NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2023-2024

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April / May 2024 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with six joints is shown in Figure 1.

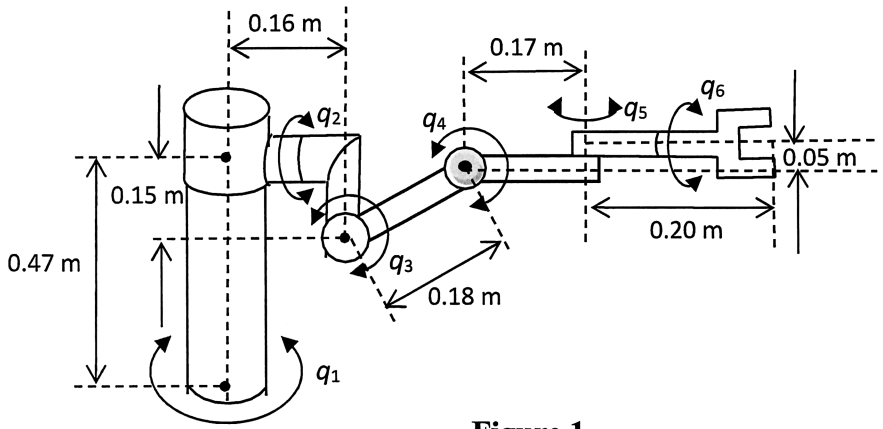

Figure 1

(a) Obtain the link coordinate diagram by using the Denavit-Hartenberg (D-H) algorithm.

(12 Marks)

(b) Derive the kinematic parameters of the robot based on the coordinate diagram obtained in part (a).

(8 Marks)

2. A Cartesian robot with two degrees of freedom is in contact with a workpiece, which is shown in Figure 2.

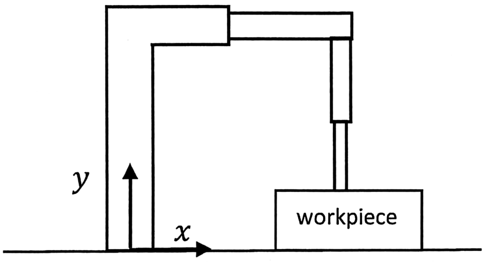

Figure 2

The dynamic equations of the robot, when it is not in contact with the workpiece, are given as follows:

$$5\ddot{x} + 10x\dot{x} + d_x = u_x$$

$$10\ddot{y} + 15y\dot{y} + 98 + d_x = u_y$$

where u_x, u_y are the control inputs and d_x, d_y are the unknown constant disturbances. The stiffness of the workpiece in any direction is given as 50 N/m and the static position of the workpiece in the y axis is 0.25 m. The system possesses unmodelled resonances at 7 rad/s and 15 rad/s.

(a) Assume that $d_x = 0$ and $d_y = 0$. Design a hybrid position and force controller for the robot. The motion control subspace should be critically damped, and the force control subspace should be overdamped with a damping ratio of 1.25. The control gains should be chosen to be as high as possible and the system should not excite all the unmodelled resonances.

(13 Marks)

(b) Assume that d_x, d_y are not negligible. Design an appropriate hybrid position and force controller and show that the steady state error can be eliminated. In case that the stiffness of the workpiece is wrongly estimated as 49 N/m, discuss its effects on the controller.

(7 Marks)

3. (a) A mobile robot platform with a shape of an equilateral triangle (i.e., all three sides have the same length) attached to a square is shown in Figure 3. The robot has one steered standard wheel, two standard wheels and one castor wheel. A local reference frame (x_r, y_r) is assigned at the mid-point between the steered standard wheel and the castor wheel. The radius of each standard wheel is 6 cm and the radius of the castor wheel is 3 cm. Let the rotational velocities of the steered standard wheel, the two standard wheels and the castor wheel be denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_{s1}$, $\dot{\varphi}_{s2}$, and $\dot{\varphi}_c$, respectively. Derive the rolling and sliding constraints of the mobile robot.

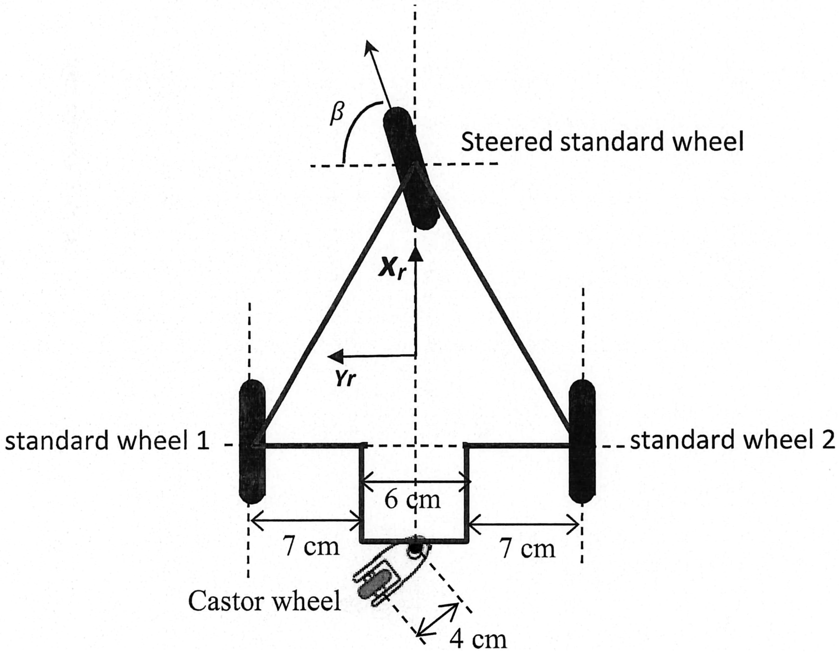

Figure 3

(10 Marks)

(b) A robot manipulator with three joint variables q_1, q_2, q_3 are mounted on a mobile robot. The link-coordinate homogeneous transformation matrix from the base coordinate to the tool coordinate of the robot manipulator is given as follows:

$$T_{base}^{tool} = \begin{bmatrix} C_1C_{23} & -C_1S_{23} & -S_1 & C_1(0.25C_{23} + 0.35C_2) \\ S_1C_{23} & -S_1S_{23} & C_1 & S_1(0.25C_{23} + 0.35C_2) \\ -S_{23} & -C_{23} & 0 & -0.25S_{23} - 0.35S_2 + 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $S_1 = \sin(q_1)$, $S_2 = \sin(q_2)$, $S_{23} = \sin(q_2 + q_3)$, $C_1 = \cos(q_1)$, $C_2 = \cos(q_2)$, $C_{23} = \cos(q_2 + q_3)$.

Solve the inverse kinematic problem using the analytic method to express $(q_1, q_2, q_3)^T$ in terms of the position of the end effector $(x, y, z)^T$.
(Note: the orientation is not required).

(10 Marks)

4. As shown in Figure 4, a moving camera takes two images of the same object at two poses. Two coordinate frames represented by F_p and F_d are attached to the projection centre of the camera at the two poses, respectively. Nine feature points O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8, and O_9 are on the same plane, with 7 groups of points on the same line, respectively, as shown in the figure. Let H denote the Euclidean homography matrix from F_p to F_d.

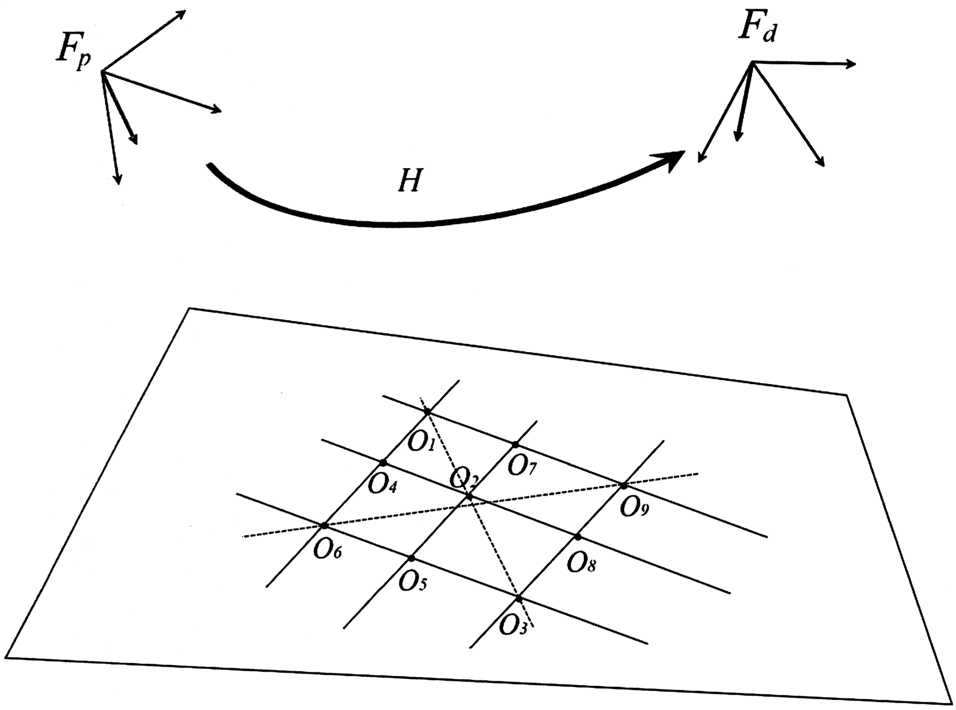

Figure 4

Seven feature points O_1, O_2, O_3, O_4, O_5, O_6, and O_7 can be detected in the image taken at the pose attached to F_p. Their corresponding normalized coordinates in F_p are given as follows:

$m_{1p} = \left[p_{1x}, p_{1y}, 1\right]^T$, $m_{2p} = \left[p_{2x}, p_{2y}, 1\right]^T$, $m_{3p} = \left[p_{3x}, p_{3y}, 1\right]^T$, $m_{4p} = \left[p_{4x}, p_{4y}, 1\right]^T$, $m_{5p} = \left[p_{5x}, p_{5y}, 1\right]^T$, $m_{6p} = \left[p_{6x}, p_{6y}, 1\right]^T$, $m_{7p} = \left[p_{7x}, p_{7y}, 1\right]^T$.

Seven feature points O_1, O_2, O_3, O_4, O_7, O_8, and O_9 can be detected in the image taken at the pose attached to F_d. Their corresponding normalized coordinates in F_d are given as follows:

$m_{1d} = \left[d_{1x}, d_{1y}, 1\right]^T$, $m_{2d} = \left[d_{2x}, d_{2y}, 1\right]^T$, $m_{3d} = \left[d_{3x}, d_{3y}, 1\right]^T$, $m_{4d} = \left[d_{4x}, d_{4y}, 1\right]^T$, $m_{7d} = \left[d_{7x}, d_{7y}, 1\right]^T$, $m_{8d} = \left[d_{8x}, d_{8y}, 1\right]^T$, $m_{9d} = \left[d_{9x}, d_{9y}, 1\right]^T$.

Note: Question No. 4 continues on page 5.

Define the scaled homography matrix as $H_n = H / h$ where h denotes the third row third column entry of the matrix H.

(a) Identify all groups of feature points that can be used to and are necessarily required to determine the scaled homography matrix.

(7 Marks)

(b) Choose one group of feature points, and determine the scaled homography matrix H_n.

(13 Marks)

5. Two sensors are used to measure a state variable x_k that can be modelled by $x_{k+1} = x_k$. The measurements of the two sensors are given by y_{1k} and y_{2k}, respectively, which are governed by the following models:

$$y_{1k} = x_k + v_{1k},\ y_{2k} = x_k + v_{2k}$$

where v_{1k} and v_{2k} are zero mean Gaussian sensor noises with variance given by a^2 and $9a^2$, respectively.

Let $\hat{x}_k$ represent the estimate of x_k and $\hat{x}_{k+1}$ represent the estimate of x_{k+1}. Let the estimation errors be $\tilde{x}_k = x_k - \hat{x}_k$ and $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error $\tilde{x}_k$ and the noise terms v_{1k} and v_{2k} are uncorrelated. Assume that $E[\tilde{x}_k] = 0$ and $E[\tilde{x}_{k+1}] = 0$. Let the estimation error variances be $p_k = E\left[\tilde{x}_k^2\right]$ and $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$, respectively.

Two estimation strategies are designed as follows:
Strategy 1:

$$\hat{x}_{k+1} = \hat{x}_k + L_{ck}\left(y_{1k} - \hat{x}_k\right) + L_{ck}\left(y_{2k} - \hat{x}_k\right)$$

Strategy 2:

$$\hat{x}_{k+1} = \hat{x}_k + L_{1k}\left(y_{1k} - \hat{x}_k\right) + L_{2k}\left(y_{2k} - \hat{x}_k\right)$$

where L_{ck}, L_{1k}, and L_{2k} represent respective Kalman gains.

(a) Design the update laws for the Kaman gains L_{ck}, L_{1k} and L_{2k} in order to minimize the corresponding estimation error variance.

(12 Marks)

(b) Compare the two strategies and discuss the advantages and disadvantages of each strategy. Justify your answer in detail.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.