NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2024-2025

EE6407 – GENETIC ALGORITHMS AND MACHINE LEARNING

Nov / Dec 2024 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 4 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

---

1. The 3-dimensional geometry of a block of material is shown in Figure 1 below. When the block is heated up, the physics model of the heat distribution within the block is described by the following equation:

$$f(x, y, z) = 5x^2 + 3y^2 + 2z^2 - 7x - 4y + 6z$$

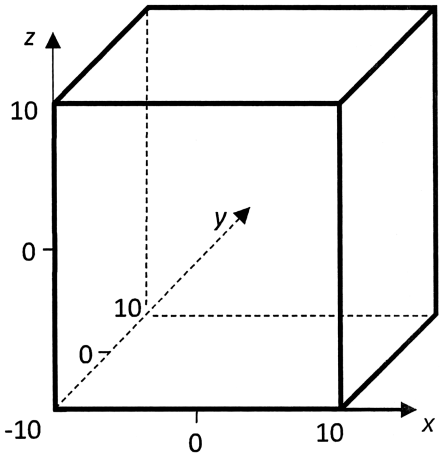

Figure 1

Note: Question No. 1 continues on page 2.

(a) The variables x, y and z are the 3-dimensional spatial coordinates while the function $f(.)$ is the temperature. We wish to apply a binary coded genetic algorithm (GA) to find the values of x, y and z such that $f(x, y, z)$ is maximum. With 12-bit binary string encoding of the chromosome, state the values of temperature for the following 12-bit binary values:

i. 000000000000
ii. 001111001001
iii. 110001110101
iv. 000000001111
v. 111111111111

State the limitation of the binary representation with respect to the optimum solution.

(15 Marks)

(b) Assume that the material as shown in Figure 1 on page 1 has a general physics model defined as follows:

$$f(x, y, z) = a_0x^2 + b_0y^2 + c_0z^2 - a_1x - b_1y + c_1z$$

It is known that the coefficients a_0, b_0, and c_0 are constants specific to the type of material and a_1, b_1, and c_1 are constants to account for variations and imperfections in the material. You are given a block of material with values of a_0, b_0, and c_0 equal to 2.5, 3.88 and 5.6, respectively. Using a thermal sensor, you obtained measurements at two sample points on the surface of the block of material as given in Table 1. Using a real-coded GA to solve for a_1, b_1, and c_1, calculate the fitness in terms of the error of estimation for chromosomes S1 = <7, 1.6, 4.8> and S2 = <-5, 6.2, 3.3>.

Table 1

x, y, z	*f(x, y, z)*
10, 5, 5	732
-10, 10, 5	688

(10 Marks)

2. A dataset consists of 500 and 300 training samples from class 1 and class 2, respectively. The maximum likelihood estimation of the mean vectors and covariance matrices for the two classes are given below:

$$\mathbf{m}_1 = \begin{bmatrix} 0.026 \\ 0.048 \end{bmatrix}, \quad \mathbf{m}_2 = \begin{bmatrix} 1.542 \\ 2.011 \end{bmatrix}$$

$$\mathbf{C}_1 = \begin{bmatrix} 0.942 & 0.012 \\ 0.012 & 1.087 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} 0.945 & 0.054 \\ 0.054 & 1.037 \end{bmatrix}$$

Assume that the samples of each class follow a multivariate normal distribution.

(a) Design a Bayes decision rule to classify samples in the two classes, and use the designed decision rule to classify the following test sample:

$$\mathbf{x}_1 = \begin{bmatrix} 0.567 \\ 1.101 \end{bmatrix}$$

(12 Marks)

(b) Design a Naïve Bayes decision rule to classify samples in the two classes and use the designed Naïve Bayes decision rule to classify the test sample $\mathbf{x}_1$ in part (a). State any assumptions made.

(13 Marks)

3. (a) In a 2-class classification problem, there are 100 training samples from each class. The mean vectors and the scatter matrices of the two classes are given below:

$$\mathbf{m}_1 = \begin{bmatrix} 0.863 \\ -1.594 \end{bmatrix}, \quad \mathbf{m}_2 = \begin{bmatrix} -1.581 \\ 1.001 \end{bmatrix}$$

$$\mathbf{S}_1 = \begin{bmatrix} 105.16 & -14.011 \\ -14.011 & 103.225 \end{bmatrix}, \quad \mathbf{S}_2 = \begin{bmatrix} 117.947 & 0.505 \\ 0.505 & 74.081 \end{bmatrix}$$

Design a Fisher linear discriminant classifier $y = \mathbf{w}^T\mathbf{x} + w_0$, and use the classifier to classify the following test sample:

$$\mathbf{x}_2 = \begin{bmatrix} -0.176 \\ 0.224 \end{bmatrix}$$

(12 Marks)

Note: Question No. 3 continues on page 4.

(b) A linear support vector machine (SVM) is used to solve a 2-class classification problem. The support vectors obtained are given by:

$$\mathbf{s}_1 = [0.287 \quad 0.0421 \quad 0.821]^T$$

$$\mathbf{s}_2 = [4.32 \quad -0.572 \quad 3.45]^T$$

$$\mathbf{s}_3 = [3.922 \quad -1.421 \quad -2.721]^T$$

$$\mathbf{s}_4 = [0.324 \quad -2.234 \quad 1.122]^T$$

The Lagrange multipliers corresponding to the support vectors are:

$$\alpha_1 = 0.3784, \quad \alpha_2 = 0.051, \quad \alpha_3 = 0.0505, \quad \alpha_4 = 0.3789$$

The class labels of the support vectors are $1, 1, -1, -1$, respectively.

Determine the linear SVM classifier $y = \mathbf{w}^T\mathbf{x} + w_0$, and use it to classify the following test sample:

$$\mathbf{x}_3 = [0.225 \quad 2.252 \quad -2.613]^T$$

(13 Marks)

4. Cluster the following samples using the agglomerative hierarchical clustering method:

$$\mathbf{z}_1 = [1.795 \quad 3.490 \quad 2.417]^T$$
$$\mathbf{z}_2 = [1.876 \quad 3.409 \quad 1.672]^T$$
$$\mathbf{z}_3 = [0.343 \quad 2.769 \quad 3.035]^T$$
$$\mathbf{z}_4 = [3.578 \quad -1.350 \quad 0.725]^T$$

(a) Assuming that the Euclidean distance and the single-linkage method are used, describe the clustering process and sketch the dendrogram at each step.

(9 Marks)

(b) Assuming that the Euclidean distance and the complete-linkage method are used, describe the clustering process and sketch the dendrogram at each step.

(7 Marks)

(c) Describe the use of Silhouette coefficient as an evaluation criterion for clustering and explain its rationale.

(9 Marks)

END OF PAPER

EE6407 GENETIC ALGORITHMS & MACHINE LEARNING

CONFIDENTIAL

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.