 r(s) + (1 - \tilde{p}_2 q_2)\,p_1 d_2(s) + \left(1 - \tilde{p}_1 p_2 q_1 + (p_2 - \tilde{p}_2)\,q_2\right) d_1(s)$$

$$y_1(s) = \frac{p_1 p_2 q_1 r(s) + (1 - \tilde{p}_2 q_2)\,p_1 d_2(s) + \left(1 - \tilde{p}_1 p_2 q_1 + (p_2 - \tilde{p}_2)\,q_2\right) d_1(s)}{\left(1 + (p_1 - \tilde{p}_1)\,p_2 q_1 + (p_2 - \tilde{p}_2)\,q_2\right)}$$

3.1.3 PID Cascade Controller Design

- Again, ignoring the saturation block, we can re-arrange the IMC cascade control block diagram as follows



Ignoring saturation block,

$u_s(s) = u(s) = u_1(s) - u_2(s)$ and

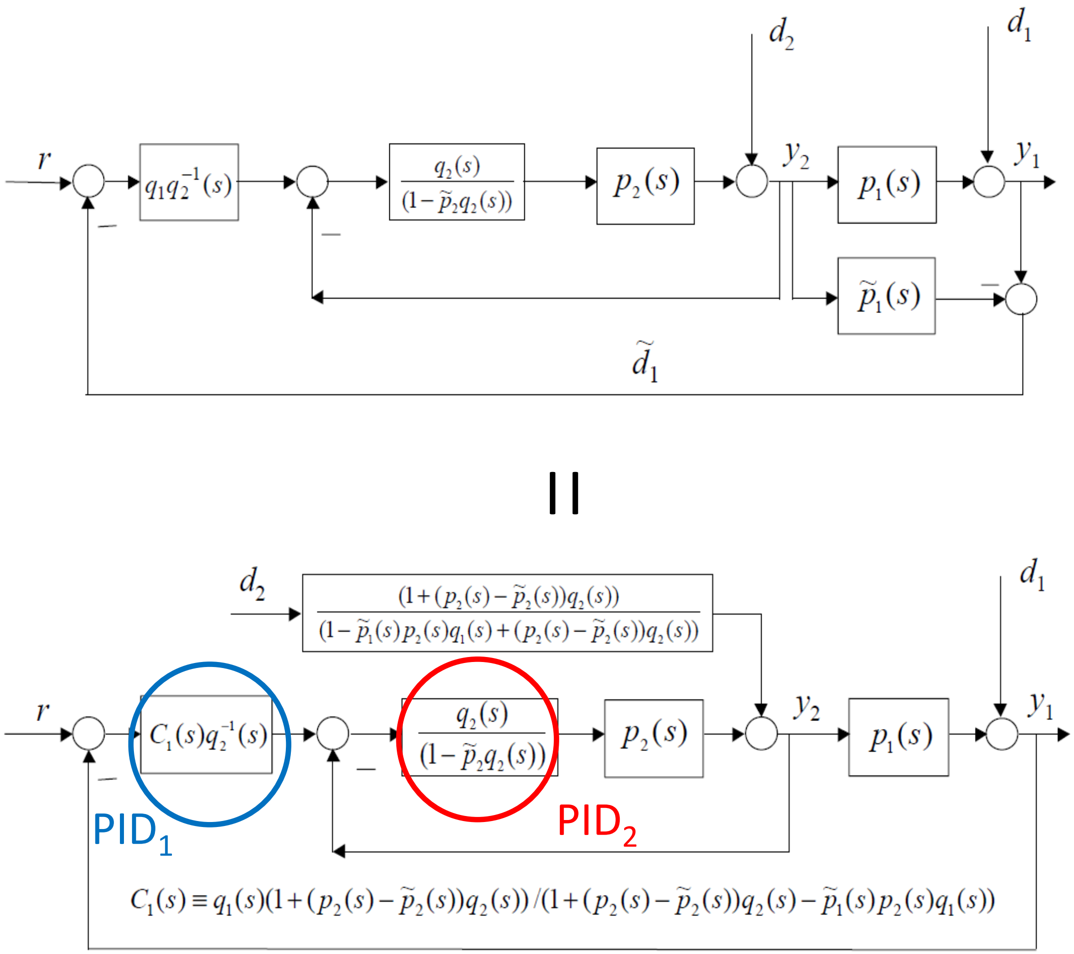
$u_2(s) = q_2\left(y_2(s) - \tilde{p}_2 u(s)\right)$

So,

$$u_2(s) = u_1(s) - u(s)$$
$$= q_2\left(y_2(s) - \tilde{p}_2 u(s)\right)$$
$$\Rightarrow (1 - q_2\tilde{p}_2)u(s) = u_1(s) - q_2 y_2(s)$$
$$u(s) = \frac{q_2}{1 - q_2\tilde{p}_2}\left(q_2^{-1}u_1(s) - y_2(s)\right)$$

3.1.3 PID Cascade Controller Design

- Equivalently,



Recall that the outer closed loop transfer function is as follows:

$$y_1(s)=\frac{p_1p_2q_1r(s)+(1-\tilde{p}_2q_2)p_1d_2(s)+(1-\tilde{p}_1p_2q_1+(p_2-\tilde{p}_2)q_2)d_1(s)}{(1+(p_1-\tilde{p}_1)p_2q_1+(p_2-\tilde{p}_2)q_2)}$$

Consider the outer closed loop transfer function from $r(s)$ to $y_1(s)$

$$\frac{y_1(s)}{r(s)}=\frac{p_1p_2q_1}{(1+(p_1-\tilde{p}_1)p_2q_1+(p_2-\tilde{p}_2)q_2)}\quad\cdots\quad(1)$$

From the lower block diagram,

$$\frac{y_1(s)}{r(s)}=\frac{C_1p_1p_2}{1+(p_2-\tilde{p}_2)q_2+C_1p_1p_2}\quad\cdots\quad(2)$$

Equating equations (1) and (2), we obtain

$$C_1=\frac{q_1(1+(p_2-\tilde{p}_2)q_2)}{1+(p_2-\tilde{p}_2)q_2-\tilde{p}_1p_2q_1}$$

If $\tilde{p}_2=p_2$, then

$$C_1=\frac{q_1}{1-\tilde{p}_1p_2q_1}$$

$$\mathrm{PID}_1(s)\approx C_1(s)q_2^{-1}(s)=\frac{q_1(s)q_2^{-1}(s)}{1-\tilde{p}_1(s)p_2(s)q_1(s)}$$

where $q_2(s)$ is first designed for inner loop and can be approximated by Taylor series expansion

3.1.3 Example: PID Cascade Controller Design

- Consider a second order process with

$$p_1(s) = \frac{K_1 e^{-T_1 s}}{\tau_1 s + 1}; \; 0.8 \le \mathrm{K}_1 \le 1.2, \quad 17.5 \le \mathrm{T}_1 \le 22.5, \quad 14 \le \tau_1 \le 16 \qquad (10.4a)$$

$$p_2(s) = \frac{K_2 e^{-T_2 s}}{\tau_2 s + 1}; \; 0.6 \le \mathrm{K}_2 \le 1.8, \quad 2 \le \mathrm{T}_2 \le 4, \quad 1 \le \tau_2 \le 3 \qquad (10.4b)$$

- We assume

$$\tilde{p}_2(s) = p_2(s)$$

$$\therefore, C_1(s) = q_1(s) / \left(1 - p_1(s)\, p_2(s)\, q_2(s)\right)$$

- Choose the following $q_2(s)$

$$q_2(s) = \frac{(s+1)}{1.8(4.18s+1)}. \qquad (10.8)$$

3.1.3 Example: PID Cascade Controller Design

Using the 1DF IMC controller for $q_2(s)$, given by Eq. (10.8) and repeated below, yields the inner loop PID controller given by Eq. (10.10a).

$$q_2(s) = \frac{(s+1)}{1.8(4.18s+1)}$$

Inner loop: $PID_2 = .134(1+1/(1.98s)+.319s/(.016s+1)).$ (10.10a)

- Choose $q_1(s)$ as follows:

$$q_1(s) = \frac{(15s+1)}{2.16(16.87s+1)}. \quad (10.5b)$$

Outer loop: $C_1(s) \cong PID_1 = .234(1+1/(23.77s)+5.35s/(.29s+1)),$ (10.9c)

The outer loop controller remains the same as in Eq. (10.9c) because $q_1(s)$ has not changed. In figures 10.18 and 10.19 the responses using Eq. (10.10a) are labeled Cascade 1. These responses show the benefits of an IMC outer loop over a PID outer loop. The outer loop PID controller in the responses in figures 10.18 and 10.19 is cascaded with the term q_2^{-1}. Since q_2^{-1} is a lead, it can be approximated by the Taylor series as the polynomial $1.8(-3.18s^2+3.18s+1)$. Multiplying this polynomial into Eq. (10.10a) and dropping terms higher than second order gives, after some rearrangement,

Outer loop: $PID_1 = .4956(1+1/(26.96s)+7.404s/(.37s+1)).$ (10.10b)

3.1.3 Example: PID Cascade Controller Design

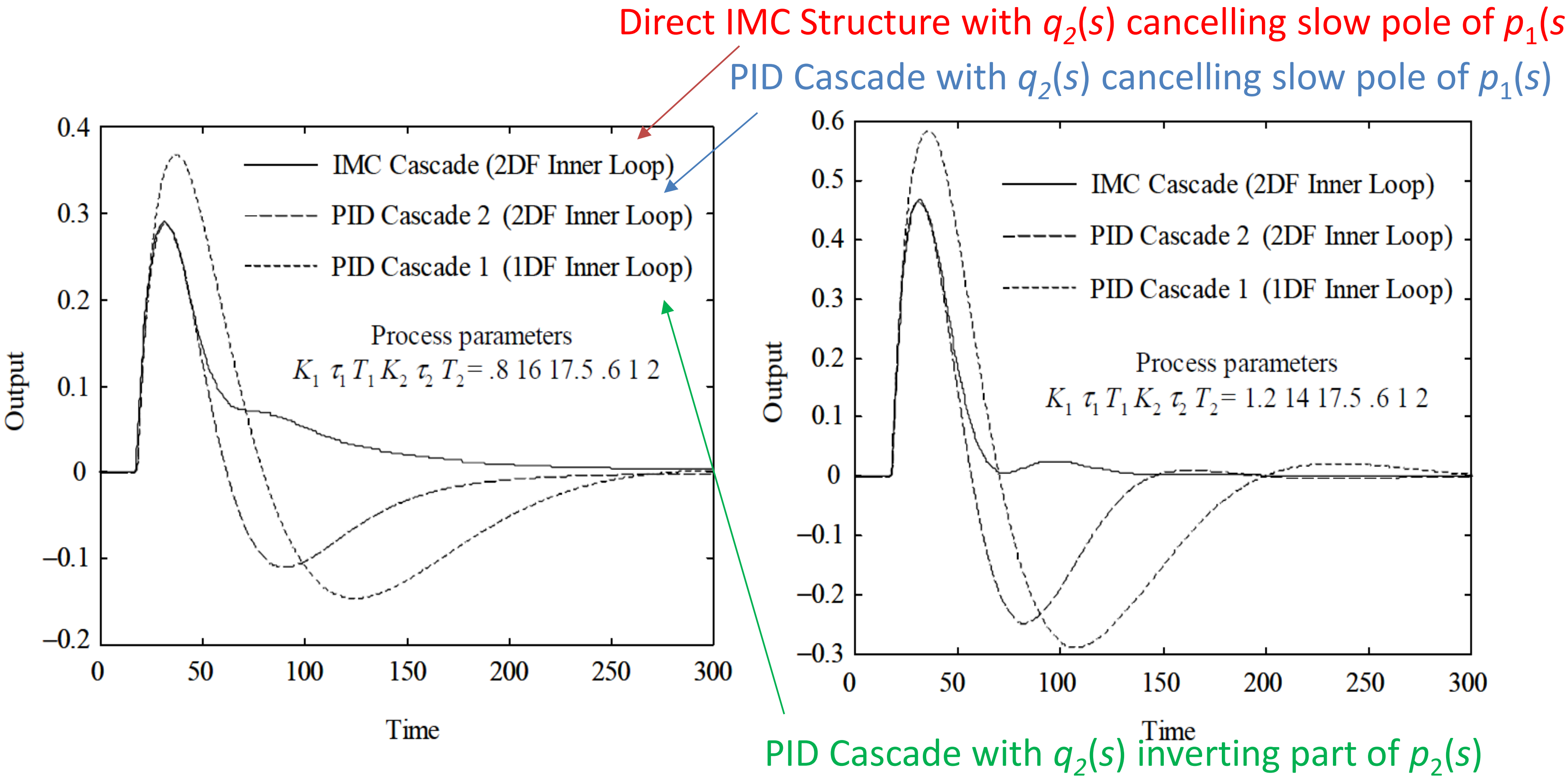
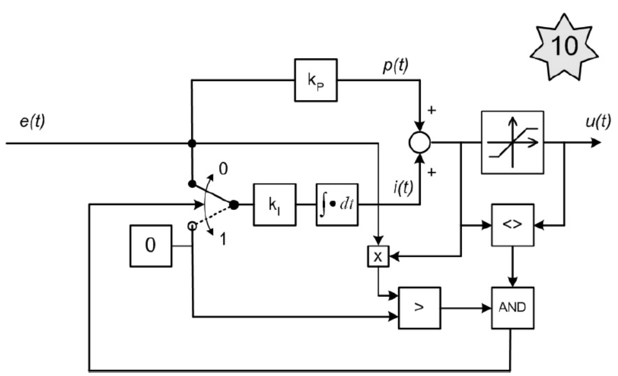
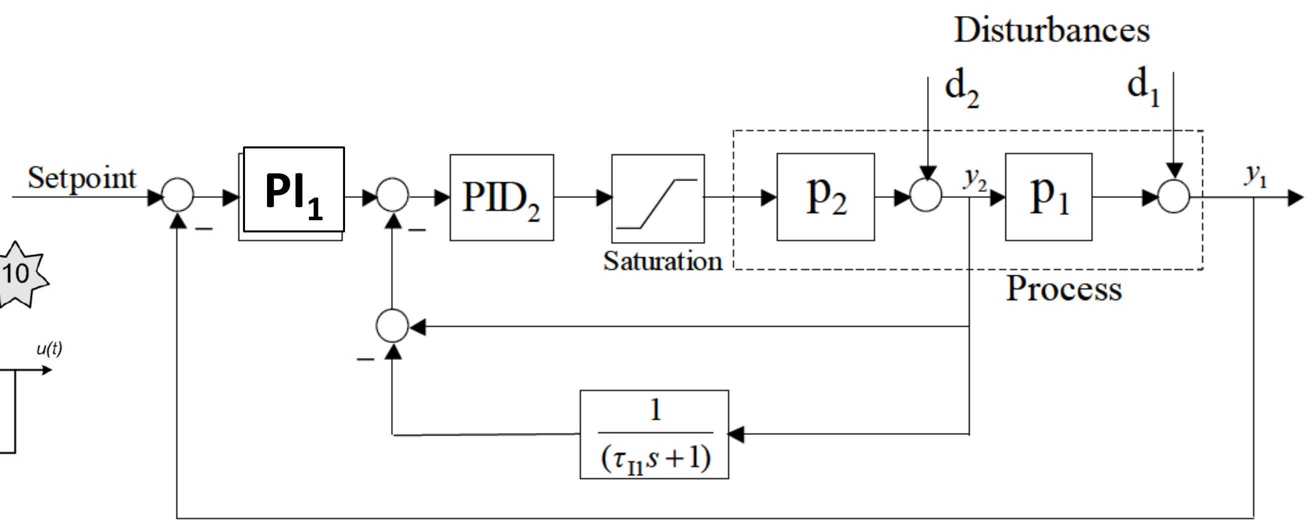


Figure 10.18 Comparison of responses to a step disturbance in the inr **Figure 10.19** Comparison of responses to a step disturbance ir outer loop.

3.1.3 Handling Saturation with PID Cascade Control

- Saturation compensation for the standard PID cascade structure is usually accomplished by either of two methods.
- The preferred method is to use logic statements that stop the integration in both the inner loop and outer loop PID controllers whenever the control effort reaches a limit, and start it again whenever the error signals change sign or the control effort comes off saturation (see bottom left figure).
- The second, and possibly more common, method is to use a standard anti-reset windup controller in the inner loop and implement the integral portion in the outer loop PID controller, as shown in below right figure.





3.1.4 Summary on Cascade Control

- In general, there may be more than 2 loops in cascade. For instance, a valve positioner can get its setpoint from a flow controller, which in turn gets its setpoint from a level controller (i.e. 3 loops in cascade)

- For effective cascade control, the inner loops should be significantly faster than the outer loops. For example, if the inner loops do not provide for faster disturbance rejection, they are not very useful.

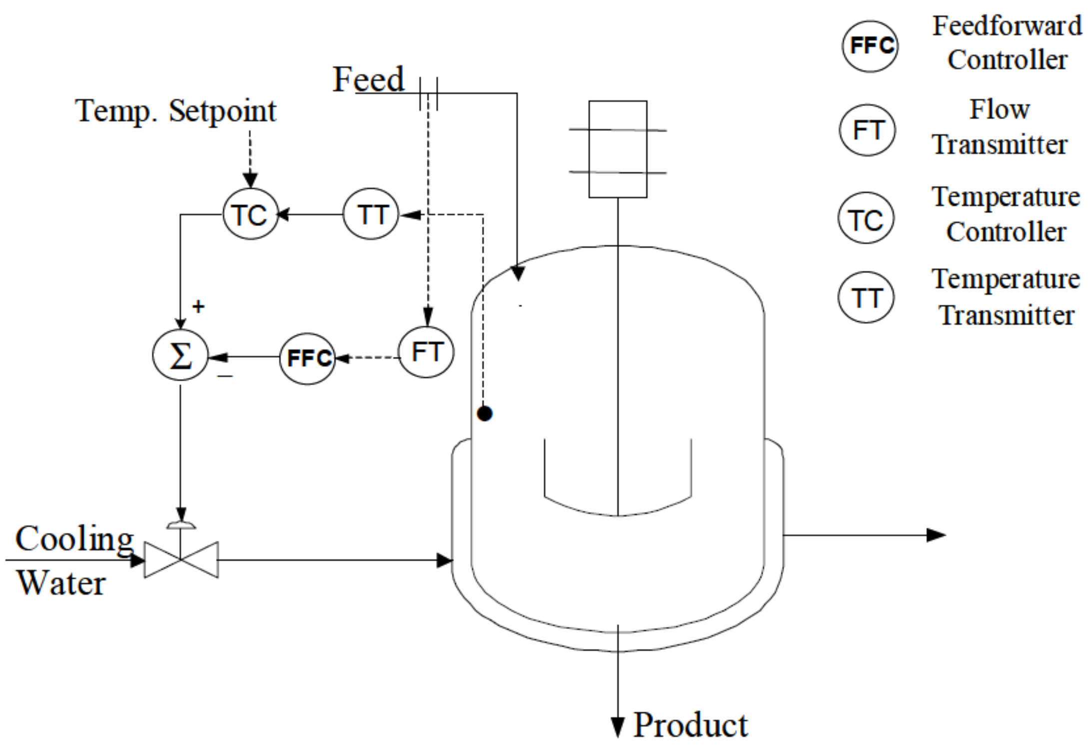
- Faster inner loops will also make the tuning of the outer loops simpler, since one then can assume that the inner loops are able to follow the setpoints from the outer loops.

3.2 Feedforward Control

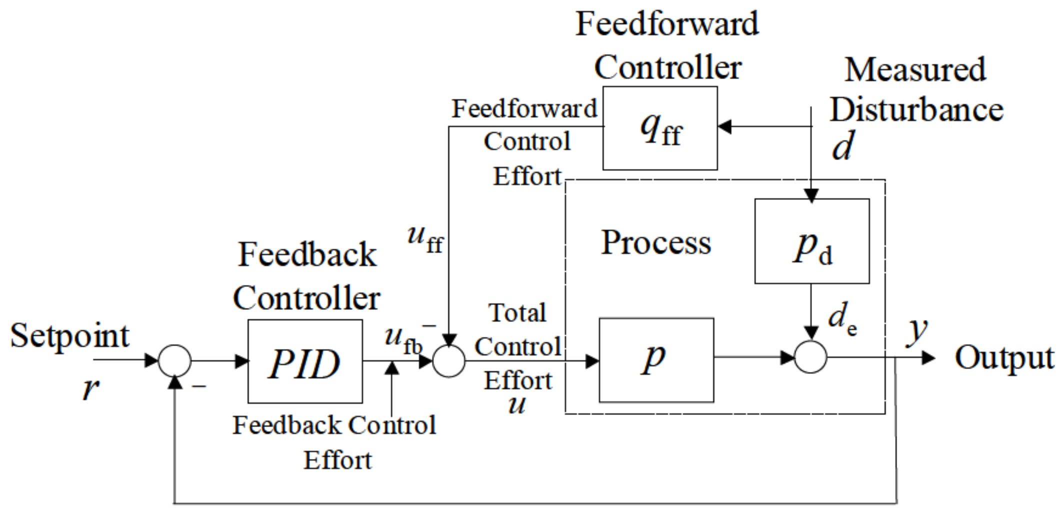
- Combined feedforward plus feedback control can significantly improve performance over simple feedback control whenever there is a major disturbance that can be measured before it affects the process output.
- Feedforward control is always used along with feedback control because a feedback control system is required to track setpoint changes and to suppress unmeasured disturbances that are always present in any real process.
- In the most ideal situation, feedforward control can entirely eliminate the effect of the measured disturbance on the process output.
- Even when there are modeling errors, feedforward control can often reduce the effect of the measured disturbance on the output better than that achievable by feedback control alone.

3.2 Example: Feedforward Control for CSTR

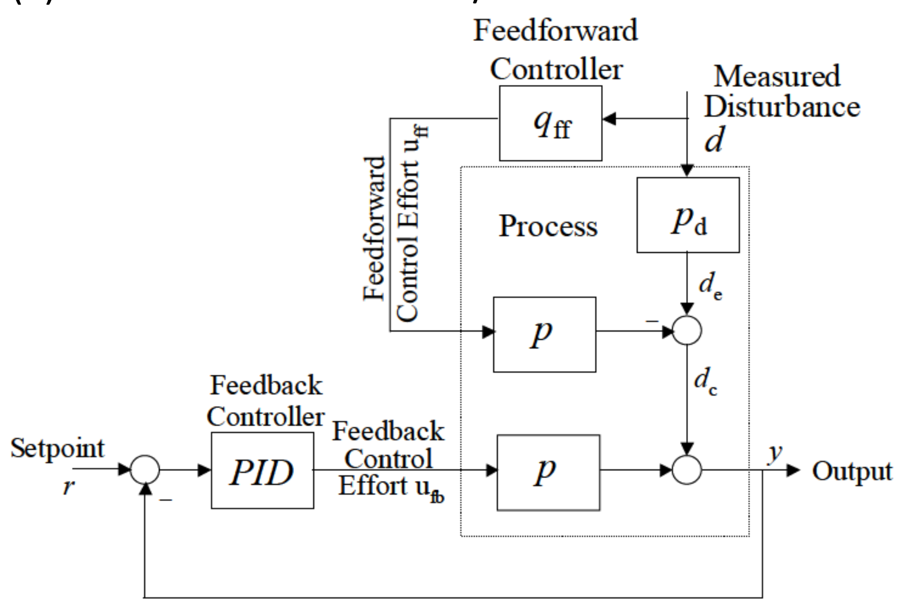
- Below figure shows a typical application of feedforward control. The continuously stirred tank reactor is under feedback temperature control. Feedforward control is used to rapidly suppress feed flow rate disturbances.



3.2.1 Feedforward/Feedback Control Structure



(a) Traditional feedforward/feedback Control Structure



(b) Equivalent block diagram showing stability is not affected by feedforward control

3.2.2 Feedforward Control Design with Perfect Compensation

1) The transfer function between the process output $y(s)$ and the measured disturbance $d(s)$

$$y(s) = \frac{d_c(s)}{1+PID(s)\times p(s)} = \frac{\left(p_d(s)-p(s)q_{ff}(s)\right)d(s)}{1+PID(s)\times p(s)} \qquad \cdots \quad (1)$$

2) To eliminate the effect of the measured disturbance, we need only choose $q_{ff}(s)$ so that

$$p_d(s)-p(s)q_{ff}(s)=0$$

3) If the deadtime and relative order of $p_d(s)$ are both greater than those of $p(s)$, and $p(s)$ has no right half plane zeros, then $q_{ff}(s)$ can be chosen as

$$q_{ff}(s)=p^{-1}(s)\,p_d(s)$$

4) Whenever the relative order of $p_d(s)$ is less than or equal to that of $p(s)$, then the noise amplification can be reduced by adding a filter $f(s)$ such that

$$q_{ff}(s)=p^{-1}(s)\,p_d(s)\,f(s), \quad f(s)=1/(\tau s+1)^r \qquad \cdots \quad (2)$$

where order r of the filter $f(s)$ is

$$r=\begin{cases} 0 & \text{, if the relative order of } p_d^{-1}(s)\,p(s) \text{ is equal or less than zero,} \\ \text{relative order of } p_d^{-1}(s)\,p(s), & \text{otherwise} \end{cases}$$

The filter time constant τ is chosen to limit noise amplification

3.2.2 Feedforward Control Design without Perfect Compensation

- When the delay (deadtime) in the disturbance lag $p_d(s)$ is less than that of the control effort lag $p(s)$, then the feedforward controller $q_{ff}(s)$, given by Eq. (2), is not realizable.
- In this case, perfect compensation is no longer possible. Much of the literature suggests designing the feedforward controller by simply dropping the unrealizable part of the controller (1 DOF design).
- A better feedforward controller can be obtained using a 2DOF design. To see why this is, we rewrite the expression for the net effect of the measured disturbance on the output from Eq. (1) as

$$d_c(s) = \left(p_d(s) - p(s)q_{ff}(s)\right)d(s) = \left(1 - p(s)q_{ff}(s)p_d^{-1}(s)\right)p_d(s)d(s) \quad \ldots (3)$$

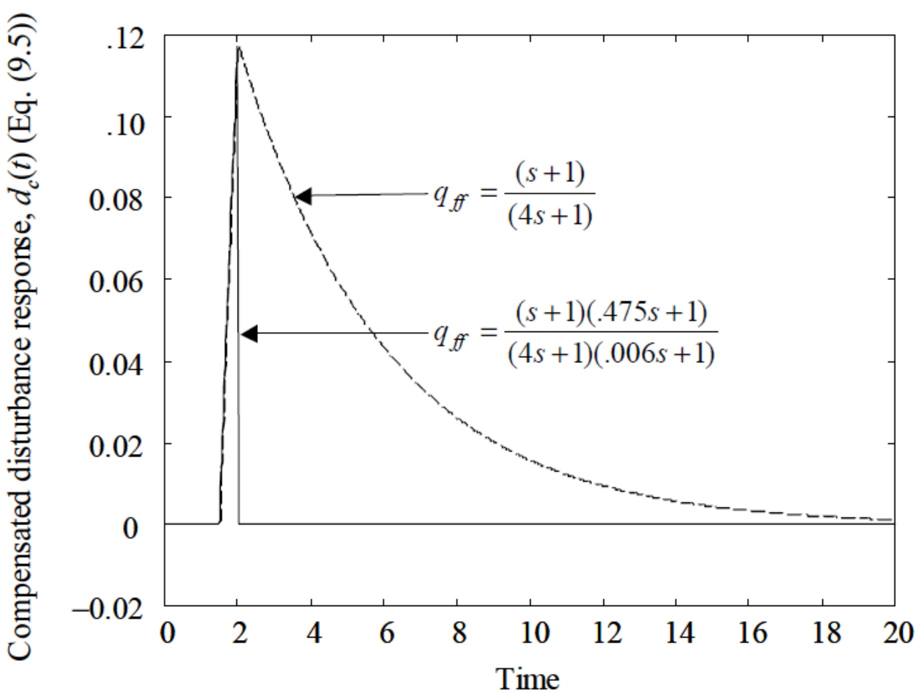
- Since, by assumption, the term contains a deadtime, there is no realizable choice of the feedforward controller $q_{ff}(s)$ that makes the term in brackets in Eq. (3) zero. Therefore, there will be a long tail to the response of $d_c(s)$ to a step disturbance $d(s)$ if the lag of $p_d(s)$ is on the order of, or larger than, the lag of $p(s)$, unless $q_{ff}(s)$ is chosen so that the zeros of $(1-p(s)\, q_{ff}(s)\, p^{-1}{}_d(s))$ cancel the poles of $p_d(s)$
- The procedure for such a choice of $q_{ff}(s)$ lies in the design of 2DOF design.

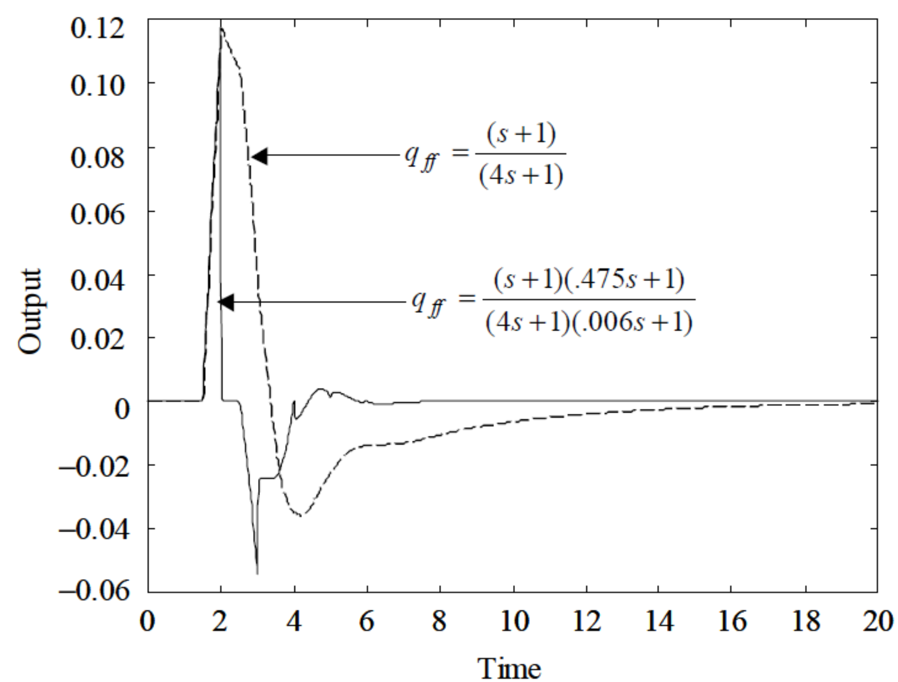
3.2.2 Example Feedforward Control to Unit Step Disturbance

- The process is as follows:

$$\widetilde{p}(s) = p(s) = e^{-s}/(s+1); \quad \widetilde{p}_d(s) = p_d(s) = e^{-0.5s}/(4s+1).$$

- 1 DOF Design $q_{ff}(s) = (s+1)/(4s+1),$
- 2 DOF Design $q_{ff}(s) = (s+1)(.473s+1)/(4s+1)(0.006s+1).$





3.2.3 Summary of Feedforward Control

- In order to implement the ideal feedforward controller $u_{ff}(s) = p^{-1}(s)p_d(s)$, both the process transfer function p(s) and the disturbance transfer function pd(s) must be known with reasonable accuracy.
- The ideal controller $u_{ff}(s)$ must also be stable and realizable, meaning $p(s)$ must not have right half plane zeros and the deadtime and relative order of $p_d(s)$ are both greater than $p(s)$.
- In practice, the feedforward control cannot be perfect, and thus feedback controller will still be needed for accurate control.
- A pure feedforward controller cannot by itself cause instability of the closed-loop system and is therefore very useful to overcome bandwidth limitations since most bandwidth limitations (excluding input/input rate constraints) will not apply to feedforward controller.
- Also, feedforward controller cannot be used to stabilize an unstable system.

3.3 Other Control Strategies

- We have discussed feed-forward and cascade-control strategies. The purpose of these control strategies is to take advantage of additional measurements to improve the disturbance rejection capability.
- Next, we shall develop strategies, based mainly on heuristics reasoning, that use more than one measurement and, in addition, manipulate more than one input.
- Ratio control, presented next, is very similar to feed-forward control where one of 2 inputs cannot be controlled and is measured so that the other inputs is applied in a specific ratio to the uncontrolled one – essentially like feedforward control.
- Split-range control is presented as a way to use more than one manipulated control to regulate a process.
- Finally, we discuss selective control, which involves the use of "selectors" to decide the proper control action to take.

Part I – Advanced Process Control

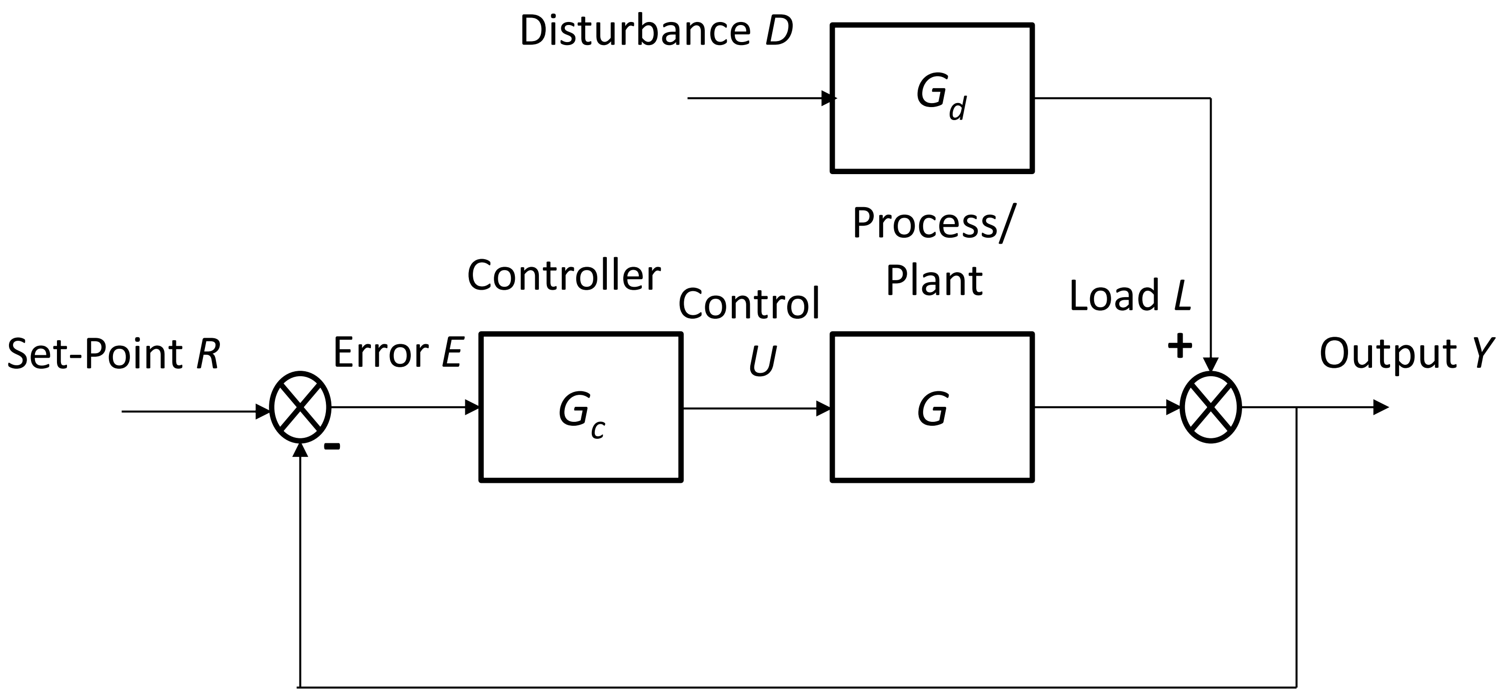
CA1 Assignment

(i) Due Date: 20 Apr 2025

(ii)Submit softcopy solution to NTULearn EE6225 Course Assignments Folder and include Your Name and Matriculation Number

1(a). Direct Synthesis Method (DSM)

- Consider the following block diagram of a feedback control system



1(a). Direct Synthesis Method (DSM)

The Second Order Plus Time Delay (SOPTD) process is given by:

$$G(s) = \frac{2e^{-s}}{(5s+1)(2s+1)}$$

The disturbance D to output Y is

$$G_d(s) = \frac{e^{-0.5s}}{(10s+1)(5s+1)}$$

Apply the Direct Synthesis Method (DSM) to design a controller using the following desired closed loop transfer function

$$G_c = \frac{1}{\tilde{G}}\left(\frac{\left(\frac{Y}{R}\right)_d}{1-\left(\frac{Y}{R}\right)_d}\right) \quad \text{where} \quad \left(\frac{Y}{R}\right)_d = \frac{e^{-s}}{\tau_c s+1}$$

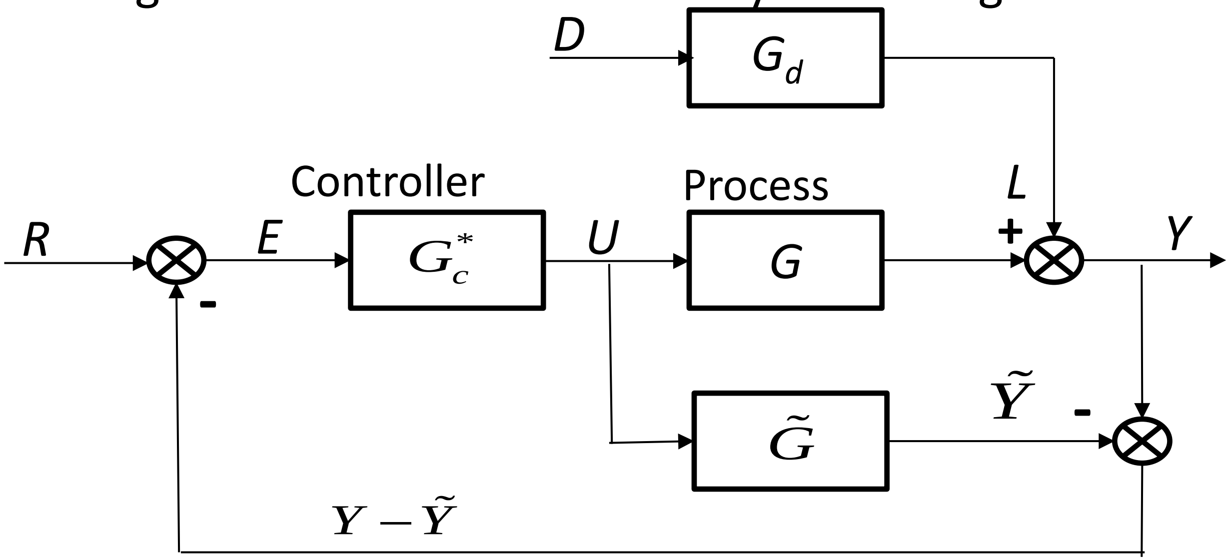
(i) Assume $\tilde{G} = G$, design a PID controller G_c for $\tau_c = 1$ sec and 10 sec.

(Hint: Using first order Taylor series to approximate $e^{-\theta s} \approx 1-\theta s$ in the denominator of G_c; the $e^{-\theta s}$ in the numerator will be cancelled by identical term in $\tilde{G}$)

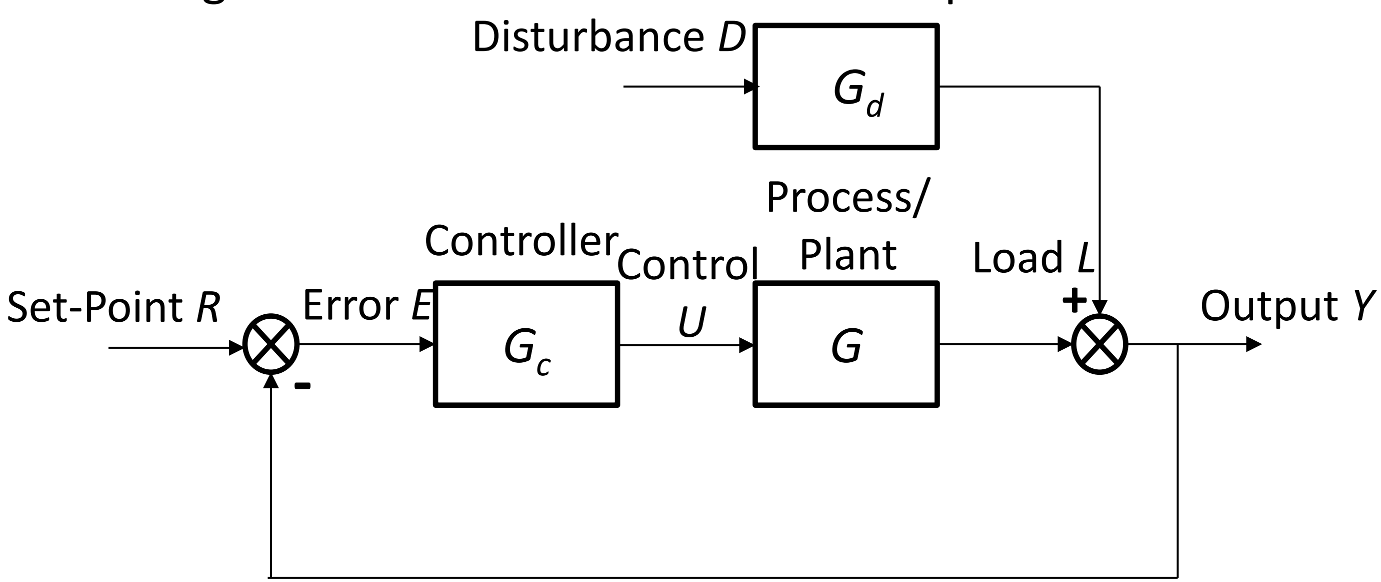
(ii) Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 60$ sec.

1(b). 1 Degree of Freedom Internal Model Control (IMC)

- (a) Block diagram of a feedback control system using IMC



- (b) Block diagram of a feedback control with equivalent standard controller



1(b). 1 Degree of Freedom Internal Model Control (IMC)

The Second Order process with right Half Plane (RHP) zero is given by:

$$G(s)=\frac{2(-s+1)}{(5s+1)(2s+1)}$$

The disturbance D to output Y is

$$G_d(s)=\frac{e^{-s}}{(10s+1)(5s+1)}$$

An IMC controller G_c^* is given by

$G_c^*=\frac{1}{\tilde{G}_-}f$ where $\tilde{G}=\tilde{G}_+\tilde{G}_-$, $\tilde{G}_+$ contains time delays and RHP zeros with $\tilde{G}_+(s=0)=1$

and $f=\frac{1}{\tau_c s+1}$

(i) Assume $\tilde{G}=G$, design an IMC Controller G_c^* for $\tau_c=5$ sec.

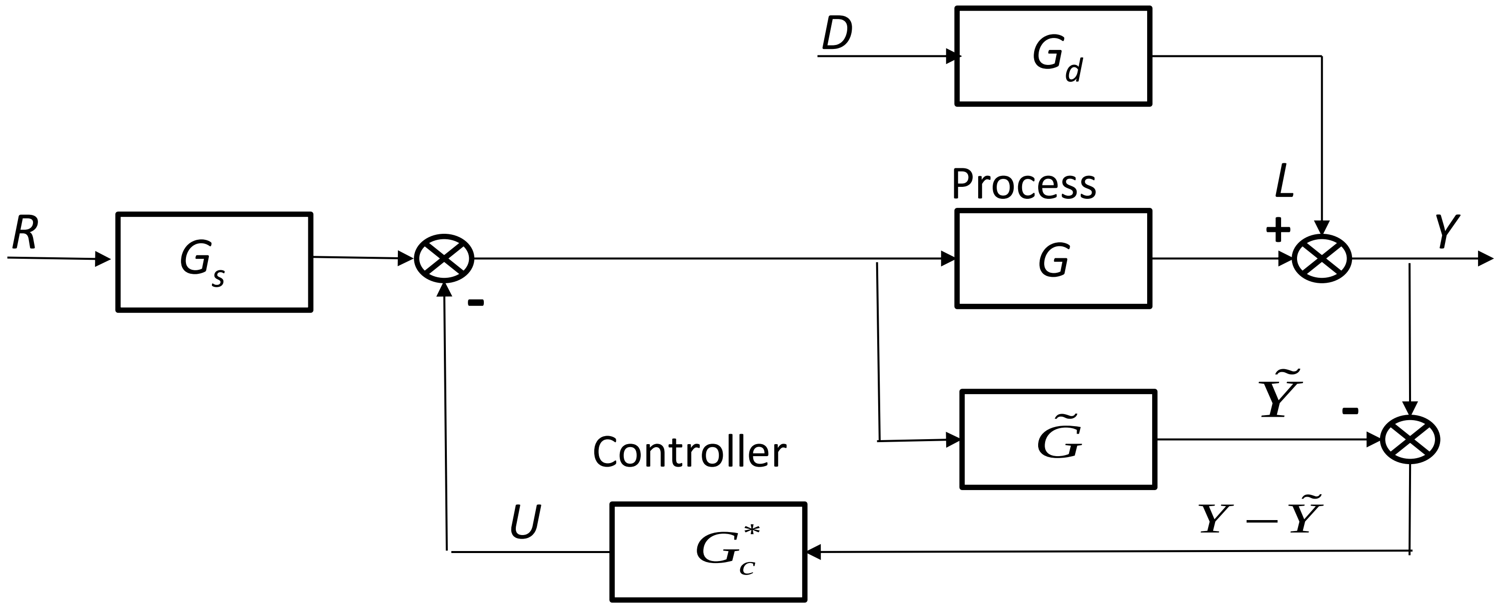
(ii) Derive the equivalent standard controller $G_c=\frac{G_c^*}{1-G_c^*\tilde{G}}$ and determine the PID parameters

$G_c=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)$. Check if you get the same G_c if you use DSM with $\left(\frac{Y}{R}\right)_d=\tilde{G}_+ f$.

(iii) Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t=0$ sec and a step disturbance input D at $t=60$ sec.

1(c). 2 Degree of Freedom Internal Model Control (IMC)

- (a) Block diagram of a feedback control system using 2 degree of freedom controller design (the controller is place in feedback path so that setpoint filter G_s can be designed independently of G_c* from $R(s)$ to $Y(s)$)



$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G_c^*(s)\left(G(s)-\tilde{G}(s)\right)}G_s(s),\quad \frac{Y(s)}{D(s)}=\frac{1-G_c^*(s)\tilde{G}(s)}{1+G_c^*(s)\left(G(s)-\tilde{G}(s)\right)}G_d(s)$$

Assume $\tilde{G}(s)=G(s)$,

$$Y(s)=G(s)G_s(s)R(s)\text{ and }Y(s)=\left(1-G_c^*(s)\tilde{G}(s)\right)G_d(s)D(s)$$

1(c). 2 Degree of Freedom Internal Model Control (IMC)

The First Order Plus Time Delay (FOPTD) is given by:

$$G(s) = G_d(s) = \frac{e^{-2s}}{(10s+1)} \quad (\text{that is, we assume that the disturbance is at the plant input})$$

An IMC controller G_c^* is given by

$$G_c^* = \frac{1}{\tilde{G}_-} f \quad \text{where } \tilde{G} = \tilde{G}_+ \tilde{G}_- \text{ and } f = \frac{\lambda s + 1}{(\tau_c s + 1)^r}$$

(i) Assume $\tilde{G} = G$, design an IMC Controller G_c^* for $\tau_c = 2$ sec, $r = 1$ and $\lambda = 0$.

Set $G_s = 1$. Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 20$ sec.

(ii) Next design another IMC Controller with $\tau_c = 2$ sec, $r = 2$ and λ is determined by setting

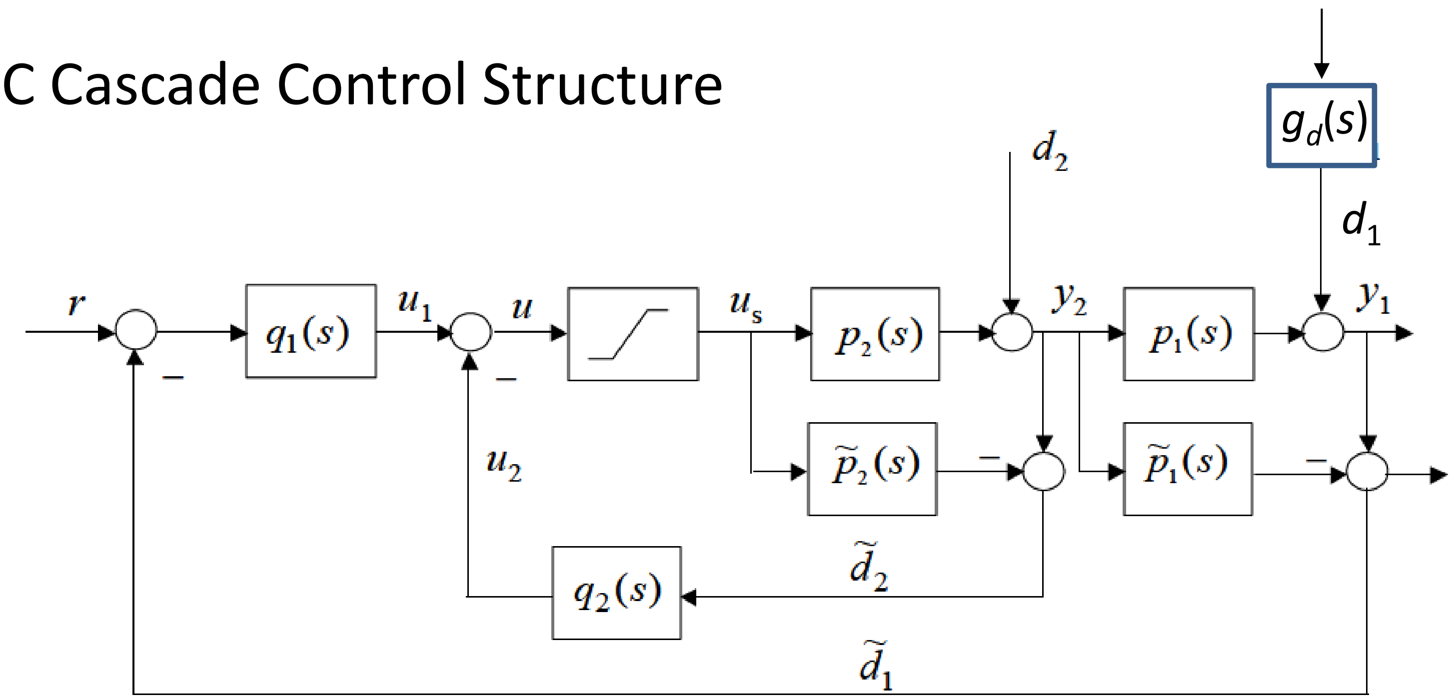
$$\left(1 - G(s) G_c^*\right) = 0 \text{ at the pole of } G_d(s) \Rightarrow \left.\left(1 - G(s) \frac{1}{\tilde{G}_-(s)} \frac{\lambda s + 1}{(\tau_c s + 1)^r}\right)\right|_{s=-1/10, \tau_c = 2, r = 2} = 0$$

Choose $G_s = \dfrac{1}{\tau_s s + 1}$ and judiciously select τ_s. Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 20$ sec.

(iii) Compare the responses obtained in (i) and (ii).

1(d). Cascade Structure and Feedforward Control

- IMC Cascade Control Structure



$$y_2(s) = \frac{p_2(s)u_1(s) + (1 - \tilde{p}_2(s)q_2(s))d_2(s)}{(1 + (p_2(s) - \tilde{p}_2(s))q_2(s)}.$$

$$y_1(s) = \frac{p_1 p_2 q_1 r(s) + (1 - \tilde{p}_2 q_2)p_1 d_2(s) + (1 - \tilde{p}_1 p_2 q_1 + (p_2 - \tilde{p}_2)q_2)d_1(s)}{(1 + (p_1 - \tilde{p}_1)p_2 q_1 + (p_2 - \tilde{p}_2)q_2)}.$$

1(d). Cascade Structure and Feedforward Control

The Second Order Plus Time Delay (SOPTD) is given by:

$$p(s) = p_1(s)\,p_2(s)$$

where by

$$p_1(s) = \frac{e^{-20s}}{15s+1} \text{ and } p_2(s) = \frac{2e^{-4s}}{s+1}$$

and

$$g_d(s) = \frac{e^{-30s}}{(10s+1)(0.3s+1)}$$

(i) Design the inner controller q_2 for $p_2(s)$ using the 1 Degree of Freedom IMC design method as in Part 1(b) with $f = \dfrac{1}{4s+1}$. Next design the outer controller q_1 for $p(s)$ using the 1 Degree of Freedom IMC design method as in Part 1(b) with $f = \dfrac{1}{(18s+1)^2}$. Using MatLab/Simulink or any simulation software, if d_1=0, plot the output response y to a step input r at time $t = 0$ sec and a step disturbance input d_2 at $t = 40$ sec.

1(d). Cascade Structure and Feedforward Control

(ii) Design the inner controller q_2 for $p_2(s)$ with $f = \dfrac{\lambda s+1}{(4s+1)^2}$ and λ is determined by setting

$$\left(1-p_2(s)q_2(s)\right)=0 \text{ at the pole of } p_1(s) \Rightarrow \left.\left(1-p_2(s)\frac{1}{\tilde{p}_{2\_}(s)}\frac{\lambda s+1}{(4s+1)^2}\right)\right|_{s=-1/15}=0$$

Use the same outer controller q_1 for $p(s)$ designed in Part 1(d)(i). Using MatLab/Simulink or any simulation software, if d_1=0, plot the output response y to a step input r at time $t=0$ sec and a step disturbance input d_2 at $t=40$ sec.

(iii) Design a feedforward controller against the distance d_1. Together with the designed IMC controllers designed in Part 1 (d)(ii), if d_1=0, plot the output response y to a step input r at time $t=0$ sec and a step disturbance input d_2 at $t=40$ sec.

(iv) Compare the responses obtained in (i), (ii) and (iii).

1(e) Automobile Cruise Control

1. In this problem, we develop a simple model of an automobile cruise control system. The control objective is to maintain a speed preset by the driver. If we Figure 2.18. Mass and damper system neglect the inertia of the wheels, and assume that friction (which is proportional to the car.s speed) is what is opposing the motion of the car, then the plant description is reduced to the simple mass and damper system shown in Figure 1.1.

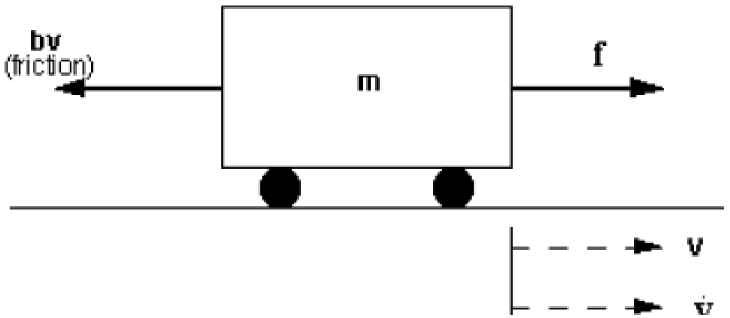


Figure 1.1

Using Newton's law of motion, the model equations for this system are

$$m\frac{dv}{dt} + bv = f$$

$$y = v$$

where f is the force produced by the engine and v is the measured velocity of the automobile. For this example, Let us assume that mass m = 1000 kg, viscous coefficient b = 50 Newton seconds/meter.

1(e) Automobile Cruise Control

The plant transfer function can be easily derived to be as follows:

$$\frac{Y(s)}{F(s)} = \frac{1}{ms+b} = \frac{0.001}{s+0.05} \tag{1.1}$$

(a) Design a Proportional-Integral-Derivative (PID) controller for the closed loop system in Figure 1.2 with plant description given in Equation (1.1). The design specifications are to achieve a rise time of 5 sec, with an allowable overshoot of 10% on the velocity and a 2% steady-state error to a step input command of 10 m/s.

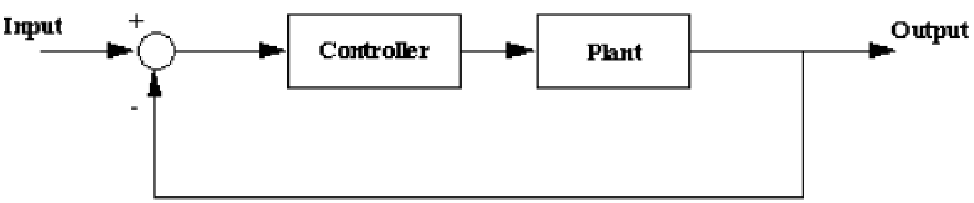


Figure 1.2

The standard PID controller is given is given as:

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \ ,$$

where K_P, K_I and K_D are the proportional, integral and derivative gains respectively which are to be determined. Plot the velocity error, the PID controller output and the actual velocity.