EE6221 Robotics and Intelligent Sensors (Part 3)

Lecture 2: Vision Sensors and Systems

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Outline

- Introduction
- Application scenarios
- Cameras
- Pinhole camera model
- Camera calibration
- For reference reading:
  - Color space representations
  - Useful image processing methods

Application Scenarios of Camera Sensors

- Vision-based control
- Vision-based estimation
- Vision-based navigation
- ……

Academic Examples

Vision-guided tracking examples

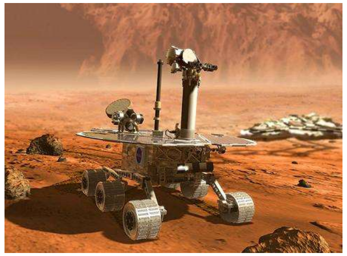
Vision-guided robot navigation

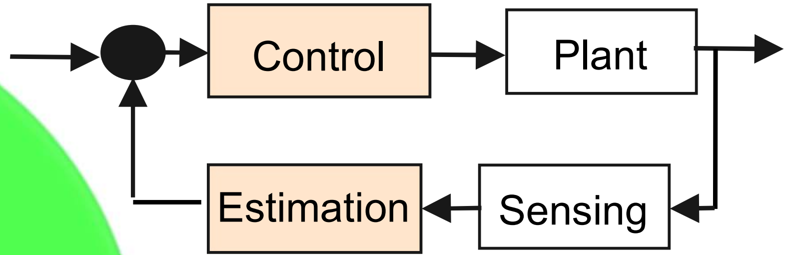
Recent Examples

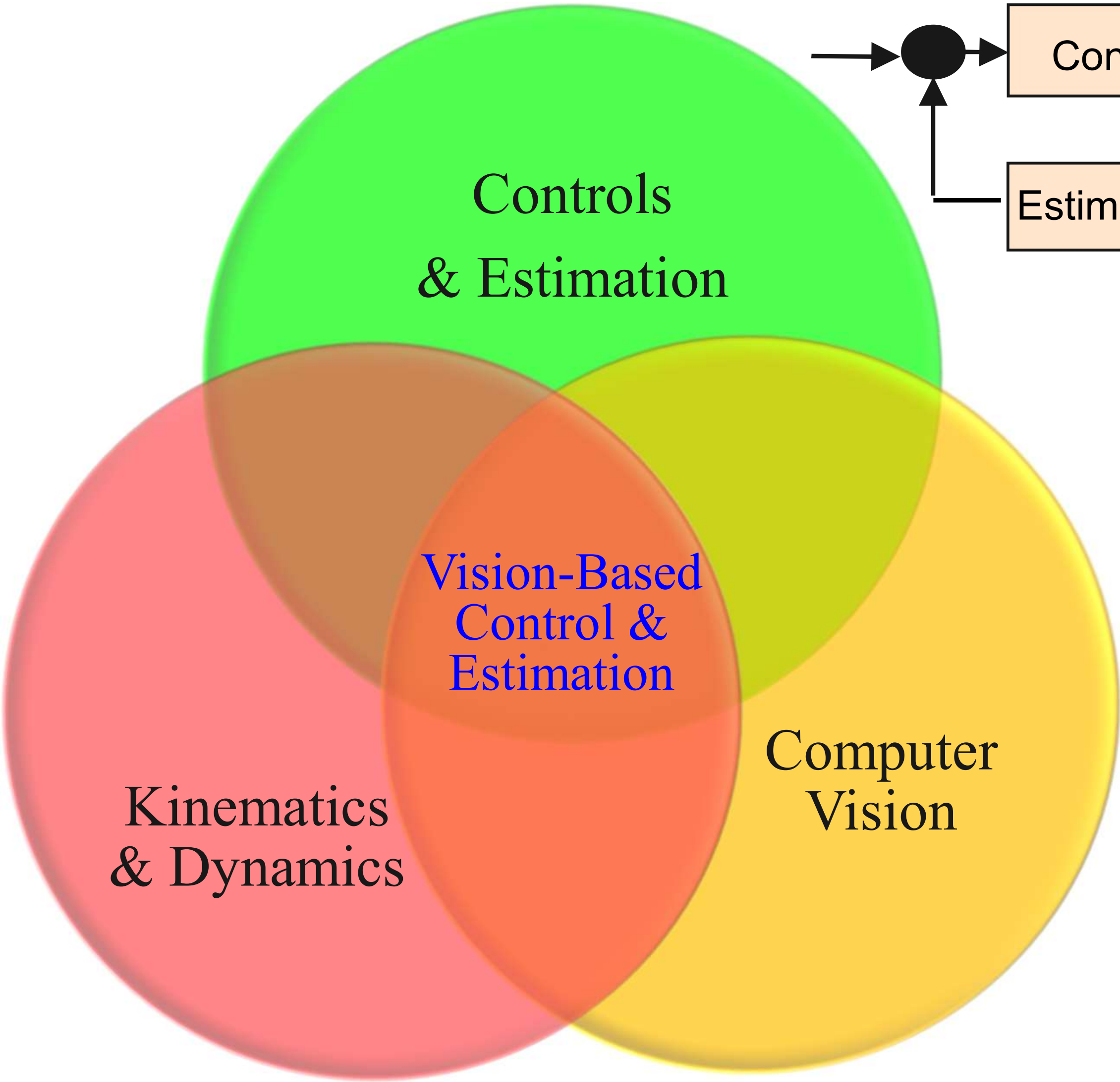
Mobile Aloha (Stanford University and Google)

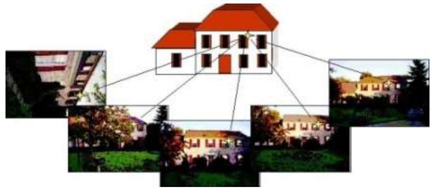
Vision-Based Control & Estimation

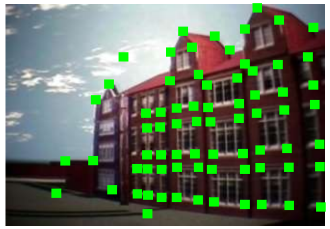
- Vision-based control and estimation: use of image data in feedback control and state estimation of moving agents (e.g., robot manipulators, mobile robots, or unmanned vehicles, etc).





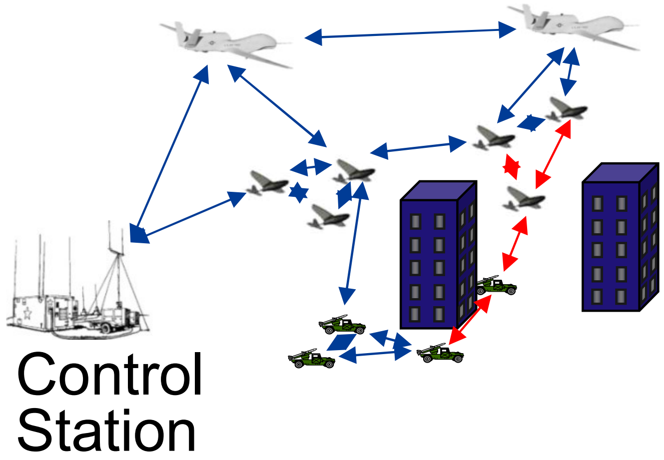


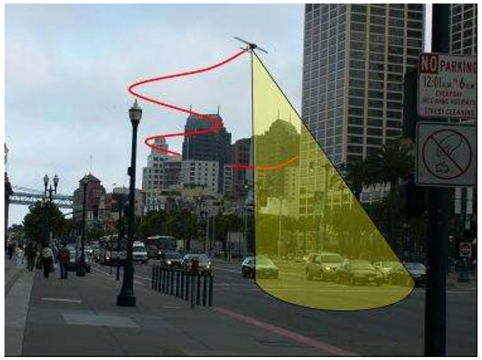


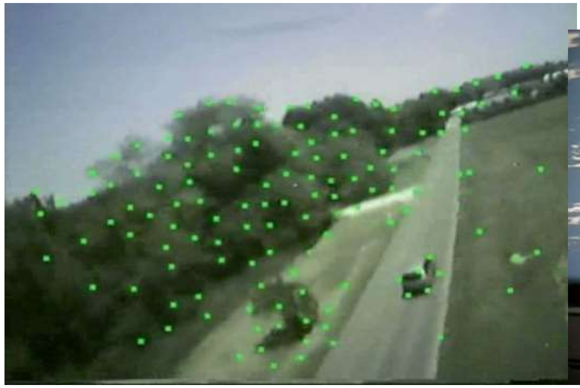


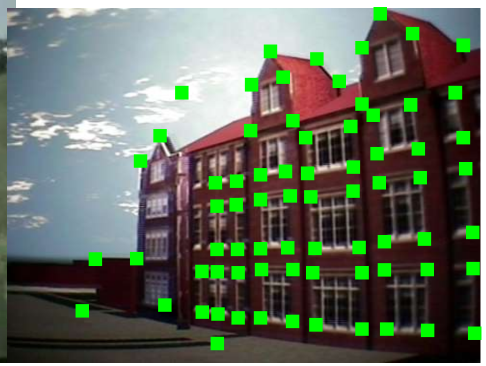
Motivation – Vision-based Control

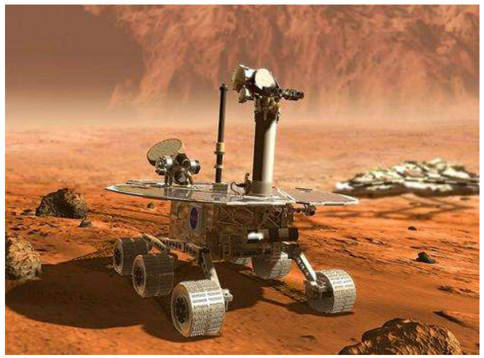
- **Why study vision-based control (also called visual servo control)?**
  - Position & orientation is typically required in navigation and control of autonomous vehicles
  - GPS may not be available, IMU has error drift
- **Advantages of vision:**
  - Vision is a rich data set, a lot of potential information
  - Vision is a passive sensor, can't be detected like sonar, radar, laser, etc.
  - Vision is intuitive to humans
  - Cameras are relatively cheap and versatile





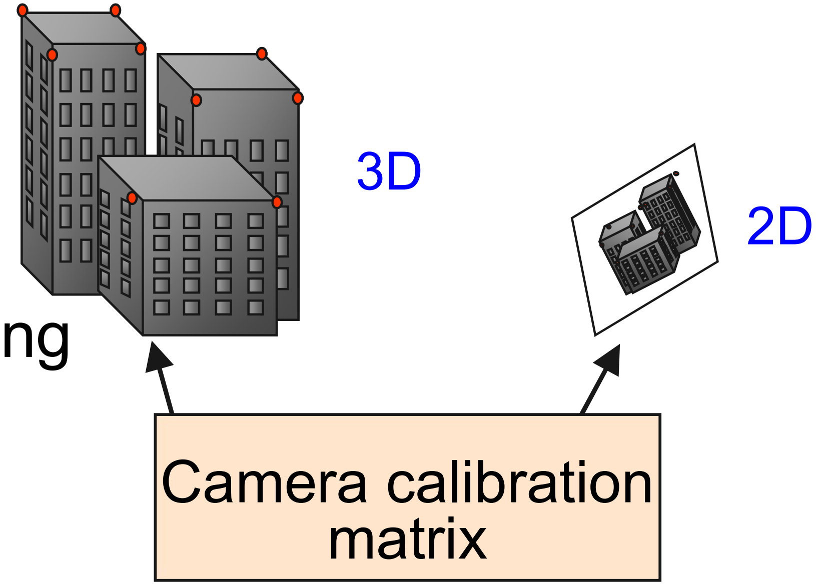


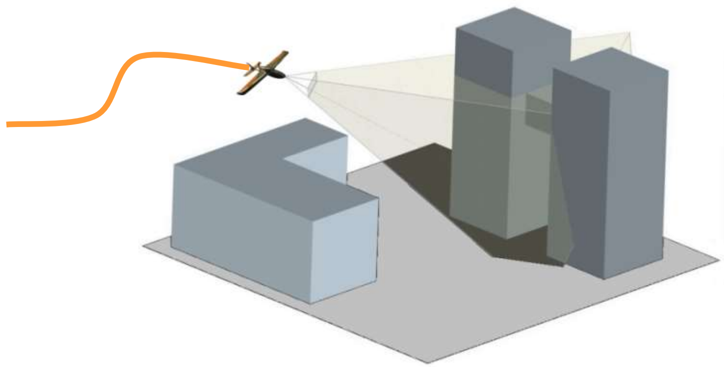


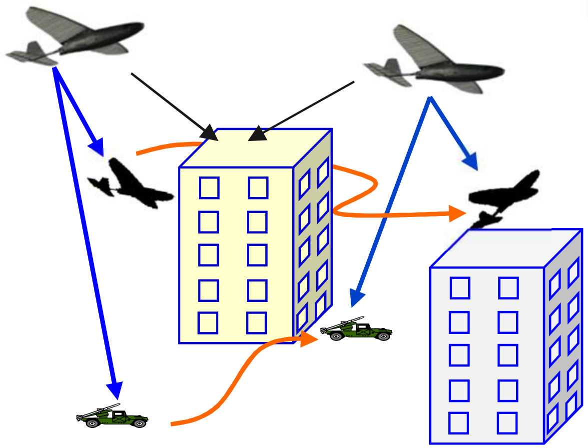


Challenges – Vision-based Control

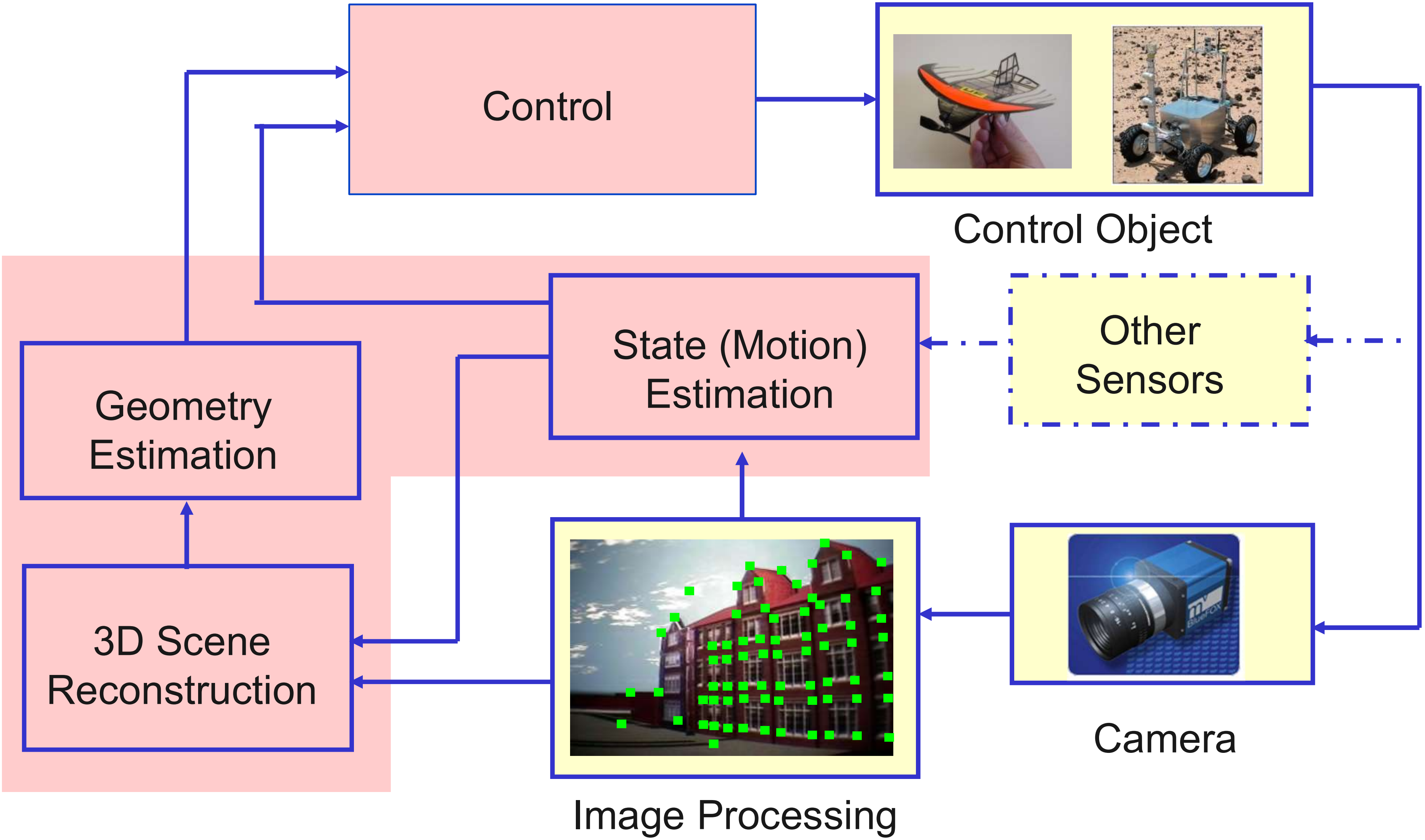
- Robust and real-time vision estimation
- Feedback through a transformation: from 3D to 2D
- Loss of depth information during imaging projection
- Camera calibration uncertainty
- Limited field of view
- Relative motion between targets
- Nonlinear, multivariable differential equations with unmeasurable states







Vision-Based Control & Estimation



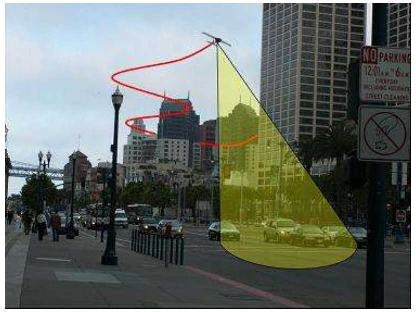
Applications

Systems:

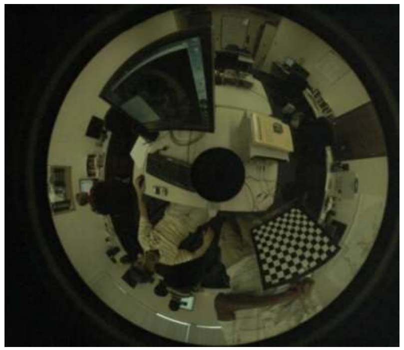
- Autonomous ground vehicles
- Smart cars
- Unmanned air vehicles
- Mobile sensor networks
- Nano/micro manipulation
- Mobile manipulators
- ……










Imaging-Introduction

- Cameras are composed of several key components
  - The lens collects and focuses light
  - Imaging surface measures intensity/frequency of light
- We will discuss simple models for imaging that are accurate especially when using quality cameras
- We will discuss camera calibration, which is essential for image based pose and structure estimation, and can increase the accuracy of simple models, even for low quality cameras
- We will discuss common, simple image processing routines that may prove useful

Imaging-Introduction

Reference Books:

- Computer Vision: A Modern Approach, by Forsyth and Ponce
- Computer and Robot Vision, by Haralick and Shapiro
- An Invitation to 3-D Vision by Ma, Soatto, Kosecka and Sastry

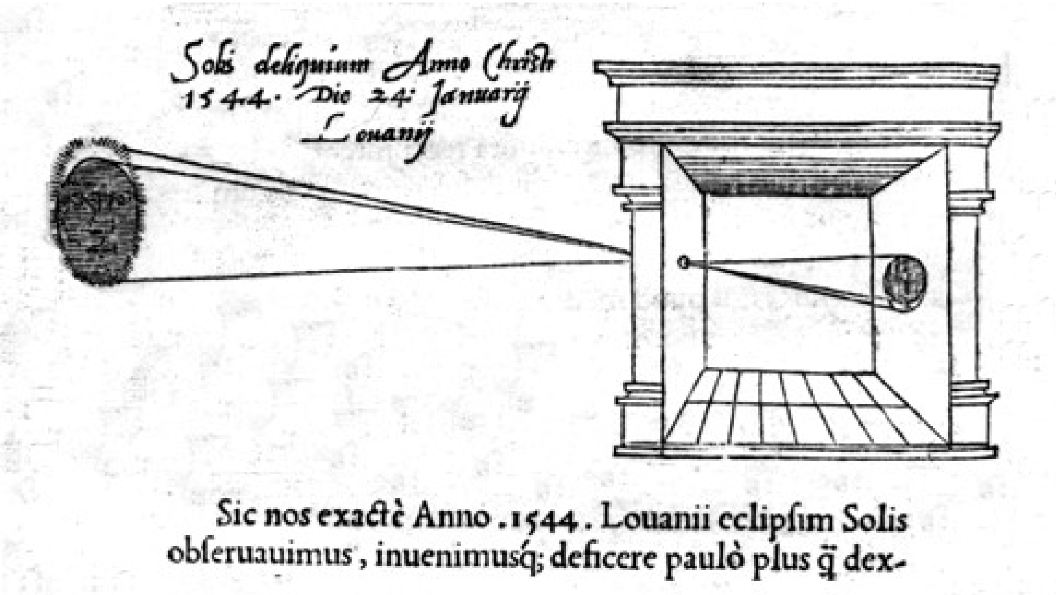
Imaging-Introduction

Free software resources for image processing:

- Intel Open Source Computer Vision Library for C++ (OpenCV)
  - http://www.intel.com/technology/computing/opencv/
- Machine Vision Toolbox for Matlab
  - http://www.petercorke.com/Machine%20Vision%20Toolbox.html
- Camera Calibration Toolbox for Matlab
  - http://www.vision.caltech.edu/bouguetj/calib_doc/
- Image Processing Toolbox for Matlab
  - Included in most full versions of Matlab
- GNU Image Manipulation Program (Image Processing)
  - http://www.gimp.org/
- Virtual Dub (Video Processing/Editing)
  - http://www.virtualdub.org/index

Camera History

- The Pinhole Camera Model was discussed by Aristitotle and Euclid in 3rd Century BC
- Ibn al-Haitham is credited with building the first pinhole camera (camera obscura) in 10th Century AD
- Giovanni Battista della Porta first added a lens to focus light in 15th Century
- Boyle and Hooke made portable models in 17th Century
- First photograph was by Niépce in 1826
- First color photo by Maxwell in 1861



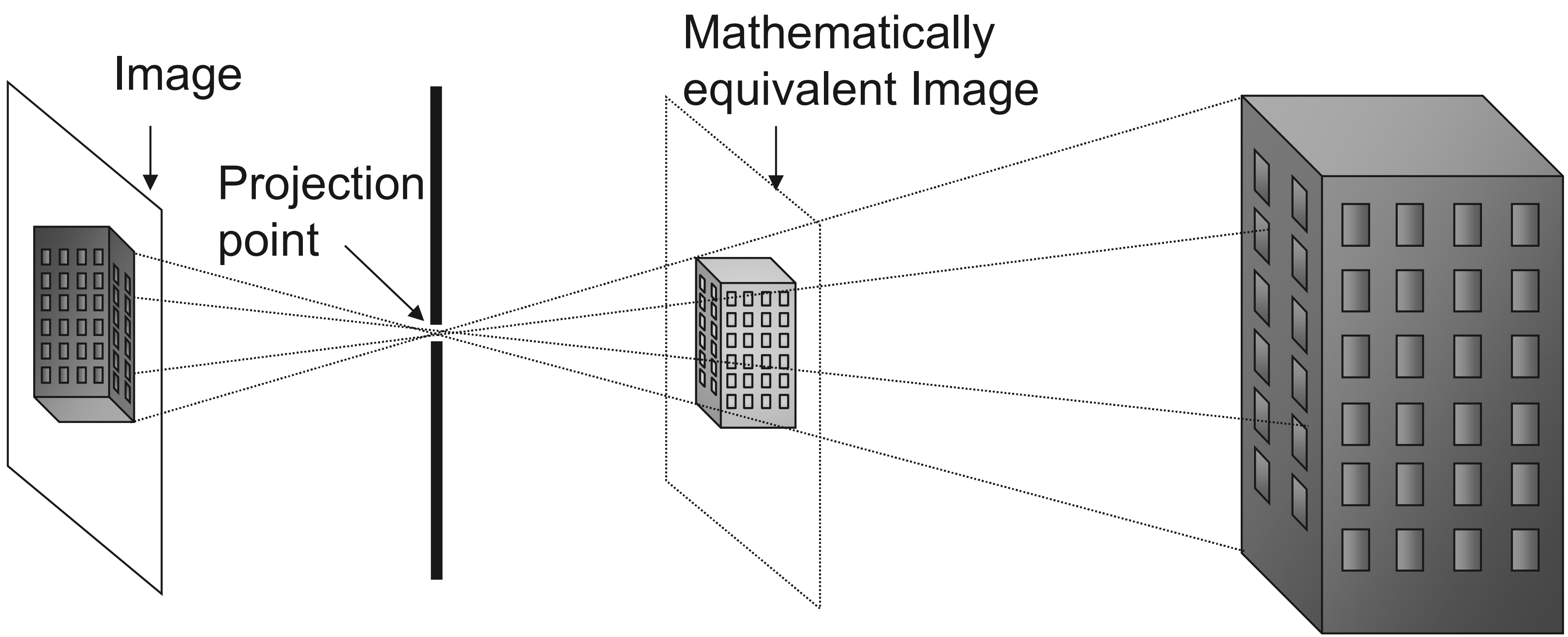
Reinerus Gemma-Frisius, 1544

Camera History

- First flexible film developed by Eastman Kodak 1885
- First electronic camera (precursor to TV cameras) developed by Farnsworth in 1927
- Video tape recorders introduced in 1951 by Bing Crosby Enterprises
- First digital camera built by Sasson w/ Eastman Kodak in 1973
- First commercial digital camera by Eastman Kodak in 1991 ($13K!)

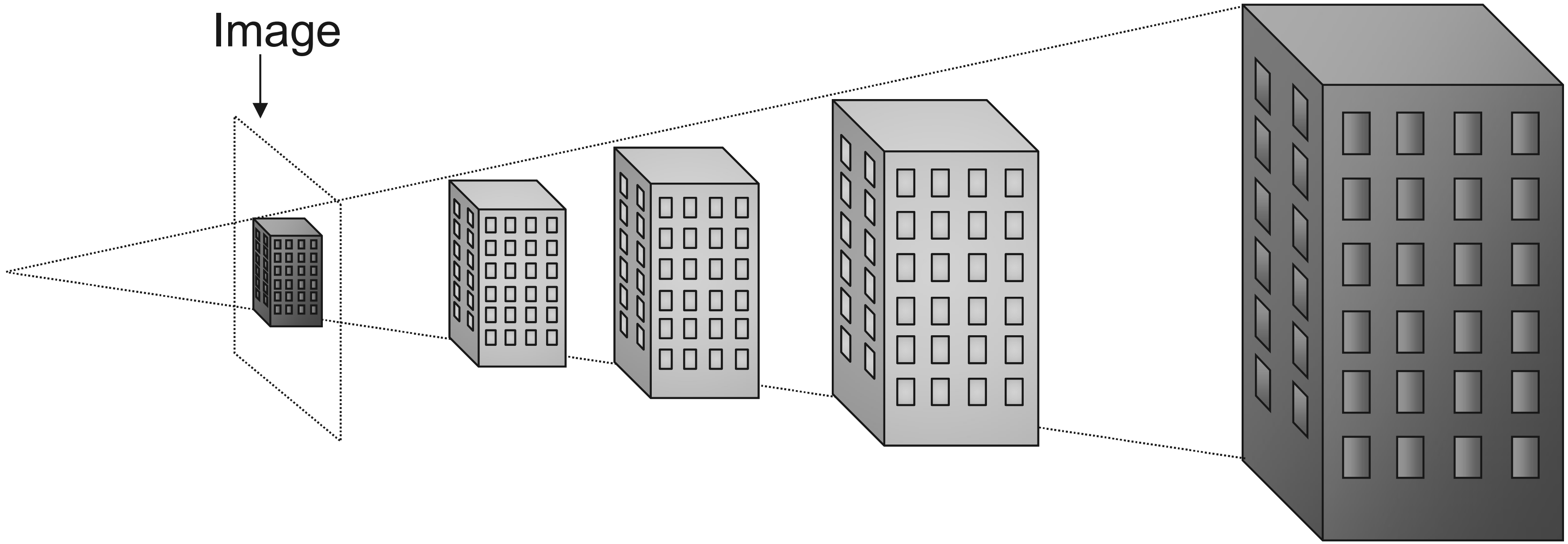
Kodak DCS-100, 1991

Pinhole Projection Model



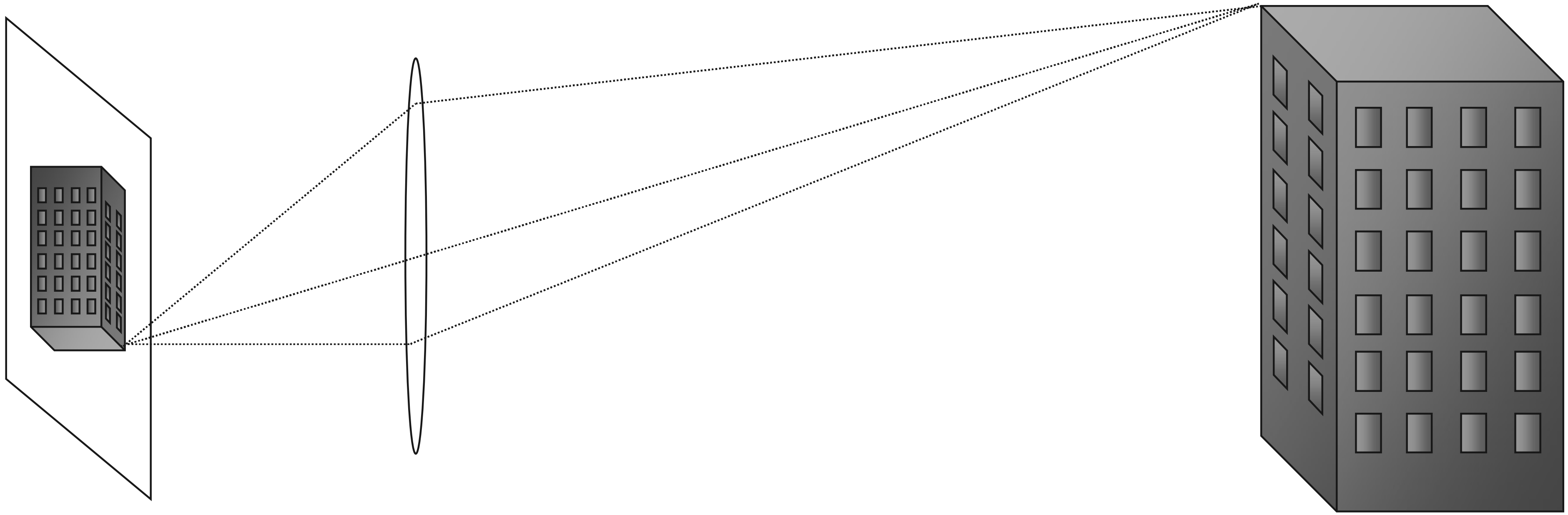
- Rays from every point in the scene intersect at the projection point and continue through to the imaging surface
- Results in an inverted image
- Mathematically, we can consider an imaging surface in front of the projection point, which is not inverted

Pinhole Projection Model



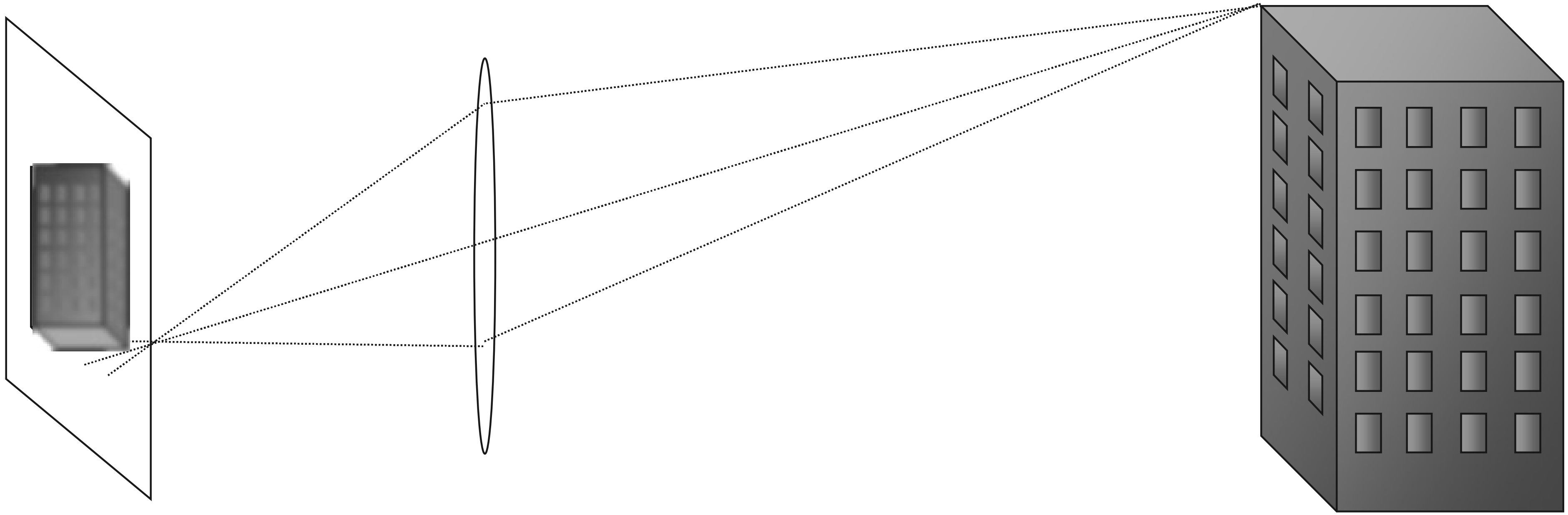
- All point along the rays intersect the image plane at the same point
- This results in a loss of size and depth information
- A priori knowledge or additional sensors/measurements can determine scale
- Fundamental drawback to imaging as a sensor

Perfect Lens Projection Model



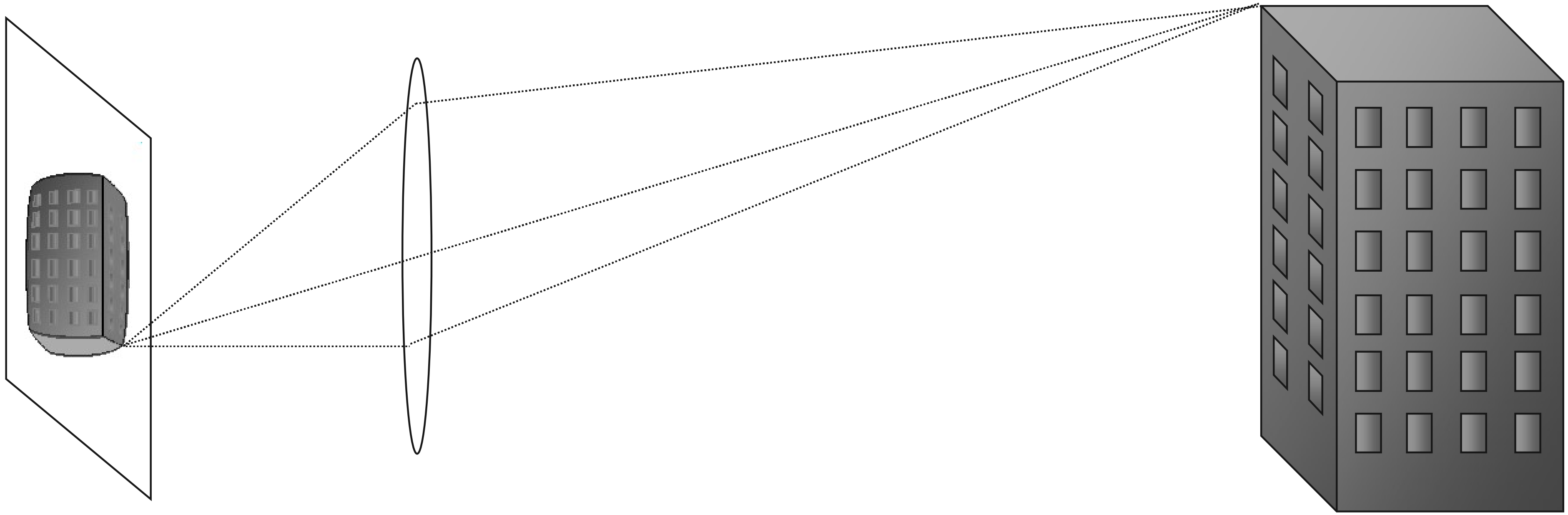
- A lens collects light by focusing many rays from a 3D point to same point on image plane
- A perfect lens is identical to the pinhole camera model at a specific focal length

Imperfect Lens Projection



- If the distance between the lens and imaging surface is not correct, rays from a 3D point will not be focused to a single point on the image
- This results in a blurred image
- Images can be sharpened, but not of much use in estimation and control

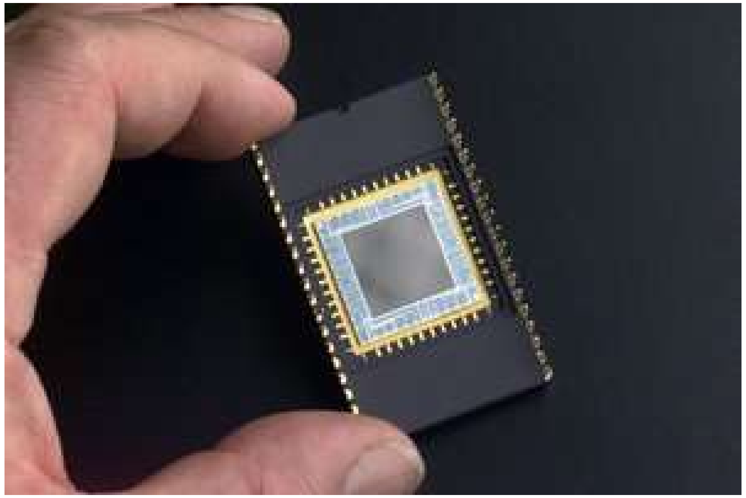
Imperfect Lens Projection



- Real lens will have imperfections which lead to misfocus (blur) or distortion
- Radial distortion occurs when 3D lines are not projected to 2D lines
- Calibration can learn distortion parameters and remove them from an image – approximating pinhole camera model

Digital Imaging

- In digital cameras, the imaging surface is an array of light sensitive elements (e.g. CCD's)
- The amount of incident light on each element is recorded as a number



- The image is discretized in two ways
  - The image is broken into discrete pixels with coordinates $[u,v]$
  - The value of each pixel takes discrete values in some range
- Video is further discretized temporally
- This results in some amount of quantization noise

Digital Imaging

[0,0]

255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	254	233	182
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	237	207	171	192
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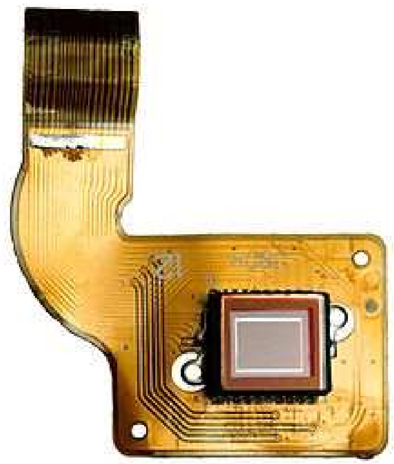
[31,31]

32x32 pixel image
Each pixel is eight bits

- For an eight-bit image, pixels take integer values in range 0…255
- Typically pixel coordinates are counted from top left to bottom right, though some cameras count from bottom left

Digital Imaging

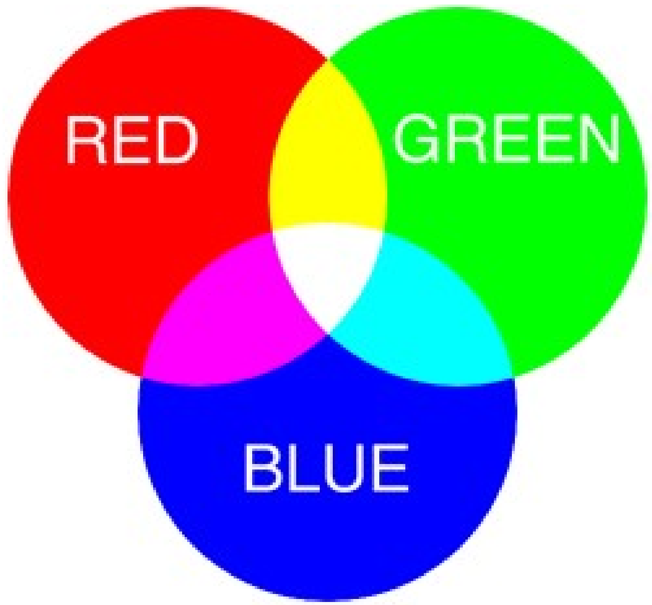
- “Charge coupled device” (CCD) and “complementary metal oxide semiconductor” (CMOS) are typical light sensing technologies
- Performance and cost of the two are converging, but typically:
  - CCD more prevalent
  - CCD delivers less noisy image than CMOS
  - Light sensitivity for CCD better than CMOS
  - CMOS requires less power than CCD
  - CMOS cheaper to manufacture than CCD



A CCD image sensor on a flexible circuit board

Digital Imaging

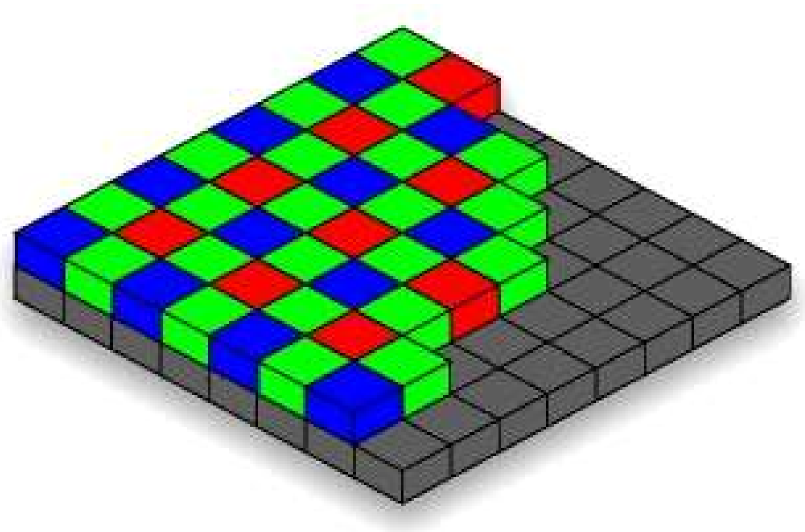
- In grayscale images, pixel has a single measurement of light intensity from pitch black to pure white
- In color images, each pixel has three measurements associated with it, typically red, green and blue (RGB), which indicate the intensity of light at that frequency

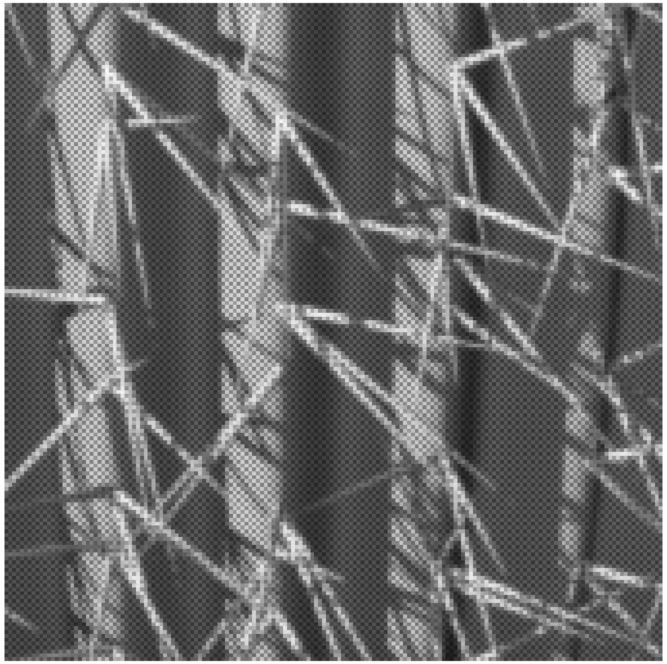


- RGB are primarily color of light and combine to form other colors
- The human eye is more sensitive to green light, so color cameras generally are more sensitive to green as well.
- RGB to grayscale conversion is not simply the norm of RGB values

Digital Imaging

- Note that most color cameras do not have separate receptors for each color
- A pattern of color filters, known as “Bayer Pattern” are placed over a single surface
- Resulting image is filtered by camera software/firmware to provide RGB image, but high speed camera may deliver raw Bayer image



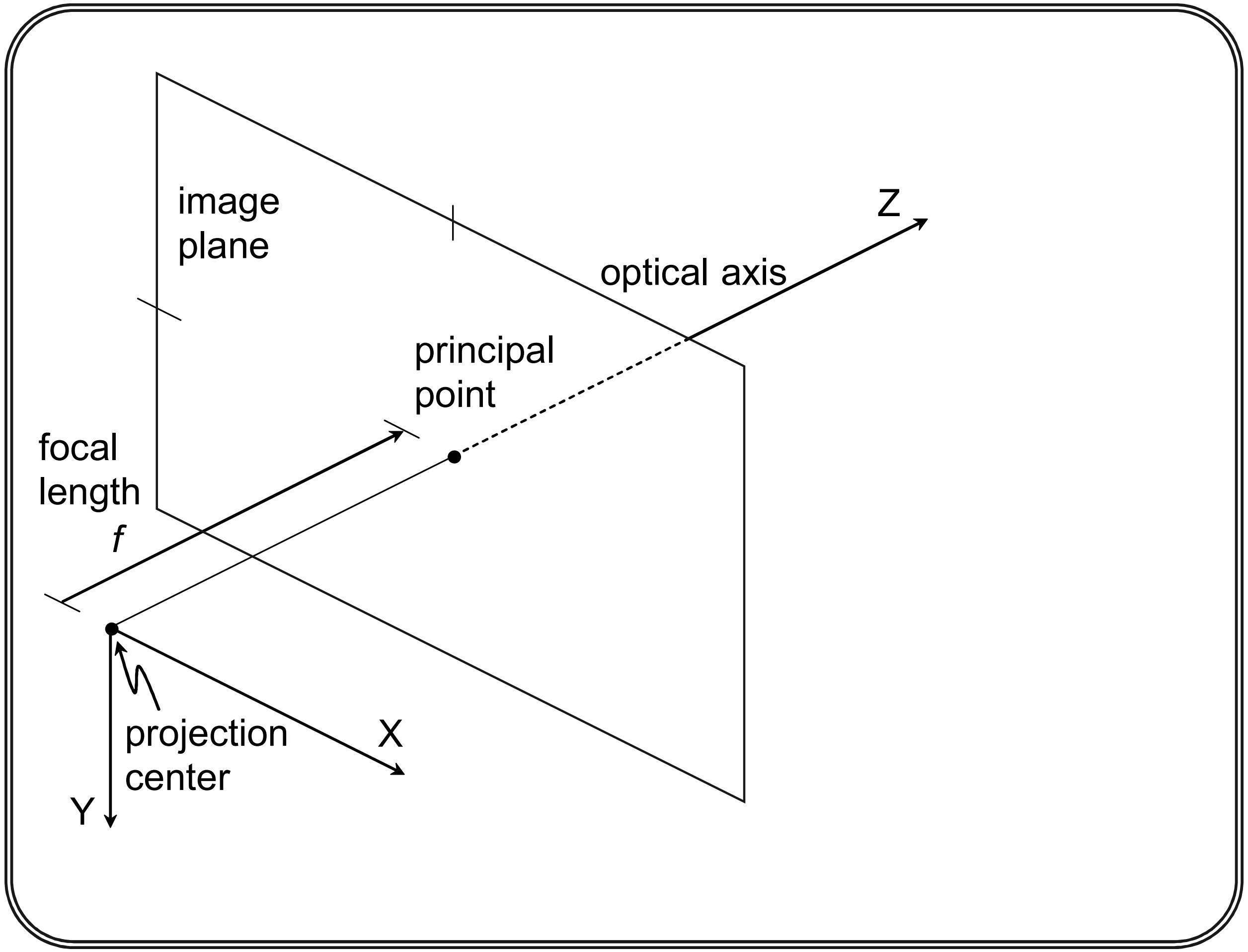


Pinhole Camera Model

- The pinhole camera model is a first order approximation of camera optics
- Widely used and essential for structure from motion algorithms
- Mathematics are simple and accurate for most quality cameras
- If higher order optical effects are pervasive, proper camera calibration can often make the first order approximation valid

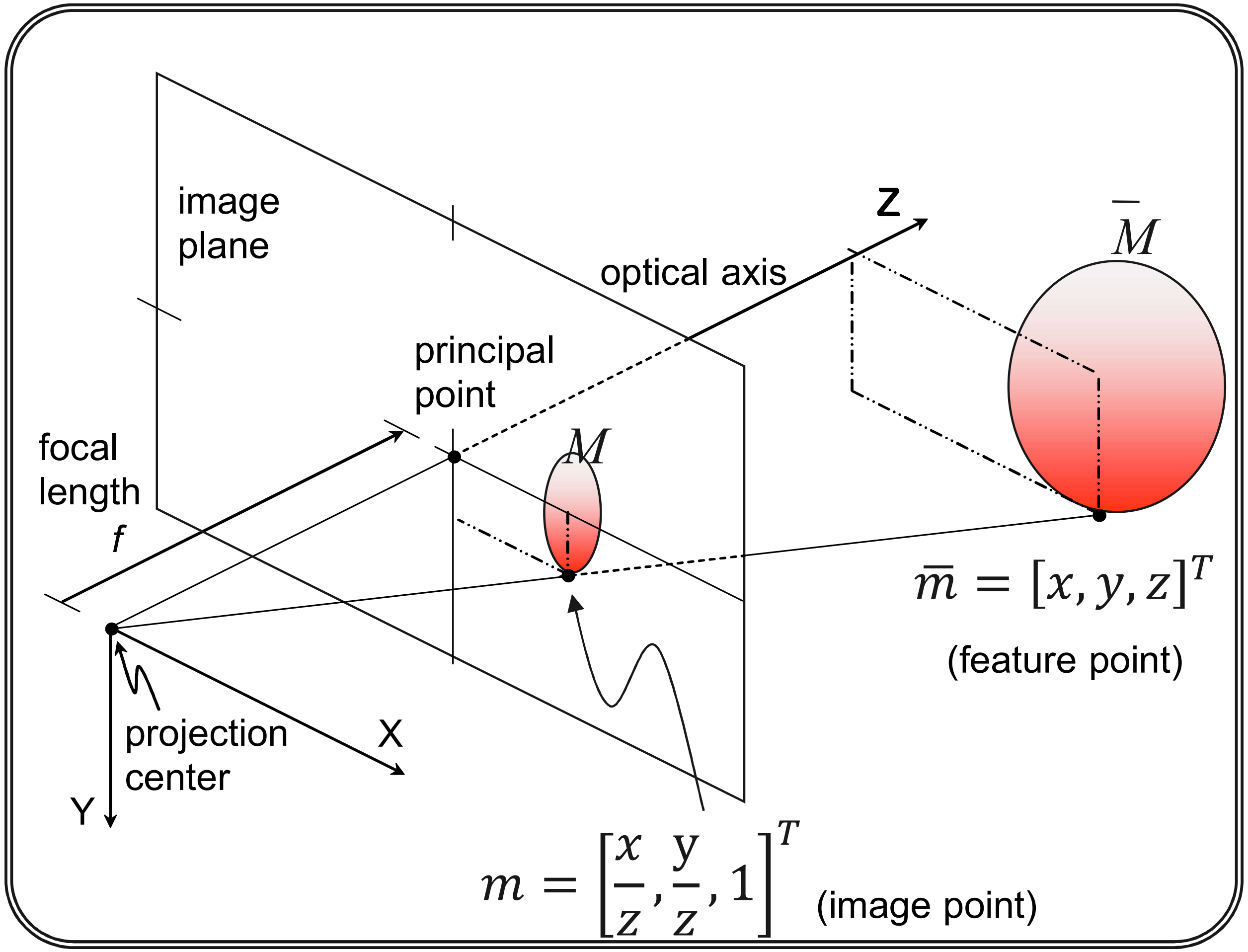
Pinhole Camera Model

- Reference frame $\mathcal{F}$ is attached to the camera.
- The origin of $\mathcal{F}$ is the "projection center"
- The imaging surface is a plane orthogonal to the *z* axis and intersects the *z* axis at a distance *f*
- Point of intersection is the "principal point"
- *f* is the "focal length" of the camera, for simplicity assume *f*=1



Pinhole Camera Model

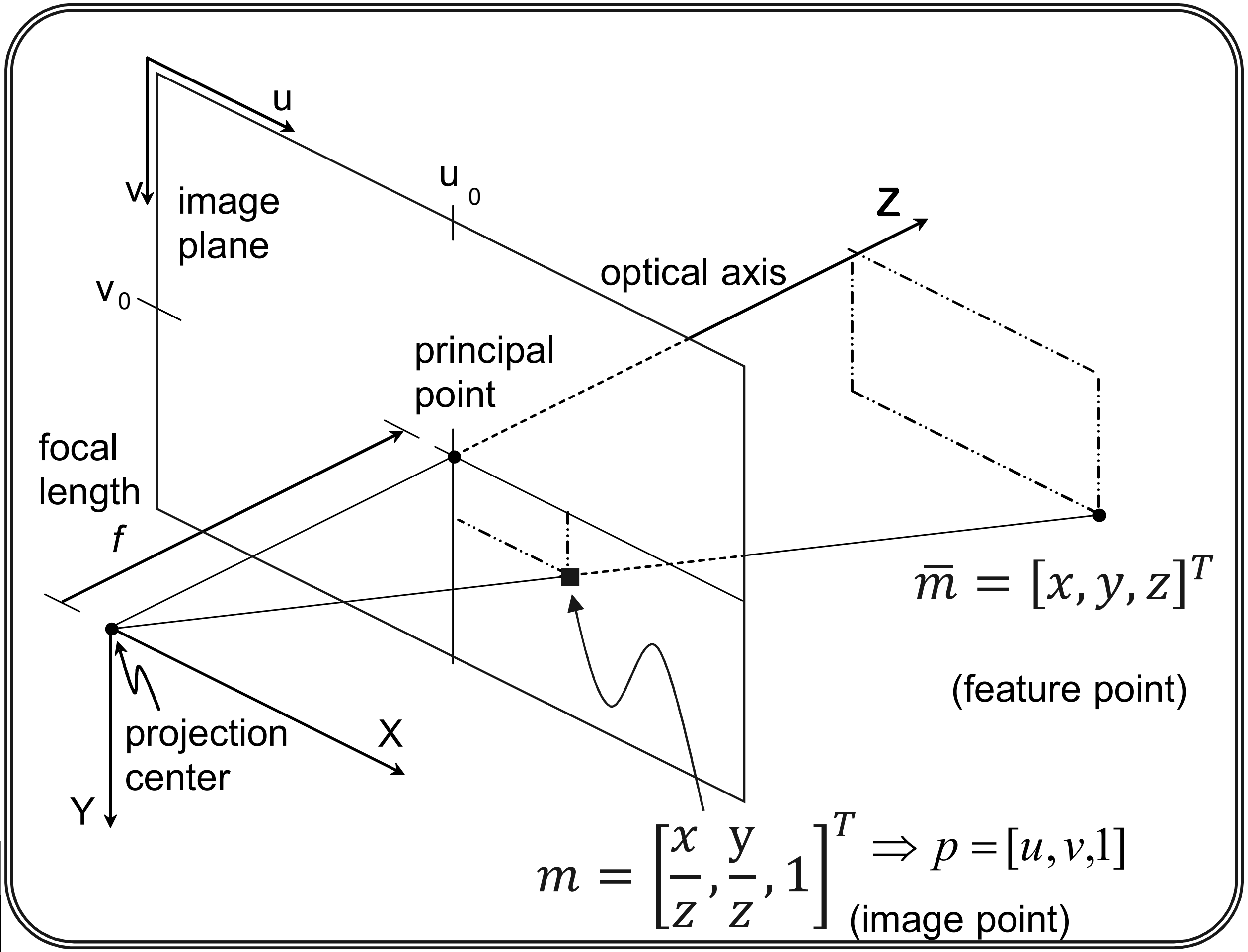
- A feature point has 3D coordinates $\bar{m} = [x, y, z]^T$ in the camera frame
- It projects to a point in the image plane with coordinates $m = \left[\frac{x}{z}, \frac{y}{z}, 1\right]^T$
- The projection of a point can be denoted by function $m = \pi(\bar{m}) = \frac{1}{z}\,\bar{m}$
- A curve or surface given as a set of 3D points $\bar{M}$ projects to $M = \pi(\bar{M})$



Pinhole Camera Model

- When dealing with a digital camera, we must account for the number of pixels, shape of pixels, and size of pixel elements

- Image point m is discretized to pixel coordinates $p=[u,v,1]^T$



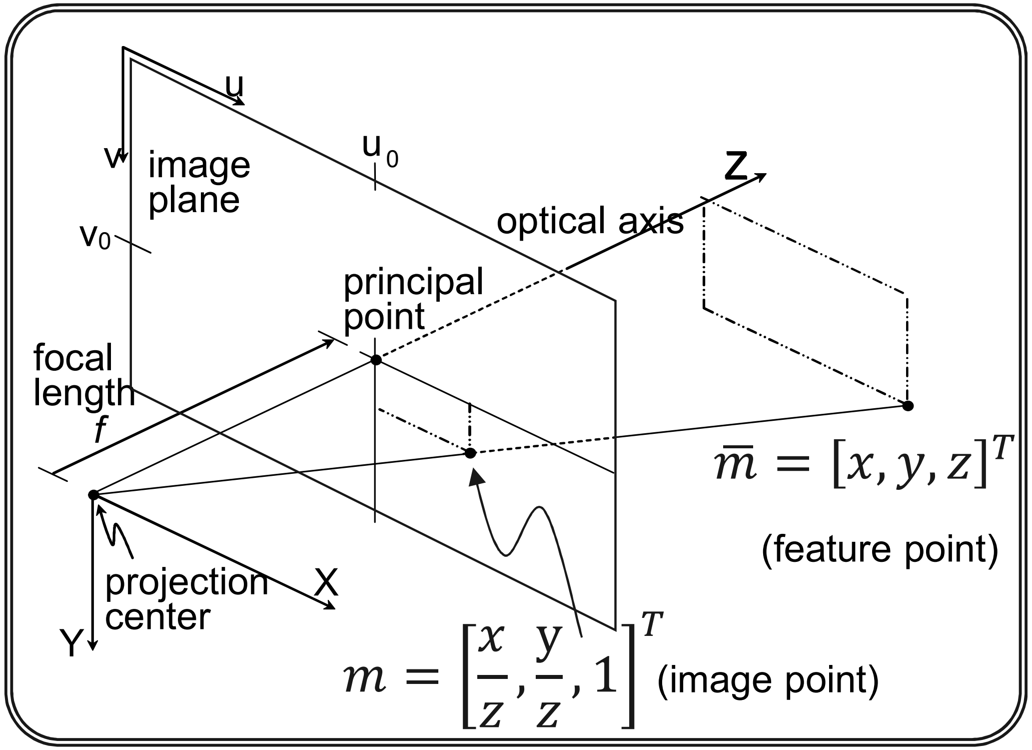
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f\sigma_x & -f\sigma_x \tan\alpha & u_0 \\ 0 & f\sigma_y \sec\alpha & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \\ 1 \end{bmatrix}$$

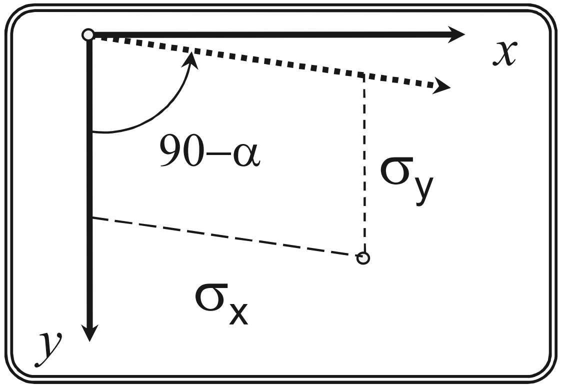
$$p = Am$$

Pinhole Camera Model

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f\sigma_x & -f\sigma_x \tan\alpha & u_0 \\ 0 & f\sigma_y \sec\alpha & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \\ 1 \end{bmatrix}$$

$$p = Am$$



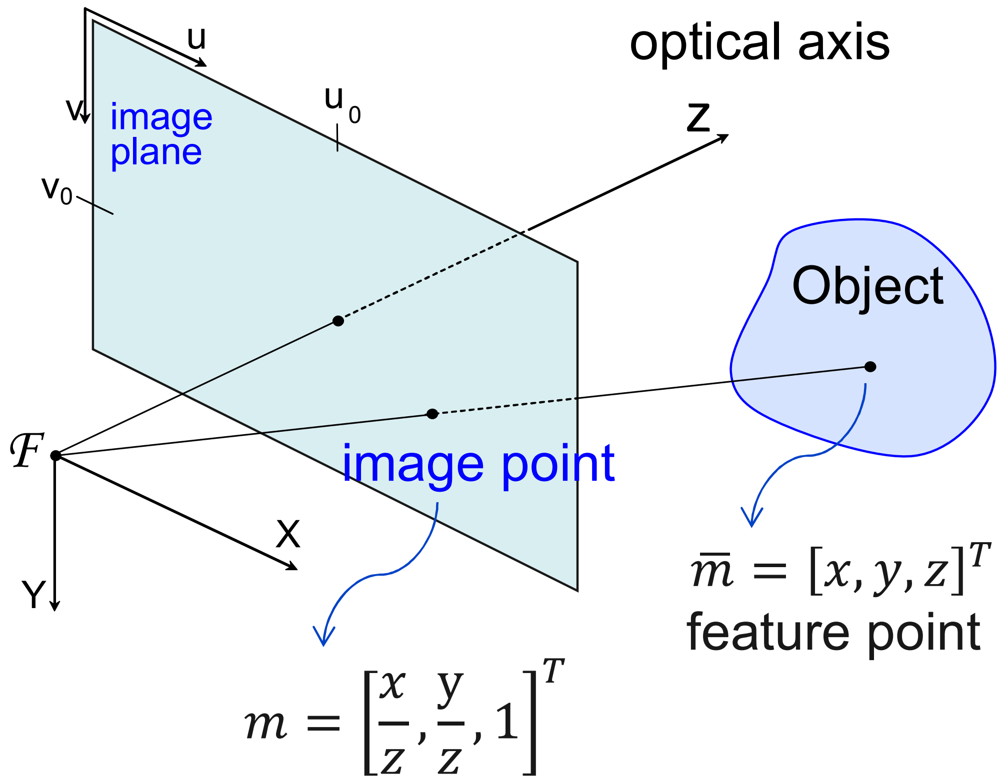


- A is intrinsic calibration matrix
- f is camera focal length
- σ_x, σ_y are size of pixels in focal lengths
- $[u_0, v_0]$ is the principal point in pixels
- α is skew angle, usually ≈0

Pinhole Camera Model (Summary)

- 3D feature point with Euclidean coordinates in camera frame $\mathcal{F}$

$$\bar{m} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



- Projected to image plane with normalized coordinates in $\mathcal{F}$

$$m = \begin{bmatrix} m_x \\ m_y \\ 1 \end{bmatrix} = \frac{\bar{m}}{z} = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$

- Mapped to pixel coordinates by Calibration Matrix

$$p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = Am = \begin{bmatrix} f\sigma_x & -f\sigma_x \tan\alpha & u_0 \\ 0 & f\sigma_y \sec\alpha & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ 1 \end{bmatrix}$$

A: camera calibration matrix

Pinhole Camera Model (Remarks)

- Many estimation and control schemes require normalized coordinates for points
- Given pixel coordinates p of a point in the image and knowledge of calibration matrix A, recover normalized coordinates

$$m = A^{-1} p$$

- There is no way to recover the Euclidean coordinates without additional information. Depth ambiguity is a consequence of imaging.

Camera Calibration

- Consider an inertial, world reference frame $\mathcal{F}_w$
- The camera frame $\mathcal{F}_c$ has translation T and rotation R w.r.t. $\mathcal{F}_w$.
- A point has coordinates $m_w=[x_w,y_w,z_w]^T$ in the world frame.
- Pixel coordinates of the point in the image are given by

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f\sigma_x & -f\sigma_x \tan\alpha & u_0 \\ 0 & f\sigma_y \sec\alpha & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Intrinsic calibration matrix}} \underbrace{\begin{bmatrix} 1/z & 0 & 0 & 0 \\ 0 & 1/z & 0 & 0 \\ 0 & 0 & 1/z & 0 \end{bmatrix}}_{\text{Projection matrix}} \underbrace{\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}}_{\text{Extrinsic calibration matrix}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

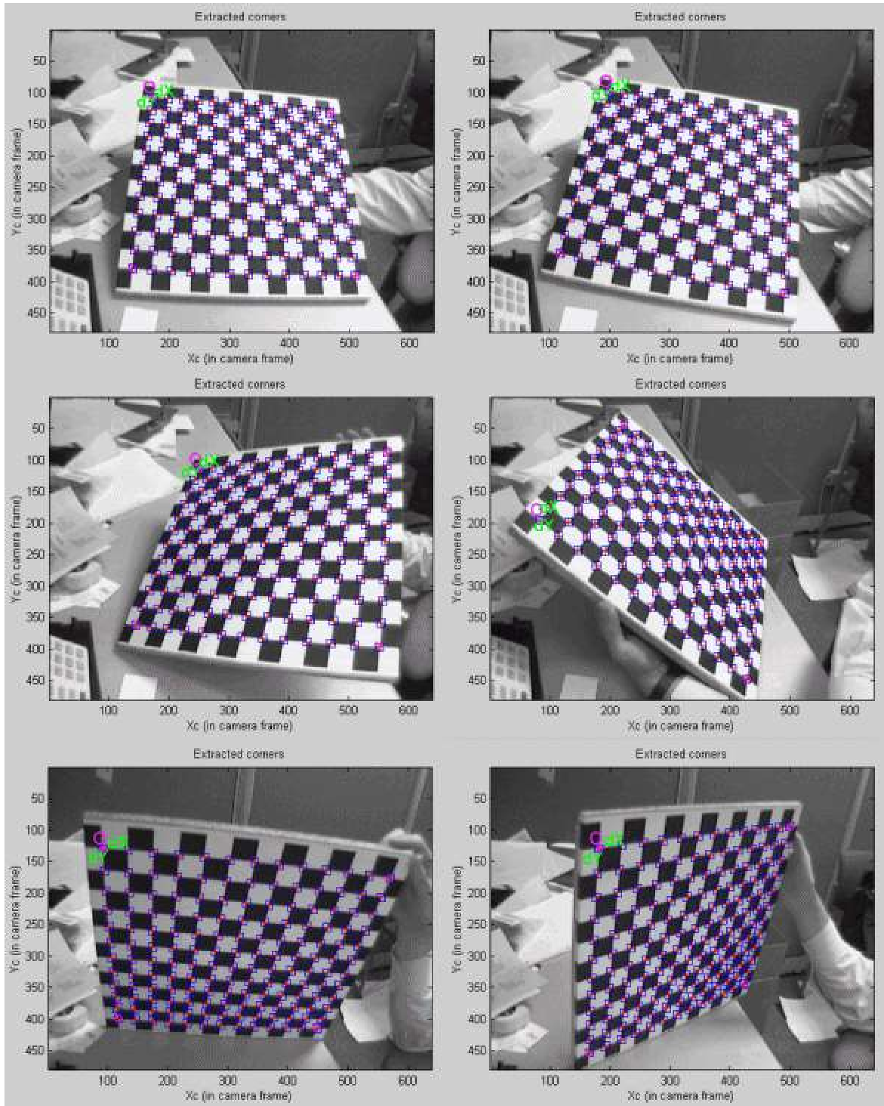
- Calibration is the processing of recovering these parameters

Camera Calibration

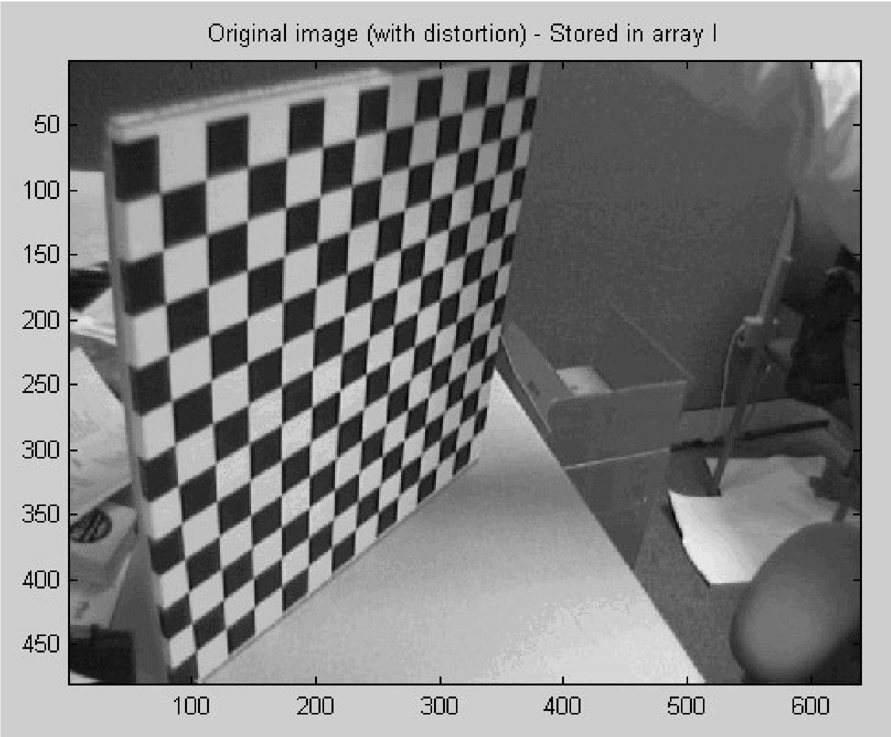
- There are many calibration techniques, and many free resources
- Intel Open Source Computer Vision Library
  - http://www.intel.com/technology/computing/opencv/
- Camera Calibration Toolbox for Matlab
  - http://www.vision.caltech.edu/bouguetj/calib_doc/
- The DLR Camera Calibration Toolbox
  - http://www.dlr.de/rm-neu/desktopdefault.aspx/tabid-3925/

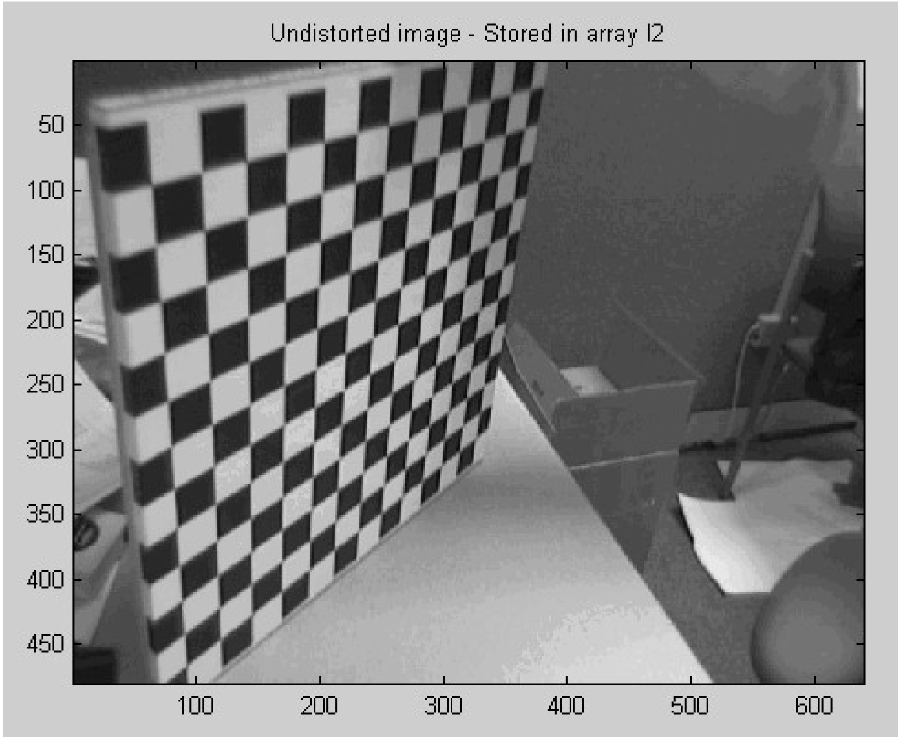
Camera Calibration

- Most calibration methods require some sort of known target such as a calibration board
- Numerous images of target are taken
- Features, such as corners, are extracted
- Comparison of known target and image points allows solving of intrinsic and extrinsic camera parameters
- Intrinsic calibration parameters are usually solved simply as the elements of the calibration matrix A



Camera Calibration





- In addition to intrinsic calibration matrix, most calibration methods also give nonlinear distortion parameters which can undistort images

$$m_d = \left(1 + k_1 r^2 + k_2 r^4 + k_5 r^6\right)\begin{bmatrix} m_x \\ m_y \end{bmatrix} + \begin{bmatrix} 2k_3 m_x m_y + k_4\left(r^2 + 2m_x^2\right) \\ 2k_4 m_x m_y + k_3\left(r^2 + 2m_y^2\right) \end{bmatrix}$$

Exercise (not mandatory)

Try one of the calibration example by yourself using matlab

There are many calibration techniques, and many free resources

Intel Open Source Computer Vision Library

http://www.intel.com/technology/computing/opencv/

Camera Calibration Toolbox for Matlab

http://www.vision.caltech.edu/bouguetj/calib_doc/

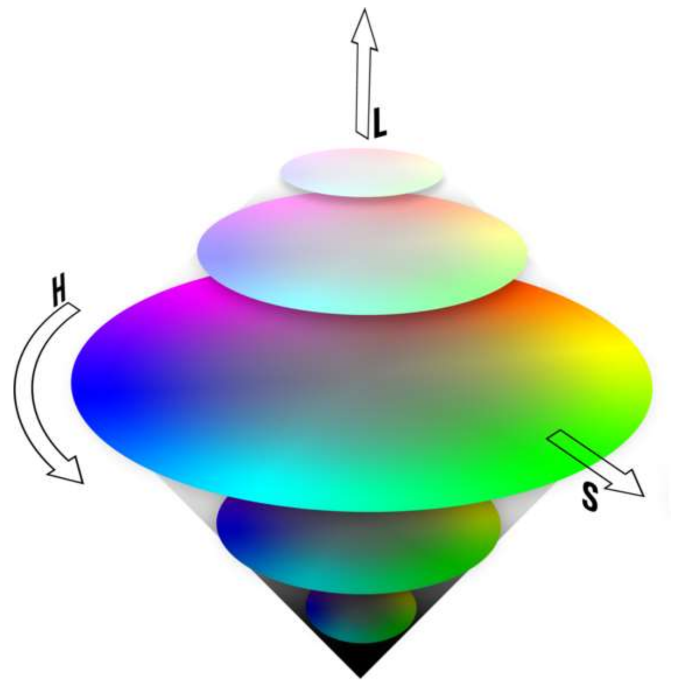
The DLR Camera Calibration Toolbox

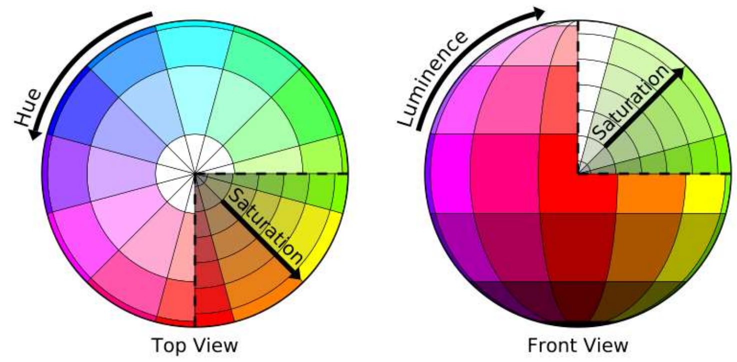
http://www.dlr.de/rm-neu/desktopdefault.aspx/tabid-3925/

- The following slides are for reference reading if interested

Useful Imaging Methods - Color Spaces

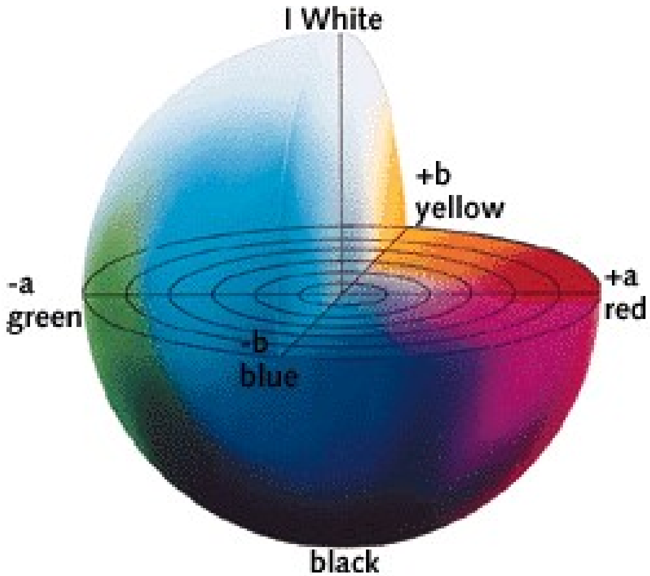
- Several useful alternatives to RGB color space
- Hue Saturation Luminescence (HSL) color space
  - Color mapped to Hue value which is cyclical
  - Light intensity (white to black) maps to Luminescence
  - Color intensity (grey to bright) maps to Saturation
- Particularly useful for finding objects of similar colors





Useful Imaging Methods - Color spaces

- CIELAB color space
  - Light intensity (white to black) maps to Luminescence
  - A and B denote color and intensity
- A "linear" color space
- Useful when "distance" between colors is needed to differentiate objects



Useful Imaging Methods - Image Processing

- In filtering a small 2D “window” or “kernel” of values is convolved with each pixel of the image
- For each pixel in the image, take the sum of products of multiplying the kernel by the pixel values

$$\frac{1}{21} * \begin{bmatrix} 1 & 3 & 1 \\ 3 & 5 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

3x3 Gaussian Kernel

255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	254	233	182
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	237	207	171	192
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	244	254	178	162
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	253	243	252
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	253	234	231	234	213
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	253	232	221	243	250
255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	252	183	181	163
255	255	255	255	255	255	255	255	255	255	254	255	255	254	254	255	255	254	254	254	254	255	255	255	255	255	255	254	252	168	181	174
255	255	255	255	255	255	255	255	255	238	222	225	214	206	200	195	183	201	202	187	197	202	203	230	253	254	255	255	237	254	161	224
255	255	255	255	255	255	255	255	255	242	202	216	214	214	216	200	197	215	215	217	205	189	197	228	255	255	255	255	246	253	153	229
255	255	255	255	255	255	255	254	255	224	201	210	206	207	224	198	207	202	193	179	206	194	189	205	254	255	255	255	252	250	163	240
253	253	251	254	254	254	255	255	251	190	210	199	220	190	200	203	212	201	223	204	184	215	202	185	246	255	252	254	250	254	156	220
254	254	255	220	183	211	215	253	189	206	214	202	220	220	202	214	211	193	191	220	212	184	182	222	209	215	193	210	118	129	158	241
255	253	254	252	181	152	160	163	171	136	125	127	126	139	113	124	117	108	111	119	132	88	141	107	127	137	136	129	107	237	159	245
163	210	247	254	238	143	130	131	119	139	140	117	121	129	127	120	124	122	105	127	128	104	127	128	84	96	89	113	253	215	150	212
157	168	160	182	185	243	130	115	111	108	103	130	118	103	101	91	96	92	94	94	94	116	101	94	85	96	96	201	255	251	151	238
95	133	126	139	187	254	237	145	138	146	130	133	153	144	147	139	146	138	153	138	135	143	128	137	128	119	225	248	251	221	137	201
107	140	119	160	134	152	169	131	133	132	117	132	127	123	119	124	118	121	120	116	116	118	108	113	122	115	135	128	105	138	123	131
85	88	104	113	127	109	185	147	109	119	139	115	138	146	128	135	131	130	137	124	129	141	136	89	109	117	82	101	70	76	88	174
99	106	94	101	109	138	206	88	75	85	85	77	86	82	82	85	77	90	69	81	79	78	73	79	73	97	139	119	95	84	147	207
97	78	116	91	101	99	200	175	102	138	130	81	82	93	87	71	94	92	128	81	75	120	123	130	77	144	188	195	139	124	88	105
98	96	123	100	93	110	190	207	127	187	179	76	90	100	91	87	102	102	160	100	91	163	177	183	97	133	220	226	119	114	115	112
103	63	70	68	111	146	207	139	141	232	237	77	98	105	96	97	110	99	189	109	92	217	215	226	101	128	253	153	82	104	153	130
60	78	99	71	70	93	233	236	148	147	125	185	180	201	203	208	208	193	198	185	190	189	135	138	135	147	138	133	126	125	106	109
57	114	50	64	89	95	133	128	155	67	242	183	183	191	202	188	198	193	209	193	195	194	193	250	159	158	173	142	159	143	134	134
63	91	70	75	86	88	133	94	109	114	104	119	130	121	130	125	146	117	131	134	126	134	105	118	123	125	97	89	127	121	113	134
45	87	109	108	116	123	67	102	100	115	117	126	104	116	109	91	90	84	113	113	102	96	122	92	97	99	115	123	139	86	118	139
59	64	103	81	119	72	56	71	84	74	64	79	72	70	86	82	116	108	67	71	71	83	73	148	117	78	86	90	86	100	90	110
81	61	93	104	91	109	120	131	129	146	153	156	163	187	187	198	189	195	201	199	187	190	206	85	71	87	75	103	77	72	39	57
139	159	153	162	160	175	174	171	181	174	173	183	177	193	182	161	184	180	175	188	176	183	183	187	165	207	212	210	194	192	187	195
157	169	170	177	187	184	178	187	172	165	171	181	202	198	202	181	201	183	189	195	174	169	172	177	163	185	170	180	203	211	205	212
173	180	177	168	165	170	166	168	170	188	185	168	193	200	187	176	171	194	203	190	185	186	165	202	192	197	175	202	184	196	193	210

What is Image Filtering?

Modify the pixels in an image based on some function of a local neighborhood of the pixels

10	5	3
4	5	1
1	1	7

Some function

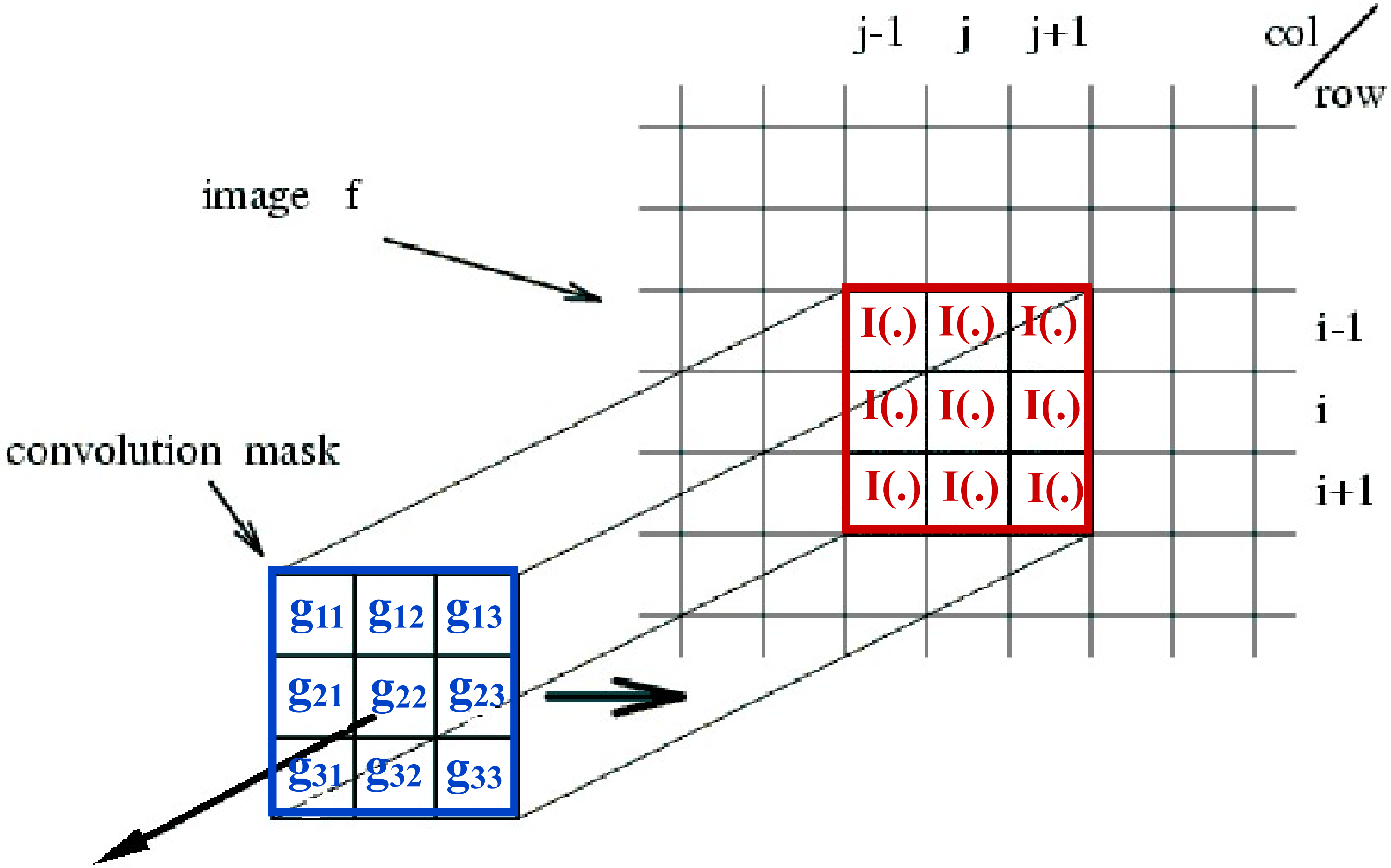
	7	

Linear Filtering

- Linear case is simplest and most useful
  - Replace each pixel with a linear combination of its neighbors.
- The prescription for the linear combination is called the convolution kernel.

$$\begin{bmatrix} 10 & 5 & 3 \\ 4 & 5 & 1 \\ 1 & 1 & 7 \end{bmatrix} \otimes \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 1.0 & 0.5 \end{bmatrix}}_{\text{kernel}} = \begin{bmatrix} & & \\ & 7 & \\ & & \end{bmatrix}$$

Linear Filter = Convolution



$$
\begin{aligned}
f(i,j) = \; & g_{11}\, I(i-1,j-1) + g_{12}\, I(i-1,j) + g_{13}\, I(i-1,j+1) + \\
& g_{21}\, I(i,j-1) + g_{22}\, I(i,j) + g_{23}\, I(i,j+1) + \\
& g_{31}\, I(i+1,j-1) + g_{32}\, I(i+1,j) + g_{33}\, I(i+1,j+1)
\end{aligned}
$$

Linear Filter = Convolution

$$f[m,n] = I \otimes g = \sum_{k,l} I[m-k, n-l]\, g[k,l]$$

with $\sum_{k,l} g[k,l] = 1$

Useful Imaging Methods - Image Processing

- Gaussian Filtering convolves a low-pass filter with the image, which gives a smoothed image
- This can reduce pixilation effects and aid in false positives for corner detection
- Too much results in blurred image which can make corner detection hard

$$\frac{1}{21} * \begin{matrix} 1 & 3 & 1 \\ 3 & 5 & 3 \\ 1 & 3 & 1 \end{matrix}$$

3x3 Gaussian Kernel

Original Image

3x3 Gaussian Filter

Image Smoothing With Gaussian

```
figure(3);
sigma = 3;
width = 3 * sigma;
support = -width : width;
gauss2D = exp( - (support / sigma).^2 / 2);
gauss2D = gauss2D / sum(gauss2D);
smooth = conv2(conv2(bw, gauss2D, 'same'), gauss2D', 'same');
image(smooth);
colormap(gray(255));

gauss3D = gauss2D' * gauss2D;
tic ; smooth = conv2(bw,gauss3D, 'same'); toc
```

Useful Imaging Methods - Image Processing

- High Pass filtering can give a simple edge detection
- Subsequent edge detection in two directions gives a simple point detection

$$\begin{matrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{matrix}$$

3x3 vertical edge detector

$$\begin{matrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{matrix}$$

3x3 horizontal edge detector

Vertical Edge

Horizontal Edge

Edge Detection With Smoothed Images

```
figure(4);
[dx,dy] = gradient(smooth);
gradmag = sqrt(dx.^2 + dy.^2);
gmax = max(max(gradmag));
imshow(gradmag);
colormap(gray(gmax));

figure(6)
edge(bw, 'sobel')
```

Useful Imaging Methods - Image Processing

- Median Filtering is a nonlinear filter where the median value in the kernel window is given to the pixel
- Removes “salt and pepper” noise and jittering
- Results in a loss of texture

Noisy Image

3x3 Gaussian Filter

3x3 Median Filter

Useful Imaging Methods - Image Processing

- In thresholding, all pixels with a value below a certain level are rounded to 0, all pixels above are rounded to max
- Thresholding is often a prerequisite to segmentation algorithms which find connected regions
- Centroids of segments make excellent feature points