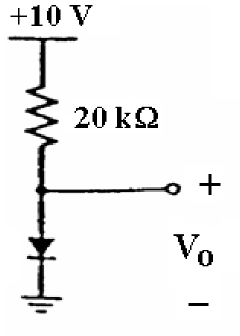
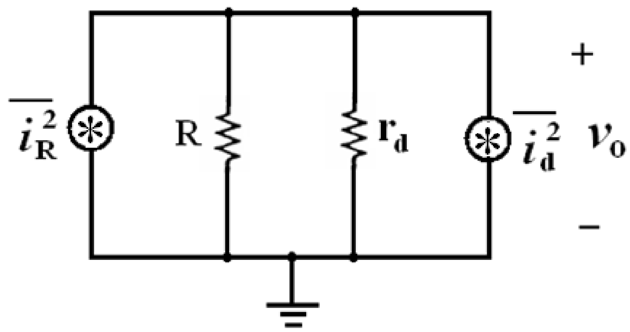
EE4341 TUTORIAL 2 SOLUTION

1.





$$I_D = \frac{10\text{V} - 0.6\text{V}}{20\text{k}\Omega} = 470\ \mu\text{A}$$

$$V_T = \frac{kT}{q} = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}} = 26\,\text{mV}$$

$$r_d = \frac{V_T}{I_D} = \frac{26\,\text{mV}}{470\ \mu\text{A}} = 55.3\,\Omega$$

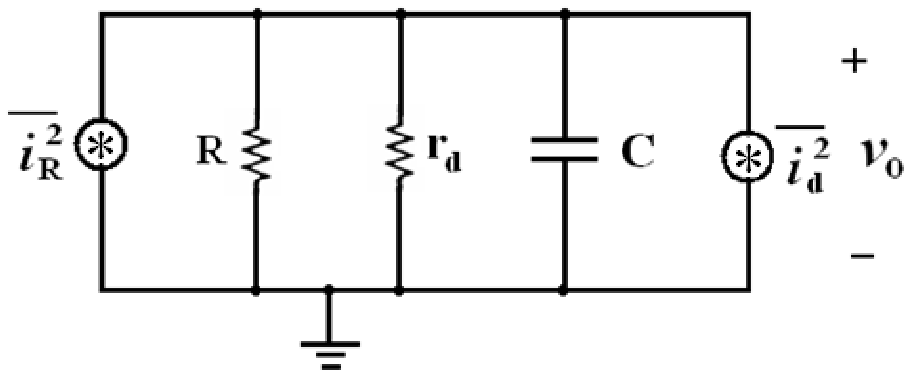
$$\overline{i_R^{\,2}} = \frac{4kT}{R} = \frac{1.656\times10^{-20}}{20\times10^{3}} = 8.28\times10^{-25}\ \text{A}^2/\text{Hz}$$

$$\overline{i_d^{\,2}} = 2qI_D = 2\times1.6\times10^{-19}\times470\mu = 1.504\times10^{-22}\ \text{A}^2/\text{Hz}$$

$$R_{eq} = R\,//\,r_d = 20k\,//\,55.3 = 55.15\,\Omega$$

$$\overline{v_o^{\,2}} = \left(\overline{i_R^{\,2}} + \overline{i_d^{\,2}}\right)R_{eq}^{\,2} = \left(1.5123\times10^{-22}\right)\left(55.15\right)^2 = 4.6\times10^{-19}\ \text{V}^2/\text{Hz}$$

$$V_o = \sqrt{\overline{v_o^{\,2}}\Delta f} = \sqrt{4.6\times10^{-19}\times100k} = 0.214\,\mu\text{V}$$



$$\overline{i_R^{\;2}} = 8.28\times10^{-25}\ \mathrm{A^2/Hz}$$

$$\overline{i_d^{\;2}} = 1.504\times10^{-22} + \frac{3\times10^{-16} I_D}{f} = 1.504\times10^{-22} + \frac{3\times10^{-16}\times 470\mu}{f}$$

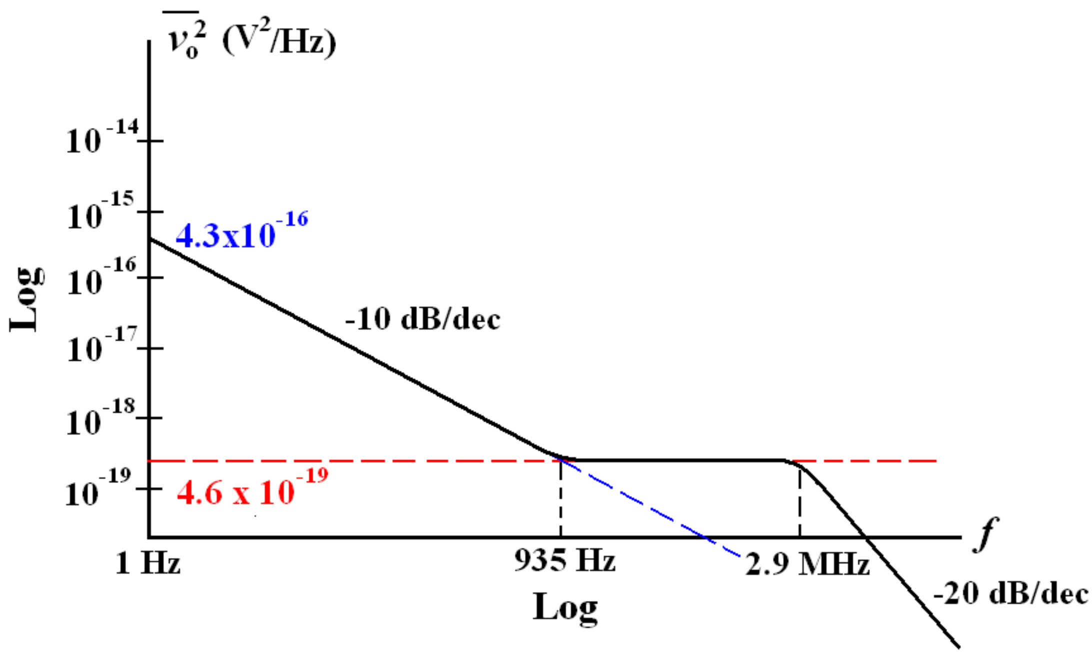
$$= 1.504\times10^{-22} + \frac{1.41\times10^{-19}}{f}\ \mathrm{A^2/Hz}$$

$$Z = R_{eq} // \left(\frac{1}{j\omega C}\right) = \frac{R_{eq}\left(\frac{1}{j\omega C}\right)}{R_{eq} + \frac{1}{j\omega C}} = \frac{R_{eq}}{1 + j\omega R_{eq} C} = \frac{R_{eq}}{1 + \frac{j\omega}{\omega_o}}$$

where $\omega_o = \frac{1}{R_{eq} C} \Rightarrow f_o = \frac{1}{2\pi R_{eq} C} = \frac{1}{2\pi\times 55.15\times 1000\,p} = 2.9\ \mathrm{MHz}$

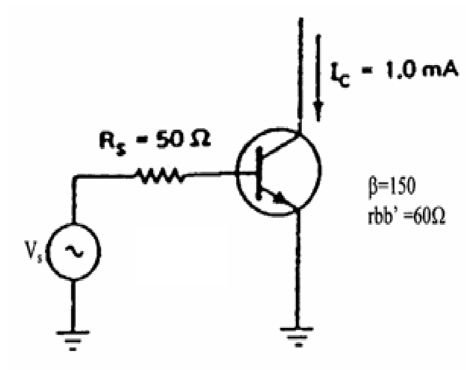
$$\overline{v_o^{\;2}} = \left(\overline{i_R^{\;2}} + \overline{i_d^{\;2}}\right)|Z|^2 = \left(1.504\times10^{-22} + 8.28\times10^{-25} + \frac{1.41\times10^{-19}}{f}\right)\frac{(55.15)^2}{1+\left(\frac{f}{f_o}\right)^2}$$

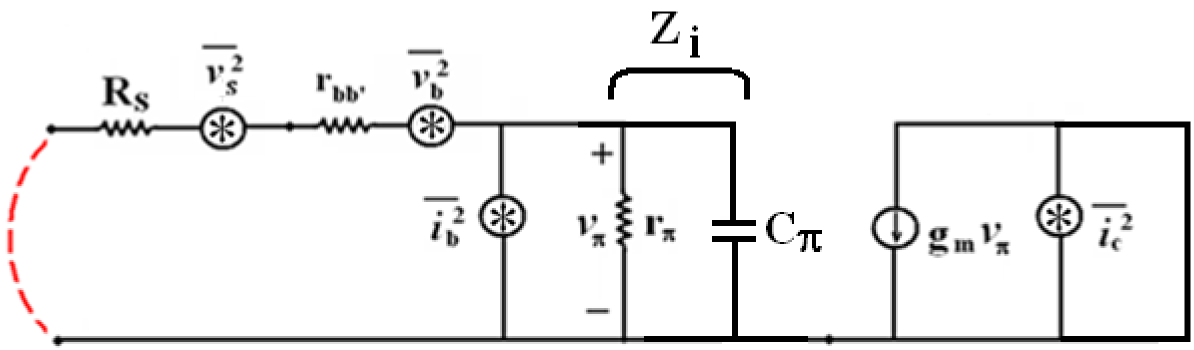
$$= \left(4.6\times10^{-19} + \frac{4.3\times10^{-16}}{f}\right)\frac{1}{1+\left(\frac{f}{f_o}\right)^2}$$

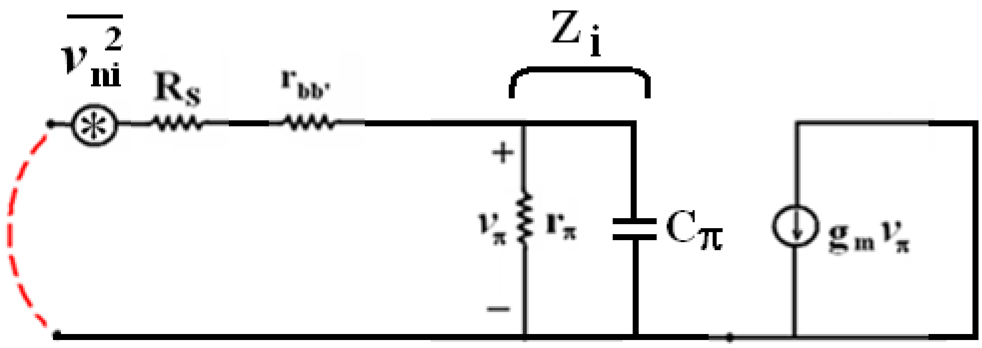


$$\overline{v_o^{\ 2}} = \left(4.6\times10^{-19} + \frac{4.3\times10^{-16}}{f}\right)\frac{1}{1+\left(\dfrac{f}{f_o}\right)^2}\ \ \mathrm{V^2/Hz}$$

2.







$$I_B = \frac{I_C}{\beta} = \frac{1\text{mA}}{150} = 6.67\,\mu\text{A}$$

$$r_\pi = \frac{V_T}{I_B} = \frac{26\text{mV}}{6.67\,\mu\text{A}} = 4\text{ k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 38.5\text{ mS}$$

$$g_m v_{ni}\left(\frac{z_i}{R_s + r_{bb'} + z_i}\right) = g_m\left[\left(v_s + v_b\right)\left(\frac{z_i}{R_s + r_{bb'} + z_i}\right) + i_b\frac{\left(R_s + r_{bb'}\right)z_i}{R_s + r_{bb'} + z_i}\right] + i_c$$

$$\because z_i >> R_s + r_{bb'} \quad \therefore \frac{z_i}{R_s + r_{bb'} + z_i} \approx 1$$

$$g_m v_{ni} = g_m\left[\left(v_s + v_b\right) + i_b\left(R_s + r_{bb'}\right)\right] + i_c$$

$$v_{ni} = v_s + v_b + i_b\left(R_s + r_{bb'}\right) + \frac{i_c}{g_m}$$

$$\overline{v_{ni}^{\;2}} = \overline{v_s^{\;2}} + \overline{v_b^{\;2}} + \overline{i_b^{\;2}}\left(R_s + r_{bb'}\right)^2 + \frac{\overline{i_c^{\;2}}}{g_m^{\;2}}$$

$$\overline{v_{ni}^{\;2}} = 4kTR_s + 4kTr_{bb'} + 2qI_B\left(R_s + r_{bb'}\right)^2 + \frac{2qI_C}{g_m^{\;2}}$$

$$= 4kT\left(R_s + r_{bb'}\right) + 2qI_B\left(R_s + r_{bb'}\right)^2 + \frac{2qI_C}{g_m^{\;2}}$$

$$= 1.82\times10^{-18} + 2.58\times10^{-20} + 2.16\times10^{-19}$$

$$= 2.06\times10^{-18}\ \ \mathrm{V^2/Hz}$$

$$\Delta f = 1.57\,f_o = 15.7\ \mathrm{kHz}$$

$$V_{ni} = \sqrt{\overline{v_{ni}^{\;2}}\,\Delta f} = \sqrt{2.06\times10^{-18}\times15.7k} = 180\ \ \mathrm{nV}$$

$$\text{For S/N} = 0\ \mathrm{dB},\ V_s = V_{ni} = 180\ \ \mathrm{nV}$$

$$\overline{v_{ni}^{\;2}} = 4kT\left(R_s + r_{bb'}\right) + 2qI_B\left(R_s + r_{bb'}\right)^2 + \frac{2qI_C}{g_m^{\;2}}$$

$$= 4kT\left[\left(R_s + r_{bb'}\right) + \frac{2qI_B}{4kT}\left(R_s + r_{bb'}\right)^2 + \frac{2qI_C}{4kTg_m^{\;2}}\right]$$

$$= 4kT\left[\left(R_s + r_{bb'}\right) + \frac{I_B}{2V_T}\left(R_s + r_{bb'}\right)^2 + \frac{I_C}{2V_T g_m^{\;2}}\right]$$

$$= 4kT\left[\left(R_s + r_{bb'}\right) + \frac{I_C}{2V_T\beta}\left(R_s + r_{bb'}\right)^2 + \frac{V_T}{2I_C}\right]$$

$$\text{Note: } V_T = \frac{kT}{q} \qquad g_m = \frac{I_C}{V_T}$$

$$\overline{v_{ni}^{\;2}} = 4kT\left[\left(R_s + r_{bb'}\right) + \frac{I_C}{2V_T\beta}\left(R_s + r_{bb'}\right)^2 + \frac{V_T}{2I_C}\right]$$

$$\frac{\partial\overline{v_{ni}^{\;2}}}{\partial I_C} = \frac{\left(R_s + r_{bb'}\right)^2}{2V_T\beta} - \frac{V_T}{2I_C^{\;2}} = 0$$

$$\therefore \frac{\left(R_s + r_{bb'}\right)^2}{2V_T\beta} = \frac{V_T}{2I_C^{\;2}} \Rightarrow I_C^{\;2} = \frac{V_T^{\;2}\beta}{\left(R_s + r_{bb'}\right)^2} \Rightarrow I_C = \frac{V_T\sqrt{\beta}}{R_s + r_{bb'}}$$

$$I_C = \frac{V_T\sqrt{\beta}}{R_s + r_{bb'}} = \frac{26m\sqrt{150}}{50+60} = 2.9 \text{ mA}$$

$$\overline{v_{ni}^{\;2}} = 4kT\left(R_s + r_{bb'}\right) + 2qI_B\left(R_s + r_{bb'}\right)^2 + \frac{2qI_C}{g_m^{\;2}}$$
$$= 1.82\times10^{-18} + 7.49\times10^{-20} + 7.46\times10^{-20}$$
$$= 1.97\times10^{-18} \text{ V}^2/\text{Hz}$$

$$V_{ni} = \sqrt{\overline{v_{ni}^{\;2}}\Delta f} = \sqrt{1.97\times10^{-18}\times15.7k} = 176 \text{ nV}$$

For S/N = 0 dB, $V_s = V_{ni} = 176 \text{ nV}$