NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2022-2023

EE6225 – MULTIVARIABLE CONTROL SYSTEMS ANALYSIS & DESIGN

April / May 2023 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 5 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. In Figure 1, $G(s)$ is a first-order process with time delay:

$$G(s) = \frac{5}{\tau s + 1} e^{-2s}, \text{ where } \tau = 3. \tag{1}$$

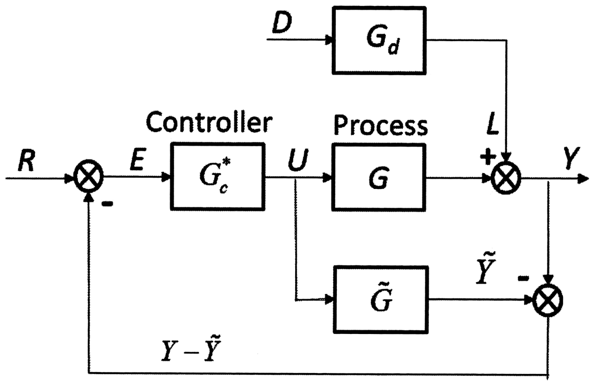

Figure 1.

(a) Assume $\tilde{G}(s) = G(s)$ and use the first order Pade approximation $e^{-\theta s} = \frac{1-0.5\theta s}{1+0.5\theta s}$. Factor $\tilde{G}(s)$ into $\tilde{G}(s) = \tilde{G}_+(s)\tilde{G}_-(s)$ where $\tilde{G}_+(s)$ contains all right half plane zeros and $\tilde{G}_+(s=0) = 1$. Find the IMC controller $G_c^*(s) = \frac{1}{\tilde{G}_-(s)} f(s)$ where $f(s) = \frac{1}{4s+1}$ is a low pass filter.

(5 Marks)

(b) Find the equivalent standard feedback controller $G_c(s) = \frac{G_c^*(s)}{1-G_c^*(s)\tilde{G}(s)}$.

(5 Marks)

(c) Determine the corresponding parameters of a PID controller

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + s\tau_D\right)$$

.

(5 Marks)

(d) If $G_d(s) = \frac{1}{(5s+1)} e^{-s}$, design a feedforward controller $G_{ff}(s)$. Suggest one way to ensure that $G_{ff}(s)$ is realizable in practice.

(5 Marks)

(e) Is the closed loop system using an IMC controller $G_c^*(s)$ internally stable if $\tau = -2$ in equation (1)? Design the equivalent standard feedback controller $G_c(s)$ with $f(s) = \frac{\lambda s+1}{(4s+1)^2}$ where the parameter λ is determined from $f\left(s = \frac{1}{2}\right) = 1$.

(5 Marks)

2. Consider the following 2×2 transfer function matrix (TFM) with time delay

$$G(s) = \begin{bmatrix} \frac{1}{(s+1)(s+2)}e^{-2s} & \frac{2}{s+1}e^{-3s} \\ \frac{-1}{s+1}e^{-s} & \frac{-2}{(s+1)(s+2)}e^{-2s} \end{bmatrix} \tag{2}$$

(a) Compute the poles and zeros of $G(s)$.

(5 Marks)

(b) Derive the relative gain array (RGA) $\Lambda(s) = G(s) \times (G^{-1}(s))^T$. Evaluate $\Lambda(s = 0)$ and determine what the best pairing is for the inputs and outputs to minimize loop interactions.

(5 Marks)

(c) Design a precompensator/decoupler $W_1(s)$ so that a diagonal controller of the form $K_1(s) = \begin{bmatrix} K_{11}(s) & 0 \\ 0 & K_{22}(s) \end{bmatrix}$, where $K_{11}(s)$ and $K_{22}(s)$ are individual controllers, can be synthesized separately for $G(s)$.

(5 Marks)

(d) Design a precompensator/decoupler $W_2(s)$ so that a diagonal controller of the form $K_2(s) = \begin{bmatrix} 0 & K_{12}(s) \\ K_{21}(s) & 0 \end{bmatrix}$, where $K_{12}(s)$ and $K_{21}(s)$ are individual controllers, can be synthesized separately for $G(s)$.

(5 Marks)

(e) List two limitations in the decoupling controller design method and comment whether it is suitable for the plant $G(s)$ described in equation (2).

(5 Marks)

3. Consider the single-input single-output discrete-time model with parameters a, b, d:

$$\frac{y_k}{u_k} = \frac{bz^{-d}}{1 - az^{-1}}.$$

The input and output of the model are denoted by u and y respectively, and the integer $d = 1, 2, \ldots$ denotes the delay.

(a) Convert the model into an equivalent state space model of the form

$$x_{k+1} = Ax_k + B\Delta u_{k-d+1}, \quad y_k = Cx_k$$

where $x_k = \begin{bmatrix} \Delta y_k & y_k \end{bmatrix}^T$, and Δ denotes the difference operator, i.e., $\Delta(\cdot)_k = (\cdot)_k - (\cdot)_{k-1}$. State clearly the matrices A, B, and C.

(5 Marks)

(b) With $d = 2, a = 0.8, b = 0.2$, compute the MPC law which minimises the cost function

$$J = \sum_{j=0}^{1}(r - y_{k+d+j})^2 + \Delta u_k \text{ with } \Delta u_{k+j} = 0, j = 1, 2, \ldots.$$

(5 Marks)

(c) For all values of d, a and b, derive a **general** expression for the state prediction x_{k+d} in the form of $x_{k+d} = Ex_k + F\overleftarrow{U} + G\Delta u_k$ where $\overleftarrow{U}$ contains the past control signals.

(5 Marks)

(d) Use your result in part (c), derive the MPC law which minimises the cost function

$$J = \sum_{j=0}^{N}(r - y_{k+d+j})^2 + \lambda\Delta u_k \text{ with } \Delta u_{k+j} = 0, j = 1, 2, \ldots.$$

(5 Marks)

(e) Does the Model Predictive Control (MPC) law you obtained in part (d) give zero offset at steady state? Justify your answer.

(5 Marks)

4. (a) A particular MPC law with the plant model

$$x_{k+1} = Ax_k + B\Delta u_k, \quad y_k = Cx_k$$

needs to respect the following constraints

$$-4 \leq \Delta u_{k+i} \leq 5, \quad i = 0, 1, 2; \quad -6 \leq \Delta y_{k+i} \leq 7, \quad i = 1, 2, 3.$$

Express these constraints in the form $\Omega\theta \leq \omega$. For simplicity, you may assume a single-input/single-output plant. State clearly the quantities Ω, θ, and ω.

(8 Marks)

(b) MPC solves online a Quadratic Program (QP) whose standard form is

$$\min_{\theta} \frac{1}{2}\theta^T H\theta + f^T\theta, \text{subject to } \Omega\theta \leq \omega.$$

Such QP can be solved by calling the function `qp(H,f,Ω,ω)`. Write, in the form of pseudo code, how would you implement the MPC law. You may assume that the quantities H and Ω is fixed at design stage, while f and ω depend on state measurements, set-point, etc and hence changes at each sampling time. In your pseudo code, explain clearly how the receding horizon principle of MPC is implemented.

(7 Marks)

(c) Given the system

$$x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k$$

derive a Moving Horizon Estimator (MHE) to estimate x_k using measurements $y_k, y_{k-1}, \ldots, y_{k-N}$. Express the MHE in the form

$$\hat{x}_k = MY + NU$$

where Y and U are vectors of past measurements y and control signals u.

(10 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.