

1. Given two polynomial matrices

$$N(s) = \begin{bmatrix} s+2 & (s+2)^2 \\ (s+2)(s+3) & (s+4)(s^2-4) \end{bmatrix} \text{ and } D(s) = \begin{bmatrix} (s+2)^2 & s+2 \\ s+2 & s^3+8 \end{bmatrix}$$

(1) Determine whether  $N(s)$  and  $D(s)$  are coprime.

(2) Calculate a unimodular matrix  $U(s)$  that can reduce the matrix  $\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}$  to a row Hermite form, and determine the greatest common right divisor of  $N(s)$  and  $D(s)$ .

(3) Let  $\hat{N}(s)$  and  $\hat{D}(s)$  be coprime matrices derived from (2). Determine whether  $\hat{D}(s)$  is column reduced. If not, find a unimodular matrix  $U(s)$  to make it column reduced.

2. Given a matrix transfer function  $G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{2}{s+4} \\ \frac{s+2}{(s+1)^2} & \frac{s+4}{(s+4)(s+1)} \\ \frac{s+2}{(s+1)(s+4)} & \frac{s+2}{(s+4)^2} \end{bmatrix}$  of a MIMO system,

(1) Find one coprime right polynomial fraction description  $G(s) = N(s)D(s)^{-1}$ .

(2) Determine all poles and transmission zeros of  $G(s)$ .

(3) Determine whether  $D(s)$  is column reduced. If not, make it column reduced.

(4) Determine a minimal realization of  $G(s)$  by using integrator coefficient matrices.

3. Given a matrix transfer function  $G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{2}{s+4} \\ \frac{s+2}{(s+1)^2} & \frac{s+4}{(s+4)(s+1)} \end{bmatrix}$ , determine a minimal realization of  $G(s)$

by using integrator coefficient matrices. Design a state feedback law  $u=r-kx$ , that assigns all system poles to  $s=-3$  by using the Lyapunov method.

4. Given an SISO system shown below, where  $A = \begin{bmatrix} -2 & 3 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $c = [1 \quad 1]$ , design feedback  $k = [k \quad k_a]$  to reject a constant disturbance  $w$  and asymptotically track a step reference  $r$ .

