EE4341/EE6341 Advanced Analog Circuits - DC-DC Converters

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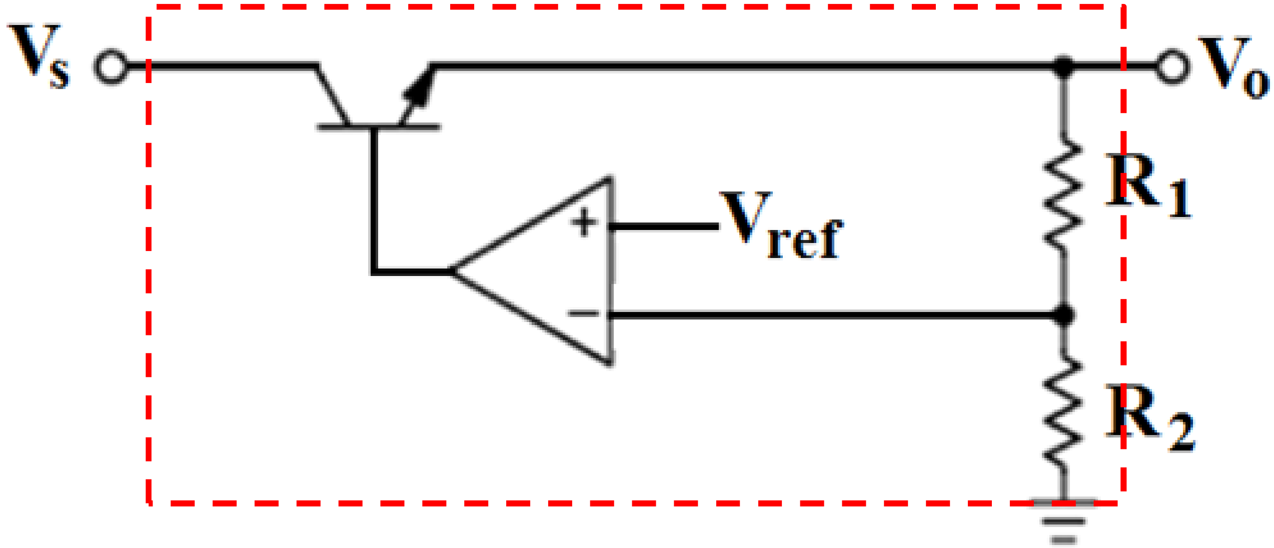
NANYANG TECHNOLOGICAL UNIVERSITY

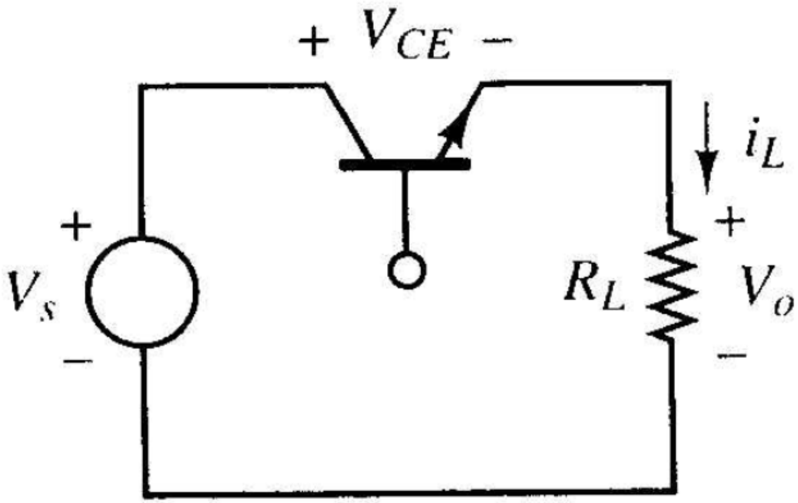
Linear Voltage Regulator

linear voltage regulator maintains a constant output voltage under varying load condition.



- To maintain a constant output, part of the output feedbacks and compares with a reference level
- Comparator output adjusts the BJT collector-emitter voltage for regulation purpose.

Linear Voltage Regular



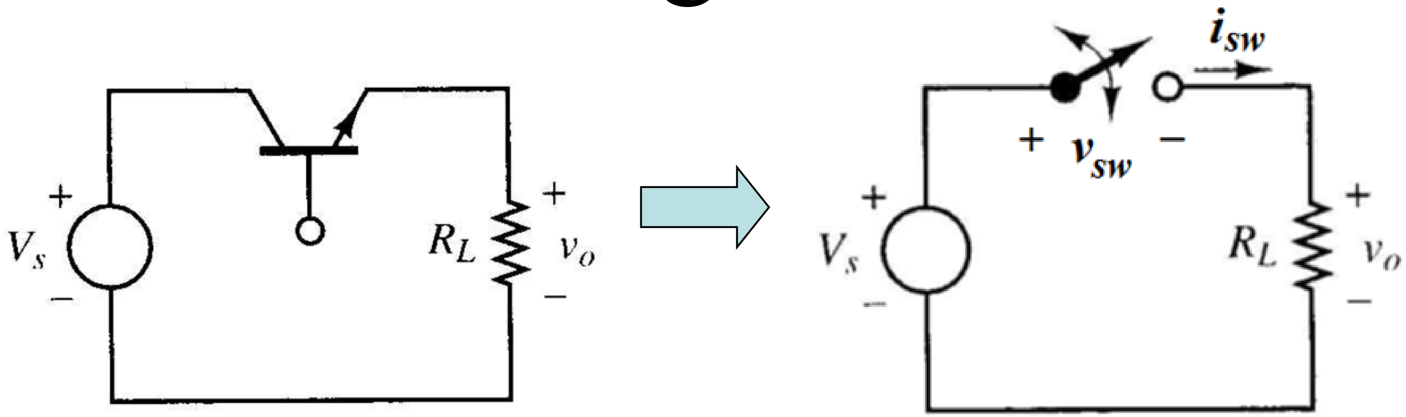
Conversion efficiency:

$$\eta = \frac{P_o}{P_s} = \frac{I_L V_o}{I_L V_s} = \frac{V_o}{V_s} \times 100\%$$

V_o	η
$0.25\ V_s$	25%
$0.5\ V_s$	50%
$0.75\ V_s$	75%

It means lower the output voltage, the lower the conversion efficiency.

Switching Converters

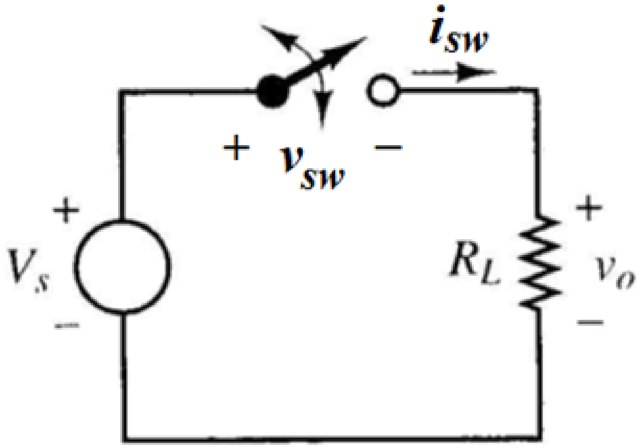


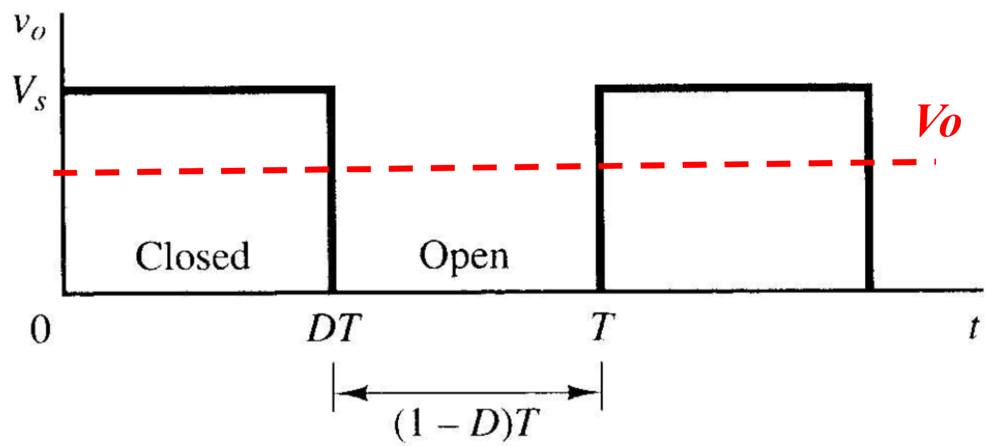
Switch is closed: $$P_{sw} = i_{sw} \times v_{sw} = \frac{V_s}{R_L} \times 0 = 0$$

Switch is opened: $$P_{sw} = i_{sw} \times v_{sw} = 0 \times V_s = 0$$

- No power dissipated in the ideal switch
- Input power = output power (efficiency is 100%)

Switching Converters





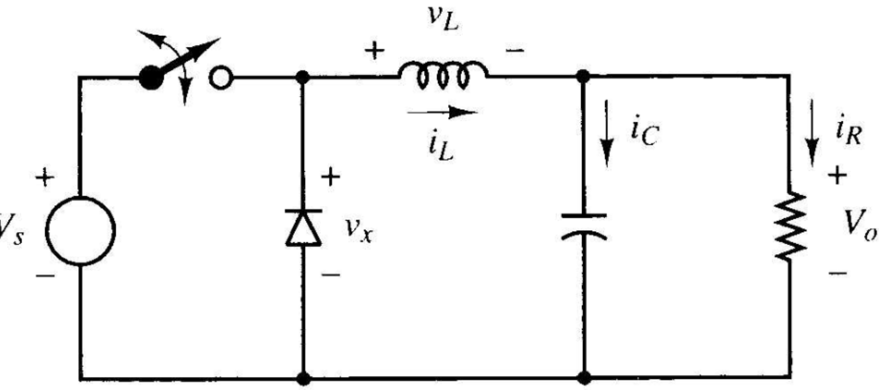
$$D = \frac{t_{on}}{T} = t_{on} f$$

Average output voltage: $$V_o = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\int_0^{DT} V_s dt = V_s D$$

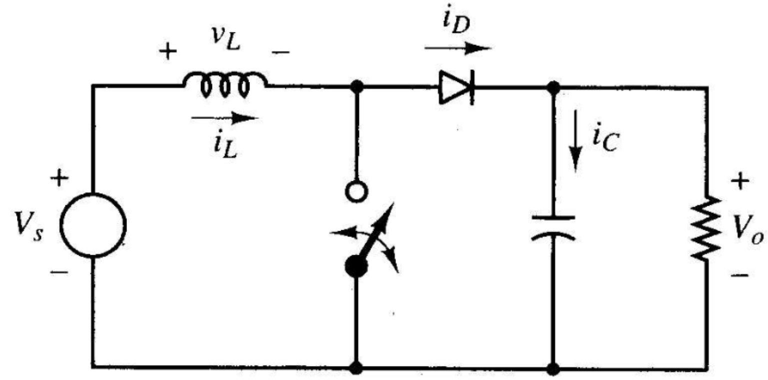
This is the average of the repetitive pulses, which is not a smooth DC!

DC-DC Converters

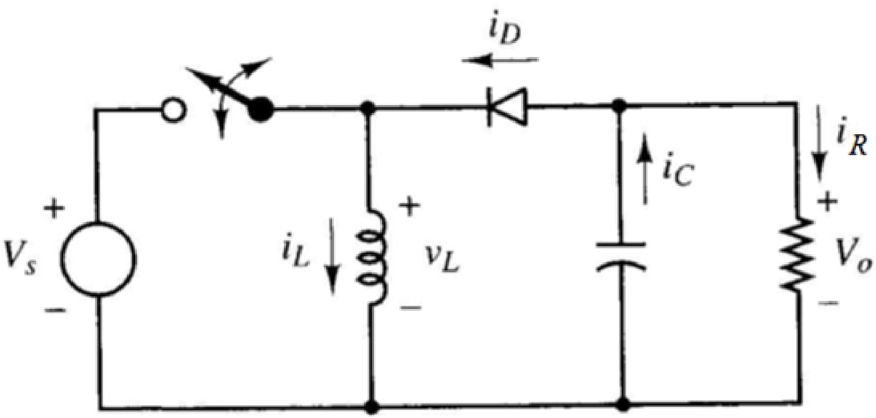
Buck Converter



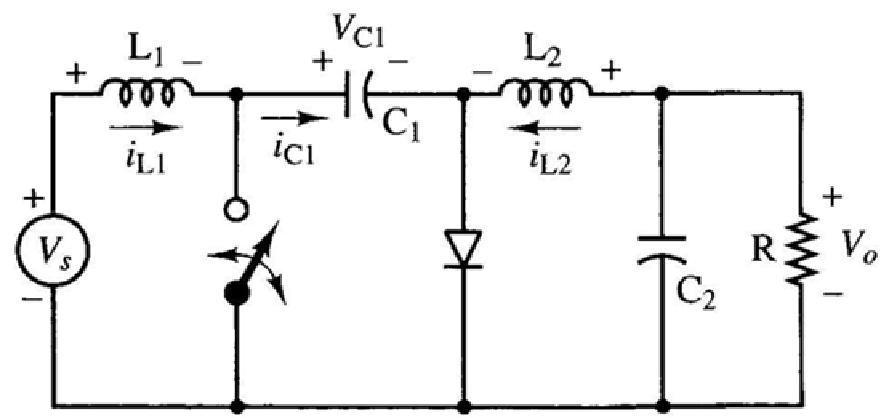
Boost Converter



Buck-Boost Converter



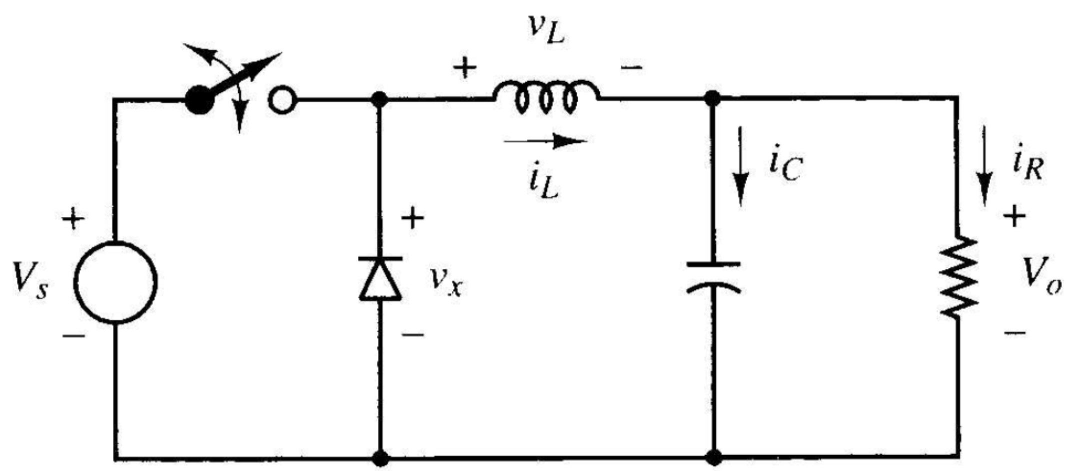
Cuk Converter



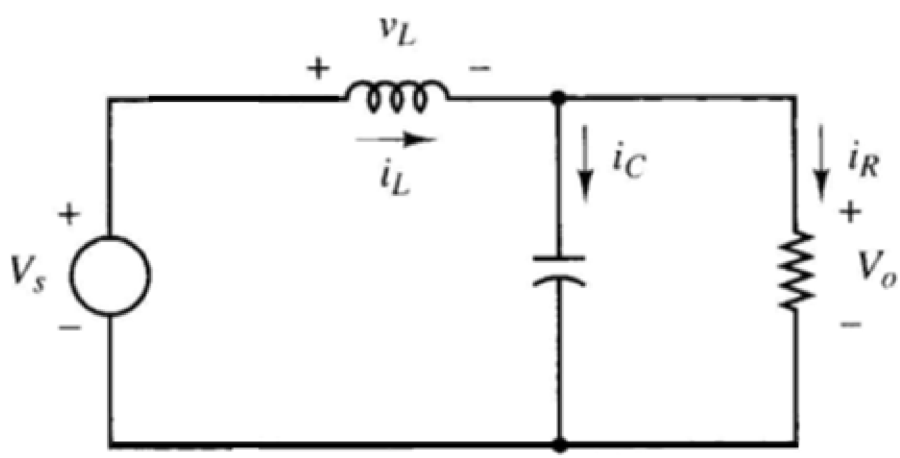
Assumptions and Symbols

- All passive and active components: *L*, *C*, diode and transistor are assumed to be ideal and lossless.
- Value of *L* is large enough so that its current is nearly constant (sufficiently large average component with ripple).
- Value of *C* is large enough so that its voltage stays nearly constant (sufficiently large average component with ripple).
- Upper-case *V* and *I* denote average voltage and current, respectively.
- Lower-case *v* and *i* denote instantaneous voltage and current, respectively.

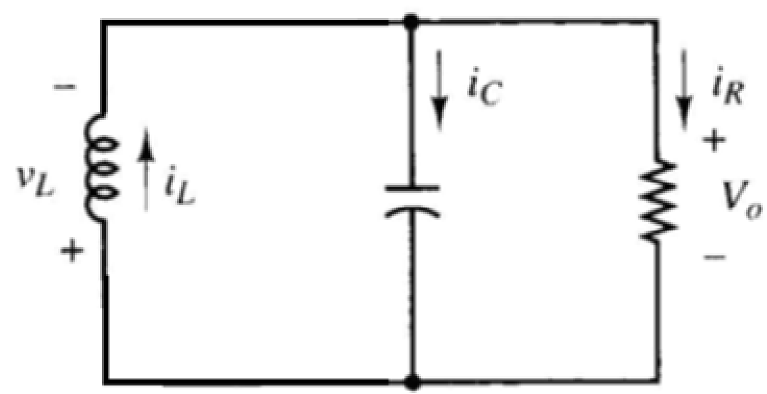
Buck Converter



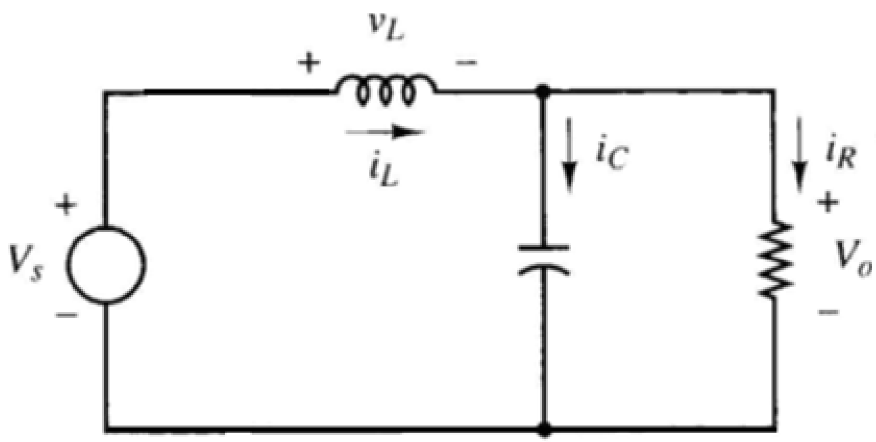
Switch closed

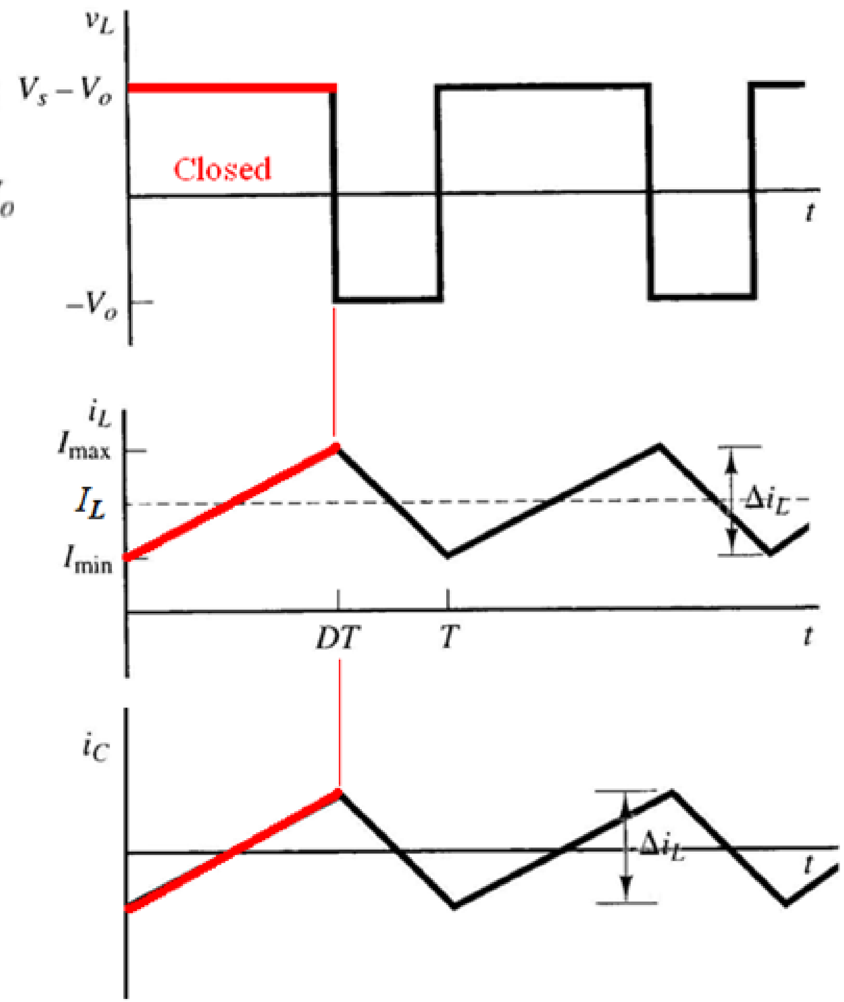


Switch opened



Buck Converter (Switch Closed)



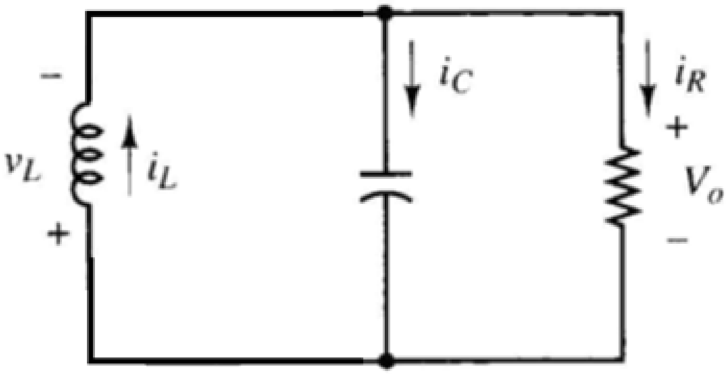


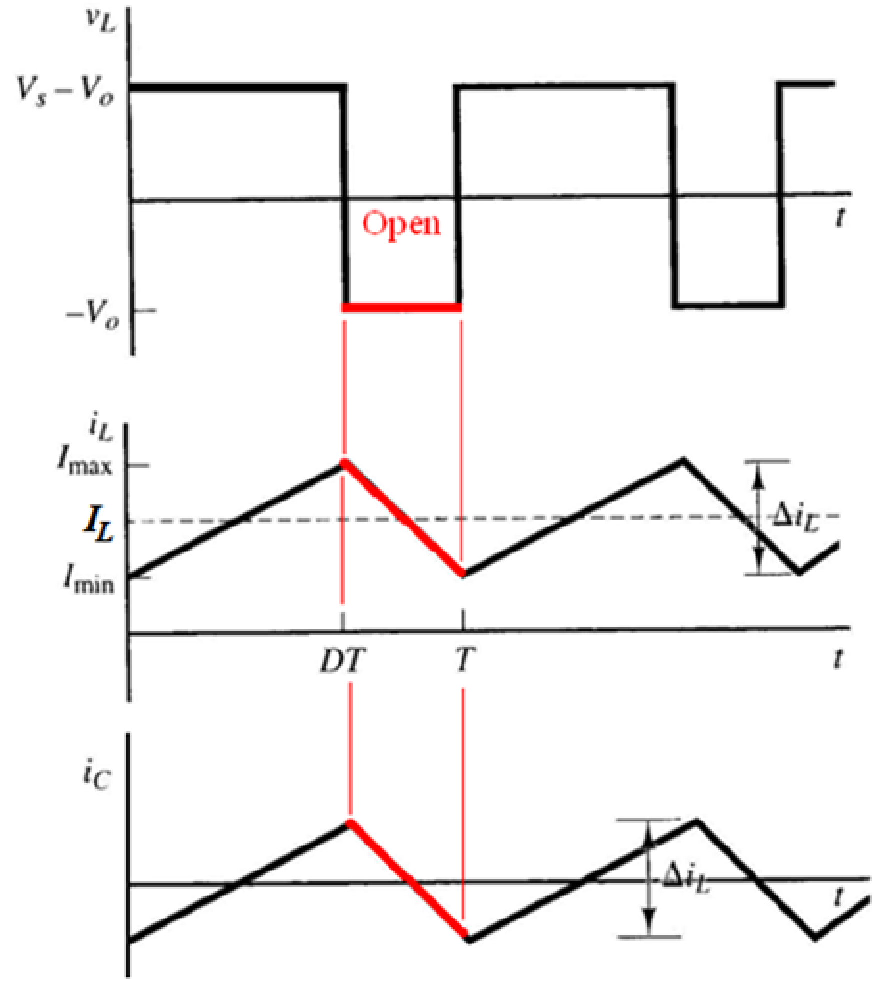
$$v_L = V_s - V_o = L\frac{di_L}{dt} = L\frac{\Delta i_L}{DT}$$

$$\frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L}$$

$$\therefore \left(\Delta i_L\right)_{closed} = \frac{\left(V_s - V_o\right)DT}{L}$$

Buck Converter (Switch Opened)



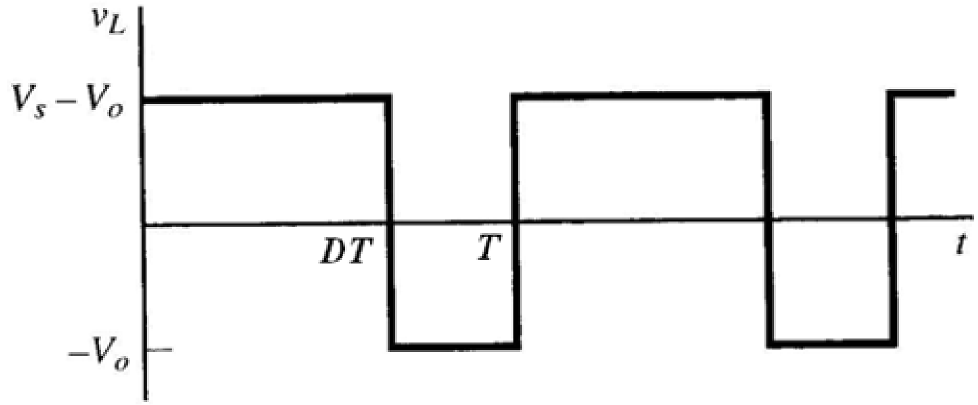


$$v_L = -V_o = L\frac{di_L}{dt} = L\frac{\Delta i_L}{(1-D)T}$$

$$\frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$\therefore (\Delta i_L)_{opened} = \frac{-V_o(1-D)T}{L}$$

Average Output Voltage



For ideal L: average inductor current $\neq$ 0, therefore average voltage across the inductor over one cycle = 0.

$$\frac{1}{T}\int_0^T v_L dt = \frac{1}{T}\left[\int_0^{DT}(V_s - V_o)dt + \int_{DT}^{T}(-V_o)dt\right] = 0$$

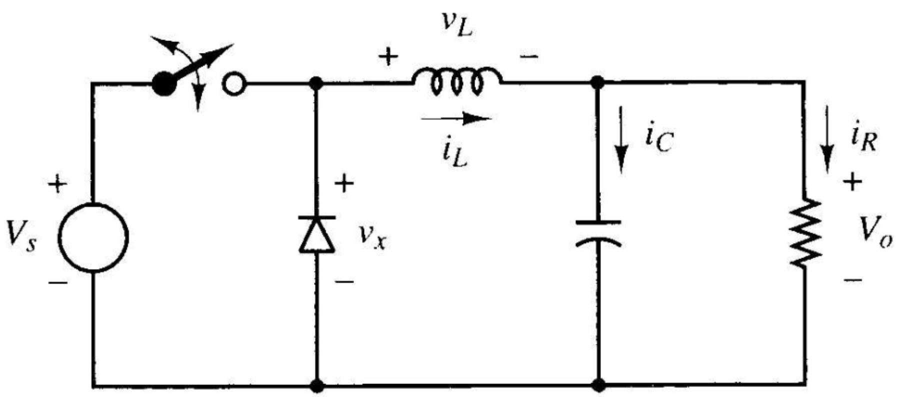
$$\frac{1}{T}\left[(V_s - V_o)DT - V_o(1-D)T\right] = 0$$

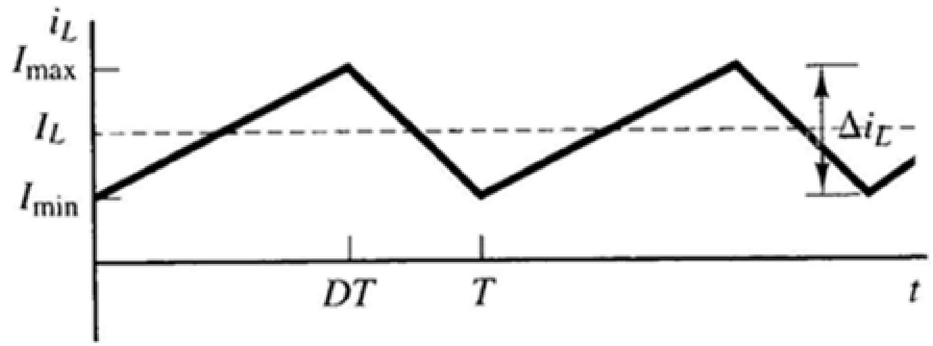
$$(V_s - V_o)D - V_o(1-D) = 0$$

$$V_o = DV_s$$

For $0 < D < 1$, $V_o < V_s$

Average Load Current





$$I_L = I_C + I_R$$

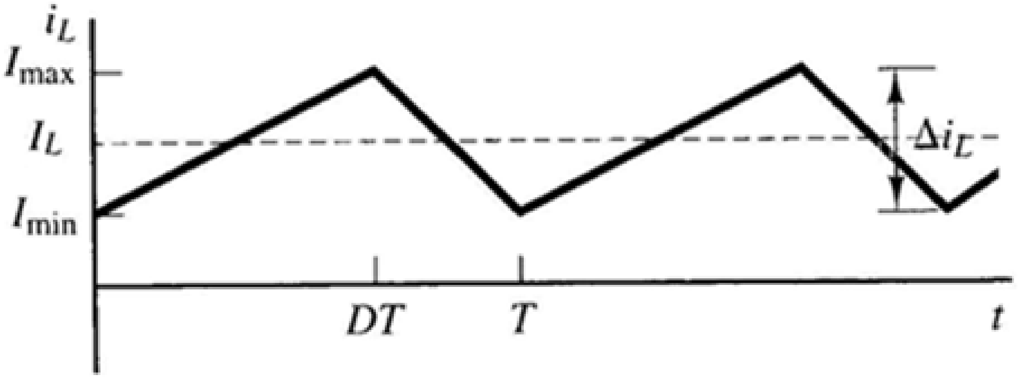
$$\because I_C = 0,\ I_L = I_R = \frac{V_o}{R}$$

$$\Delta i_L = \frac{(V_s - V_o)DT}{L} = \frac{V_o(1-D)T}{L}$$

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o(1-D)T}{L}\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o(1-D)T}{L}\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

Condition for Continuous Current



For continuous current operation, the minimum inductor current cannot be zero.

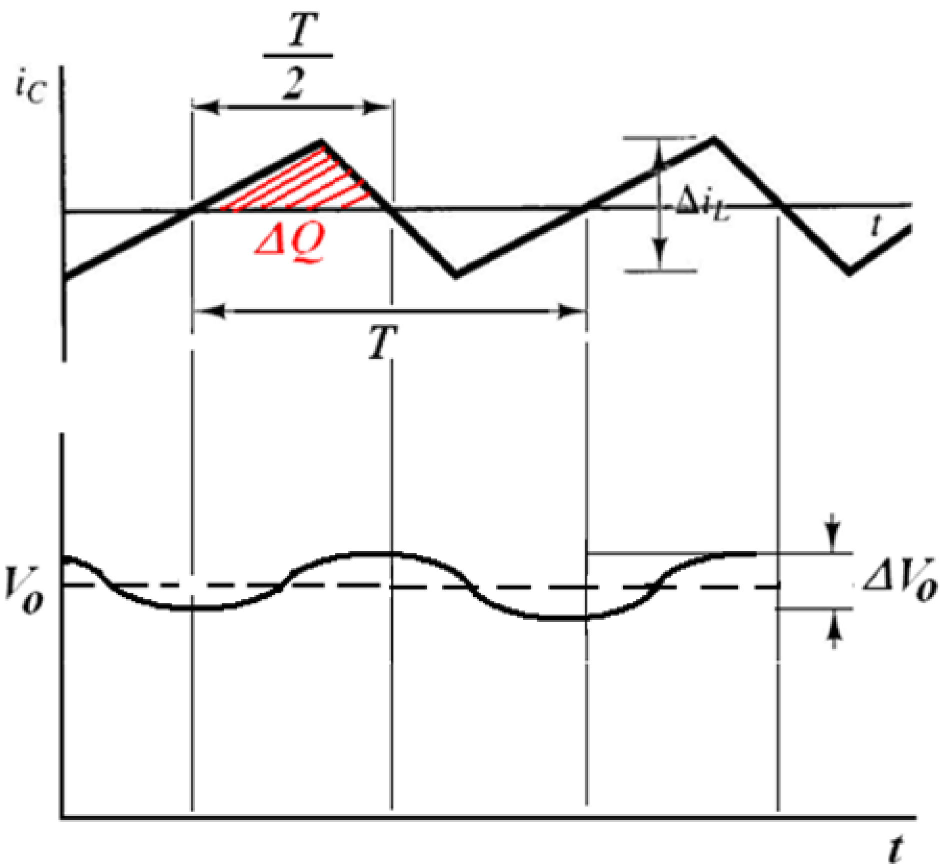
$$I_{min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right) = 0 \Rightarrow (Lf)_{min} = \frac{(1-D)R}{2}$$

By choosing a desired switching frequency f, L_{min} for continuous current operation can be found by:

$$L_{\min} = \frac{(1-D)R}{2f}$$

Note: higher f leads to lower L.

Output Ripple Voltage



$$\Delta Q = C\Delta V_o \Rightarrow \Delta V_o = \frac{\Delta Q}{C}$$

$$\Delta Q = \int i_c dt$$

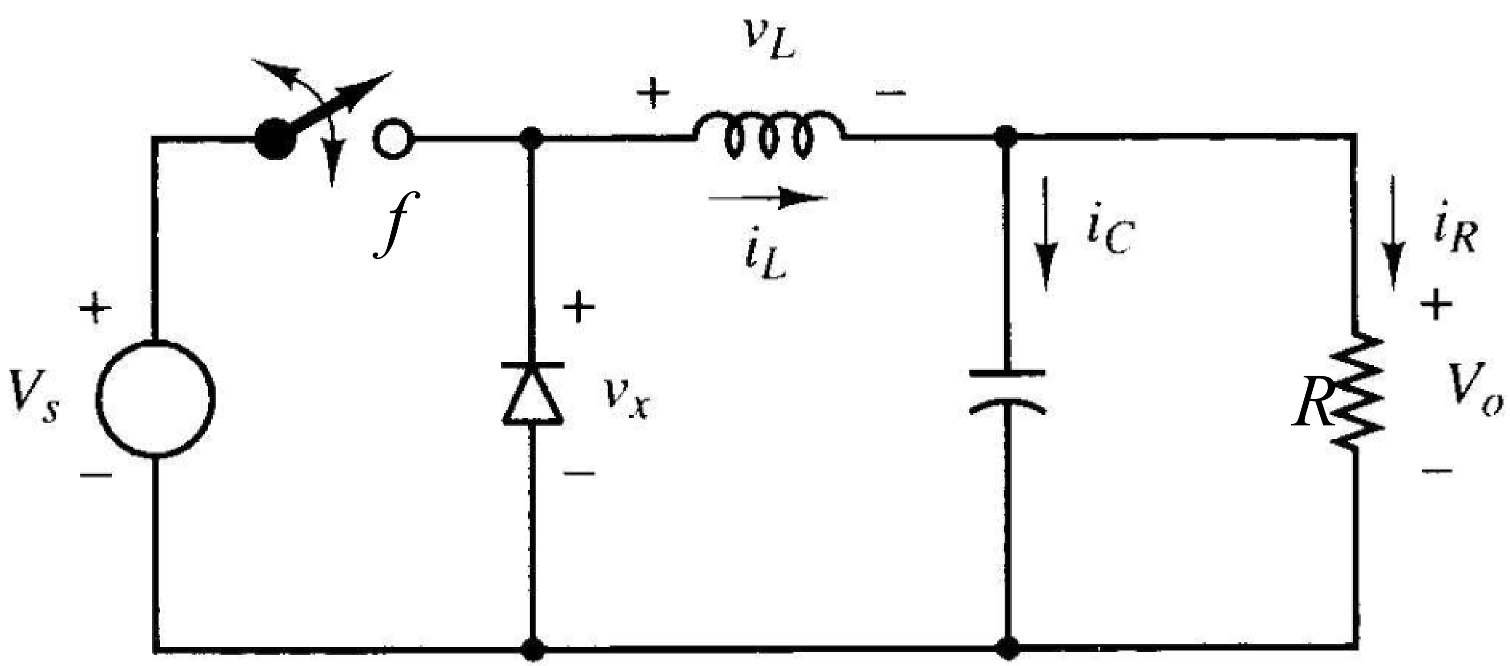
ΔQ is approximately:

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{\Delta i_L}{8f}$$

$$\Delta V_o = \frac{\Delta i_L}{8fC}$$

$$C = \frac{\Delta i_L}{8f\Delta V_o} = \frac{V_o(1-D)T}{8f\Delta V_o} = \frac{(1-D)}{8f^2\left(\frac{\Delta V_o}{V_o}\right)}$$

Buck Converter



$$V_o = DV_s$$

$$L_{\min} = \frac{(1-D)R}{2f}$$

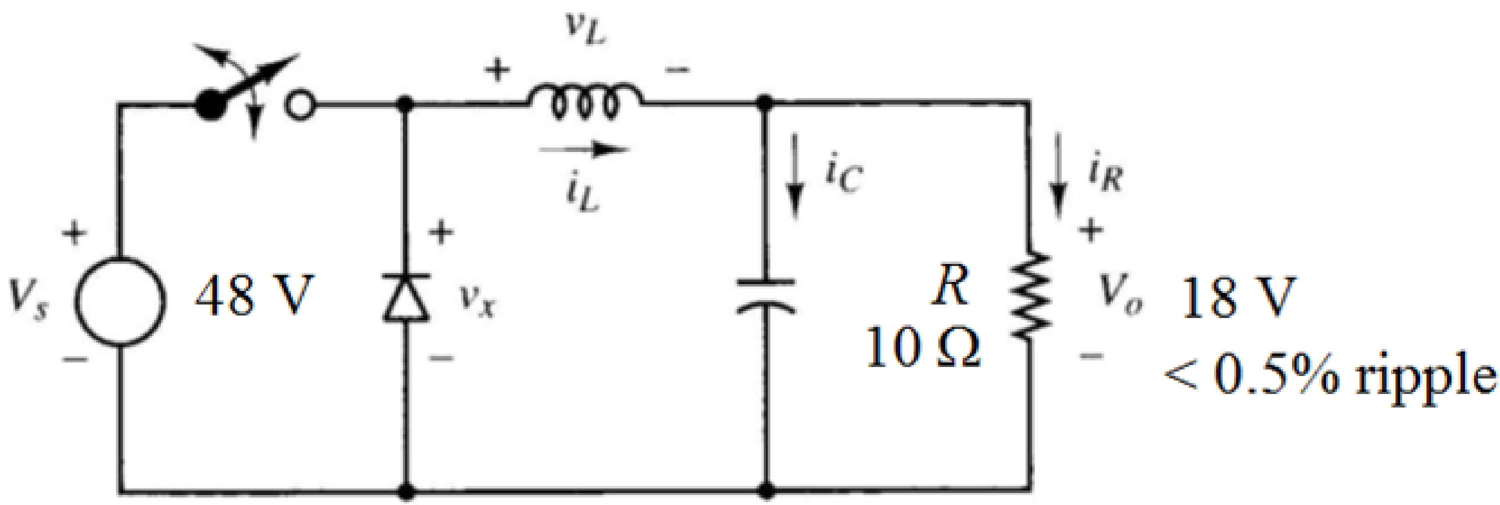
$$C = \frac{1-D}{8Lf^2\left(\frac{\Delta V_o}{V_o}\right)}$$

Exercise #1: Design a Buck converter with the following specifications:

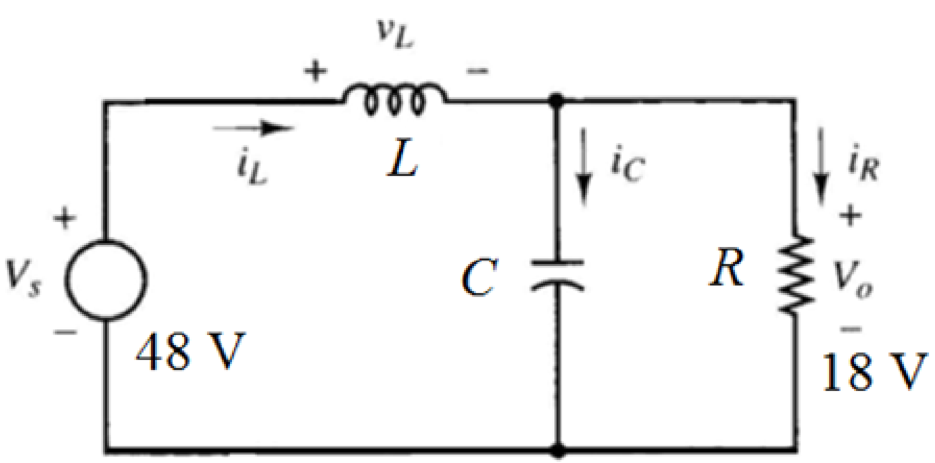
Input DC Voltage	48 V
Output DC Voltage	18 V
Output Ripple	< 0.5%
Load Resistance	10 Ω

If the switching frequency is 40 kHz, determine the required duty ratio, the values of the inductor and capacitor and their rms voltage and current ratings. Also, determine the peak inverse voltage (PIV) of the diode and transistor switch.

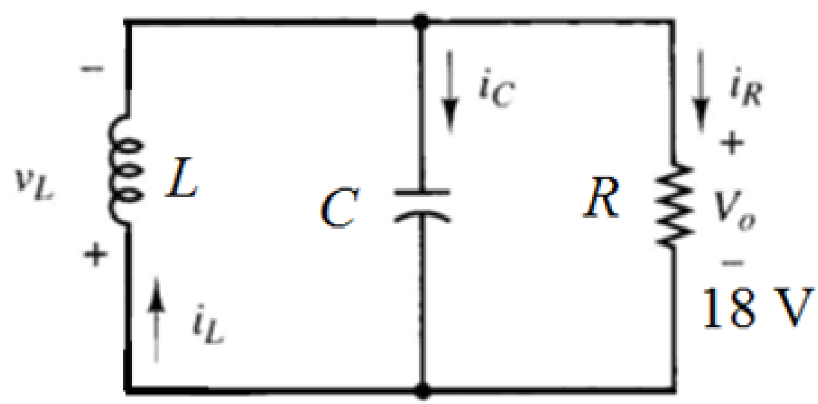
Note: To ensure continuous inductor current, choose the inductor value to be 25% larger than the minimum value.



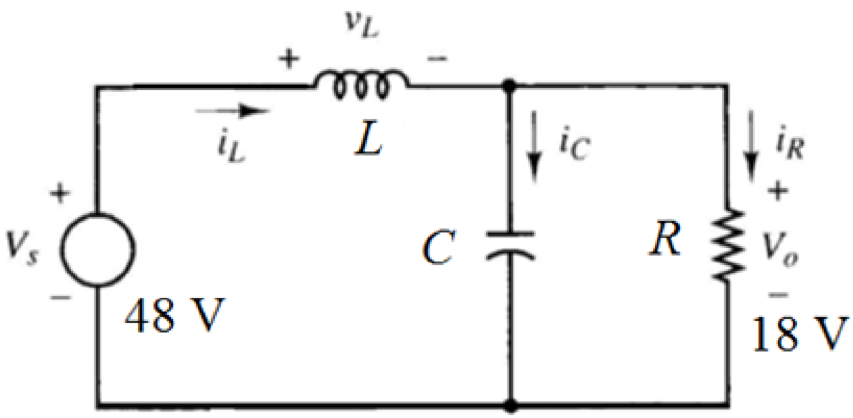
Switch closed:



Switch opened:

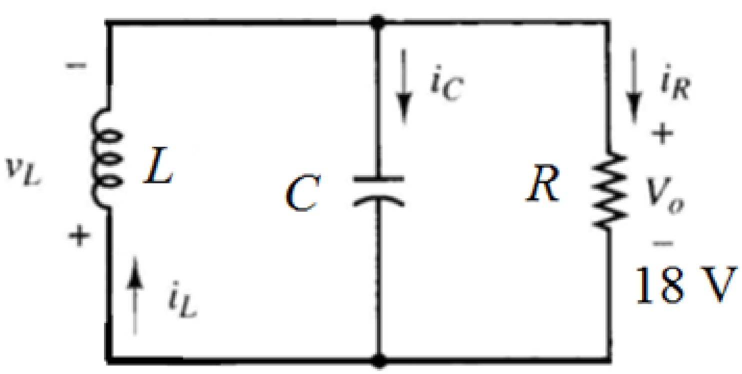


Switch closed:

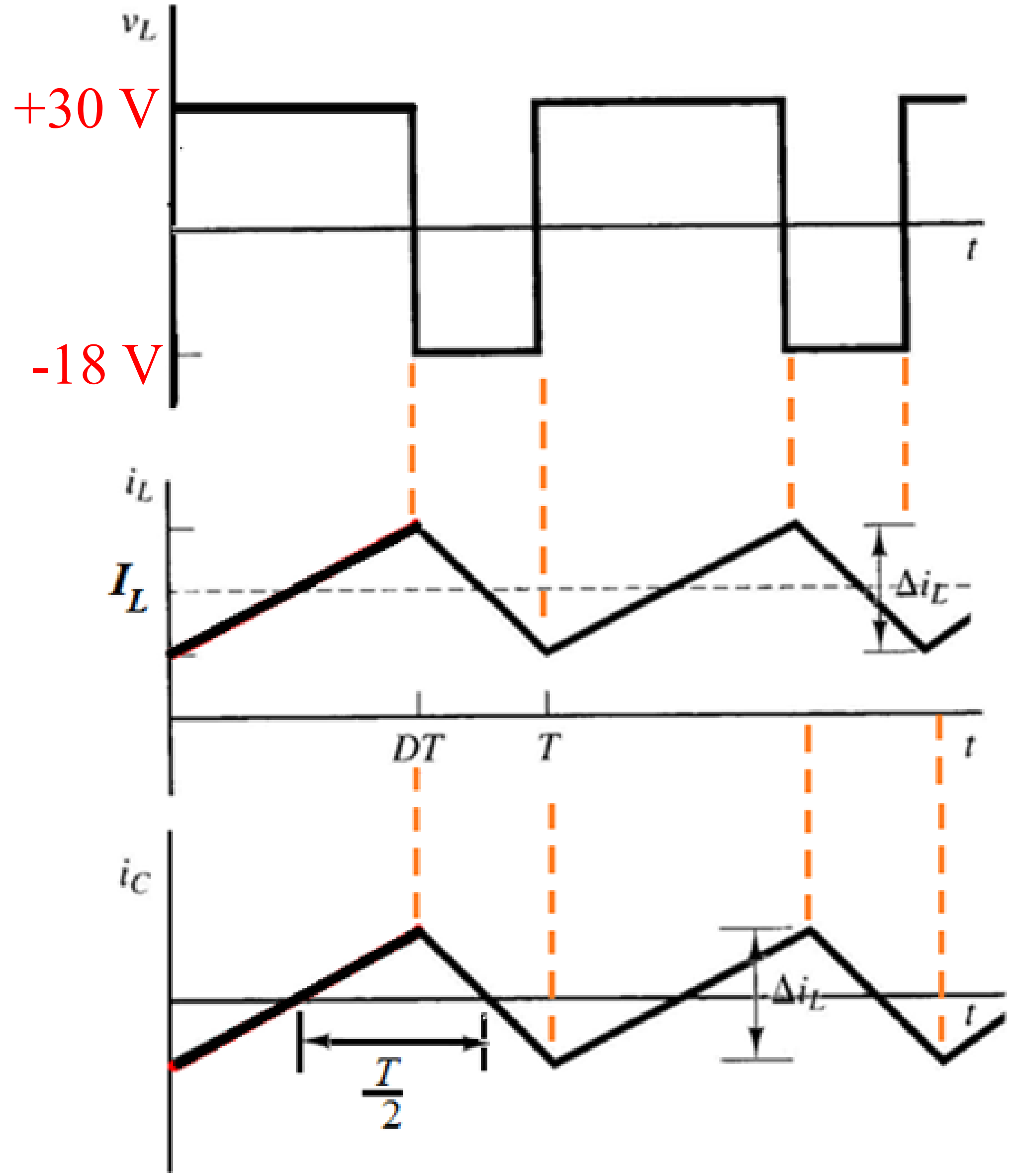


$$v_L = V_S - V_o = 48 - 18 = 30\text{ V}$$

Switch opened:



$$v_L = -V_o = -18\text{ V}$$



$$D = \frac{V_o}{V_S} = \frac{18}{48} = 0.375$$

$$v_L = L\frac{\Delta i_L}{DT} = 30$$

$$\therefore \Delta i_L = \frac{30DT}{L} = \frac{30 \times 0.375}{Lf} = \frac{11.25}{Lf}$$

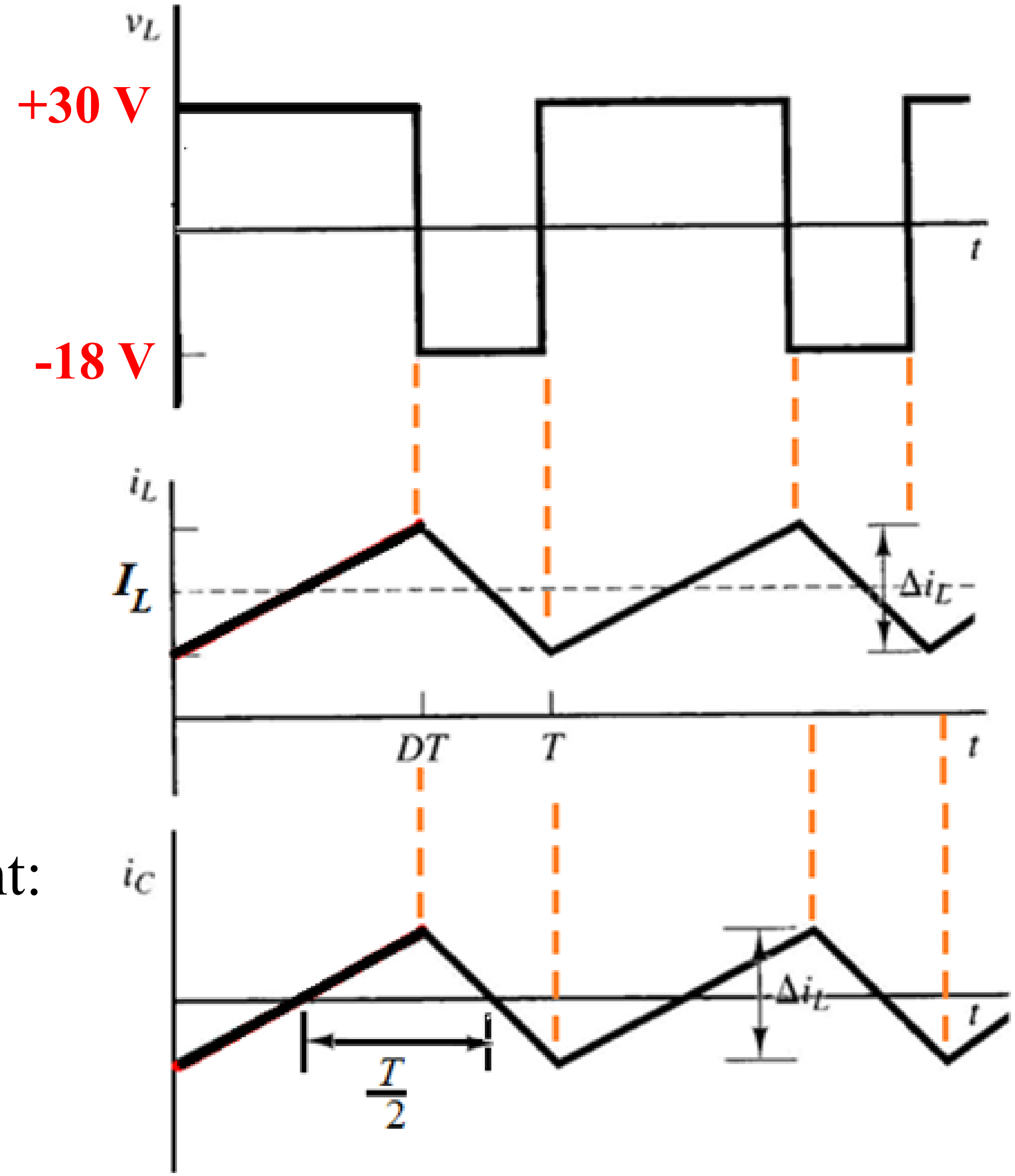
$$I_L = I_R = \frac{V_o}{R} = \frac{18}{10} = 1.8 \text{ A}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = 1.8 - \frac{5.625}{Lf}$$

For continuous inductor current:

$$I_{min} = 1.8 - \frac{5.625}{Lf} = 0$$

$$\therefore (Lf)_{min} = 3.125$$



$f = 40$ kHz, the min. inductance for continuous inductor current:

$$L_{min} = \frac{3.125}{40k} = 78\ \mu\text{H}$$

Choose the inductor value to be 25% larger than the minimum value:

$$L = 1.25 L_{\min} = 1.25 \times 78 = 97.5\ \mu\text{H}$$

$$\Delta i_L = \frac{11.25}{Lf} = \frac{11.25}{97.5\mu \times 40k} = 2.88\ \text{A}$$

The rms current rating of the inductor:

$$I_{L,rms} = \sqrt{I_L{}^2 + \left(\frac{\Delta i_L / 2}{\sqrt{3}}\right)^2} = \sqrt{1.8^2 + \left(\frac{2.88/2}{\sqrt{3}}\right)^2} = 1.98\ \text{A}$$

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right)$$

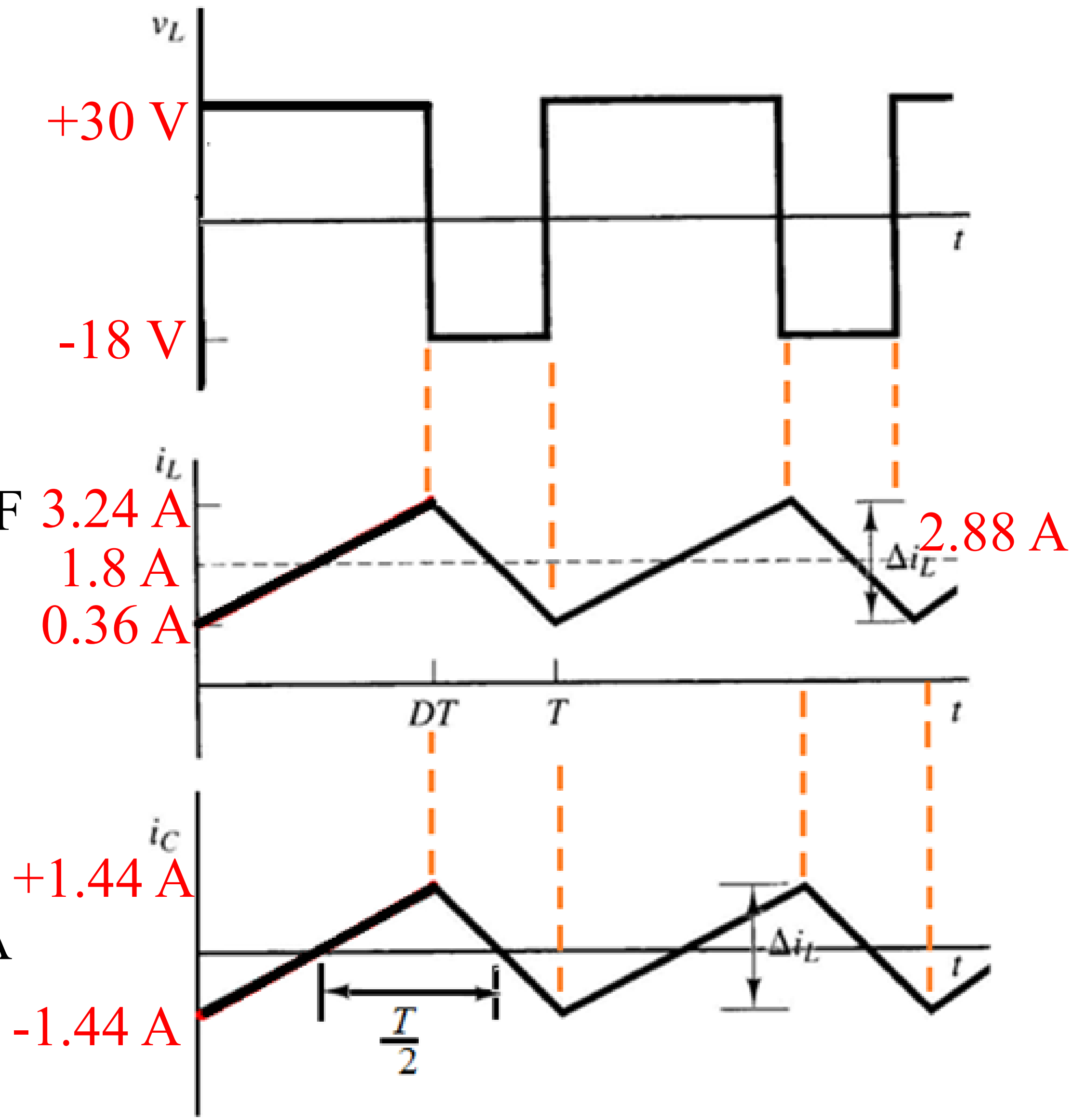
$$= \frac{\Delta i_L}{8f} = \frac{2.88}{8 \times 40k} = 9 \times 10^{-6} \text{ C}$$

$$\Delta Q = C \Delta V_o$$

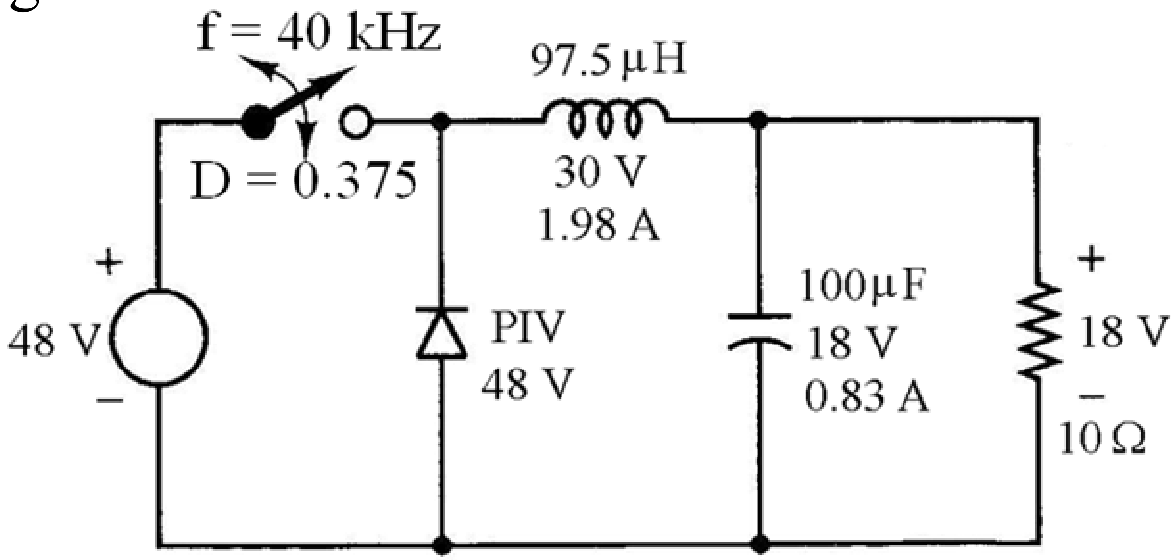
$$\therefore C = \frac{\Delta Q}{\Delta V_0} = \frac{9 \times 10^{-6}}{0.005 \times 18} = 100\,\mu\text{F}$$

The rms current rating of the capacitor:

$$I_{C,rms} = \frac{\Delta i_L / 2}{\sqrt{3}} = \frac{2.88/2}{\sqrt{3}} = 0.83 \text{ A}$$



Final design:



$D = 0.375$

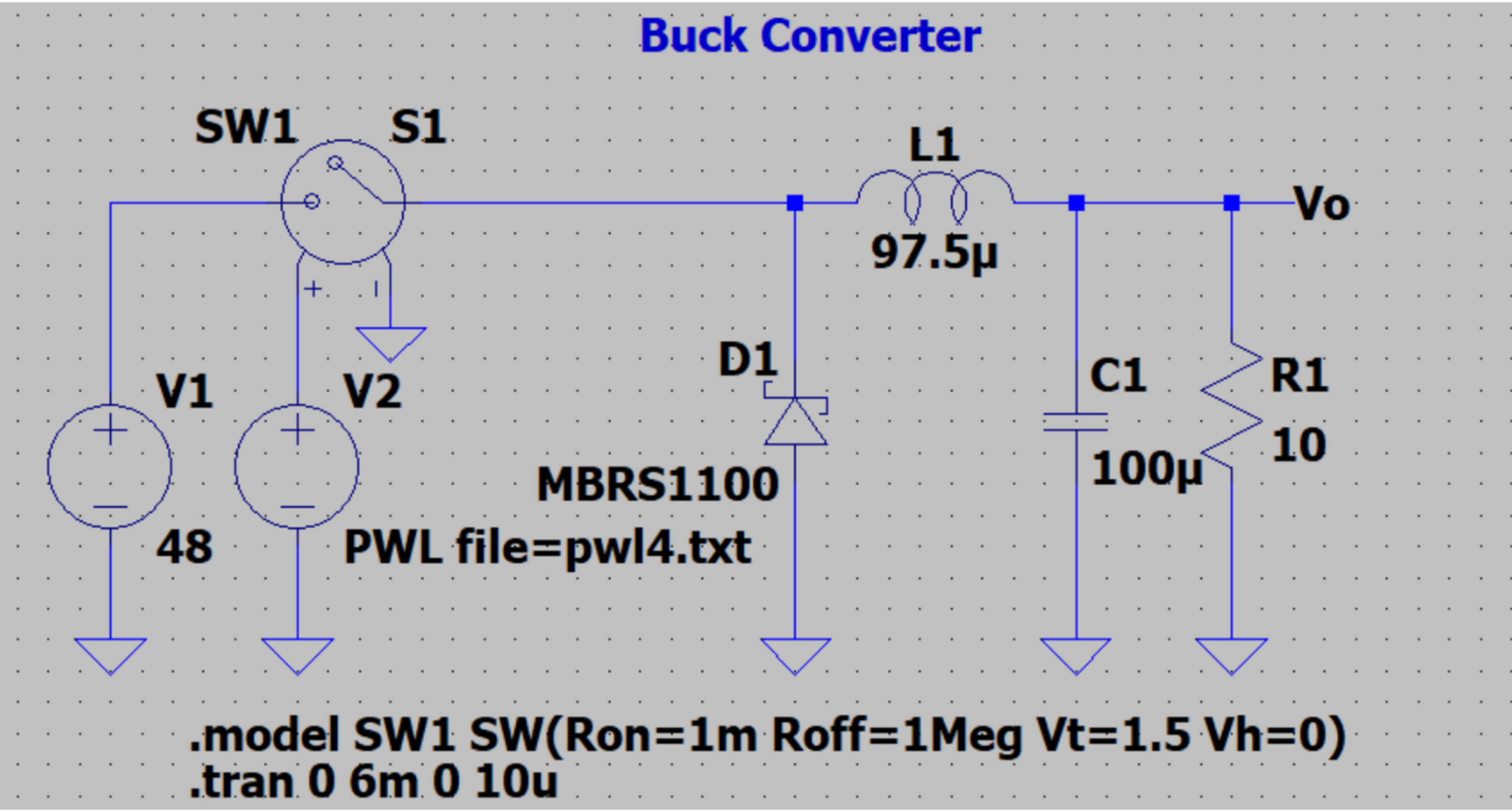
L = 97.5 μH, current rating ≥ 1.98 A, voltage rating ≥ 30 V

C = 100 μF, current rating ≥ 0.83 A, voltage rating ≥ 18 V

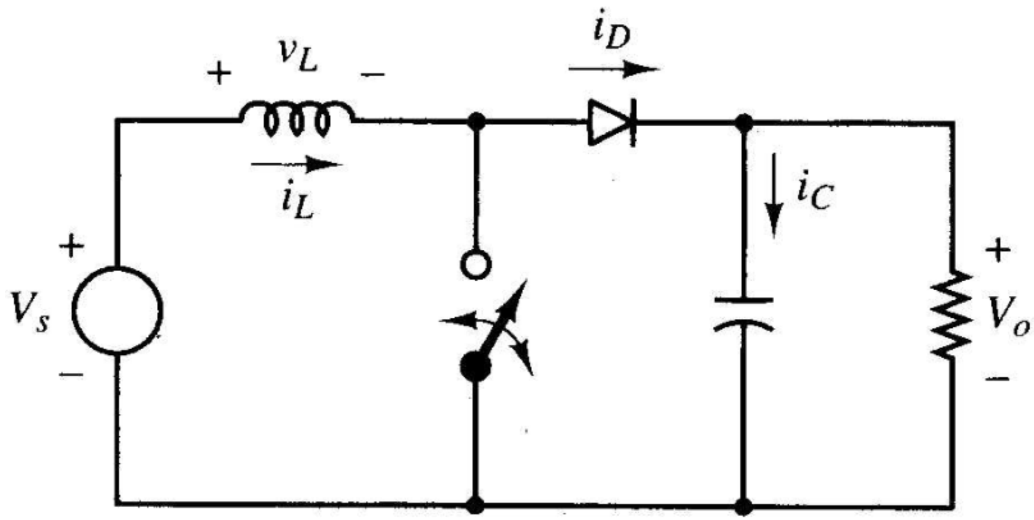
Diode PIV ≥ 48 V

Transistor PIV ≥ 48 V

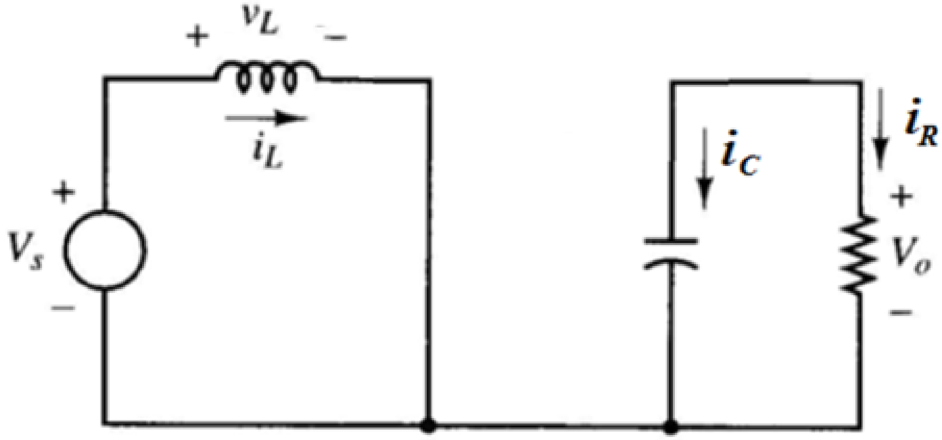
LTSpice Simulation:



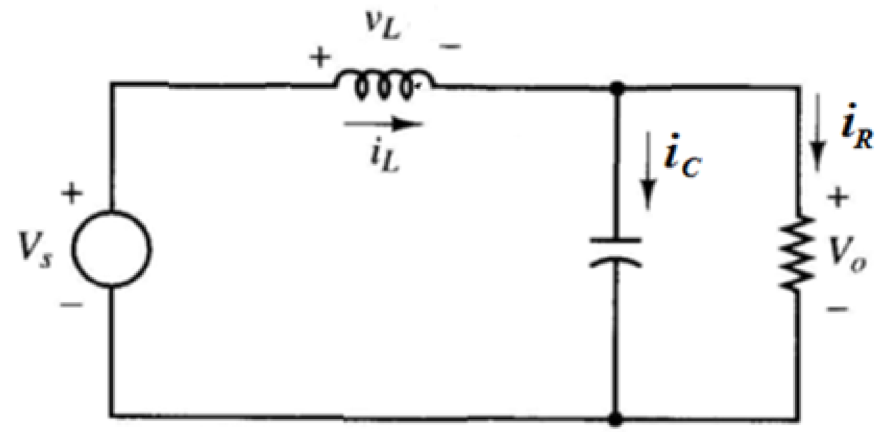
Boost Converter



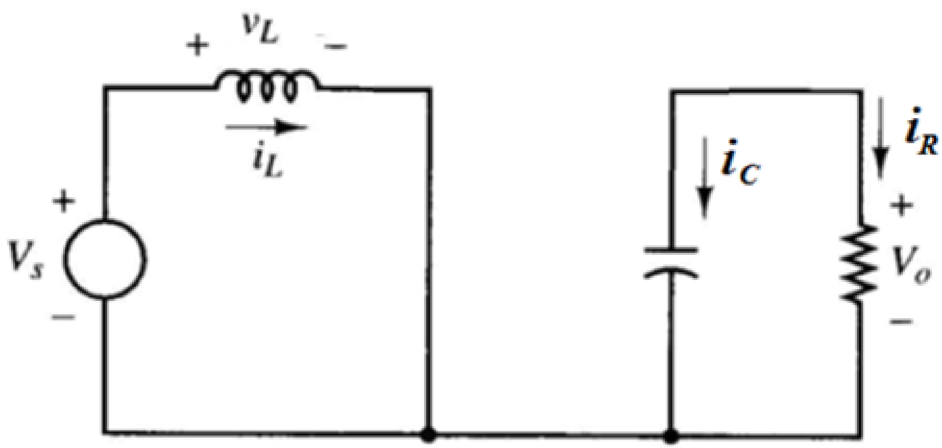
Switch closed

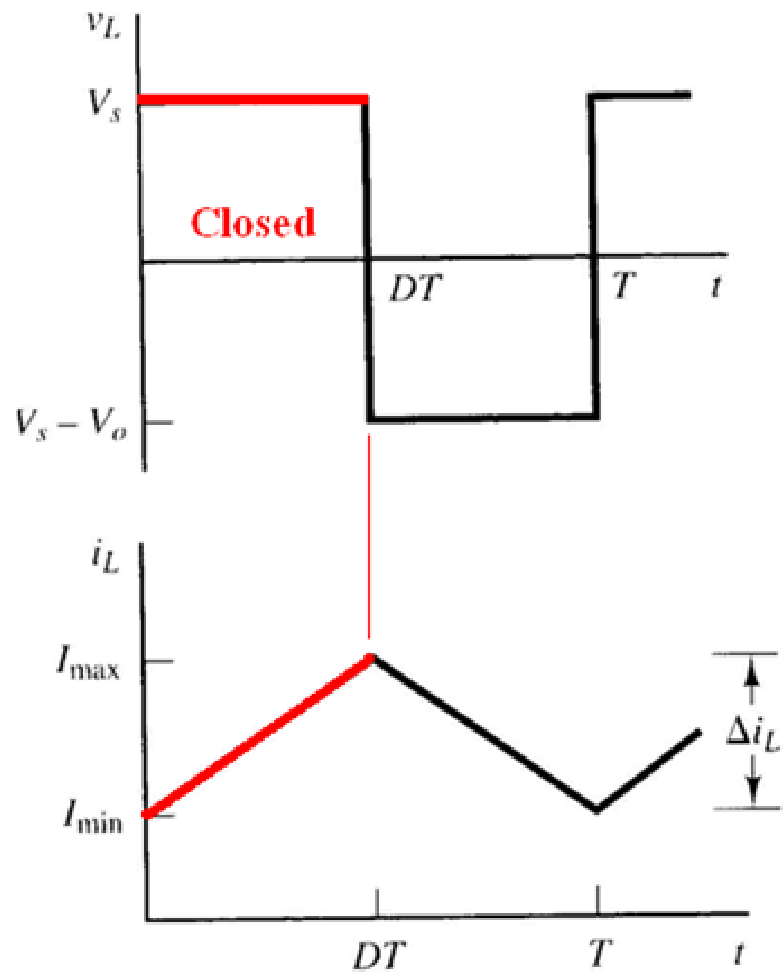


Switch opened



Boost Converter (Switch Closed)



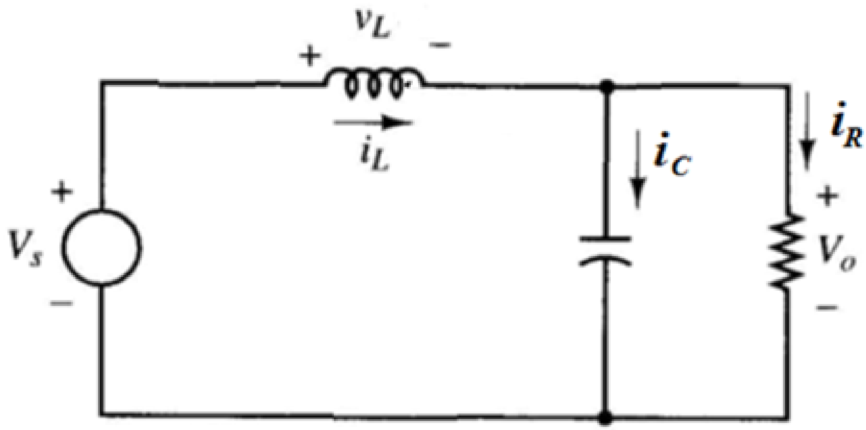


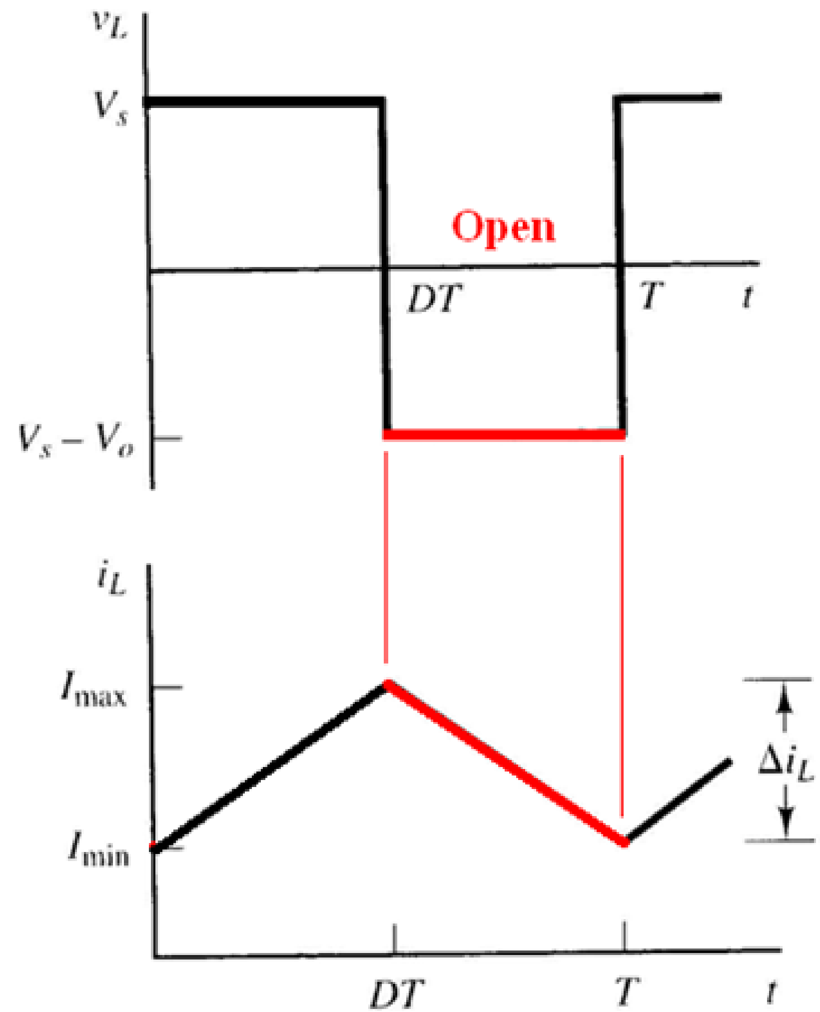
$$v_L = V_s = L\frac{di_L}{dt} = L\frac{\Delta i_L}{\Delta t}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L}$$

$$\therefore (\Delta i_L)_{closed} = \frac{DTV_s}{L}$$

Boost Converter (Switch Opened)



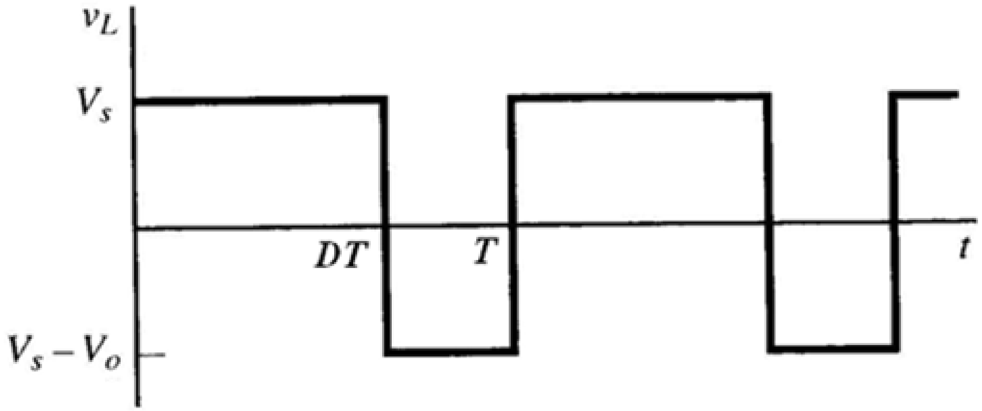


$$v_L = V_s - V_o = L\frac{di_L}{dt} = L\frac{\Delta i_L}{(1-D)T}$$

$$\frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{open} = \frac{(V_s - V_o)(1-D)T}{L}$$

Average Output Voltage



Under steady-state condition, the average voltage across L over one cycle is zero.

$$\frac{1}{T}\int_0^T v_L dt = \frac{1}{T}\left[\int_0^{DT} V_s dt + \int_{DT}^{T} (V_s - V_o) dt\right] = 0$$

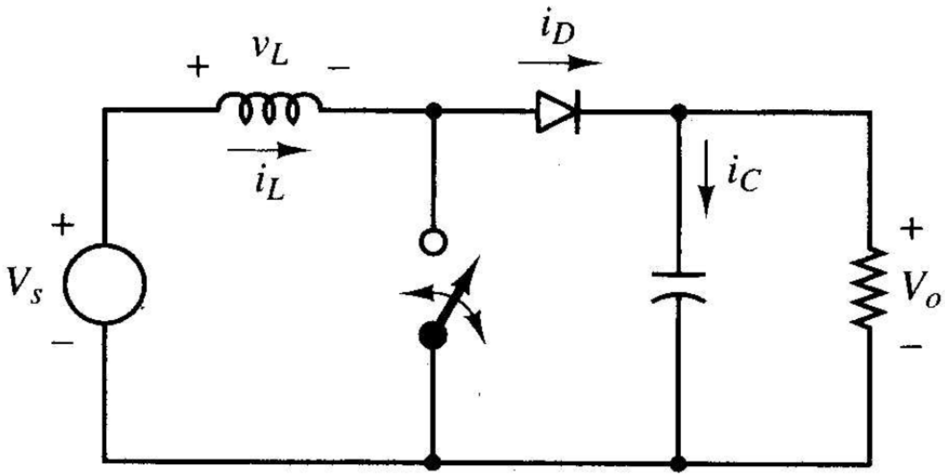
$$\frac{1}{T}\left[V_s DT + (V_s - V_o)(1-D)T\right] = 0$$

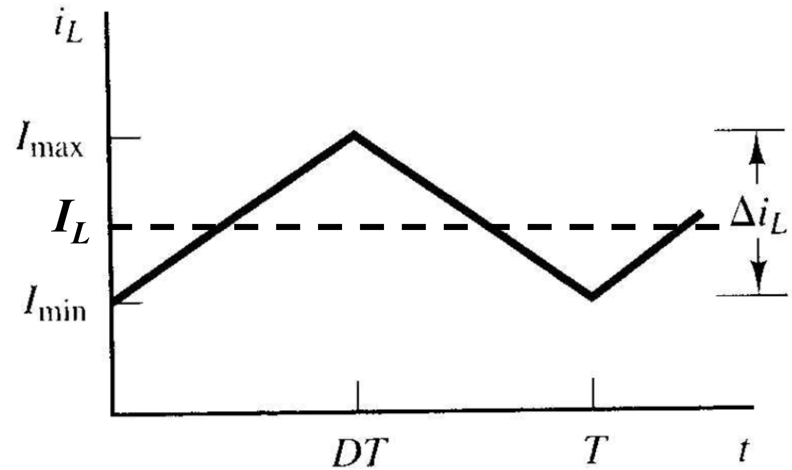
$$V_s D + (V_s - V_o)(1-D) = 0$$

$$V_o = \frac{V_s}{1-D}$$

For $0 < D < 1$, $V_o > V_s$

Inductor Current





For a lossless converter, input power = output power:

$$V_s I_L = \frac{V_o^{\,2}}{R} = \frac{\left(\frac{V_s}{1-D}\right)^2}{R} = \frac{V_s^{\,2}}{(1-D)^2 R}, \quad \therefore I_L = \frac{V_s}{(1-D)^2 R}$$

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

Continuous Inductor Current

$$I_{\min} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} = 0$$

$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

$$(Lf)_{\min} = \frac{D(1-D)^2 R}{2}$$

For a known switching frequency, minimum L for continuous current is:

$$L_{\min} = \frac{D(1-D)^2 R}{2f}$$

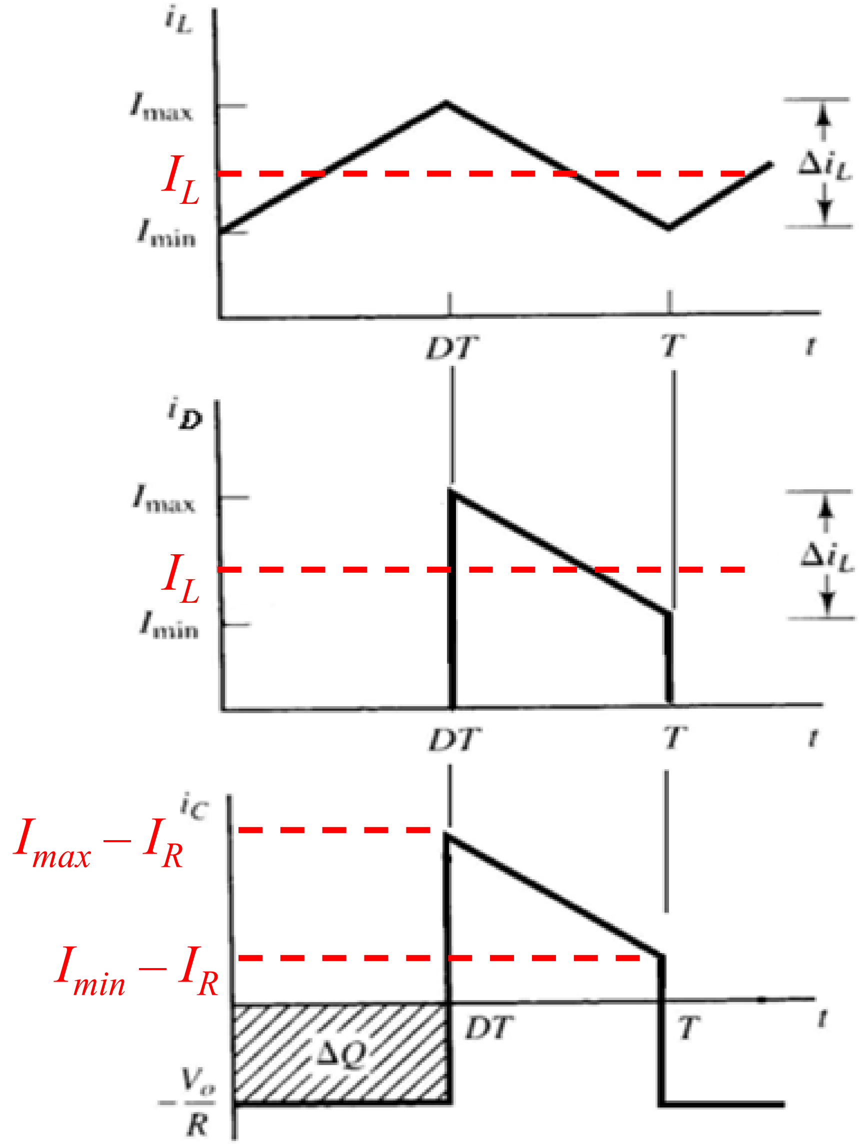
Output Ripple Voltage

$$|\Delta Q| = \left(\frac{V_o}{R}\right)DT = C\Delta V_o$$

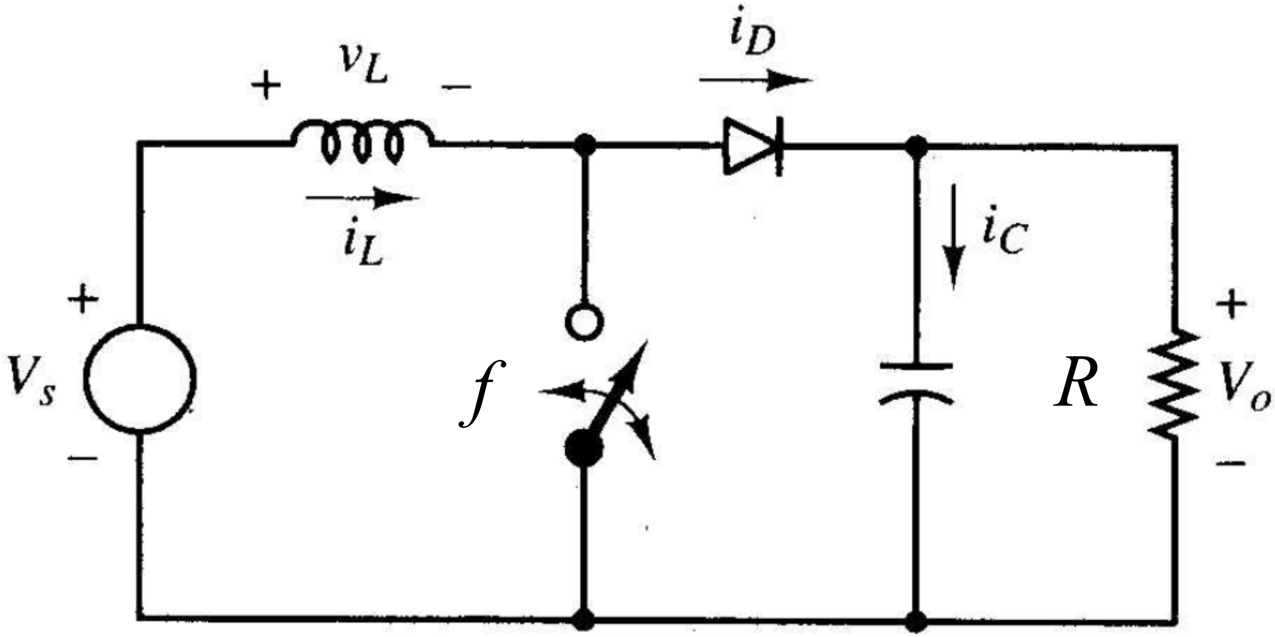
$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

$$C = \frac{D}{Rf\left(\frac{\Delta V_o}{V_o}\right)}$$



Boost Converter



$$V_o = \frac{V_s}{1-D}$$

$$L_{\min} = \frac{D(1-D)^2 R}{2f}$$

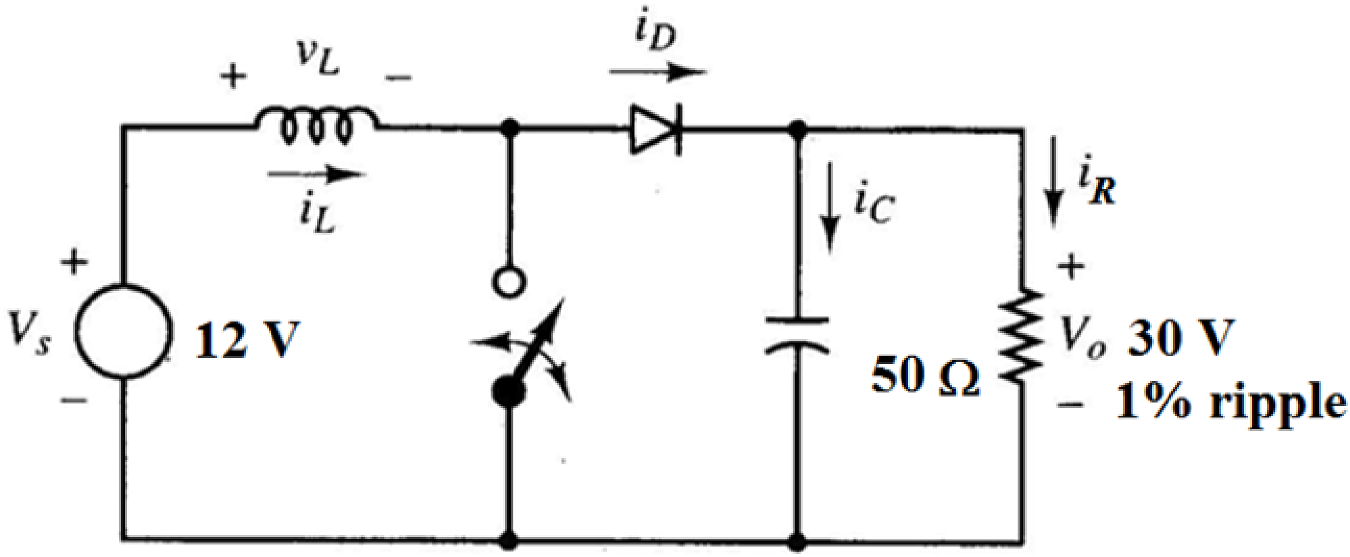
$$C = \frac{D}{Rf\left(\frac{\Delta V_o}{V_o}\right)}$$

Exercise #2: Design a Boost converter to meet the following specifications:

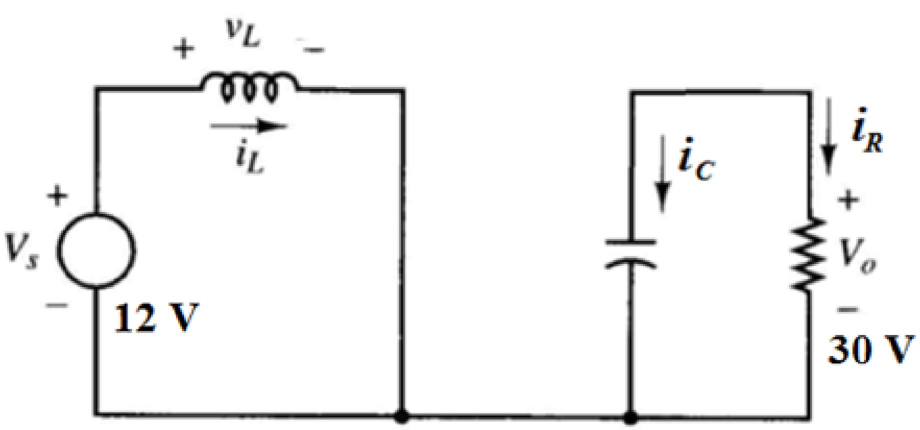
Input DC Voltage	12 V
Output DC Voltage	30 V
Output Ripple	< 1 %
Load Resistance	50 Ω

If the switching frequency is 25 kHz, determine the required duty ratio, the values of the inductor and capacitor and their rms voltage and current ratings. Also, determine the peak inverse voltage (PIV) of the diode and transistor switch.

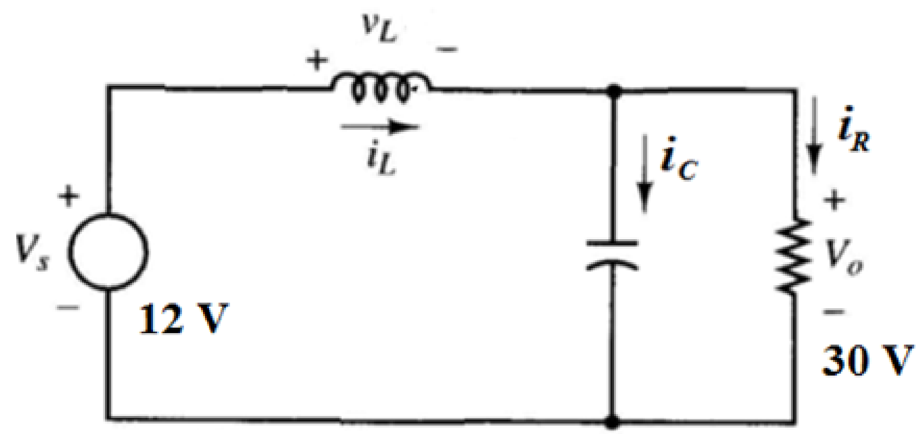
Note: The inductor value must be larger than the minimum value for continuous current operation.



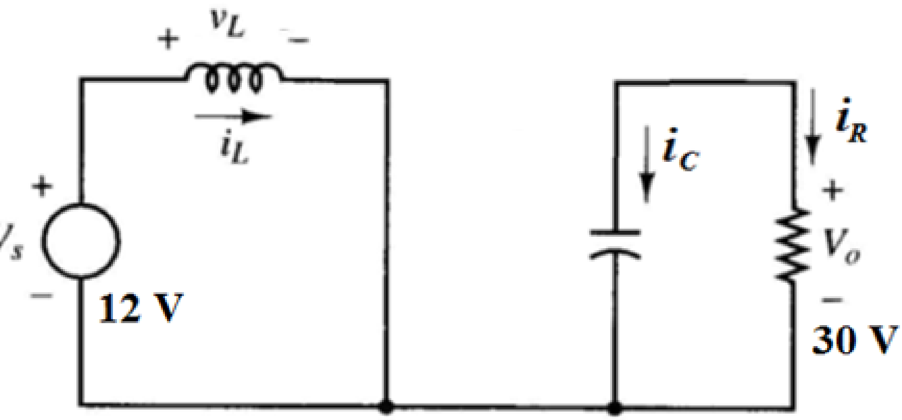
Switch closed:



Switch opened:

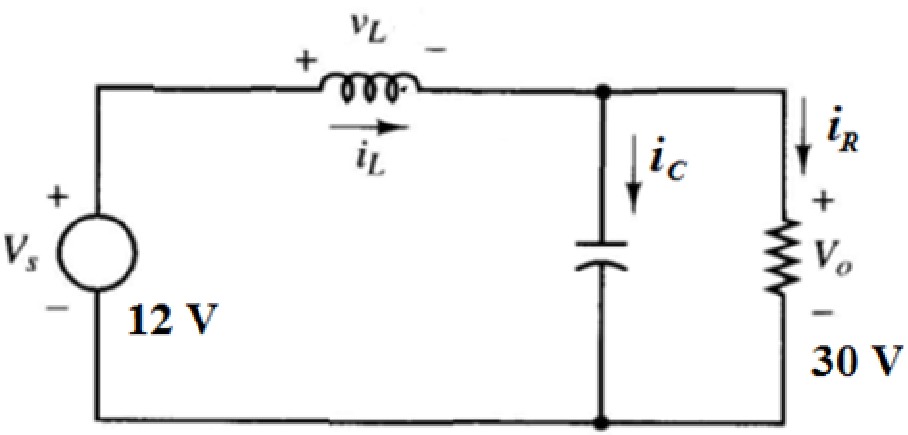


Switch closed:

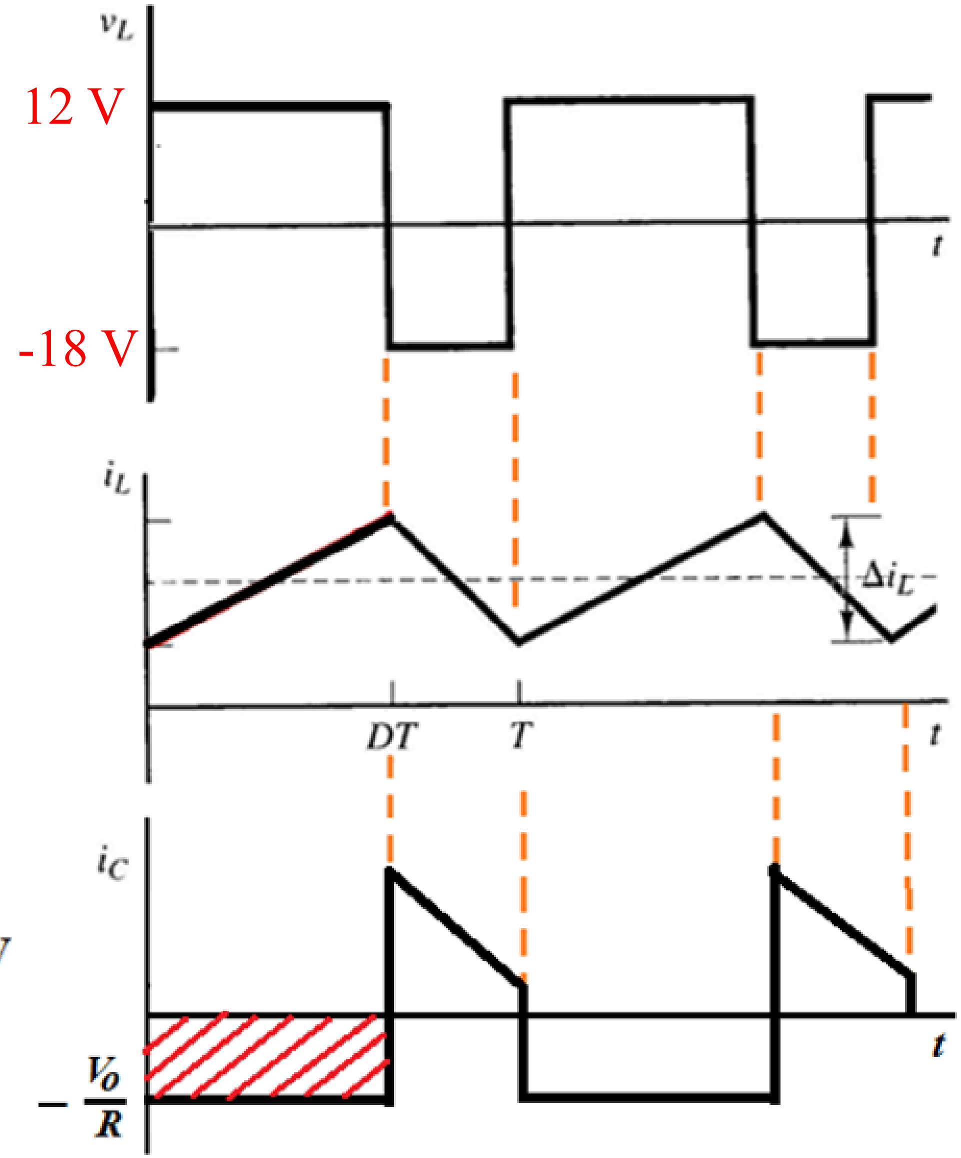


$$v_L = V_S = 12\ \text{V}$$

Switch opened:



$$v_L = V_S - V_o = 12 - 30 = -18\ \text{V}$$



The duty ratio:

$$V_o = \frac{V_S}{1 - D}$$

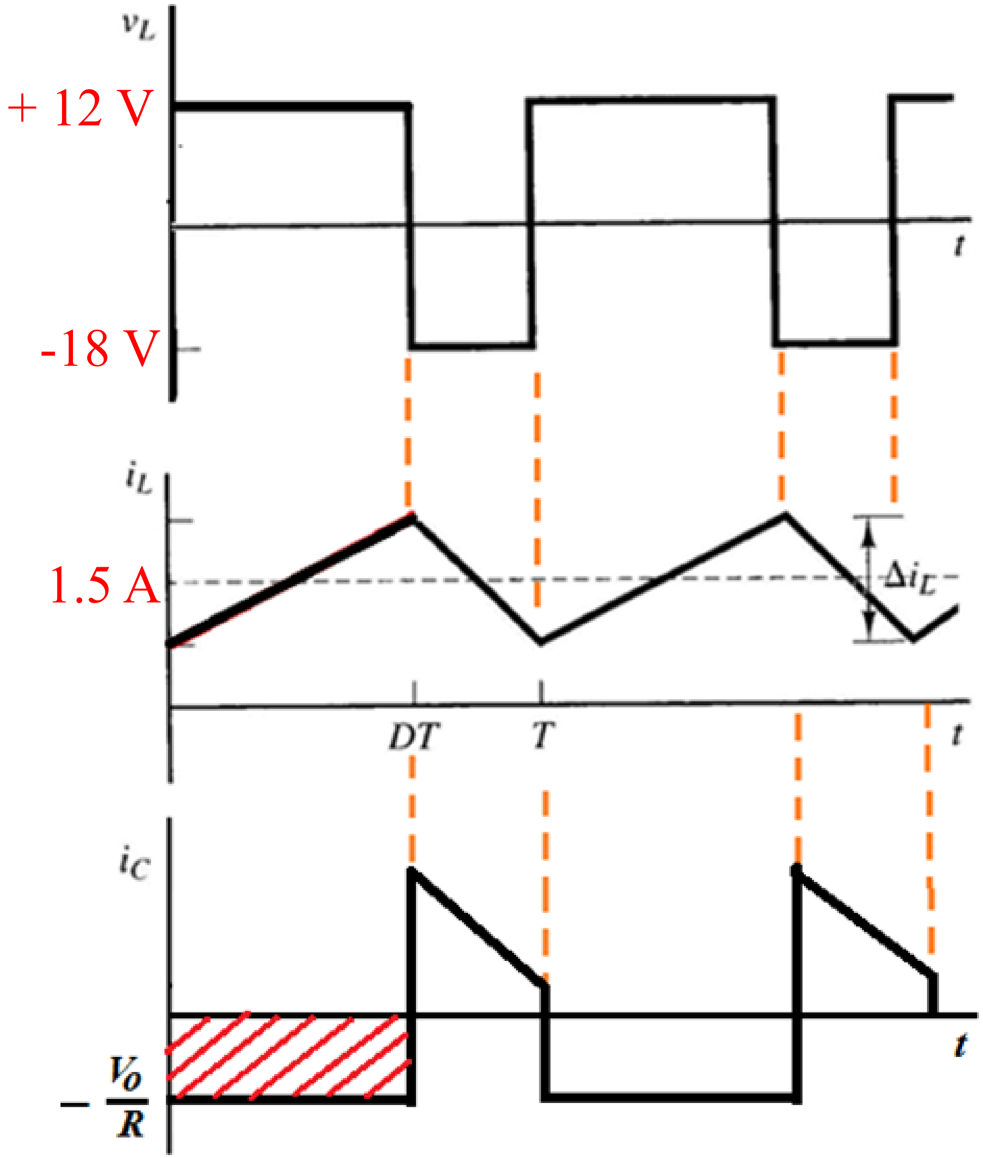
$$\therefore\ D = 1 - \frac{V_S}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$I_L V_S = \frac{V_o^{\ 2}}{R}$$

$$\therefore\ I_L = \frac{V_o^{\ 2}}{V_S R} = \frac{30^{\ 2}}{12 \times 50} = 1.5\ \mathrm{A}$$

$$v_L = V_s = L\frac{\Delta i_L}{DT}$$

$$\therefore \Delta i_L = \frac{DV_s}{Lf} = \frac{0.6 \times 12}{Lf} = \frac{7.2}{Lf}$$



$$I_{min} = I_L - \frac{\Delta i_L}{2} = 1.5 - \frac{3.6}{Lf} = 0$$

$$\therefore (Lf)_{min} = \frac{3.6}{1.5} = 2.4$$

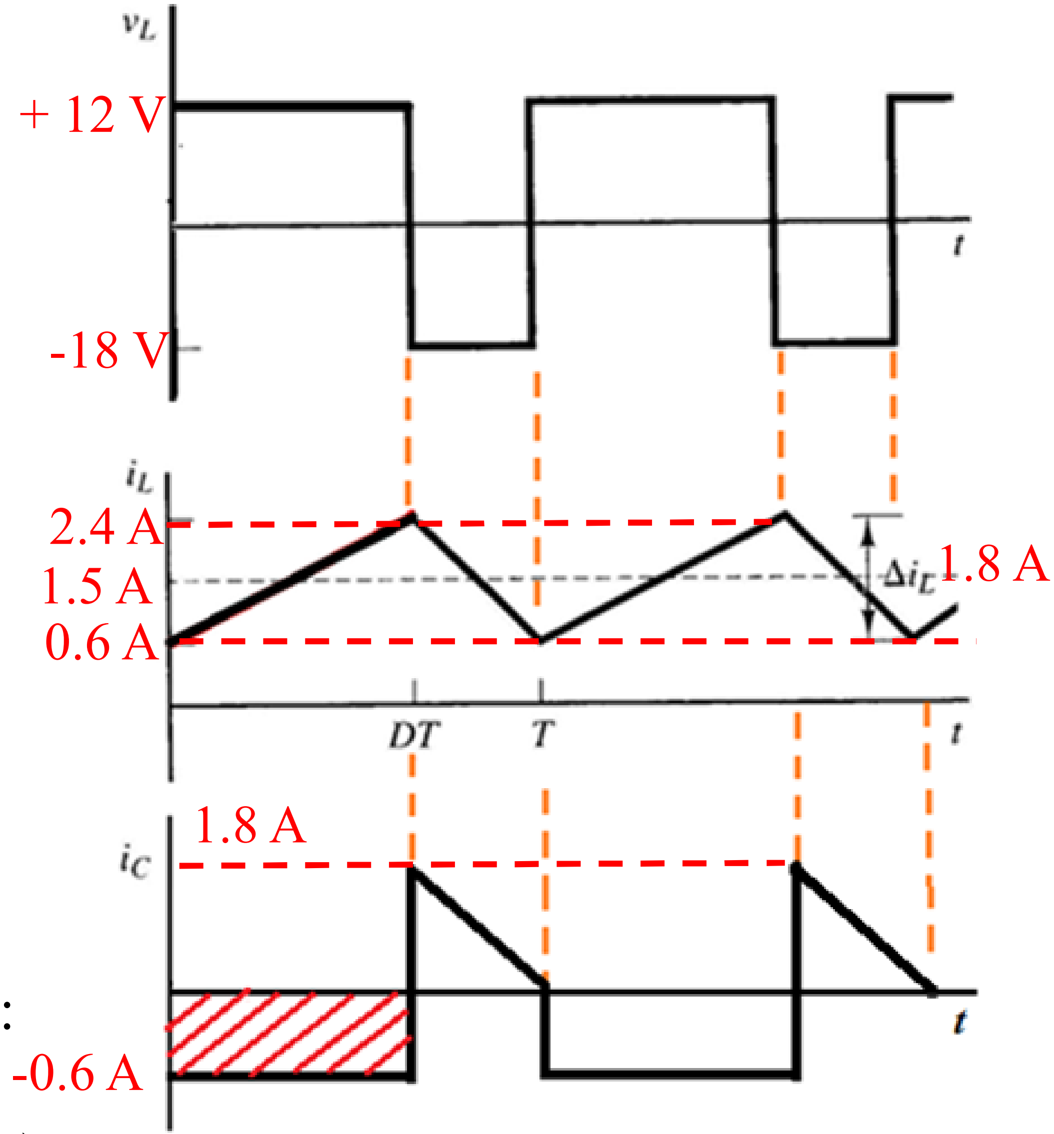
$f = 25$ kHz, the min. inductance:

$$L_{\min} = \frac{2.4}{f} = \frac{2.4}{25k} = 96\ \mu\text{H}$$

When the switch is closed, C provides the load current:

$$I_R = \frac{V_o}{R} = \frac{30}{50} = 0.6\ \text{A}$$

When the switch is opened, C is charging by the current through L:



$$i_L - I_R = i_c = \Delta i_L = 2\times(1.5-0.6) = 1.8\ \text{A}$$

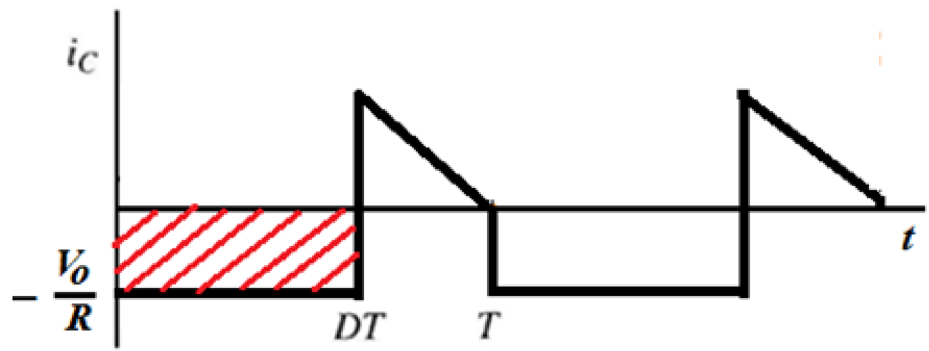
$$\Delta i_L = 1.8 = \frac{7.2}{Lf} = \frac{7.2}{L \times 25k}$$

$$\therefore L = \frac{7.2}{1.8 \times 25k} = 160\ \mu\text{H}$$

$$I_{L,rms} = \sqrt{I_L{}^2 + \left(\frac{\Delta i_L / 2}{\sqrt{3}}\right)^2} = \sqrt{1.5^2 + \left(\frac{1.8/2}{\sqrt{3}}\right)^2} = 1.59\ \text{A}$$

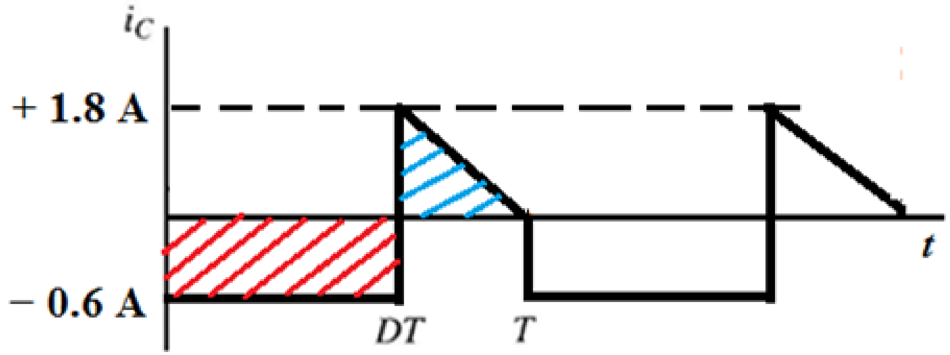
$$|\Delta Q| = \frac{V_o}{R} \times DT$$

$$= \frac{DV_o}{fR} = \frac{0.6 \times 30}{25k \times 50} = 14.4 \times 10^{-6}$$



$$\Delta Q = C \Delta V_o$$

$$\therefore C = \frac{\Delta Q}{\Delta V_o} = \frac{14.4 \times 10^{-6}}{0.01 \times 30} = 48\ \mu\text{F}$$



$$I_{C,rms} = \sqrt{\frac{1}{T}\int_0^T i_c^{\,2}(t)dt} = \sqrt{\frac{1}{T}\left[\int_0^{DT}(0.6)^2 dt + \int_0^{(1-D)T}\left(1.8 - \frac{1.8t}{(1-D)T}\right)^2 dt\right]}$$

$$\text{First term } = \int_0^{DT}(0.6)^2 dt = (0.6)^2 DT = 0.36 \times 0.6T = 0.216T$$

$$\text{Second term } = \int_0^{(1-D)T}\left(1.8 - \frac{1.8t}{(1-D)T}\right)^2 dt = \int_0^{(1-D)T}\left(3.24 - \frac{6.48t}{(1-D)T} + \frac{3.24t^2}{(1-D)^2T^2}\right)dt$$

$$= \left[3.24t - \frac{3.24t^2}{(1-D)T} + \frac{1.08t^3}{(1-D)^2T^2}\right]_0^{(1-D)T}$$

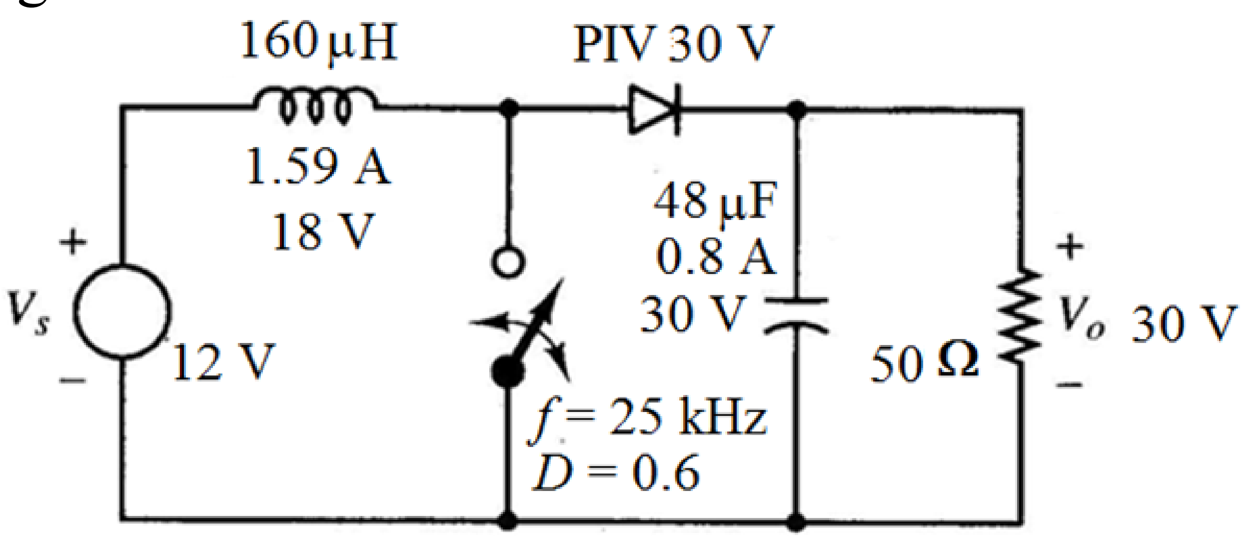
$$\text{Second term } = \left[3.24t - \frac{3.24t^2}{(1-D)T} + \frac{1.08t^3}{(1-D)^2T^2}\right]_0^{(1-D)T}$$

$$= 3.24(1-D)T - 3.24(1-D)T + 1.08(1-D)T$$

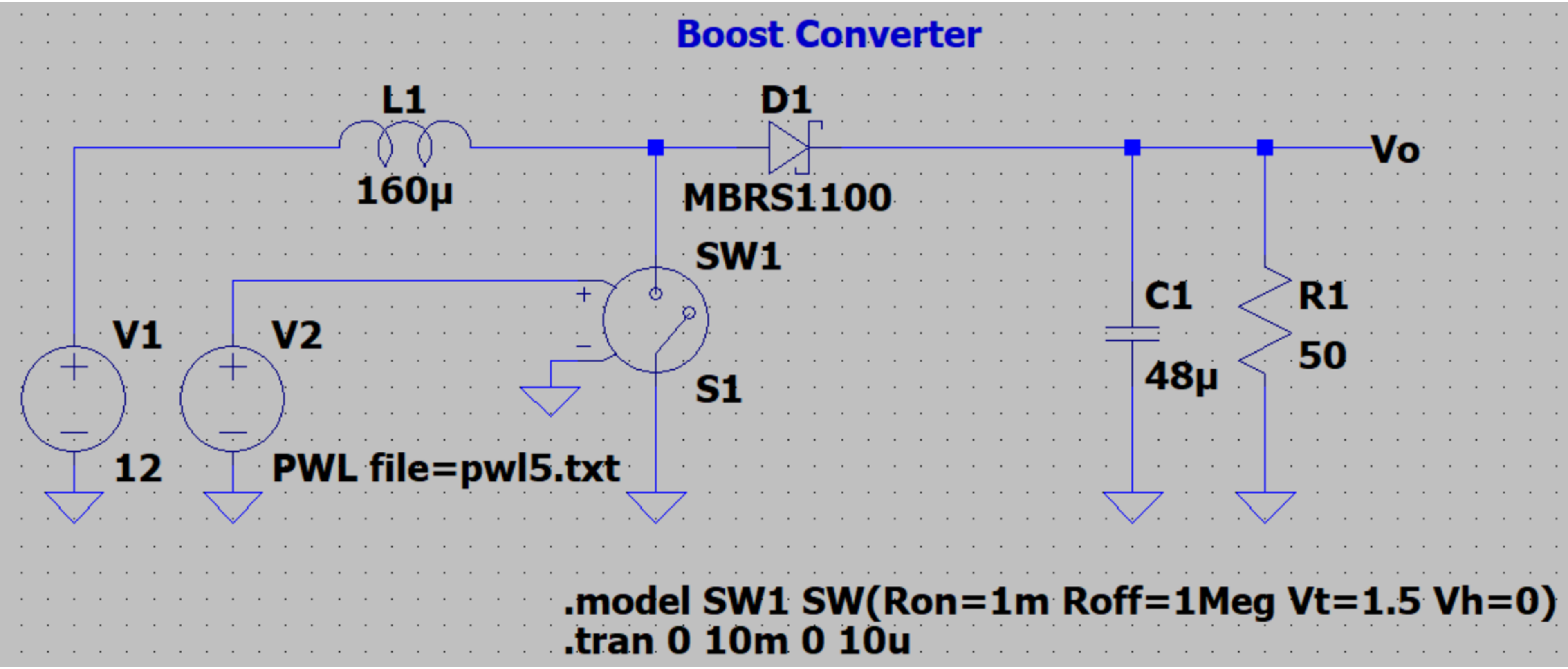
$$= 1.296T - 1.296T + 0.432T = 0.432T$$

$$I_{C,rms} = \sqrt{\frac{1}{T}\left(0.216T + 0.432T\right)} = \sqrt{0.216 + 0.432} = 0.8\ \text{A}$$

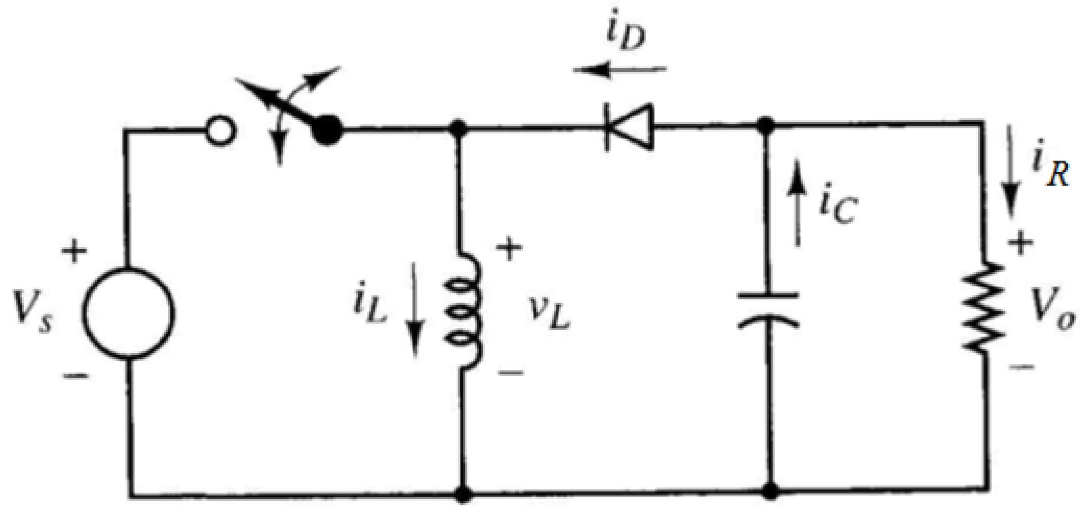
Final design:



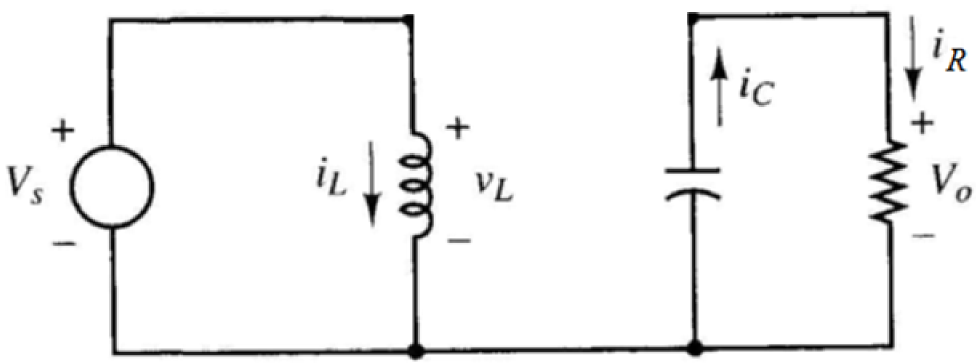
LTSpice Simulation:



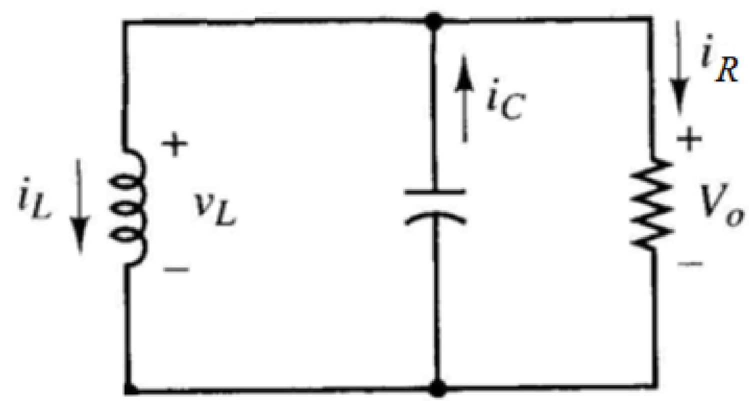
Buck-Boost Converter



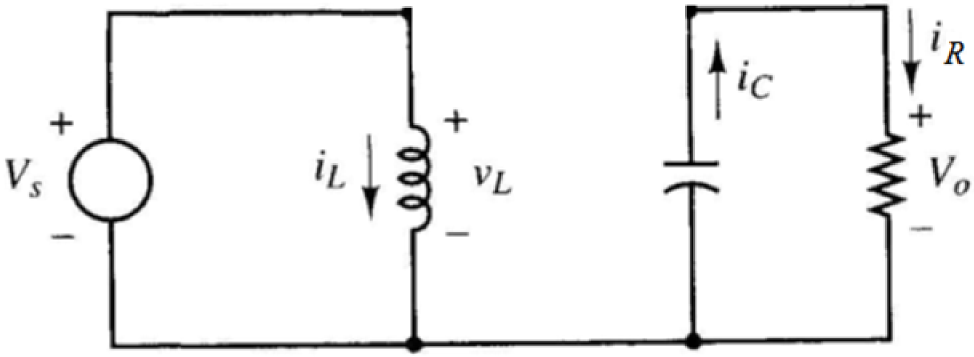
Switch closed

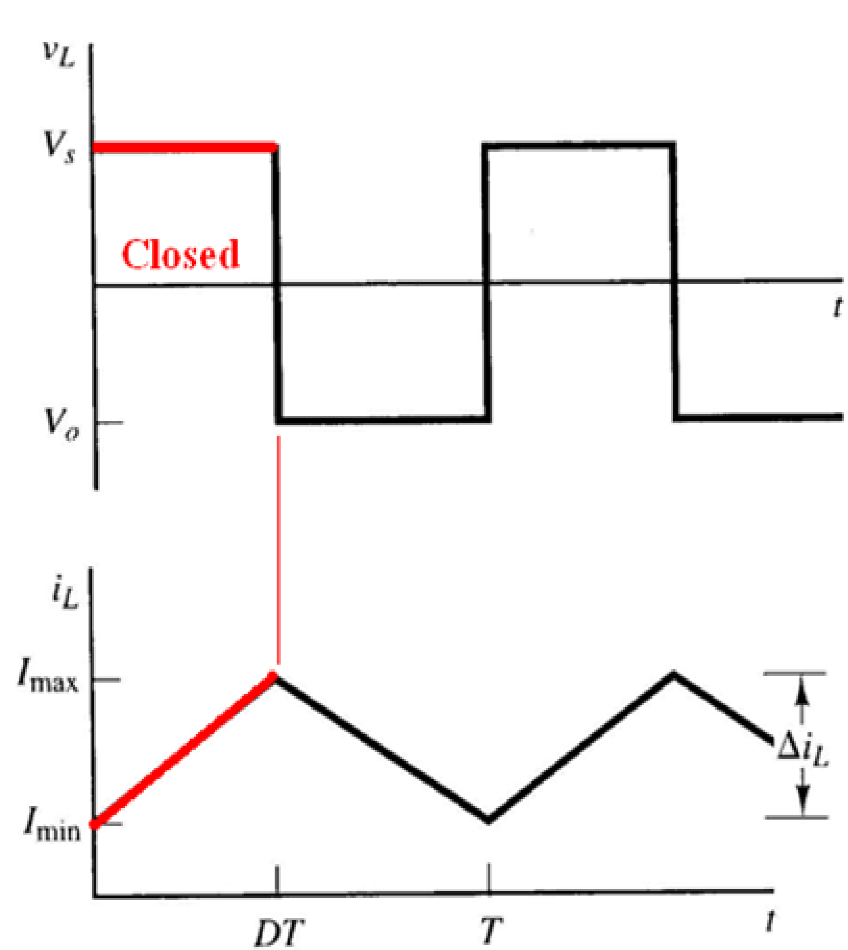


Switch opened



Buck-Boost Converter (Switch Closed)



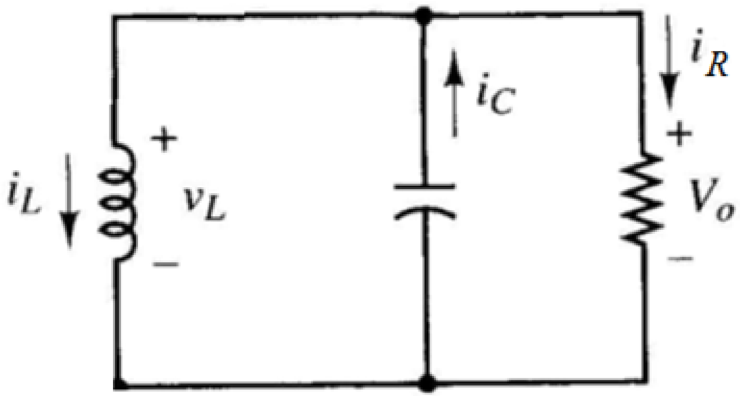


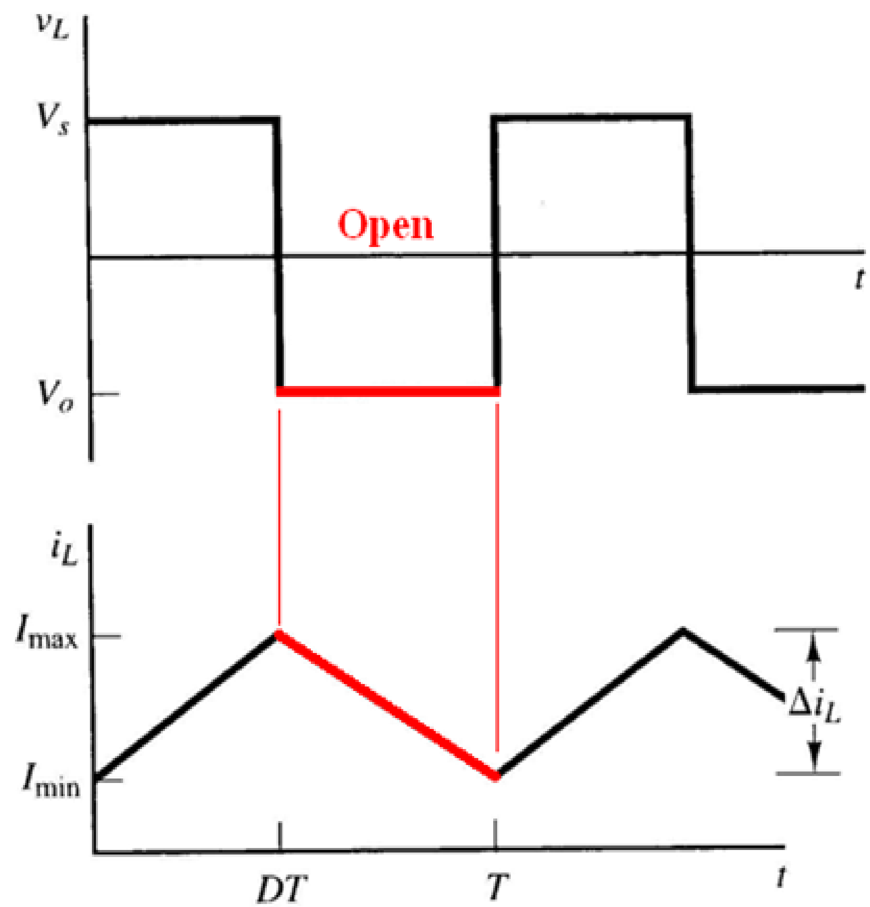
$$v_L = V_s = L\frac{di_L}{dt} = L\frac{\Delta i_L}{DT}$$

$$\frac{\Delta i_L}{DT} = \frac{V_s}{L}$$

$$(\Delta i_L)_{closed} = \frac{DTV_s}{L}$$

Buck-Boost Converter (Switch Open)





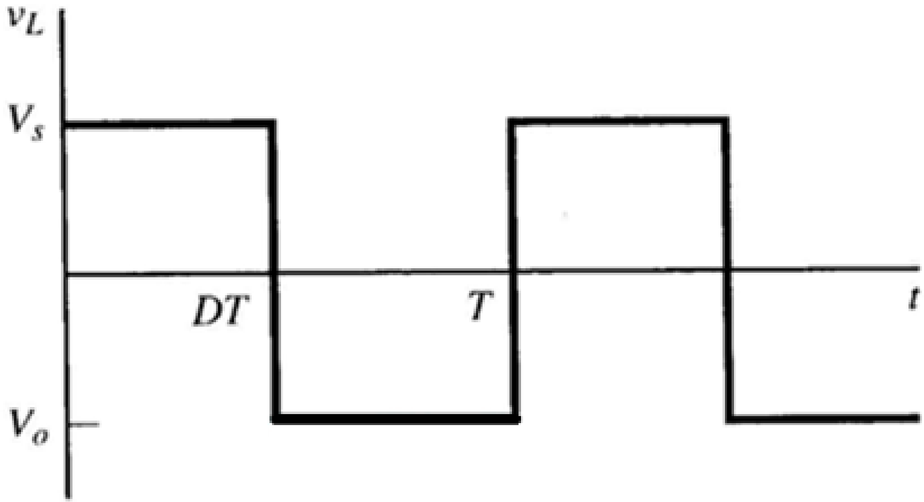
$$v_L = V_o = L\frac{di_L}{dt} = L\frac{\Delta i_L}{(1-D)T}$$

$$\frac{\Delta i_L}{(1-D)T} = \frac{V_o}{L}$$

$$(\Delta i_L)_{open} = \frac{(1-D)TV_o}{L}$$

Average Output Voltage

$$\frac{1}{T}\int_0^T v_L dt = \frac{1}{T}\left[\int_0^{DT} V_s dt + \int_{DT}^{T} V_o dt\right] = 0$$

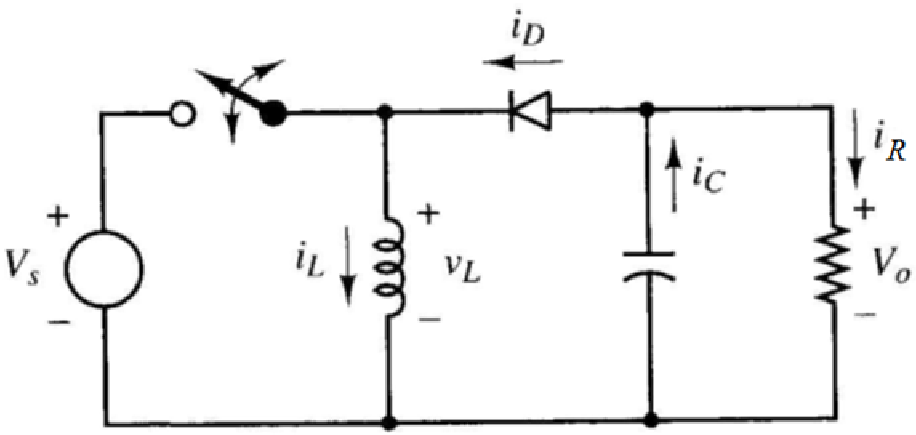


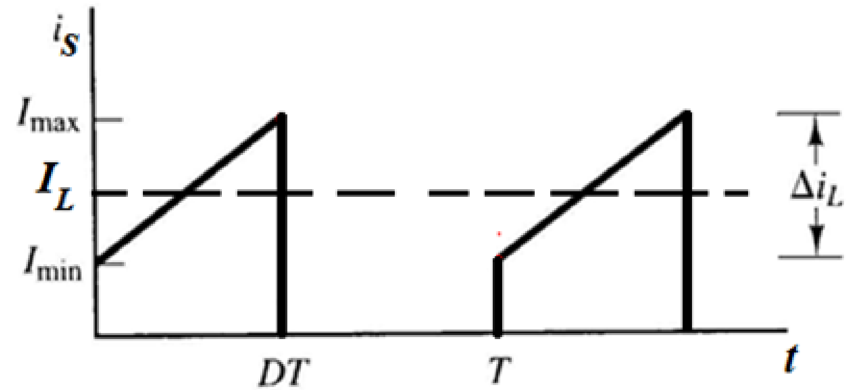
$$V_s DT + V_o(1 - D)T = 0$$

$$\therefore V_o = -V_s\left(\frac{D}{1-D}\right)$$

- Output voltage is of opposite polarity of input voltage.
- For $0 < D < 1$, $|V_o|$ can be either higher or lower than V_s
- If $D > 0.5$, $|V_o| > V_s$; If $D < 0.5$, $|V_o| < V_s$

Minimum Inductor Value





$$V_s I_s = \frac{V_o^{\ 2}}{R}$$

$$\because I_s = I_L D \qquad \therefore V_s I_L D = \frac{V_o^{\ 2}}{R}$$

$$I_L = \frac{V_o^{\ 2}}{D V_s R} = \left(\frac{V_s D}{1-D}\right)^2 \frac{1}{D V_s R} = \frac{V_s^{\ 2} D^2}{(1-D)^2 D V_s R} = \frac{V_s D}{(1-D)^2 R}$$

Minimum Inductor Value

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s D}{(1-D)^2 R} + \frac{V_s DT}{2L}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s D}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

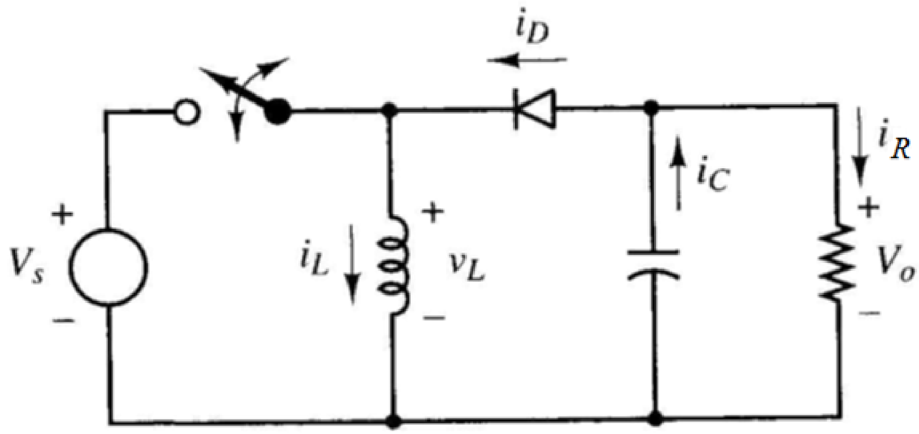
By setting $I_{min} = 0$:

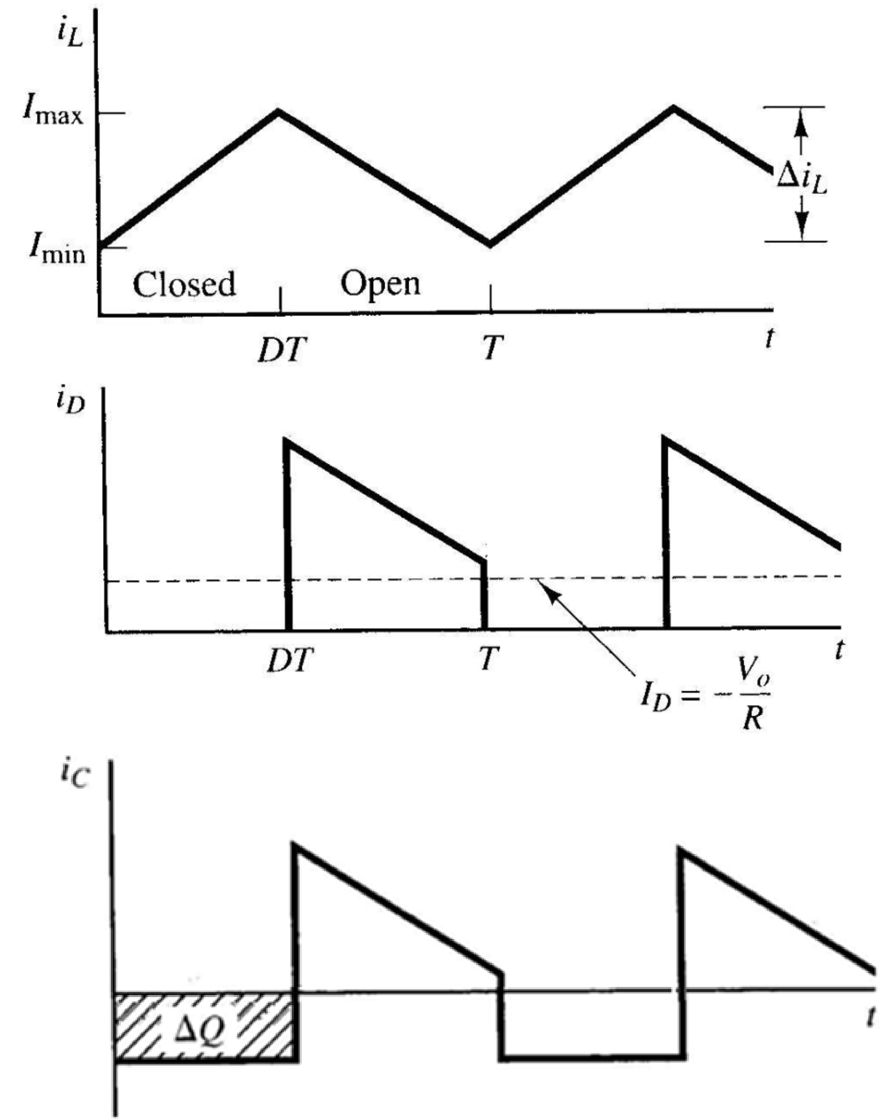
$$\frac{V_s D}{(1-D)^2 R} - \frac{V_s DT}{2L} = 0$$

$$(Lf)_{min} = \frac{(1-D)^2 R}{2}$$

$$L_{min} = \frac{(1-D)^2 R}{2f}$$

Output Ripple Voltage



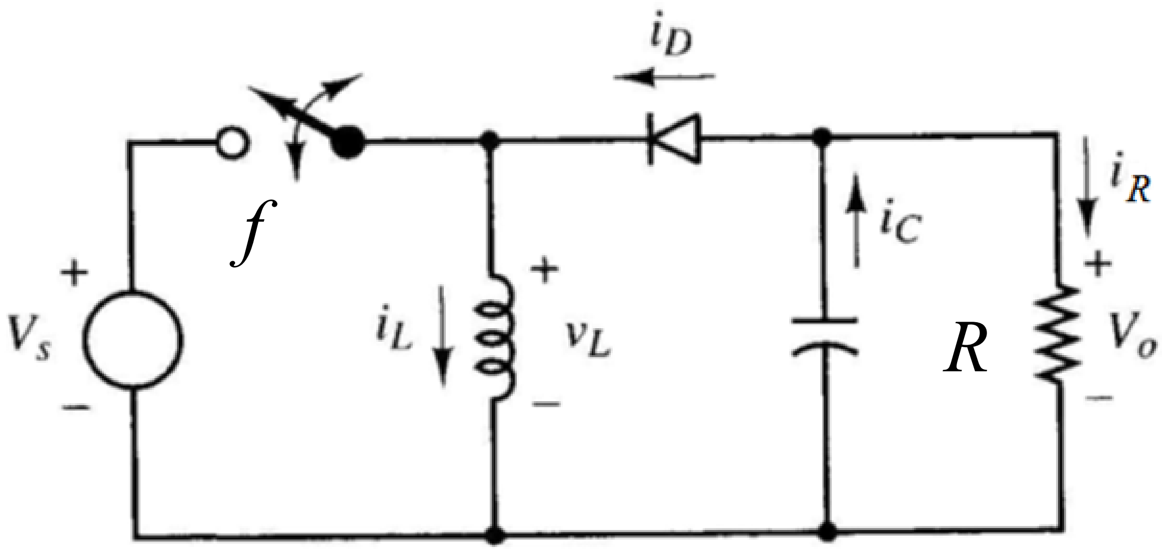


$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \qquad C = \frac{D}{Rf\left(\frac{\Delta V_o}{V_o}\right)}$$

Buck-Boost Converter

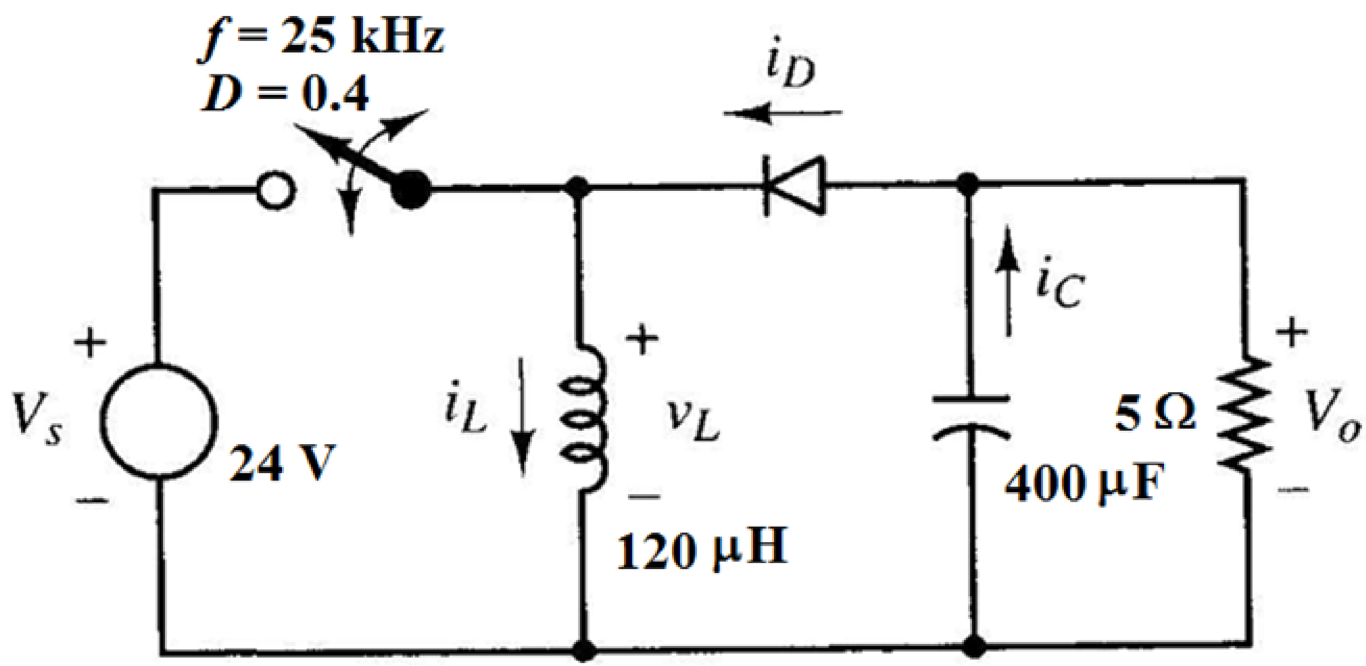


$$V_o = -\frac{DV_s}{1-D}$$

$$L_{\min} = \frac{(1-D)^2 R}{2f}$$

$$C = \frac{D}{Rf\left(\frac{\Delta V_o}{V_o}\right)}$$

Exercise #3: Determine the output voltage, inductor current and output ripple of the following Buck-Boost converter.



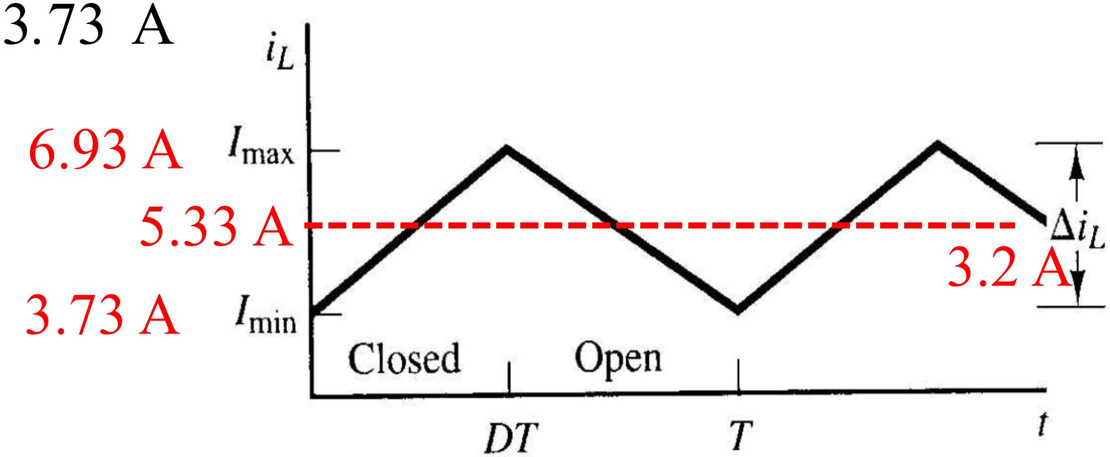
$$V_o = -V_S\left(\frac{D}{1-D}\right) = -24\left(\frac{0.4}{1-0.4}\right) = -16 \text{ V}$$

$$DI_L V_S = \frac{V_o^{\ 2}}{R} \quad \therefore I_L = \frac{V_o^{\ 2}}{DV_S R} = \frac{16^{\ 2}}{0.4 \times 24 \times 5} = 5.33 \text{ A}$$

$$v_L = V_s = L\frac{\Delta i_L}{DT} = \frac{Lf\Delta i_L}{D} \quad \therefore \Delta i_L = \frac{DV_s}{Lf} = \frac{0.4 \times 24}{120\mu \times 25k} = 3.2\text{A}$$

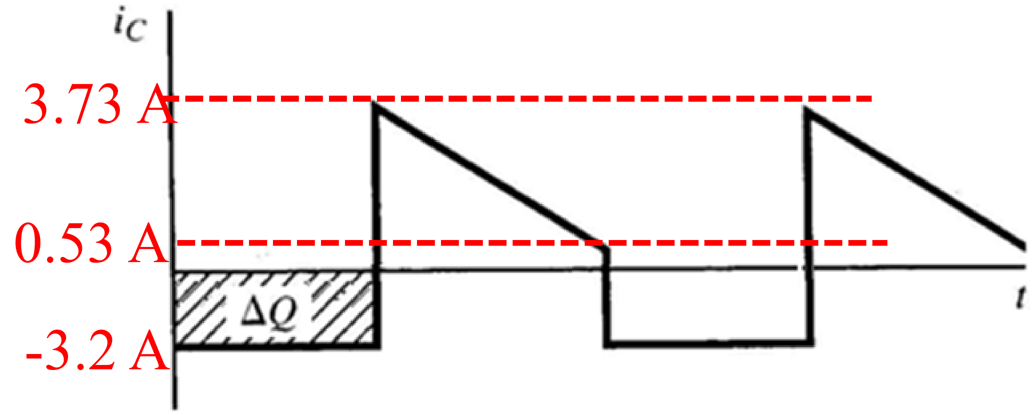
$$I_{max} = I_L + \frac{\Delta i_L}{2} = 5.33 + 1.6 = 6.93 \text{ A}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = 5.33 - 1.6 = 3.73 \text{ A}$$



$$\left|\Delta Q\right| = \frac{V_0}{R} \times DT = \frac{DV_o}{fR}$$

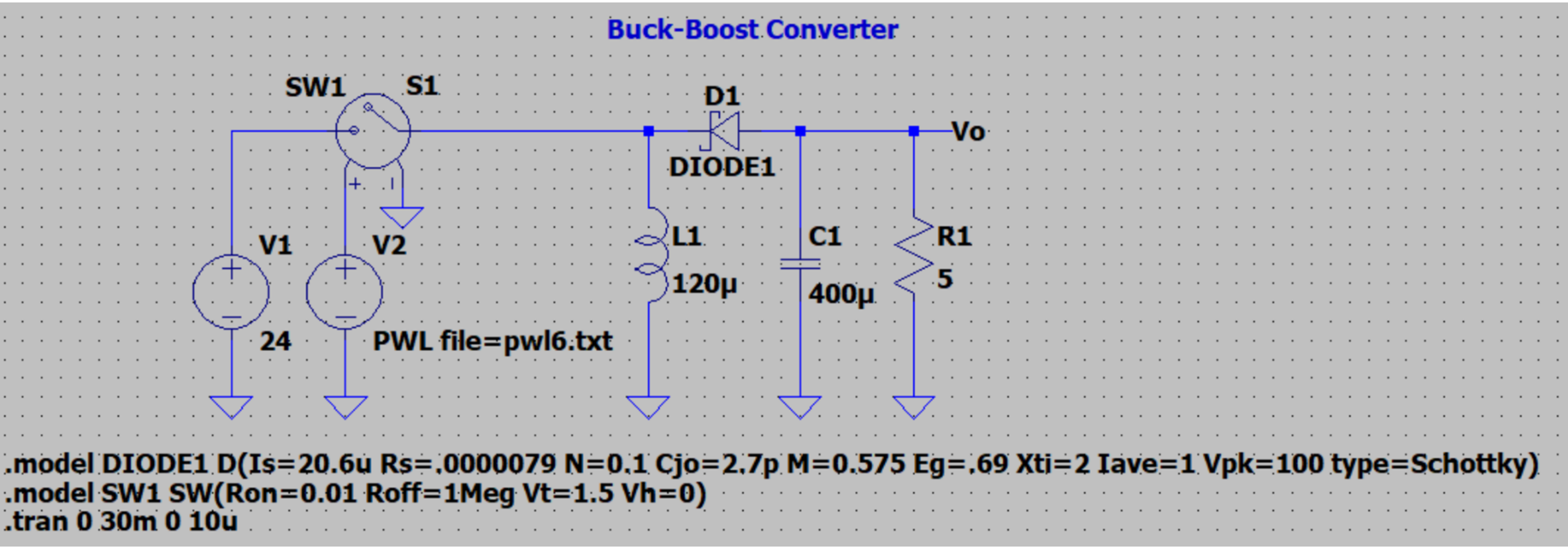
$$= \frac{0.4 \times 16}{25k \times 5} = 51.2 \times 10^{-6}$$



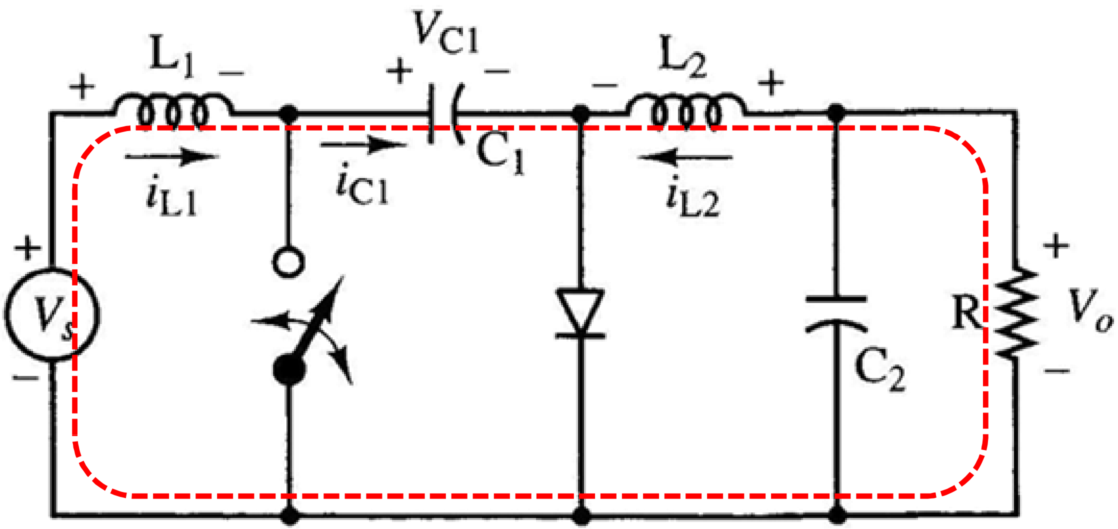
$$\Delta Q = C\Delta V_o \quad \therefore \Delta V_o = \frac{\Delta Q}{C} = \frac{51.20^{-6}}{400\mu} = 0.128\,\mathrm{V}$$

$$\frac{\Delta V_o}{V_o} = \frac{0.128}{16} \times 100 = 0.8\%$$

LTSpice simulation:



Cuk Converter



Regardless switch is closed or opened, applying KVL around the outermost loop:

$$V_s = V_{L1} + V_{C1} - V_{L2} + V_o$$

Average voltage across inductors for steady-state operation = 0.

$$V_{L1} = 0 \text{ and } V_{L2} = 0$$

The average voltage across C_1 :

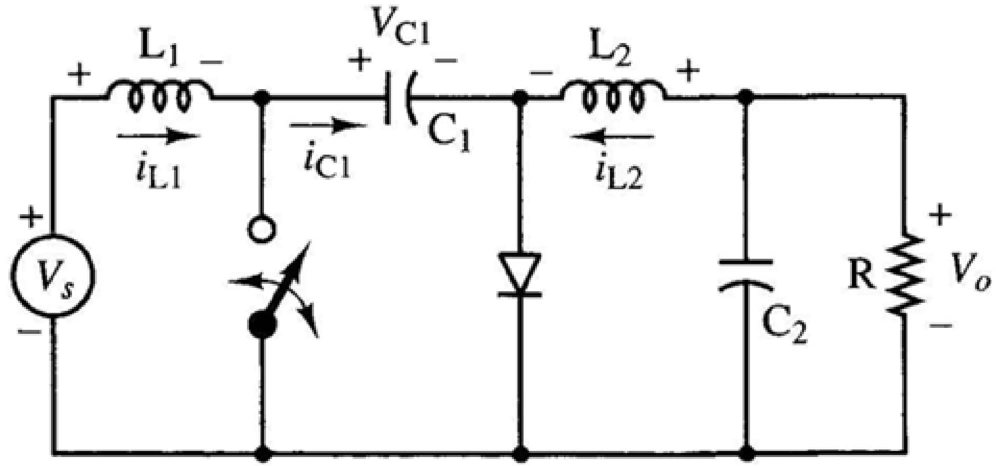
$$V_s = V_{C1} + V_o$$

$$\therefore V_{C1} = V_s - V_o$$

For lossless converter, input power = output power:

$$V_s I_{L1} = V_o\left(-I_{L2}\right)$$

$$\frac{V_o}{V_s} = -\frac{I_{L1}}{I_{L2}}$$



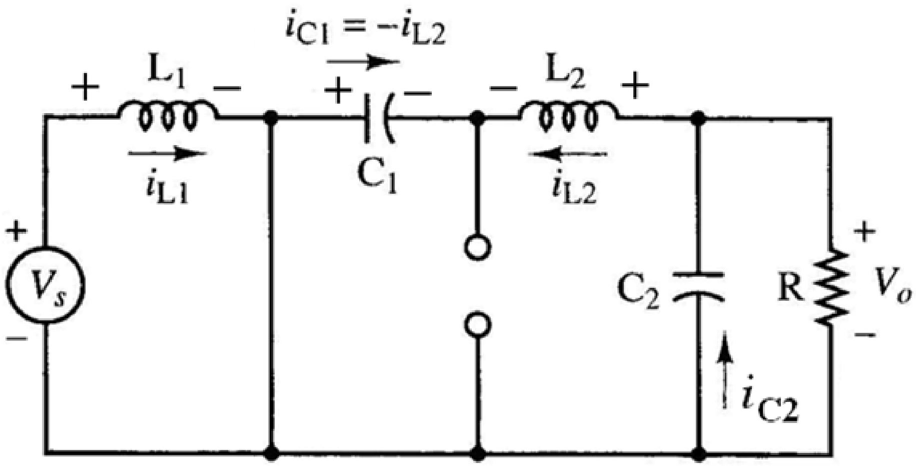
For steady state operation, average capacitor current over one cycle = 0:

$$-I_{L2}DT + I_{L1}(1-D)T = 0$$

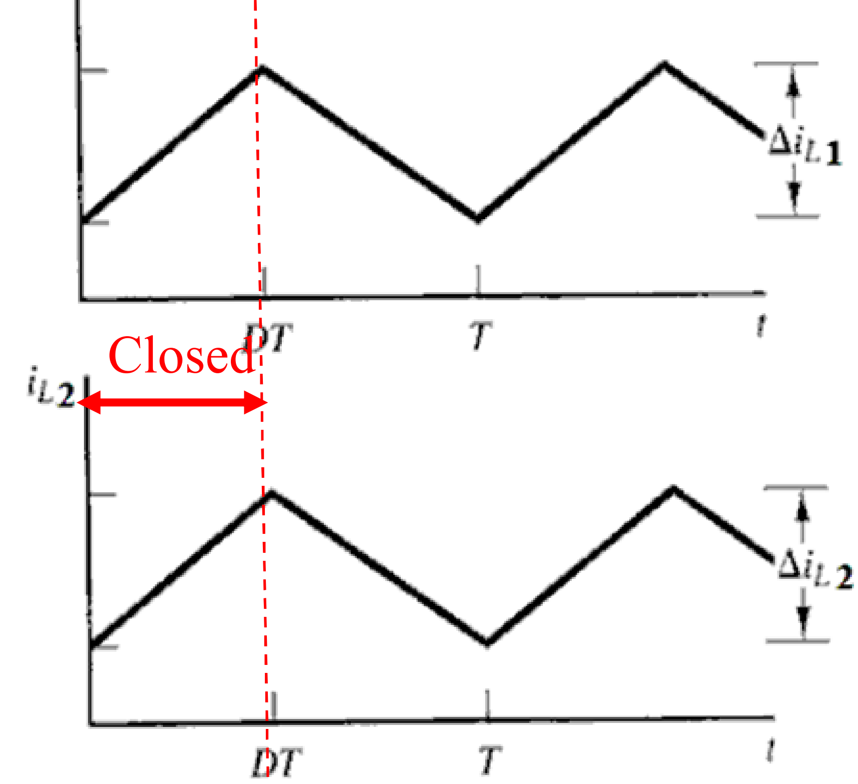
$$\frac{I_{L1}}{I_{L2}} = \frac{D}{1-D}$$

$$\frac{V_o}{V_s} = -\frac{I_{L1}}{I_{L2}} = -\left(\frac{D}{1-D}\right) \qquad \therefore V_o = -\left(\frac{D}{1-D}\right)V_s$$

Cuk Converter (Switch Closed)



Note: $V_{C1} = V_s - V_o$



L_1 receives energy from power source and L_2 receives energy from C_1.

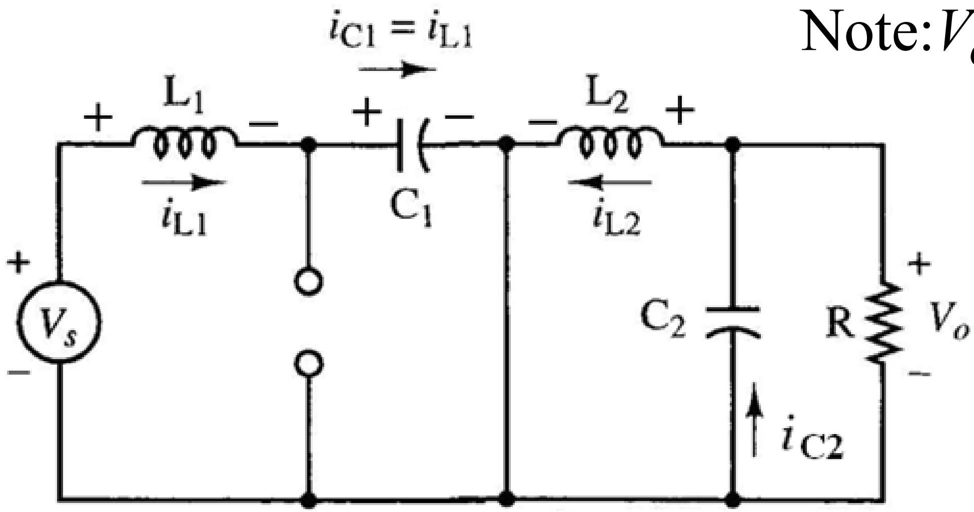
$$v_{L1} = V_s = L_1 \frac{\Delta i_{L1}}{DT} \Rightarrow \Delta i_{L1} = \frac{V_s D}{f L_1}$$

$$V_{C1} - v_{L2} + V_o = 0$$

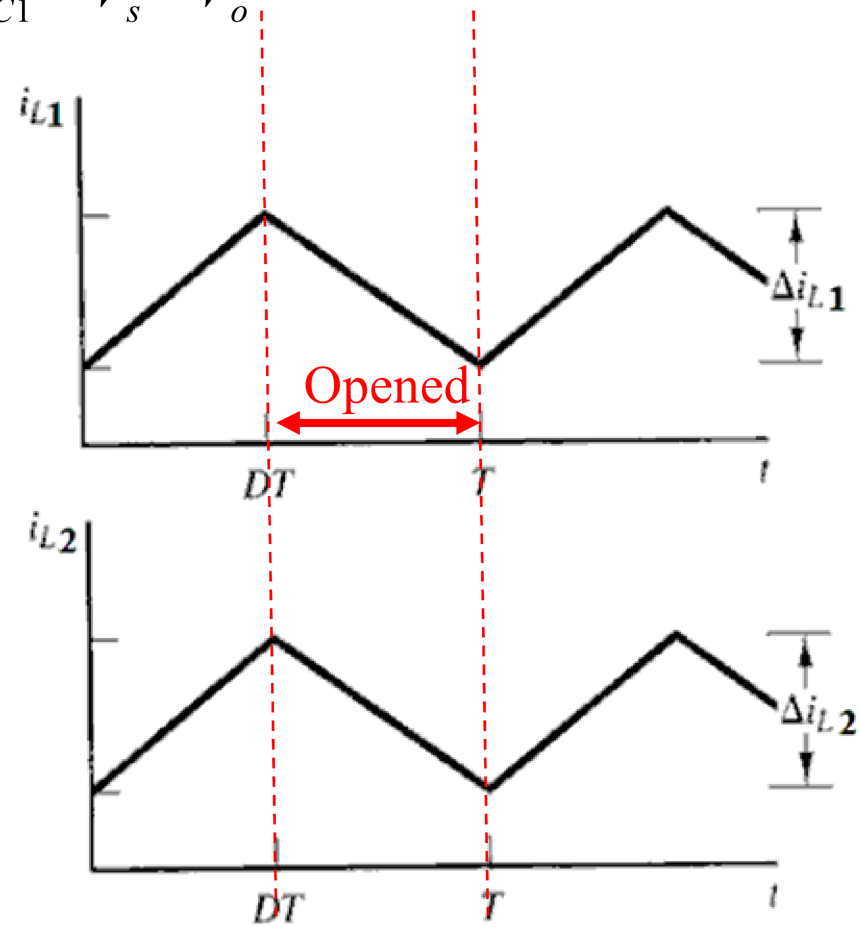
$$\therefore v_{L2} = V_{C1} + V_o = V_s - V_o + V_o = V_s$$

$$v_{L2} = V_s = L_2 \frac{\Delta i_{L2}}{DT} \Rightarrow \Delta i_{L2} = \frac{V_s D}{f L_2}$$

Cuk Converter (Switch Opened)



Note: $V_{C1} = V_s - V_o$



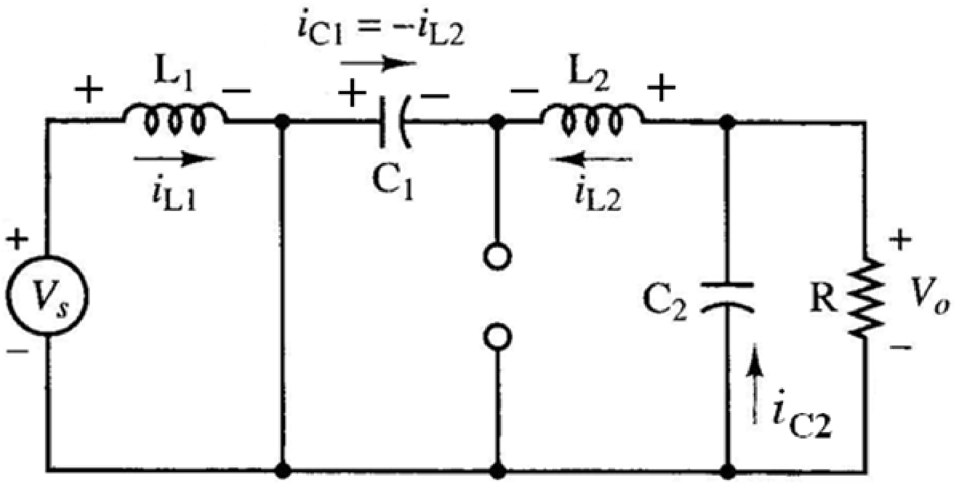
L_1 releases energy to charge C_1 and L_2 releases energy to R.

$$-V_s + v_{L1} + V_{C1} = 0$$

$$\therefore v_{L1} = V_s - V_{C1} = V_s - (V_s - V_o) = V_o$$

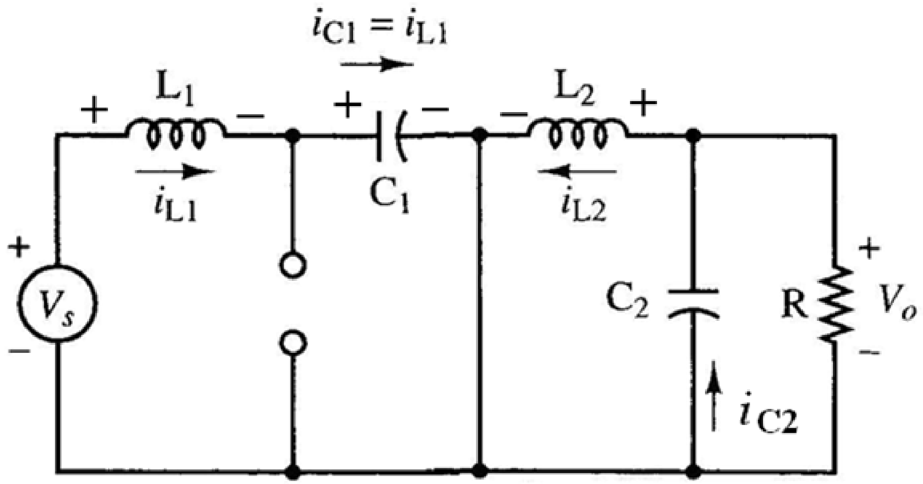
$$v_{L2} = V_o$$

Switch closed

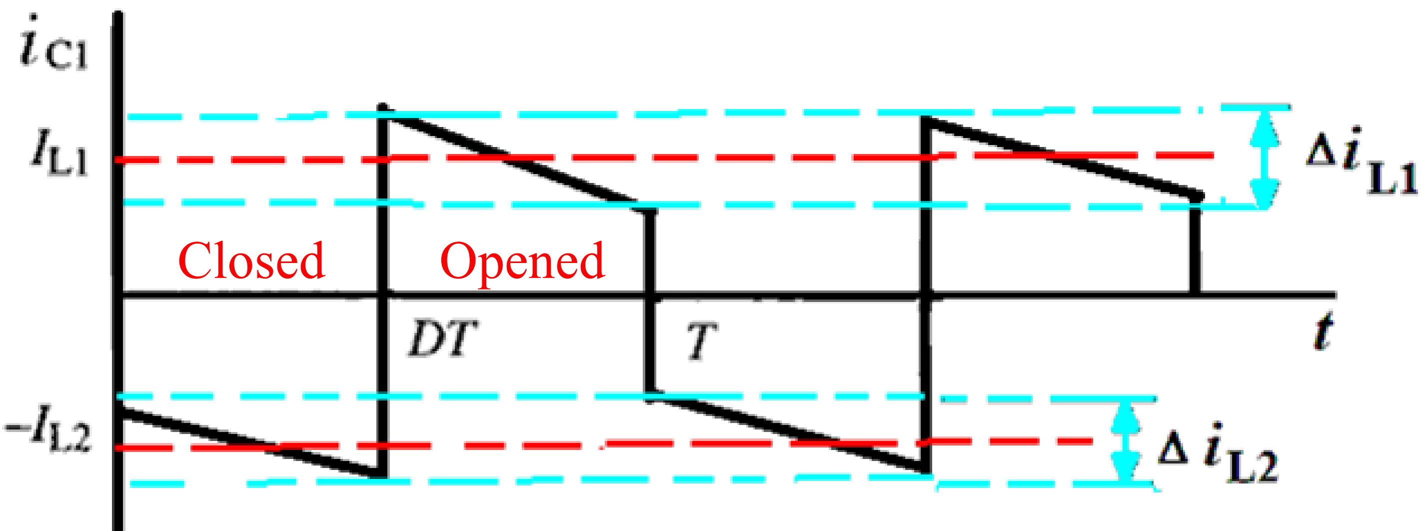


$$\left(i_{C1}\right)_{closed} = -i_{L2}$$

Switch opened



$$\left(i_{C1}\right)_{opened} = i_{L1}$$



For continuous current in L_1, the minimum inductor current > 0:

$$I_{L1} - \frac{1}{2}\Delta i_{L1} = 0,$$

$$\therefore I_{L1} = \frac{1}{2}\Delta i_{L1}$$

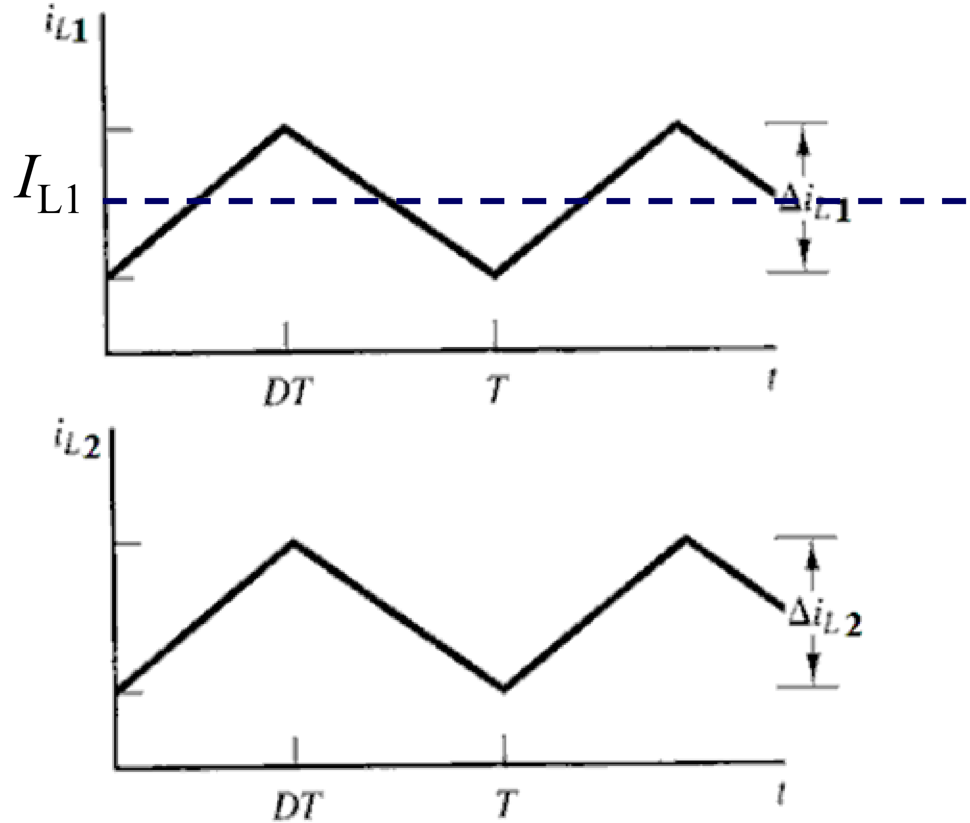
$$\left(\frac{D}{1-D}\right) I_{L2} = \frac{V_s D}{2L_1 f}$$

$$\left(\frac{D}{1-D}\right)\left(\frac{-V_o}{R}\right) = \frac{V_s D}{2L_1 f}$$

$$\left(\frac{D}{1-D}\right)^2 \left(\frac{V_s}{R}\right) = \frac{V_s D}{2L_1 f}$$

$$L_{1,min} = \frac{(1-D)^2 R}{2Df}$$

Note: $\frac{I_{L1}}{I_{L2}} = \frac{D}{1-D}, \quad \Delta i_{L1} = \frac{V_s D}{f L_1}$



For continuous current in L_2, the minimum inductor current > 0:

Note: $\Delta i_{L2} = \dfrac{V_s D}{f L_2}$, $I_{L2} = -\dfrac{V_o}{R}$

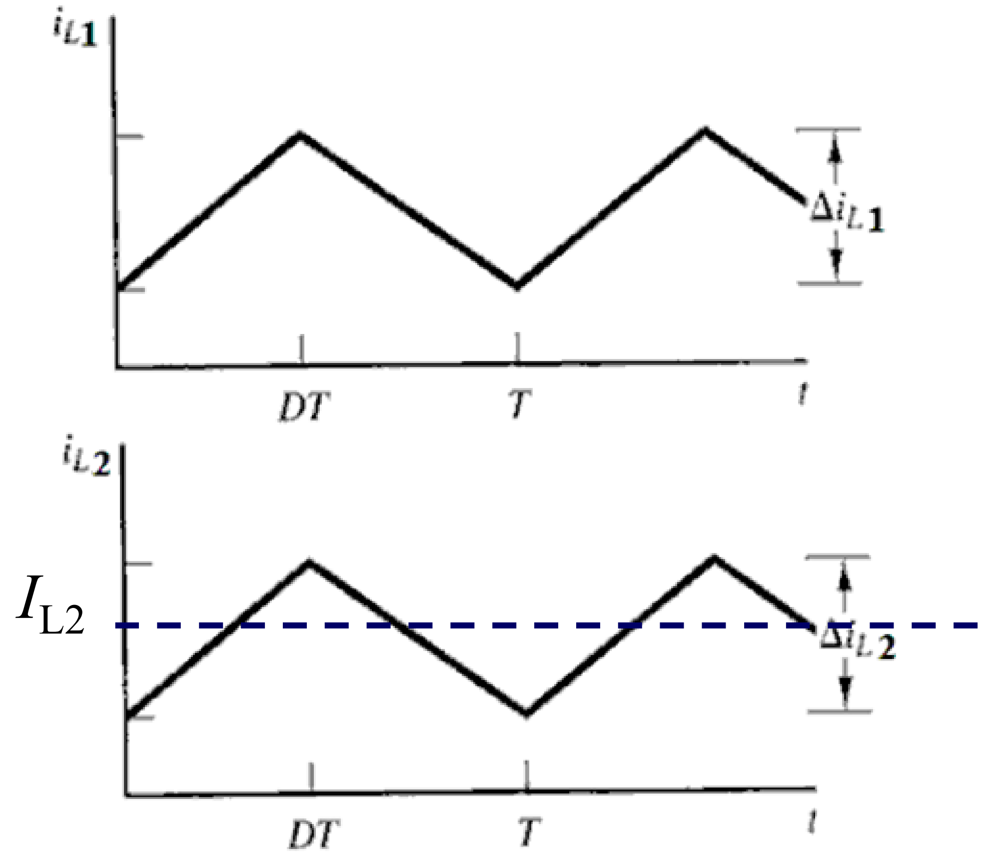
$$I_{L2} - \frac{1}{2}\Delta i_{L2} = 0$$

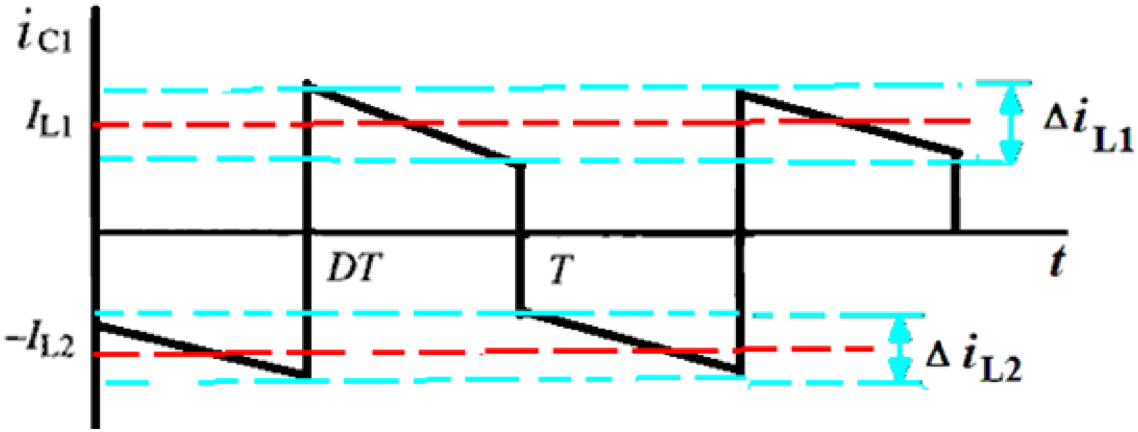
$$\therefore I_{L2} = \frac{1}{2}\Delta i_{L2}$$

$$-\frac{V_o}{R} = \frac{V_s D}{2 L_2 f}$$

$$\left(\frac{D}{1-D}\right)\frac{V_s}{R} = \frac{V_s D}{2 L_2 f}$$

$$L_{2,min} = \frac{(1-D)R}{2f}$$



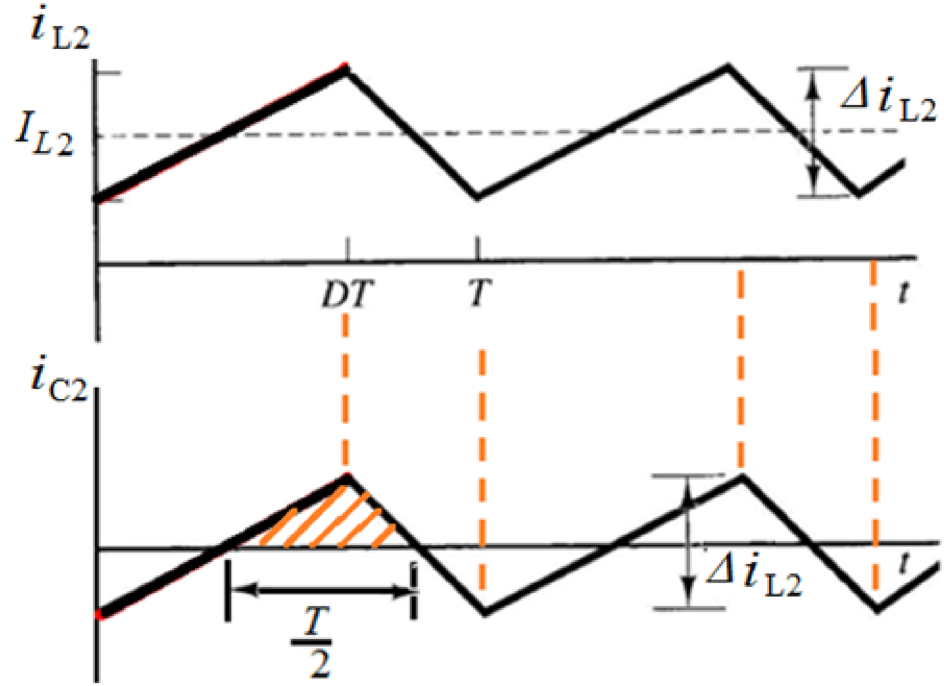


$$\Delta Q = \int_{DT}^{T} i_{L1}\,dt = I_{L1}(1-D)T \qquad \because I_{L1} = \left(\frac{D}{1-D}\right) I_{L2} \text{ and } I_{L2} = \frac{-V_o}{R}$$

$$\Delta Q = \left(\frac{D}{1-D}\right)\left(-\frac{V_o}{R}\right)(1-D)T = \frac{-DV_o}{Rf}$$

$$C_1 = \frac{\Delta Q}{\Delta V_{C1}} = \frac{-DV_o}{\Delta V_{C1} Rf} = \frac{-D}{\left(\frac{\Delta V_{C1}}{V_o}\right) Rf}$$

$$\Delta Q = \frac{1}{2}\left(\frac{\Delta i_{L2}}{2}\right)\left(\frac{T}{2}\right) = \frac{\Delta i_{L2}}{8f}$$



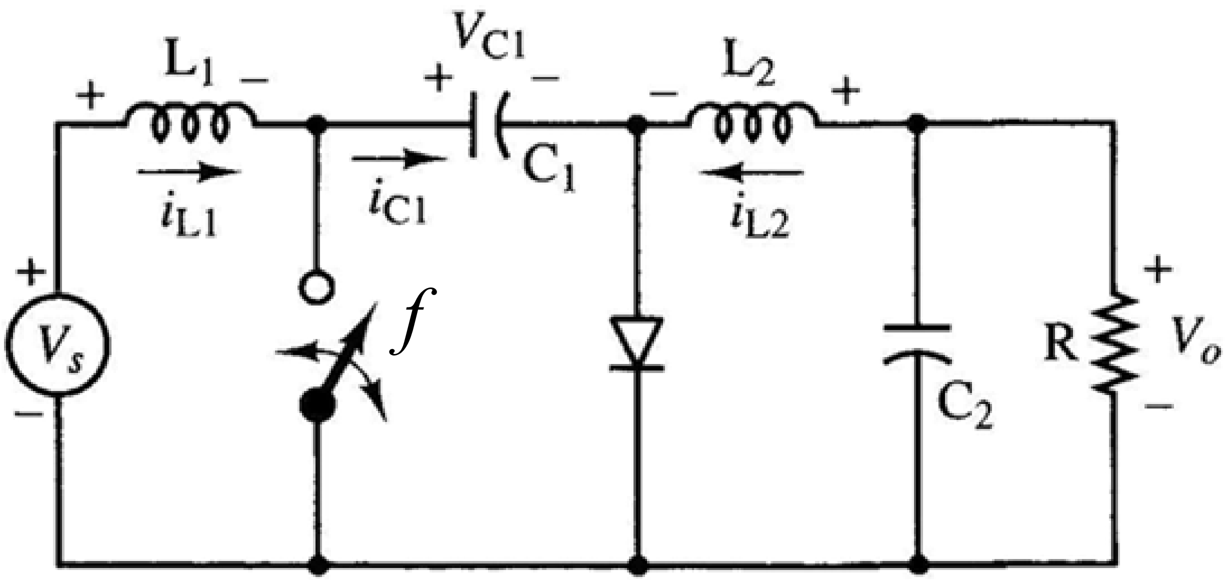
$$\Delta Q = C_2 \Delta V_{C2}$$

$$C_2 = \frac{\Delta Q}{\Delta V_{C2}} = \frac{\Delta i_{L2}}{\Delta V_{C2} 8f}$$

$$\because \Delta i_{L2} = \frac{V_s D}{f L_2}$$

$$\therefore C_2 = \frac{V_s D}{8 f^2 L_2 \Delta v_{C2}} = \frac{D}{\left(\frac{\Delta V_{C2}}{V_s}\right) 8 f^2 L_2}$$

Cuk Converter



$$V_o = -\frac{DV_s}{1-D}$$

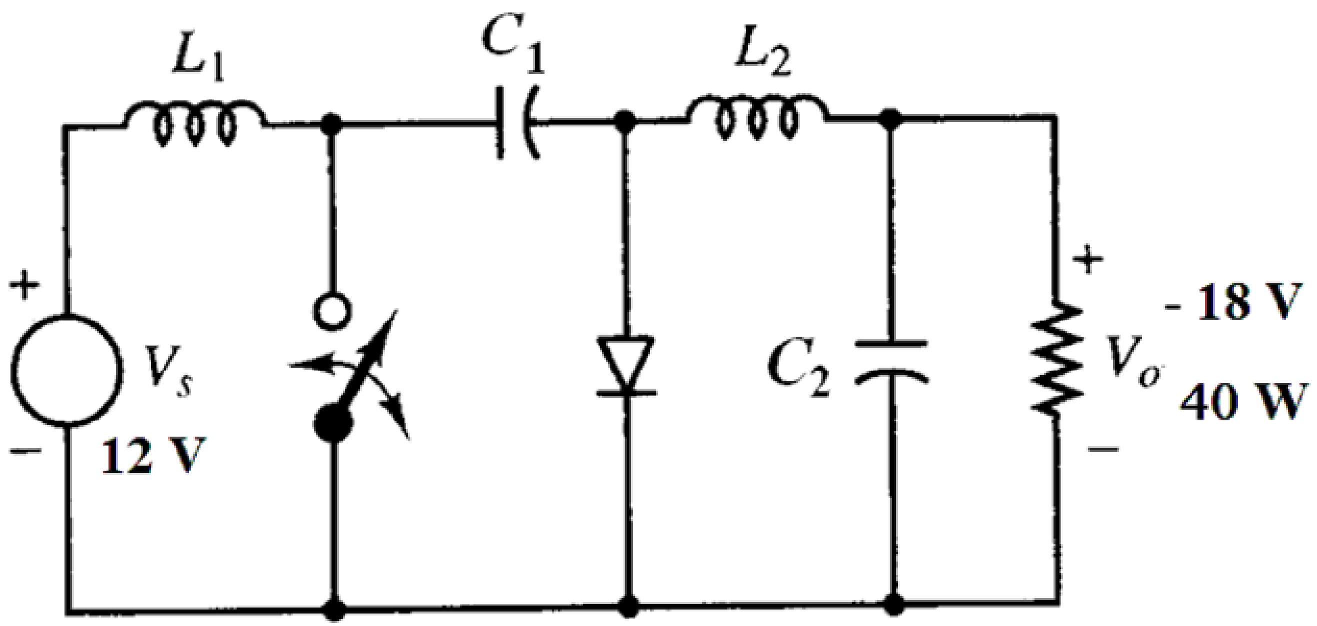
$$L_{1\min} = \frac{(1-D)^2 R}{2Df}$$

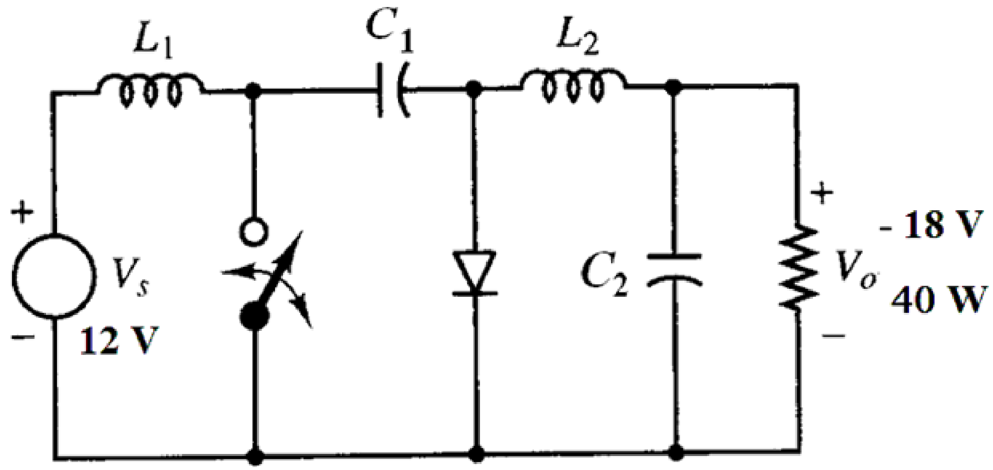
$$L_{2\min} = \frac{(1-D)R}{2f}$$

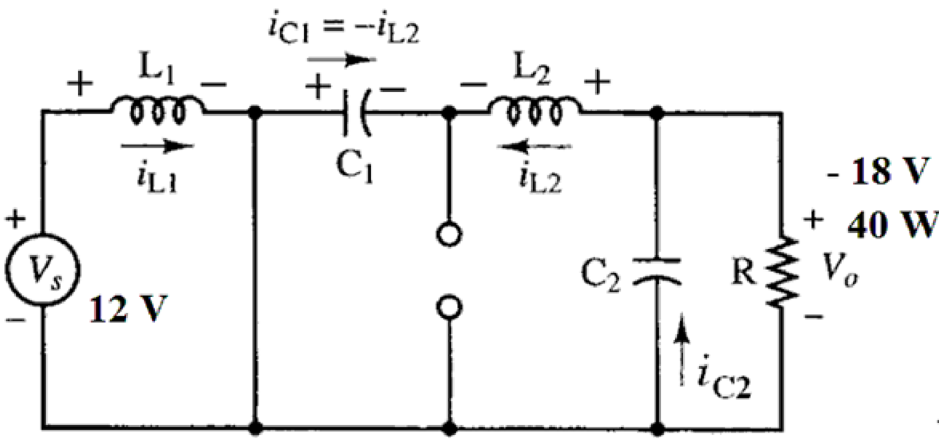
$$C_1 = \frac{-D}{\left(\frac{\Delta V_{C1}}{V_o}\right)Rf}$$

$$C_2 = \frac{D}{\left(\frac{\Delta V_{C2}}{V_s}\right)8f^2 L_2}$$

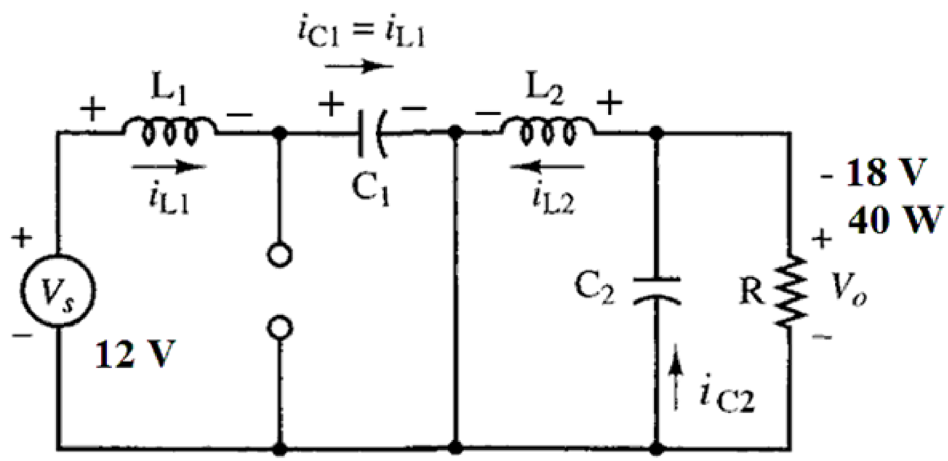
Exercise #4: For the following Cuk converter, the switching frequency is 50 kHz. Determine the duty ratio, and the values of inductors and capacitors so that the change in inductor currents is less than 10% of the average inductor currents, the output ripple is less than 1% and the ripple voltage across C_1 is less than 5%







Switch closed



Switch opened

The duty ratio:

$$\frac{V_o}{V_S} = -\frac{D}{1-D} = \frac{-18}{12} \quad \therefore D = 0.6$$

The average inductors' currents:

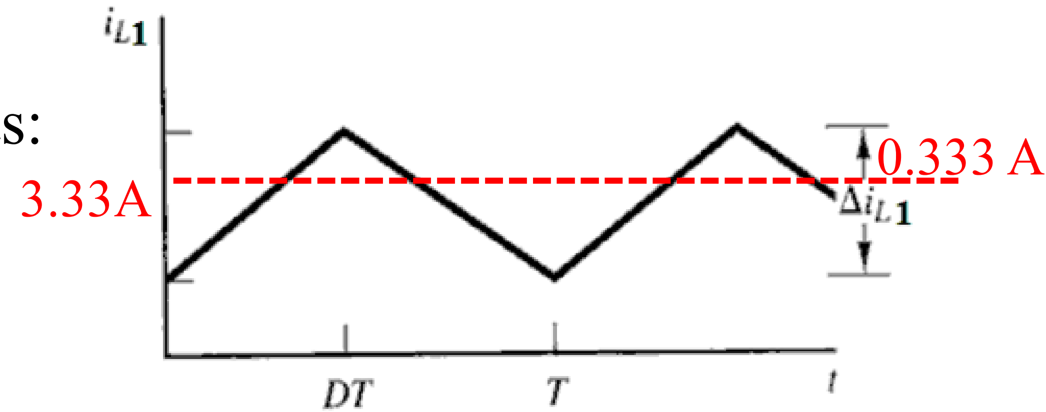
$$I_{L1} = \frac{P_s}{V_s} = \frac{40}{12} = 3.33 \quad \text{A}$$

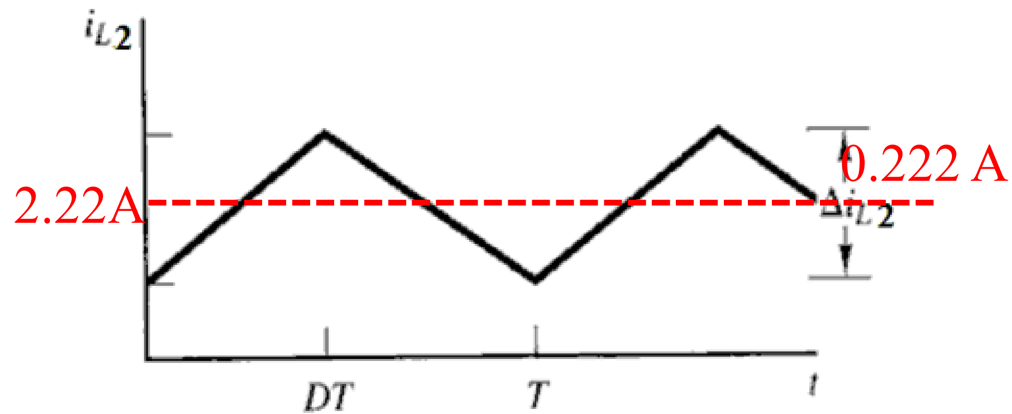
$$I_{L2} = \frac{P_o}{-V_o} = \frac{40}{18} = 2.22 \quad \text{A}$$

The current ripples of the inductors' currents:

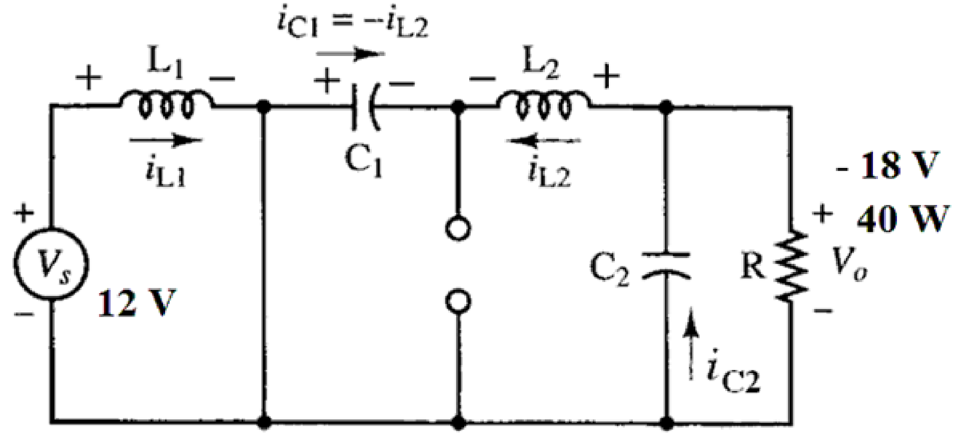
$$\Delta i_{L2} = (0.1)(2.22) = 0.222 \text{ A}$$

$$\Delta i_{L1} = (0.1)(3.33) = 0.333 \text{ A}$$





When the switch is closed:



$$V_s = L_1 \frac{\Delta i_{L1}}{DT}$$

$$L_1 = \frac{V_s DT}{\Delta i_{L1}} = \frac{V_s D}{f \Delta i_{L1}}$$

$$= \frac{12 \times 0.6}{50\text{k} \times 0.333} = 432 \ \mu\text{H}$$

$$L_2 = \frac{V_s D}{\Delta i_{L2} f} = \frac{12 \times 0.6}{0.222 \times 50\text{k}} = 649 \ \mu\text{H}$$

The value of capacitor C_2

$$\Delta Q = \frac{1}{2}\left(\frac{\Delta i_{L2}}{2}\right)\left(\frac{T}{2}\right) = \frac{\Delta i_{L2}}{8f} = \frac{0.222}{8 \times 50k} = 5.55 \times 10^{-7} \text{ C}$$

$$C_2 = \frac{\Delta Q}{\Delta v_{C2}} = \frac{5.55 \times 10^{-7}}{0.01 \times 18} = 3.08 \ \mu\text{F}$$

Average voltage and the ripple voltage across C_1

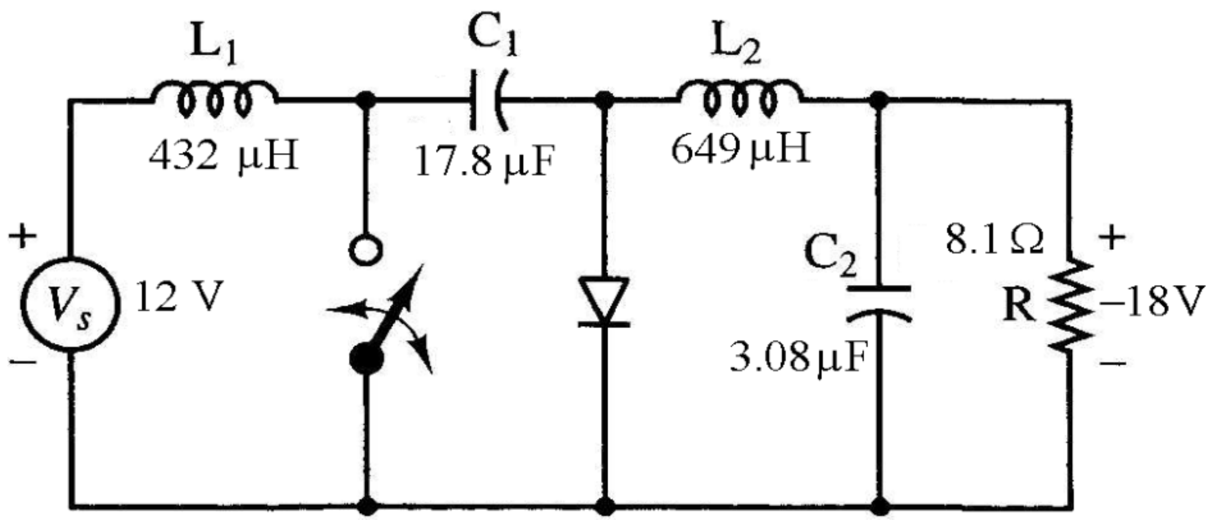
$$V_{C1} = V_s - V_o = 12 - (-18) = 30 \text{ V}$$

$$\Delta v_{C1} = (30)(0.05) = 1.5 \text{ V}$$

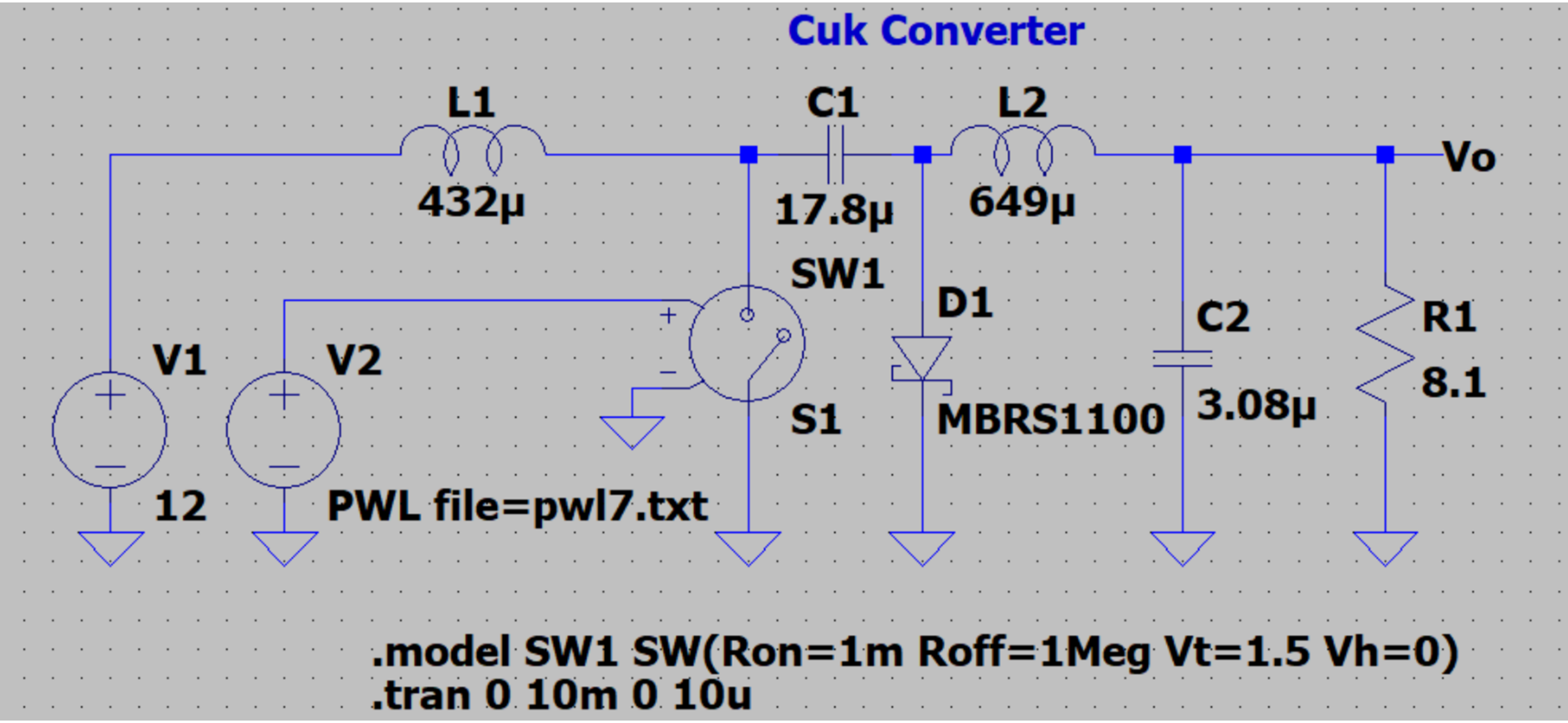
$$\Delta Q = I_{L1}(1-D)T = \frac{I_{L1}(1-D)}{f} = \frac{3.33(1-0.6)}{50k} = 2.664 \times 10^{-5} \text{ C}$$

$$C_1 = \frac{\Delta Q}{\Delta v_{C1}} = \frac{2.664 \times 10^{-5}}{1.5} = 17.8\ \mu\text{F}$$

Final design:



LTSpice simulation:

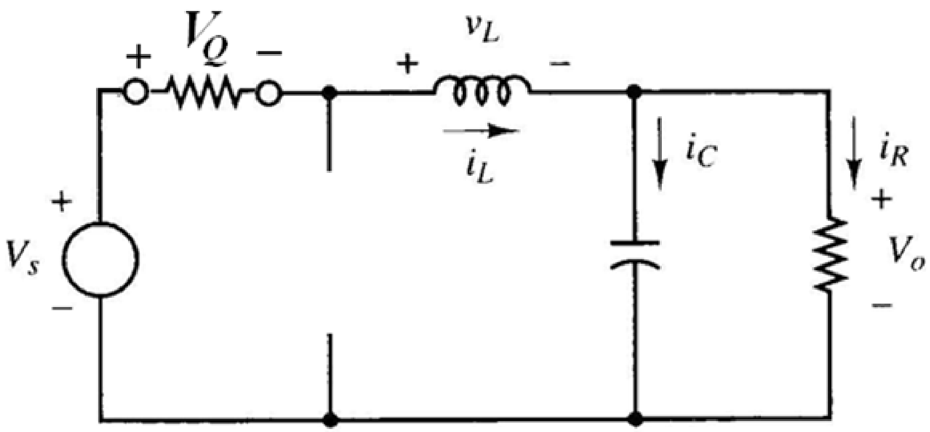


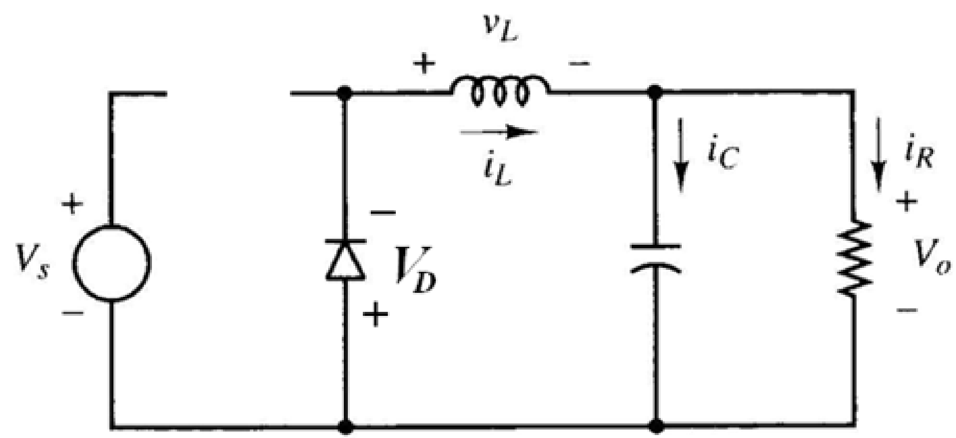
Impacts of Non-ideal Effects

- In the earlier analyses, all the calculations have assumed that both passive and active components are ideal and lossless.
- There will be finite voltage drops across conducting switch and diode
- L and C have finite resistances that contribute to power loss.
- All these effects have impacts on the performance of a DC-DC converter.

Finite Turn-On Voltage

Using Buck converter as an example:





When the switch closed

$$v_L = V_s - V_Q - V_o$$

When the switch open

$$v_L = -V_D - V_o$$

Average voltage across inductor

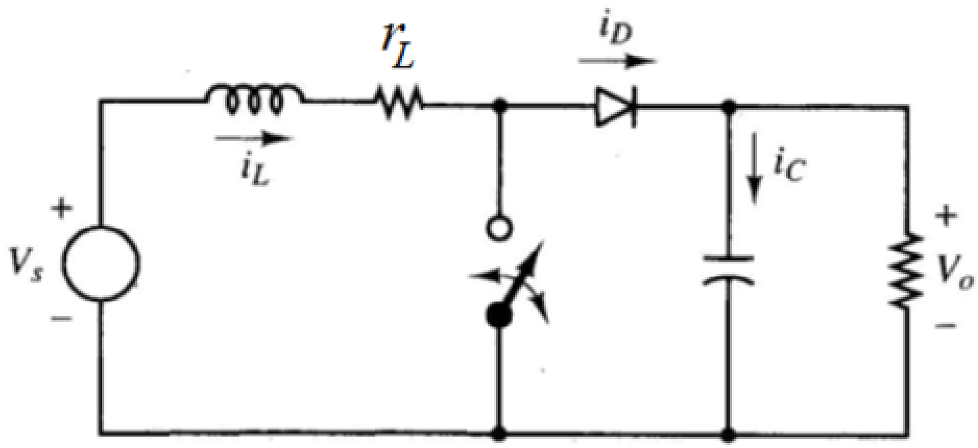
$$V_L = \left(V_s - V_Q - V_o\right)D + \left(-V_D - V_o\right)\left(1 - D\right) = 0$$

$$V_o = V_s D - V_Q D - V_D\left(1 - D\right)$$

Note: Lower than $V_o = V_s D$ for ideal case

Non-ideal Inductor

Using boost converter as example:



$$P_s = P_o + P_{r_L}$$

$$V_s I_L = V_o I_D + I_L^2 r_L$$

$$I_D = I_L(1-D)$$

$$V_s I_L = V_o I_L(1-D) + I_L^2 r_L$$

$$V_s = V_o(1-D) + I_L r_L$$

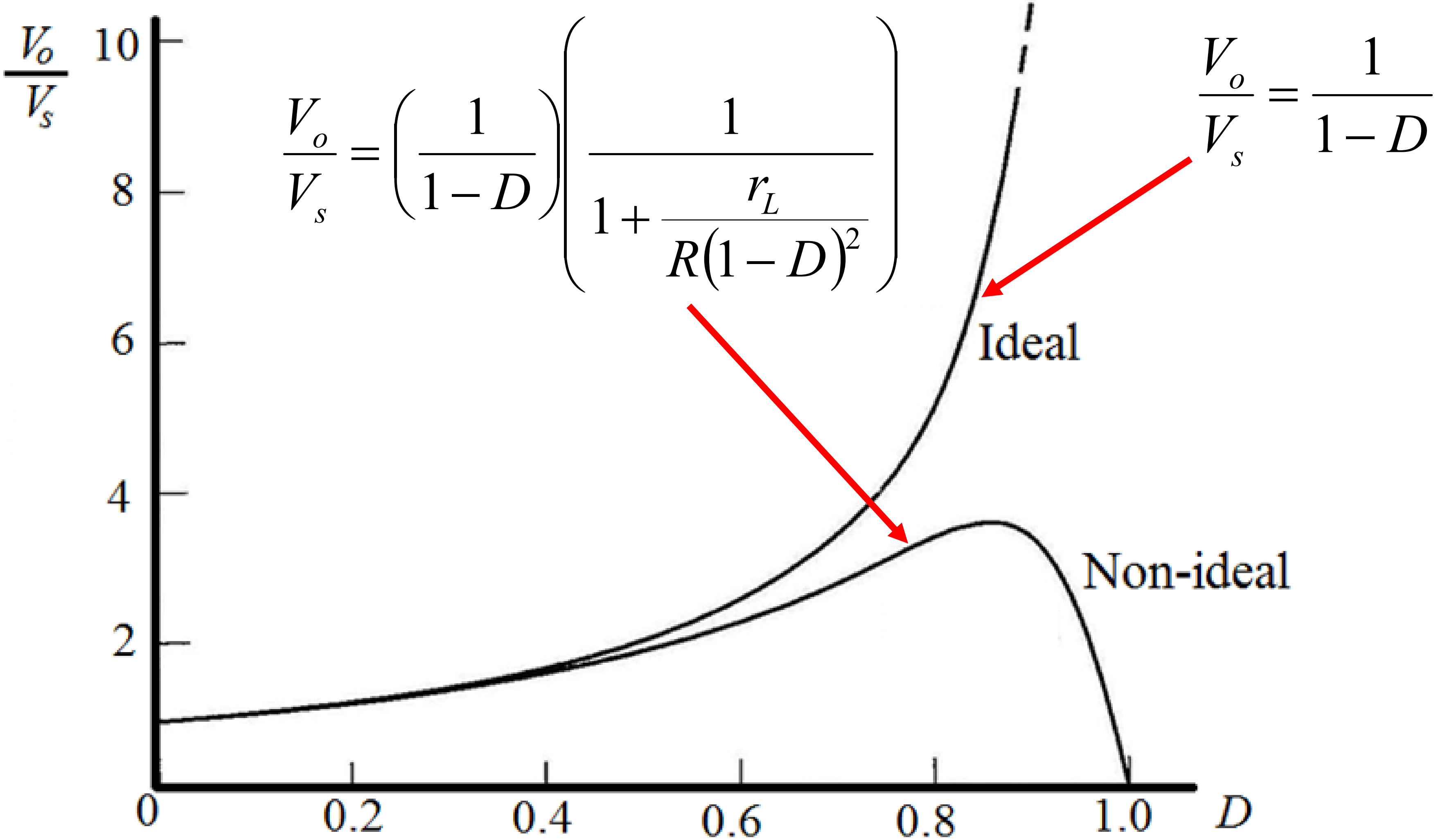
$$\because I_L = \frac{I_D}{1-D} = \left(\frac{V_o}{R}\right)\left(\frac{1}{1-D}\right) \quad \therefore V_s = V_o(1-D) + \left(\frac{r_L V_o}{R}\right)\left(\frac{1}{1-D}\right)$$

$$V_s = V_o(1-D) + \left(\frac{r_L V_o}{R}\right)\left(\frac{1}{1-D}\right)$$

$$V_s = V_o\left[(1-D) + \frac{r_L}{R(1-D)}\right]$$

$$\frac{V_o}{V_s} = \frac{1}{(1-D) + \frac{r_L}{R(1-D)}}$$

$$\frac{V_o}{V_s} = \left(\frac{1}{1-D}\right)\left(\frac{1}{1 + \frac{r_L}{R(1-D)^2}}\right)$$



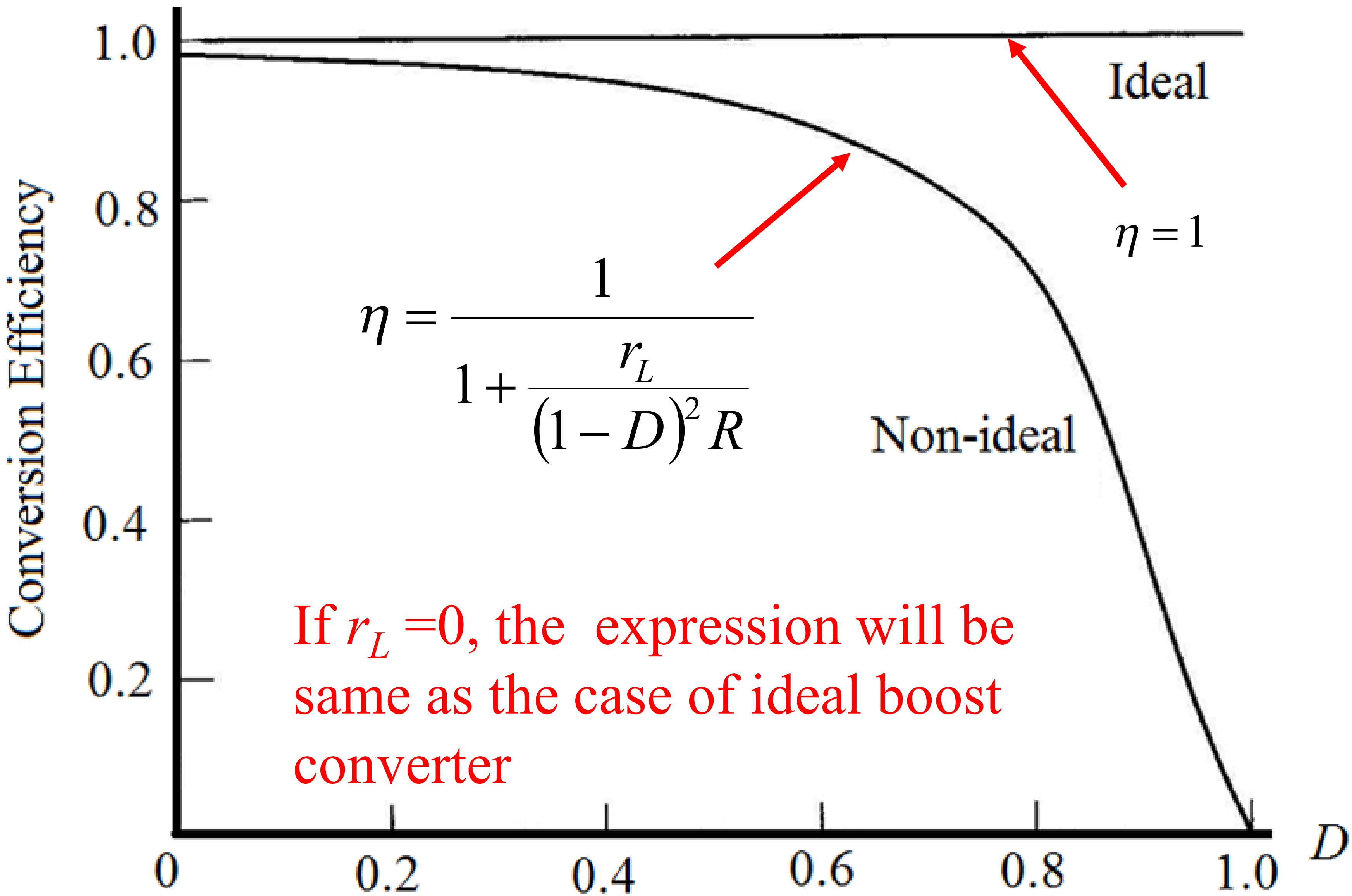
If r_L =0, the expression will be same as the case of ideal boost converter

Conversion Efficiency

$$\eta = \frac{P_o}{P_o + I_L{}^2 r_L} = \frac{\dfrac{V_o{}^2}{R}}{\dfrac{V_o{}^2}{R} + I_L{}^2 r_L}$$

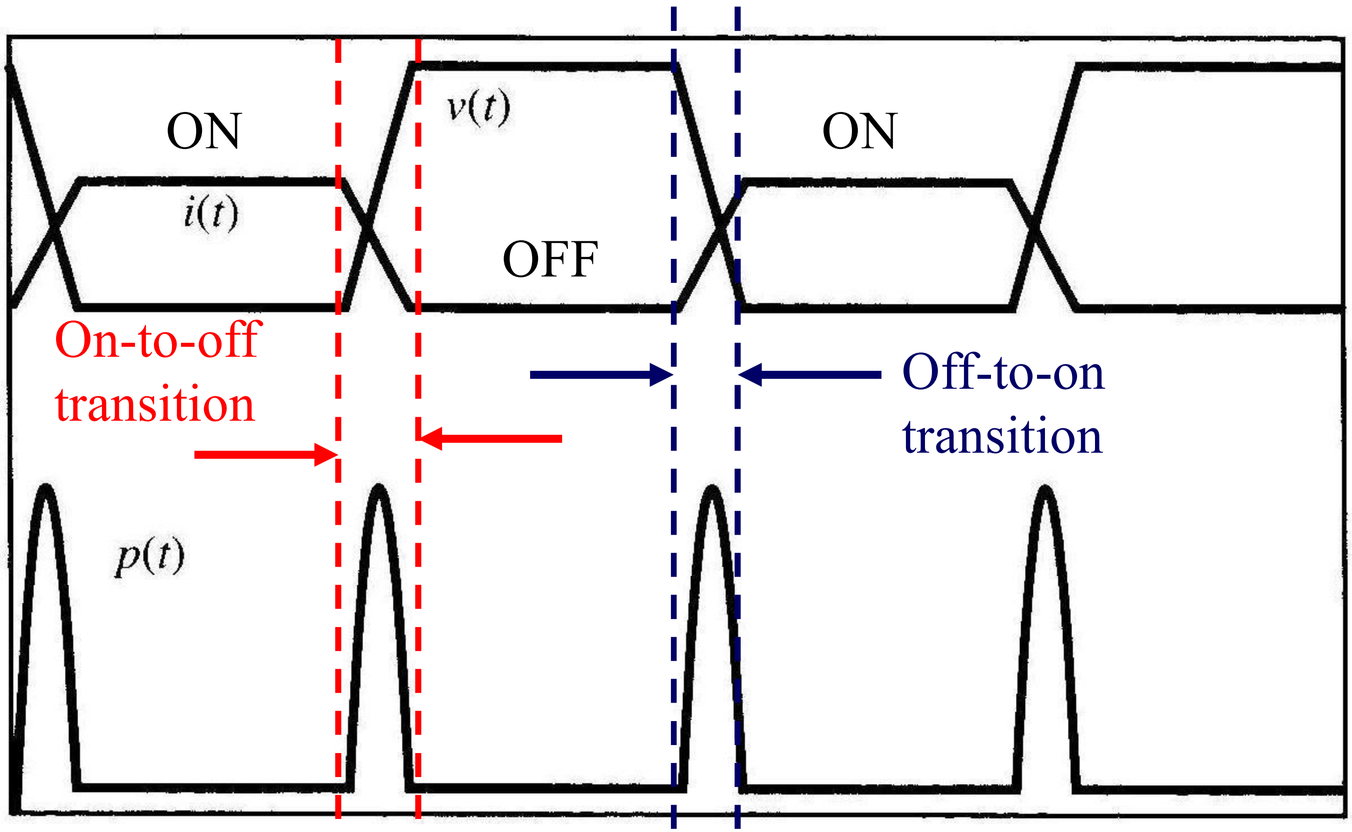
$$= \frac{\dfrac{V_o{}^2}{R}}{\dfrac{V_o{}^2}{R} + \left(\dfrac{V_o}{R} \times \dfrac{1}{1-D}\right)^2 r_L}$$

$$= \frac{1}{1 + \dfrac{r_L}{(1-D)^2 R}}$$



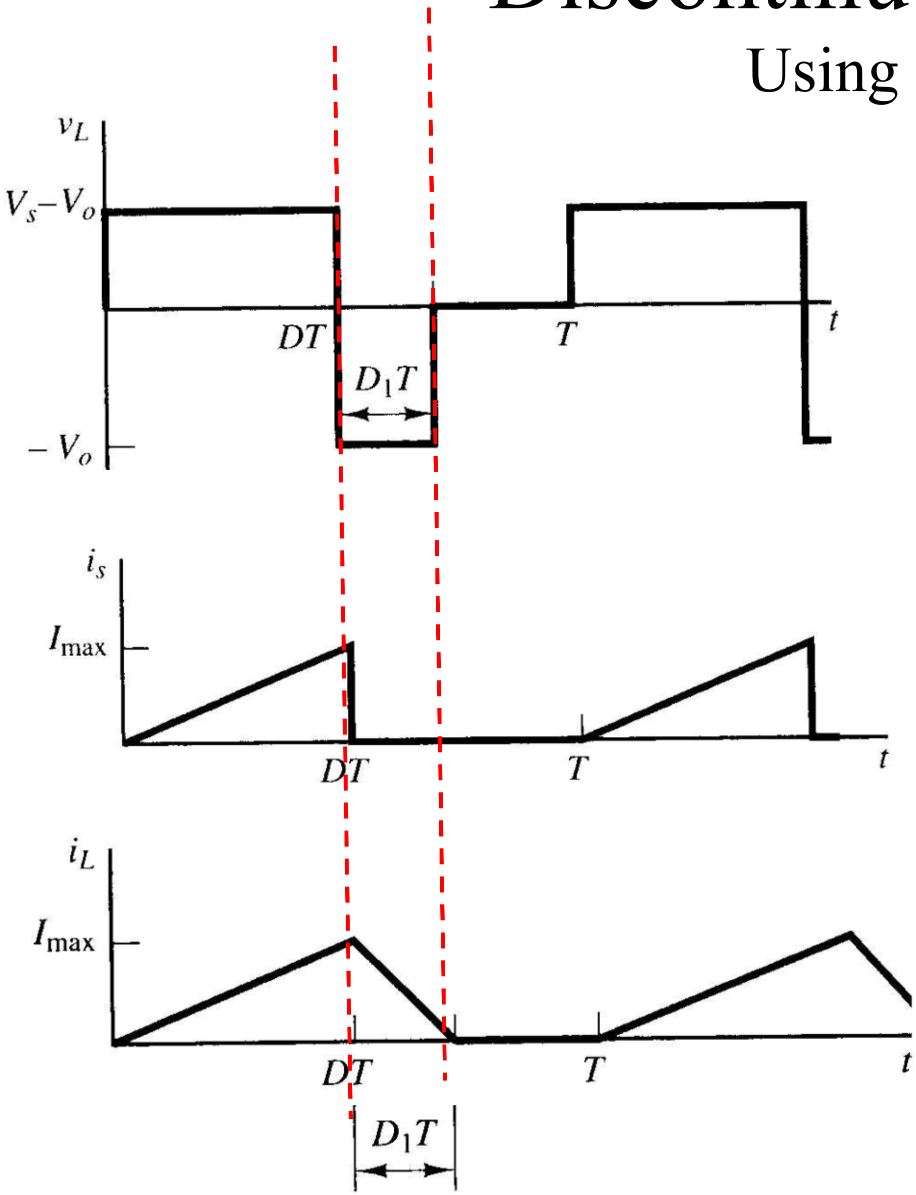
Conversion Efficiency
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
D
Ideal
$\eta = 1$
$\eta = \dfrac{1}{1 + \dfrac{r_L}{(1-D)^2 R}}$
Non-ideal
If r_L =0, the expression will be same as the case of ideal boost converter

Switching Losses



- Power dissipation happens in each switching transition.
- Average power dissipated in over one period and therefore higher switching frequency results in higher switching losses.

Discontinuous Current



Using Buck converter as an example:

$$(V_s - V_o)DT - V_o D_1 T = 0$$

$$(V_s - V_o)DT = V_o D_1 T$$

$$V_o = V_S\left(\frac{D}{D + D_1}\right)$$

Average inductor current = average load current:

$$I_L = I_R = \frac{V_o}{R}$$

$$I_L = \frac{1}{T}\left(\frac{1}{2} I_{max} DT + \frac{1}{2} I_{max} D_1 T\right)$$

$$= \frac{1}{2} I_{max}(D + D_1) = \frac{V_o}{R}$$

Switch is closed:

$$v_L = V_s - V_o = L\frac{\Delta i_L}{DT} = L\frac{I_{\max}}{DT}$$

$$I_{\max} = \left(\frac{V_s - V_o}{L}\right)DT = \frac{V_o D_1 T}{L}$$

Substituting the above I_{max} expression into following expression:

$$\frac{1}{2} I_{\max}\left(D + D_1\right) = \frac{1}{2}\left(\frac{V_o D_1 T}{L}\right)\left(D + D_1\right) = \frac{V_o}{R}$$

$$D_1^{\ 2} + DD_1 - \frac{2L}{RT} = 0$$

$$D_1 = \frac{-D \pm \sqrt{D^2 + \frac{8L}{RT}}}{2}$$

Take +ve value:

$$D_1 = \frac{-D + \sqrt{D^2 + \frac{8L}{RT}}}{2}$$

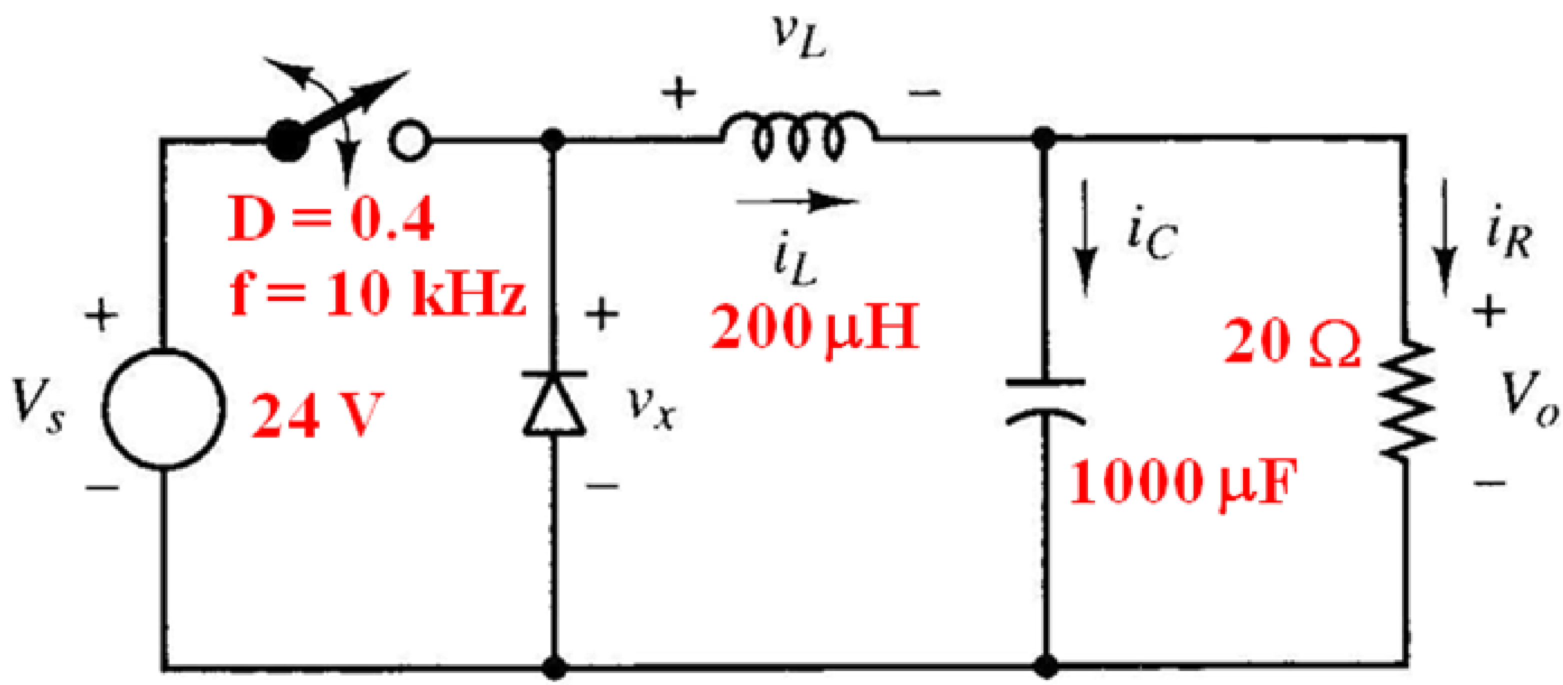
$$V_o = V_s\left(\frac{D}{D + D_1}\right)$$

$$= V_s\left(\frac{2D}{D + \sqrt{D^2 + \frac{8L}{RT}}}\right)$$

Boundary between continuous and discontinuous current:

$$D_1 = 1 - D$$

Exercise #5: Is the given Buck converter in continuous current operation? Determine its output voltage.



$$D_1 = \frac{-D + \sqrt{D^2 + \frac{8L}{RT}}}{2} = \frac{1}{2}\left(-0.4 + \sqrt{0.4^2 + \frac{8 \times 200\mu \times 10k}{20}}\right) = 0.29$$

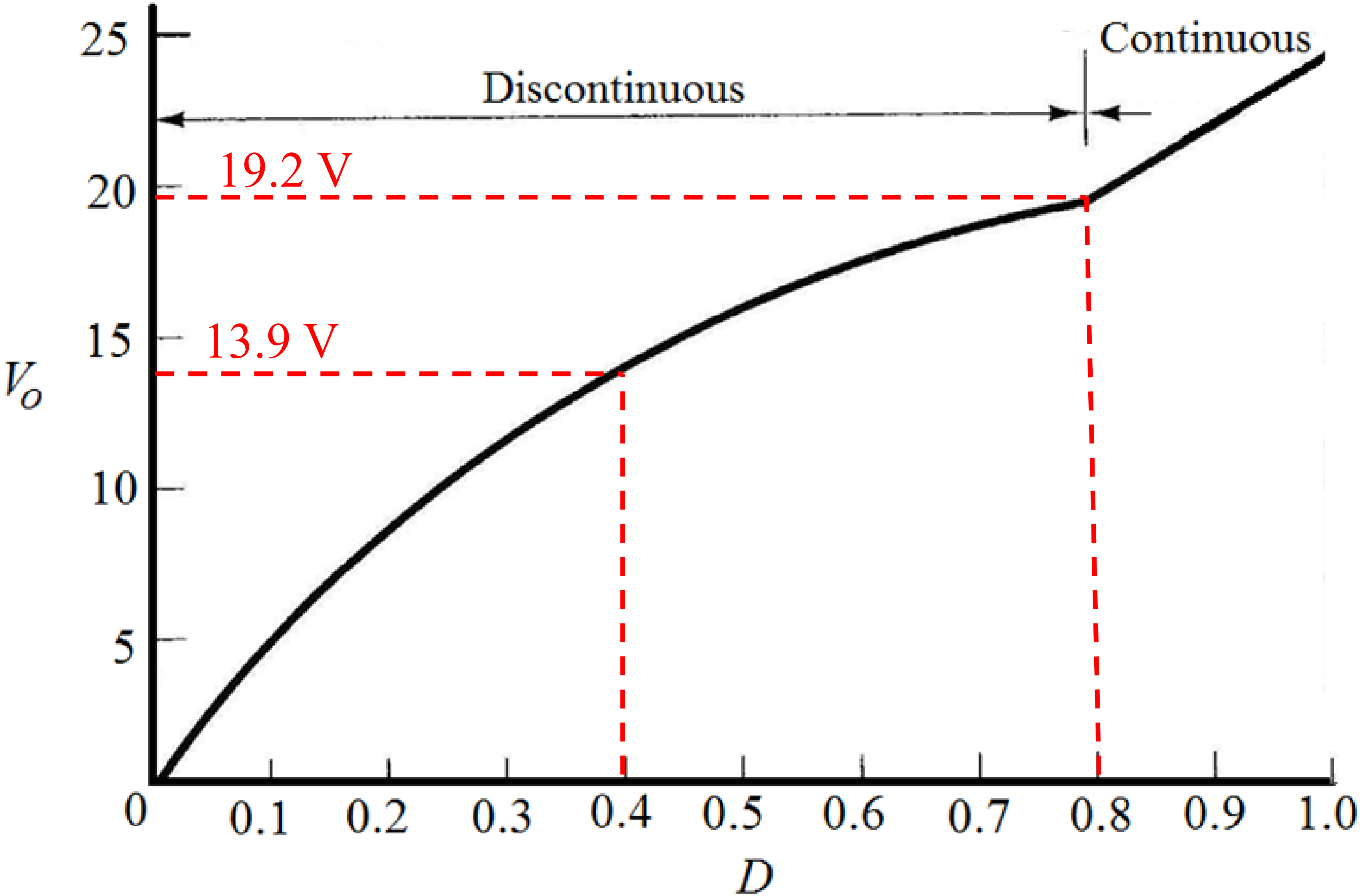
$D_1 = 0.29 < (1–D) = 0.6$, the inductor current is discontinuous.

$$V_o = V_s\left(\frac{D}{D + D_1}\right) = 24\left(\frac{0.4}{0.4 + 0.29}\right) = 13.9\text{V}$$

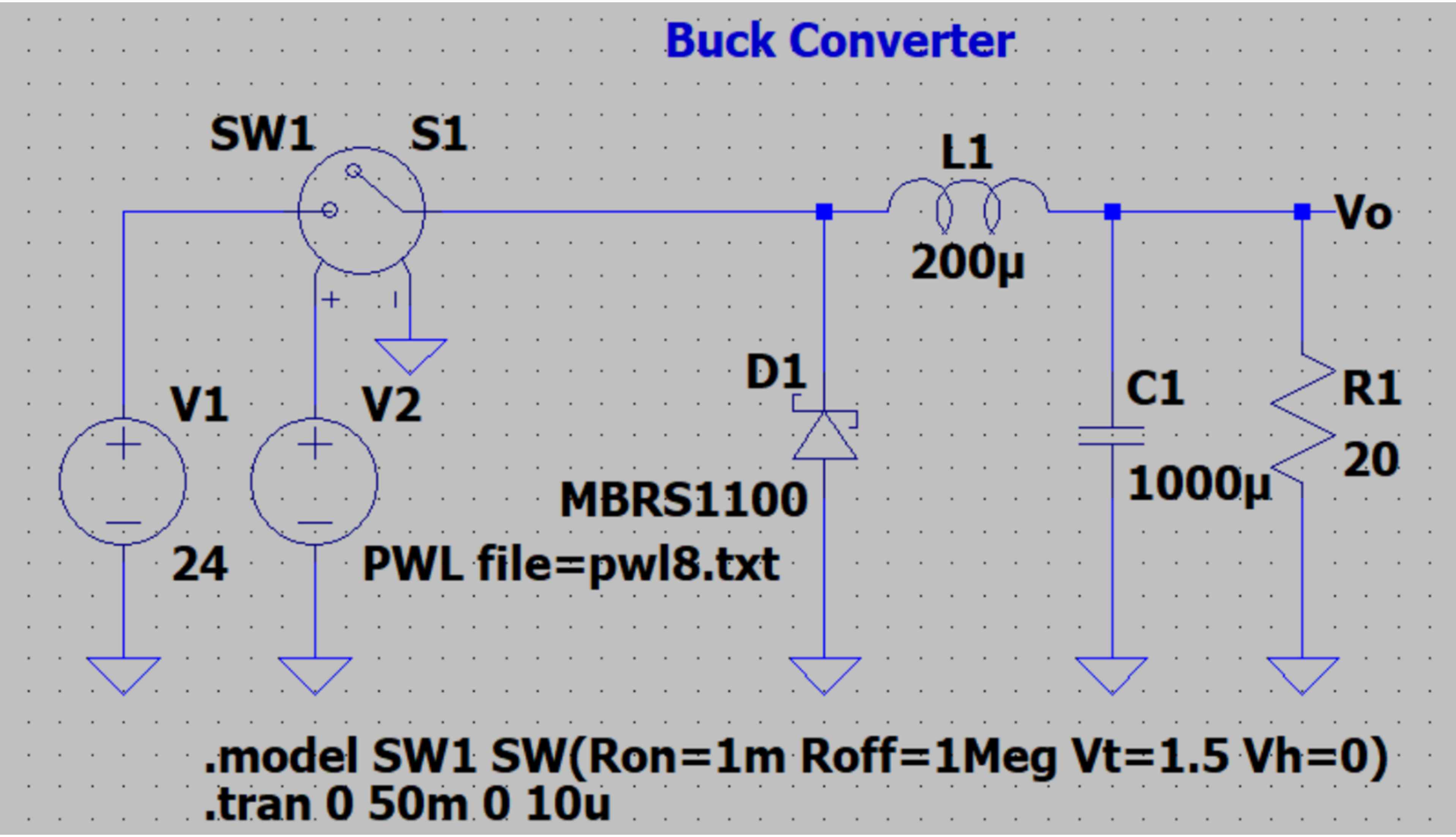
The boundary occurs when $D_1 = 1–D$

$$1 - D = \frac{-D + \sqrt{D^2 + \frac{8L}{RT}}}{2}$$

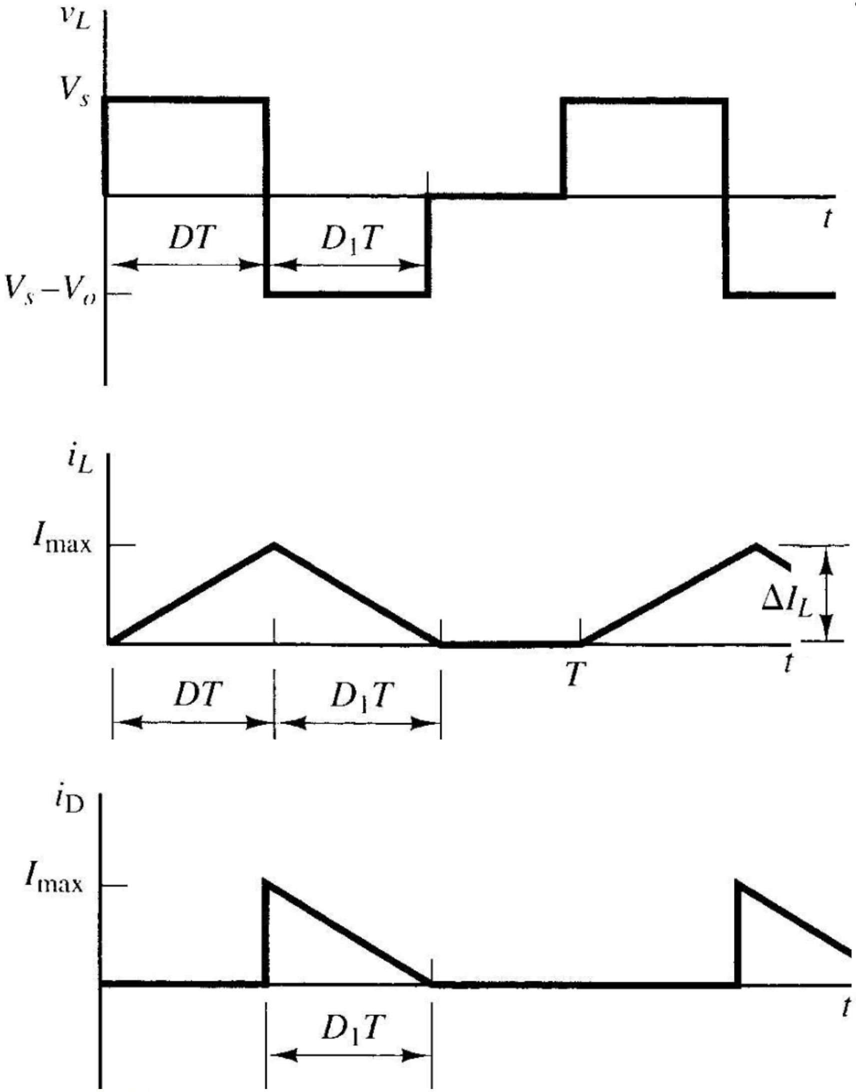
$$D = 1 - \frac{2Lf}{R} = 1 - 0.2 = 0.8 \qquad V_o = DV_s = 0.8 \times 24 = 19.2 \text{ V}$$


25
20
15
10
5
0
Discontinuous
Continuous
19.2 V
13.9 V
V_o
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
D

LTSpice simulation:



Discontinuous Current



Using Boost converter as an example:

$$V_s DT + (V_s - V_o) D_1 T = 0$$

$$V_s DT + V_s D_1 T = V_o D_1 T$$

$$V_o = V_S \left(\frac{D + D_1}{D_1} \right)$$

Average diode current:

$$I_D = \frac{1}{T}\left(\frac{1}{2} I_{max} D_1 T\right) = \frac{1}{2} I_{max} D_1$$

$$v_L = V_s = L\frac{\Delta i_L}{DT} \Rightarrow \Delta i_L = \frac{V_s DT}{L} = I_{max}$$

$$I_D = \frac{1}{2}\left(\frac{V_s DT}{L}\right) D_1 = \frac{V_o}{R}$$

$$D_1 = \left(\frac{V_o}{V_s}\right)\left(\frac{2L}{RDT}\right)$$

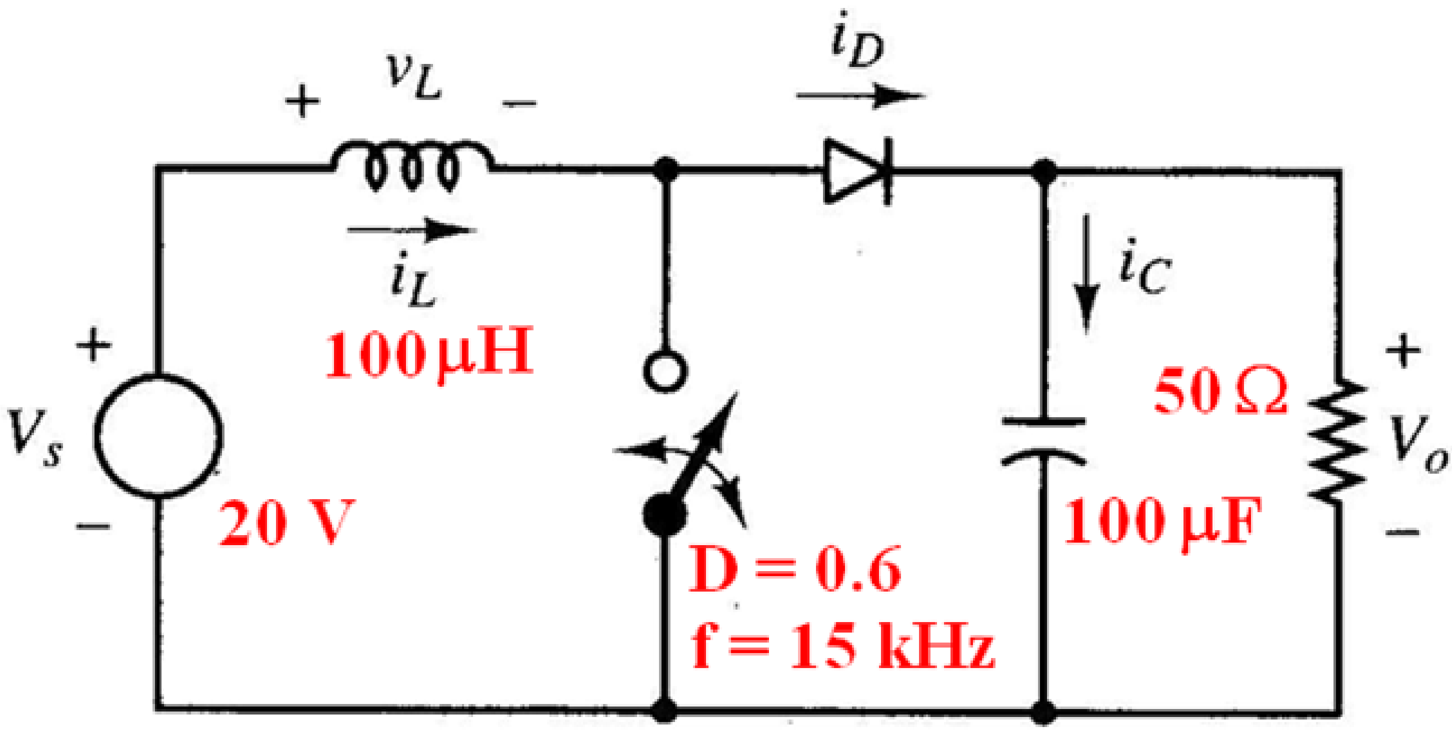
Substituting $D_1 = \left(\frac{V_o}{V_s}\right)\left(\frac{2L}{RDT}\right)$ into $V_o = V_S\left(\frac{D+D_1}{D_1}\right)$

$$V_o = V_S\left(\frac{D+\frac{2LV_o}{RDTV_s}}{\frac{2LV_o}{RDTV_s}}\right) \Rightarrow \left(\frac{V_o}{V_s}\right)^2 - \left(\frac{V_o}{V_s}\right) - \frac{D^2RT}{2L} = 0$$

$$\frac{V_o}{V_s} = \frac{1}{2}\left(1+\sqrt{1+\frac{2D^2RT}{L}}\right)$$

The boundary between continuous and discontinuous current: $D_1 = 1 - D$.

Exercise #6: For the given Boost converter, verify if the inductor current is discontinuous. Determine the output voltage and the maximum inductor current.



Assume that the inductor current is continuous:

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{20}{(1-0.6)^2(50)} = 2.5 \text{ A}$$

$$\Delta i_L = \frac{V_s D}{fL} = \frac{(20)(0.6)}{(15\times 10^3)(100\times 10^{-6})} = 8 \text{ A}$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = 2.5 - 4 = -1.5 \text{ A}$$

Since I_{min} is negative, the inductor current is discontinuous.

Maximum inductor current,

$$I_{max} = \Delta i_L = 8 \text{ A}$$

$$V_o = \frac{V_s}{2}\left(1+\sqrt{1+\frac{2D^2RT}{L}}\right) = \frac{20}{2}\left(1+\sqrt{1+\frac{2\times 0.6^2\times 50}{100\mu\times 15k}}\right) = 60 \text{ V}$$

For continuous inductor current:

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{20}{(1-D)^2(50)} = \frac{0.4}{(1-D)^2}$$

$$\Delta i_L = \frac{V_s D}{fL} = \frac{20D}{(15\times 10^3)(100\times 10^{-6})} = 13.33D$$

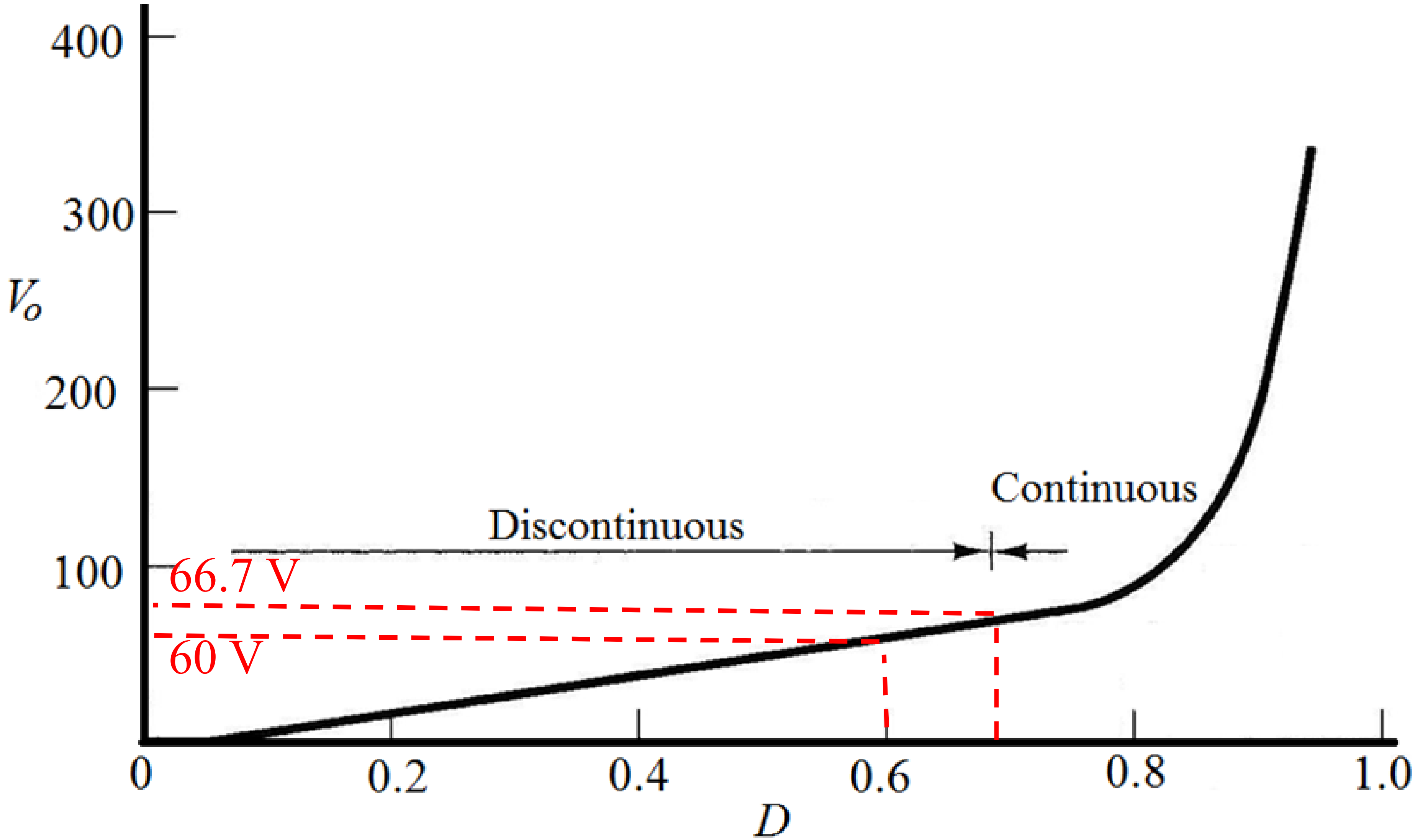
$$I_L = \frac{\Delta i_L}{2} \Rightarrow \frac{0.4}{(1-D)^2} = \frac{13.33D}{2}$$

$$\therefore D(1-D)^2 = 0.06$$

Cannot solve directly. Use successive approximation (i.e. choose any value of D and look for convergence).

$$D \approx 0.7$$

$$V_o = \frac{V_s}{1-D} = \frac{20}{1-0.7} = 66.7 \text{ V}$$


400
300
200
100
V_o
Continuous
Discontinuous
66.7 V
60 V
0
0.2
0.4
0.6
0.8
1.0
D

LTSpice simulation: