Part I – Advanced Process Control

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Course Outline

1.Dynamic Model, MIMO, Poles and Zeros, Trade-Offs and Limits of Performance

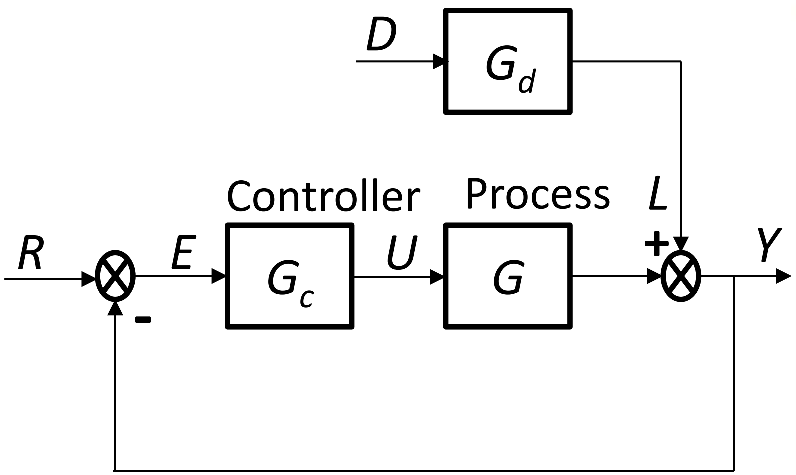
2.SISO - PID, DSM, IMC, IMC-Based PID, Smith Predictor

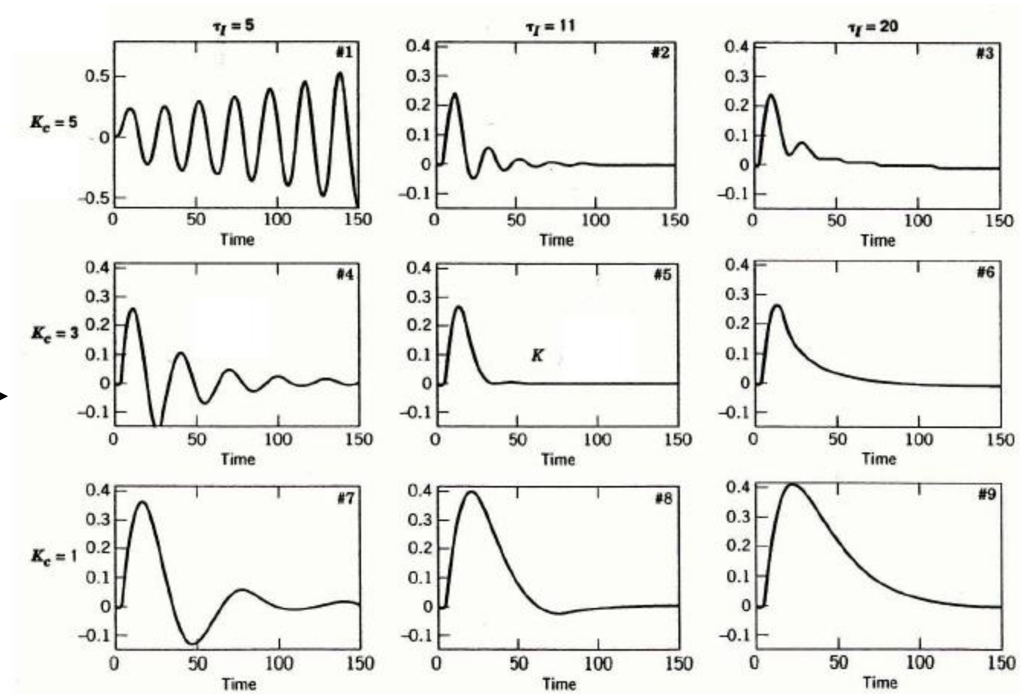
3.SISO - Cascade, Feedforward, Ratio, Split Range, Selective Control

4,5. MIMO – MultiLoop Interaction Dynamics, RGA, Decoupling, Multivariable Feedback Control

6.System Identification, Linear Regression, RLS, 4SID

2. PID Controller Performance and Tuning





- Unit Step Disturbance Response for Different PI Controller Tuning

Fig. 2.1

Outline of Lecture 2

2.1 PID Control

- 2.1.1 PID Structure – 1 and 2 degree of freedom Design
- 2.1.2 Trial and Error Tuning
- 2.1.3 Continuous Cycling Method Tuning
- 2.1.4 Relay Auto Tuning
- 2.1.5 Integral Error Criteria for Tuning

2.2 Direct Synthesis Method

- 2.2.1 First Order Process
- 2.2.2 First Order Plus Time Delay (FOPTD)
- 2.2.3 Second Order Plus Time Delay (SOPTD)

2.3 Internal Model Control (IMC)

- 2.3.1 IMC Design Strategy
- 2.3.2 IMC-Based PID
- 2.3.3 Smith Predictor

2. Learning Objectives

- Introduce fundamental concepts of PID control design, tuning methods for the proportional, integral and derivative gains
- Direct Synthesis Method as an introduction to the model-based techniques
- Use Internal Model Controllers to realize classical feedback (PID) controllers
- Smith Predictors for process plant with significant transportation delay

2.1 Performance Criteria for Closed-Loop Control System

- The function of a feedback control system is to ensure that the closed loop system has desirable dynamic and steady-state response characteristics.
- Ideally, we would like the closed-loop system to satisfy the following performance criteria:
  - The closed-loop system must be stable.
  - The effects of disturbances are minimized, providing good disturbance rejection.
  - Rapid, smooth responses to set-point changes are obtained, that is, good set-point tracking.
  - Steady-state error (offset) is eliminated.
  - Excessive control action is avoided.
  - The control system is robust, that is, insensitive to changes in process conditions and to inaccuracies in the process model.

Stability

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Robust
Stability
&
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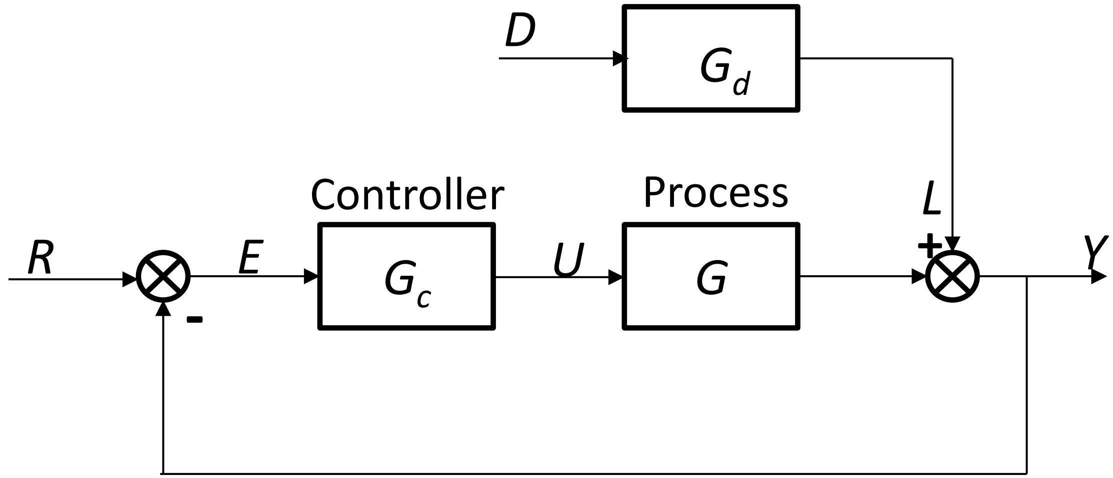
2.1 PID Control

- PID (Proportional-Integral-Derivative) controller provides an efficient solution to real-world control problems
- It is simple and effective and is the most popular controller in industrial applications – petrochemical, pharmaceutical, food, chemical, semiconductor, aerospace, building HVAC systems etc.
- They can be designed to be reasonably robust to changes in the process plant model and disturbance

2.1.1 PID Controller with 1 Degree of Freedom



We define the error

$$E(s) = R(s) - Y(s)$$

The transfer function of a (parallel) PID controller is commonly expressed as

$$U(s) = G_c(s)E(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)E(s) \quad \left[\text{i.e. } u(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(\tau)d\tau + \tau_D\frac{de(t)}{dt}\right)\right]$$

where $U(s)$ is the control signal acting on the error signal $E(s)$, K_C is the proportional gain, τ_I is the integral time constant, τ_D is the derivative time constant and s is the argument of the Laplace Transform.

Equivalently, we can also expressed the (parallel) PID controller as

$$G_c(s) = K_C + K_I\frac{1}{s} + K_D s$$

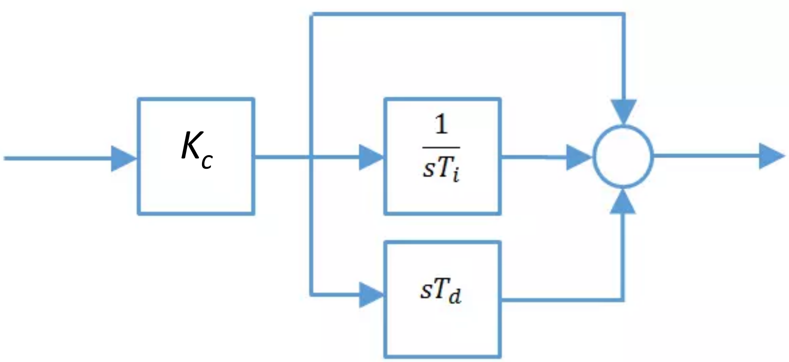
where $K_I = K_C/\tau_I$ is the integral gain and $K_D = K_C\tau_D$ is the derivative gain

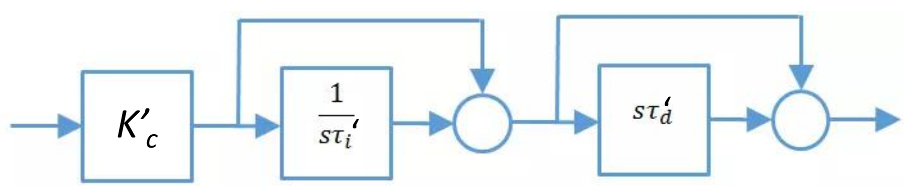
2.1.1 Parallel and Serial PID

Table 12.2 Equivalent PID Controller Settings for the Parallel and Series Forms

Parallel Form	Series Form
$G_c(s) = K_c\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right)$	$G_c(s) = K_c'\left(1 + \dfrac{1}{\tau_I' s}\right)(1 + \tau_D' s)^{\dagger}$
$K_c = K_c'\left(1 + \dfrac{\tau_D'}{\tau_I'}\right)$	$K_c' = \dfrac{K_c}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_I = \tau_I' + \tau_D'$	$\tau_I' = \dfrac{\tau_I}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_D = \dfrac{\tau_D'\tau_I'}{\tau_I' + \tau_D'}$	$\tau_D' = \dfrac{\tau_I}{2}\left(1 - \sqrt{1 - 4\tau_D/\tau_I}\right)$

†These conversion equations are only valid if $\tau_D/\tau_I \le 0.25$.

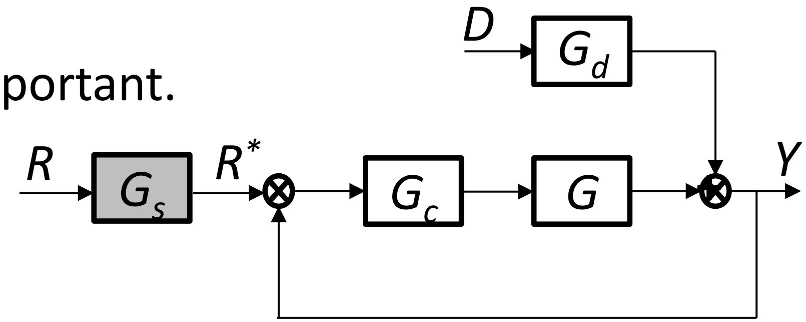




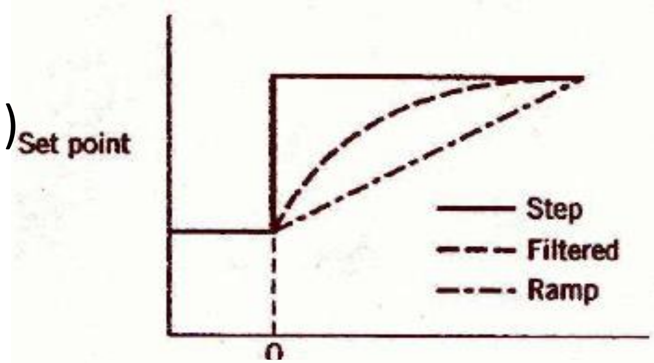
$$K_c'\left(1+\frac{\tau_D'}{\tau_I'}\right)\left(1+\frac{1}{\left(\tau_I'+\tau_D'\right)s}+s\frac{\tau_D'\tau_I'}{\tau_I'+\tau_D'}\right)$$

2.1.1 PID Controller with 2-Degree of Freedom

- Trade-off between set-point tracking and disturbance rejection
- In general, disturbance rejection is more important. Tune the controller for satisfactory disturbance rejection.



- Controllers with two degrees of freedom
- Strategies to adjust set-point tracking and disturbance rejection independently
  - Gradual change in set point (ramp or filtered)

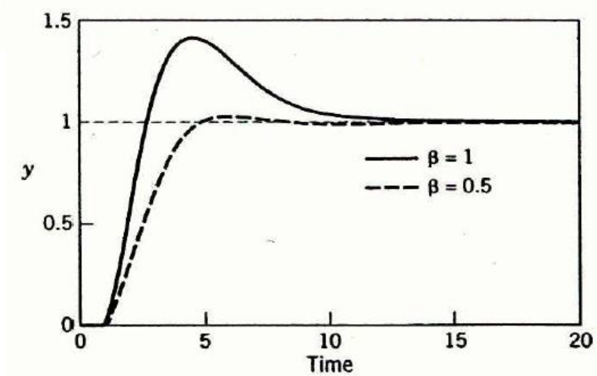


$$\frac{R^*}{R} = \frac{1}{\tau_f s + 1} \quad \text{(first order filter)}$$

  - Modification of PID Control Law

$$u(t) = K_C(\beta\, r - y) + K_C\left(\frac{1}{\tau_I}\int_0^t e(\tau)\,d\tau + \tau_D \frac{de(t)}{dt}\right)$$

$0<\beta<1$, output response becomes faster with more overshoot as β increases



2.1.1 PID Controller Design

In PID control system design, controller parameters tuning is non-trivial. They are tuned so that the closed-loop system meets the following five objectives:

1) stability usually measured in the frequency domain

Stability

2) transient response, including rise time, overshoot, and settling time

3) steady-state accuracy

4) disturbance attenuation and robustness against environmental uncertainty, often at steady state

Perfor
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5) robustness against plant modeling uncertainty, usually measured in the frequency domain.

Robust
Stability
&
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2.1.1 Effects of P, I and D Tuning

The three-term functionalities include:

1) The proportional term provides an overall control action proportional to the error signal through the all pass gain factor.

2) The integral term reduces steady-state errors through low-frequency compensation.

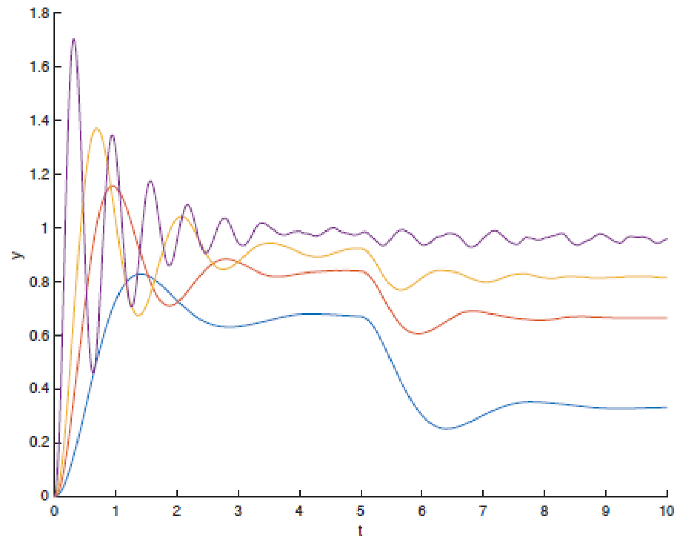
3) The derivative term improves transient response through high-frequency compensation.

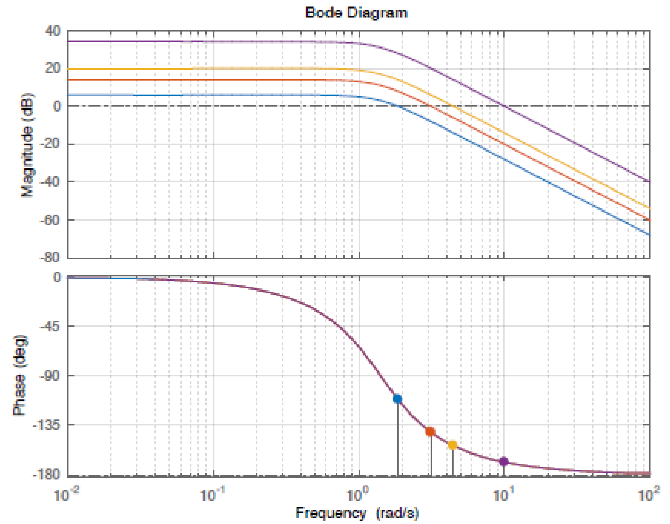
For optimum performance, K_C, K_I (or τ_I) and K_D (or τ_D) must be tuned jointly, although the individual effects of these three parameters on the closed-loop performance of stable plants are summarized in below table.

	Rise Time	**Overshoot**	**Settling Time**	**Steady-State Error**	**Stability**
Increasing K_c	Decrease	Increase	Small Increase	Decrease	Degrade
Increasing k_I	Small Decrease	Increase	Increase	Large Decrease	Degrade
Increasing k_D	Small Decrease	Decrease	Decrease	Minor Change	Improve

2.1.1 Effects of K_c on Response and PM

- Second Order Process Plant





As the proportional gain increases,

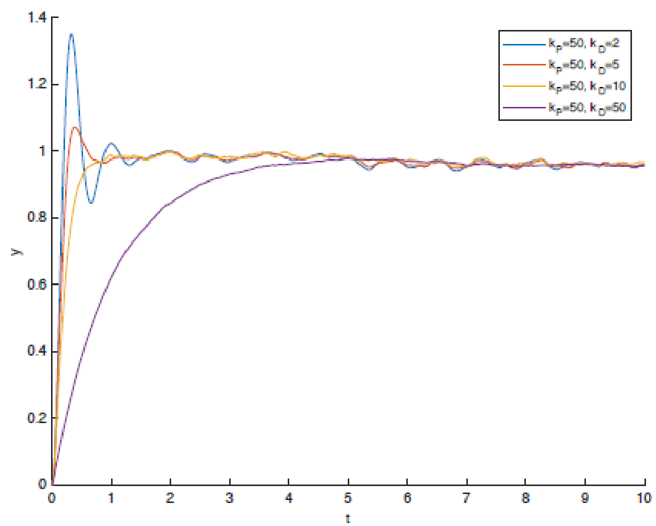
- The closed-loop system become more oscillatory (warning!);
- The steady-state error decreases;
- The response becomes faster;
- The sensitivity to noise increases.

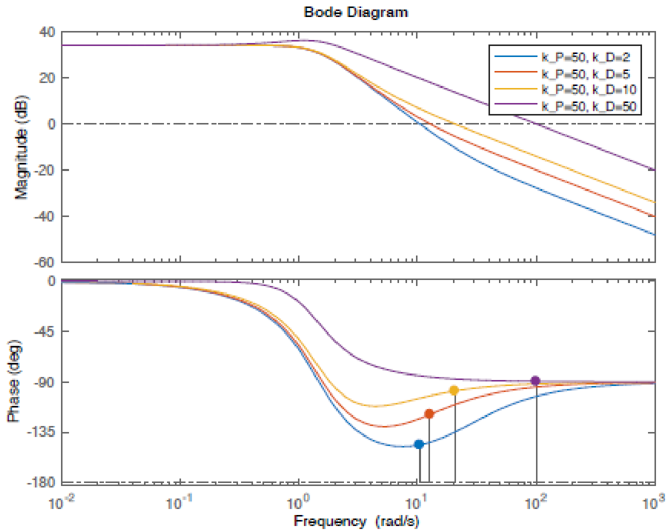
As the proportional gain increases,

- Phase margin gets smaller and smaller!
- The crossover frequency increases;
- The low-frequency gain increases;
- The high-frequency gain increases;

2.1.1 Effects of K_D on Response and PM

- Second Order Process Plant





As the derivative gain increases,

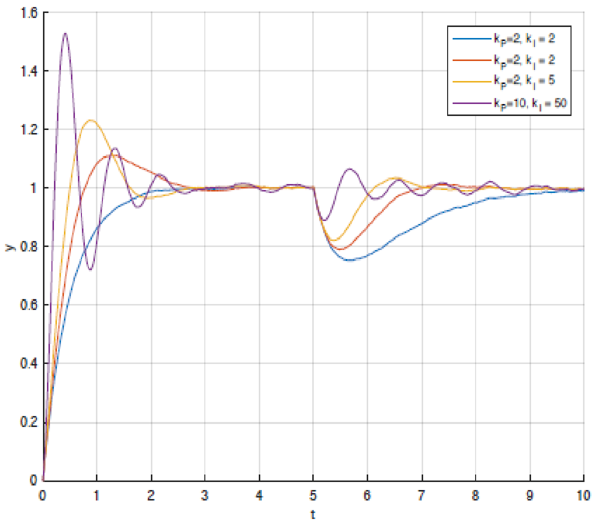
- The steady-state error not affected;
- The response becomes less oscillatory, but potentially slower
- The sensitivity to noise increases!

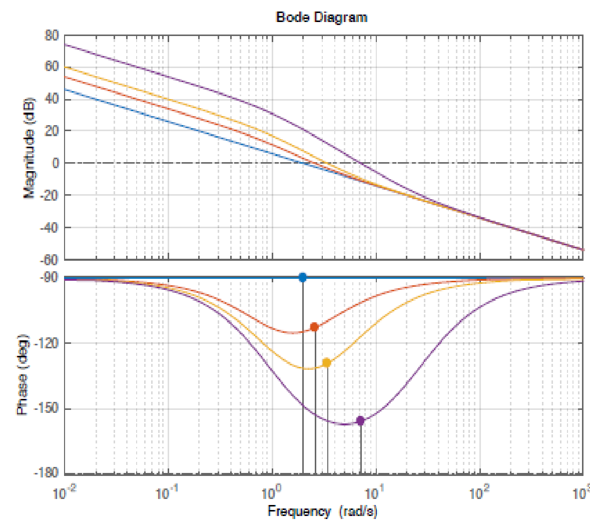
As the derivative gain increases,

- Phase margin increases;
- The crossover frequency increases;
- The low-frequency gain does not change;
- The high-frequency gain increases.

2.1.1 Effects of K_I on Response and PM

- First Order Process Plant





As the integral gain increases,

- The steady-state error is zero (as long as k_I is not zero)
- The response becomes more oscillatory (warning!)
- The sensitivity to noise does not change!

As the integral gain increases,

- Phase margin decreases;
- The crossover frequency increases;
- The low-frequency gain increases — but goes to infinity near 0 in all cases ;
- The high-frequency gain does not change.

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2.1.1 PID Controller Tuning Methods

- Trial and Error Tuning
- Continuous Cycling Method Tuning/Ziegler Nichols Tuning Relation
- Relay Auto Tuning
- Integral Error Criteria
- Direct Synthesis Method (DSM)
- Internal Model Control (IMC) Method

2.1.2 Trial and Error Tuning

- <u>Step1: With P-only controller</u>
  – Start with low K_c value and increase it (K_{cu}) until the response has a sustained oscillation (continuous cycling) for a small set point or load change.
  – Set K_c = 0.5 K_{cu}.
- <u>Step2: Add I mode</u>
- – Decrease the reset time until sustained oscillation occurs. (τ_{IU})
- – Set $\tau_I = 3\tau_{IU}$.
- – If a further improvement is required, proceed to Step 3.
- <u>Step3: Add D mode</u>
  – Increase the derivative time until sustained oscillation occurs. (τ_{DU})
- – Set $\tau_D = \tau_{DU}/3$.

(The sustained oscillation should not be caused by the controller saturation)

2.1.3 Continuous Cycling Method Tuning

Over 60 years ago, Ziegler and Nichols (1942) published a classic paper that introduced the *continuous cycling method* for controller tuning. It is based on the following trial-and-error procedure:

Step 1. After the process has reached steady state (at least approximately), eliminate the integral and derivative control action by setting τ_D to zero and τ_I to the largest possible value.

Step 2. Set K_c equal to a small value (e.g., 0.5) and place the controller in the automatic mode.

Step 3. Introduce a small, momentary set-point change so that the controlled variable moves away from the set point. Gradually increase K_c in small increments until continuous cycling occurs. The term *continuous cycling* refers to a sustained oscillation with a constant amplitude. The numerical value of K_c that produces

2.1.3 Continuous Cycling Method Tuning

continuous cycling (for proportional-only control) is called the *ultimate gain*, K_{cu}. The period of the corresponding sustained oscillation is referred to as the *ultimate period*, P_u.

Step 4. Calculate the PID controller settings using the Ziegler-Nichols (Z-N) tuning relations in Table 12.6.

Step 5. Evaluate the Z-N controller settings by introducing a small set-point change and observing the closed-loop response. Fine-tune the settings, if necessary.

The continuous cycling method, or a modified version of it, is frequently recommended by control system vendors. Even so, the continuous cycling method has several major disadvantages:

1. It can be quite time-consuming if several trials are required and the process dynamics are slow. The long experimental tests may result in reduced production or poor product quality.

2.1.3 Continuous Cycling Method Tuning

2. In many applications, continuous cycling is objectionable because the process is pushed to the stability limits.
3. This tuning procedure is not applicable to integrating or open-loop unstable processes because their control loops typically are unstable at both high and low values of K_c, while being stable for intermediate values.
4. For first-order and second-order models without time delays, the ultimate gain does not exist because the closed-loop system is stable for all values of K_c, providing that its sign is correct. However, in practice, it is unusual for a control loop not to have an ultimate gain.

2.1.3 Continuous Cycling Method Tuning

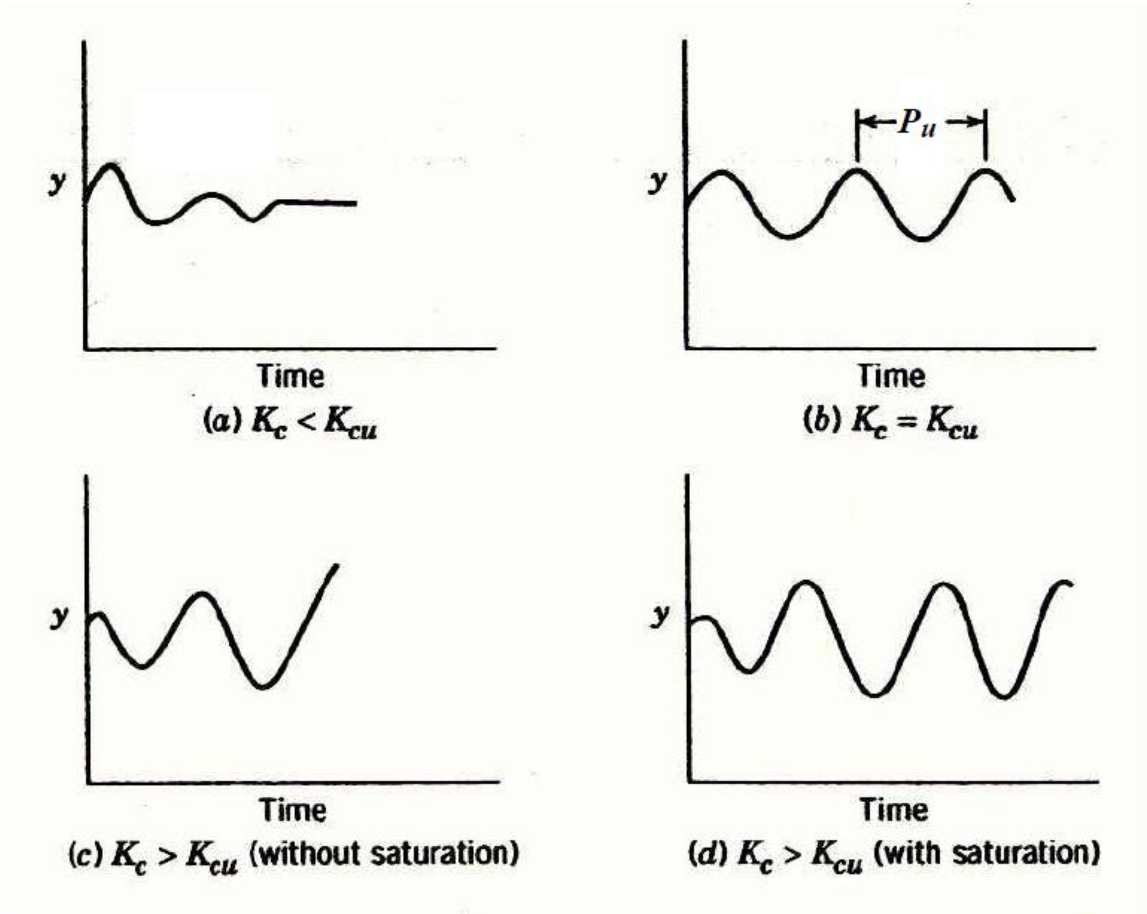


Figure 12.12 Experimental determination of the ultimate gain K_{cu}.

2.1.3 Ziegler-Nichols Tuning Rule

Table 12.6 Controller Settings based on the Continuous Cycling Method

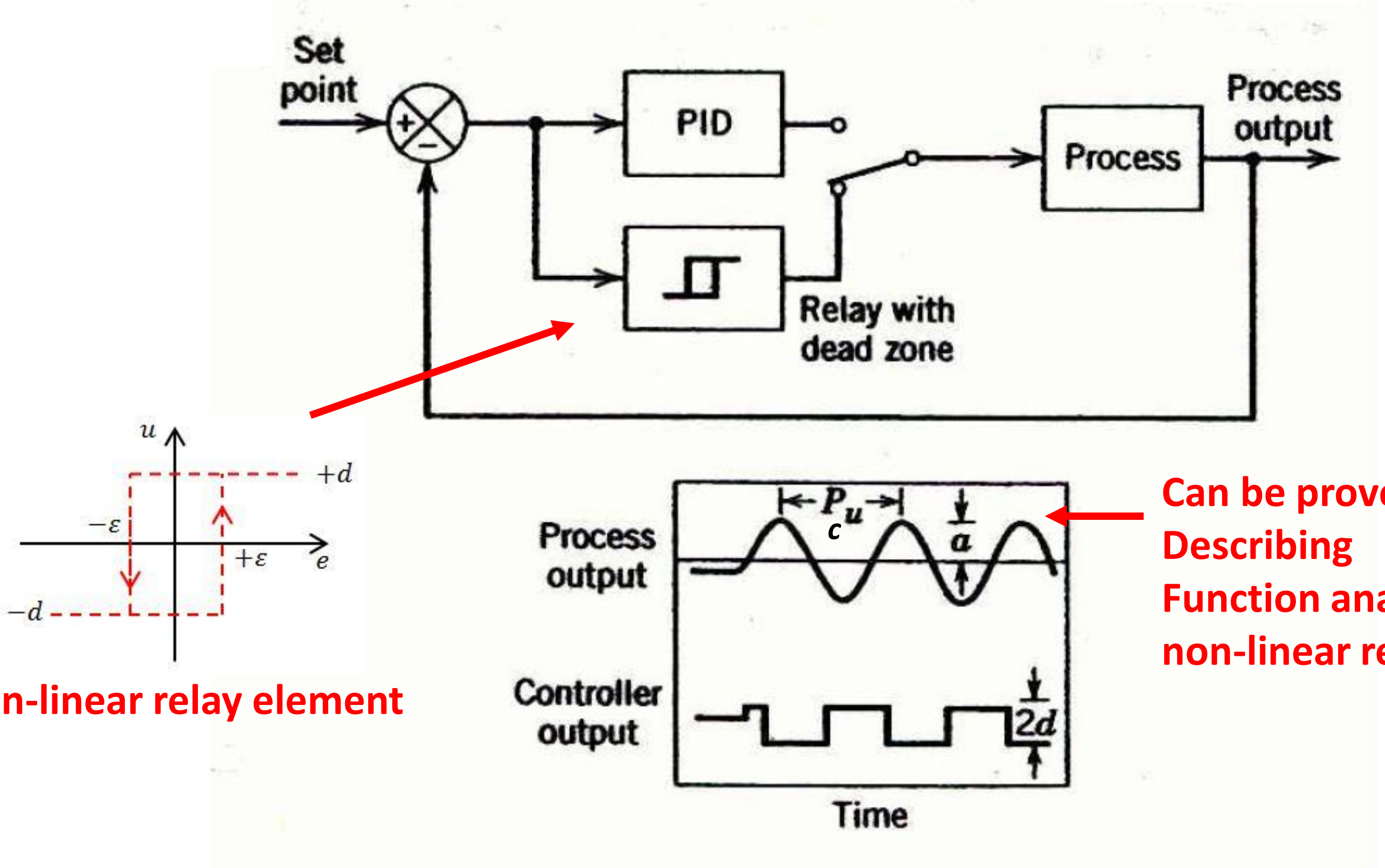
Ziegler-Nichols	K_c	τ_I	τ_D
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$
Tyreus-Luyben†	K_c	τ_I	τ_D
PI	$0.31K_{cu}$	$2.2P_u$	—
PID	$0.45K_{cu}$	$2.2P_u$	$P_u/6.3$

† Luyben and Luyben (1997).

2.1.4 Relay Auto Tuning

- Åström and Hägglund (1984) have developed an attractive alternative to the continuous cycling method.
- In the *relay auto-tuning* method, a simple experimental test is used to determine K_{cu} and P_u.
- For this test, the feedback controller is temporarily replaced by an on-off controller (or *relay*).
- After the control loop is closed, the controlled variable exhibits a sustained oscillation that is characteristic of on-off control (cf. Section 8.4). The operation of the relay auto-tuner includes a *dead band* as shown in Fig. 12.14.
- The dead band is used to avoid frequent switching caused by measurement noise.

2.1.4 Relay Auto Tuning



Non-linear relay element

Can be proved by Describing Function analysis of non-linear relay element

Figure 12.14 Auto-tuning using a relay controller.

2.1.4 Relay Auto Tuning

- The relay auto-tuning method has several important advantages compared to the continuous cycling method:

  1. Only a single experiment test is required instead of a trial-and-error procedure.
  2. The amplitude of the process output a can be restricted by adjusting relay amplitude d.
  3. The process is not forced to a stability limit.
  4. The experimental test is easily automated using commercial products.
  5. Determine P_C and calculate $K_{CU} = \dfrac{4d}{\pi a}$
  6. Use Ziegler-Nichols Tuning Rules to tune PID parameters

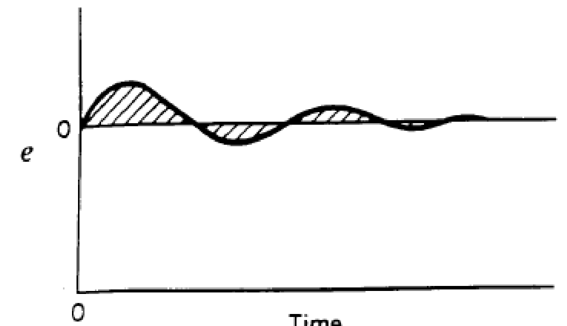
2.1.5 PID Tuning Based on Integral Error Criteria

- Controller tuning relations have been developed that optimized the closed-loop response for a simple process model and a specified disturbance or setpoint change
- The optimum settings minimize an integral error criterion
- Three popular integral error criteria are:

1. Integral of the absolute value of the error (IAE)

$$\text{IAE} = \int_0^\infty |e(t)|\, dt$$

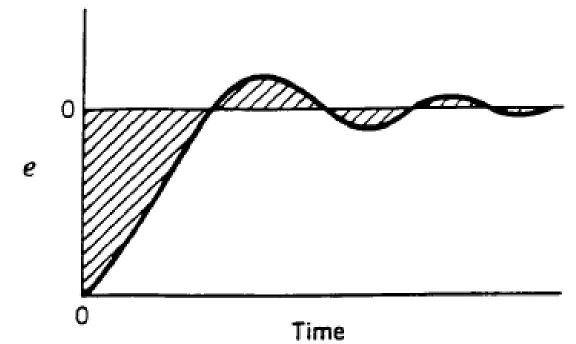
where $e(t)$ is the difference between setpoint r and output y



(a) Load change

2. Integral of the squared error (ISE)

$$\text{ISE} = \int_0^\infty e^2(t)\, dt$$



(b) Set-point change

3. Integral of the time-weighted absolute error (ITAE)

$$\text{ITAE} = \int_0^\infty t|e(t)|\, dt$$

2.1.5 ITAE for FOPTD Process

- PID tuning relation based on ITAE for FOPTD process $\dfrac{Ke^{-\theta s}}{s\tau+1}$

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6–8][a]

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

- Similar tuning relations for IAE and ISE for other process models are available in the literature

2.1.5 PI Design Example using ITAE Criteria

$$G(s) = \frac{10e^{-s}}{2s+1}$$

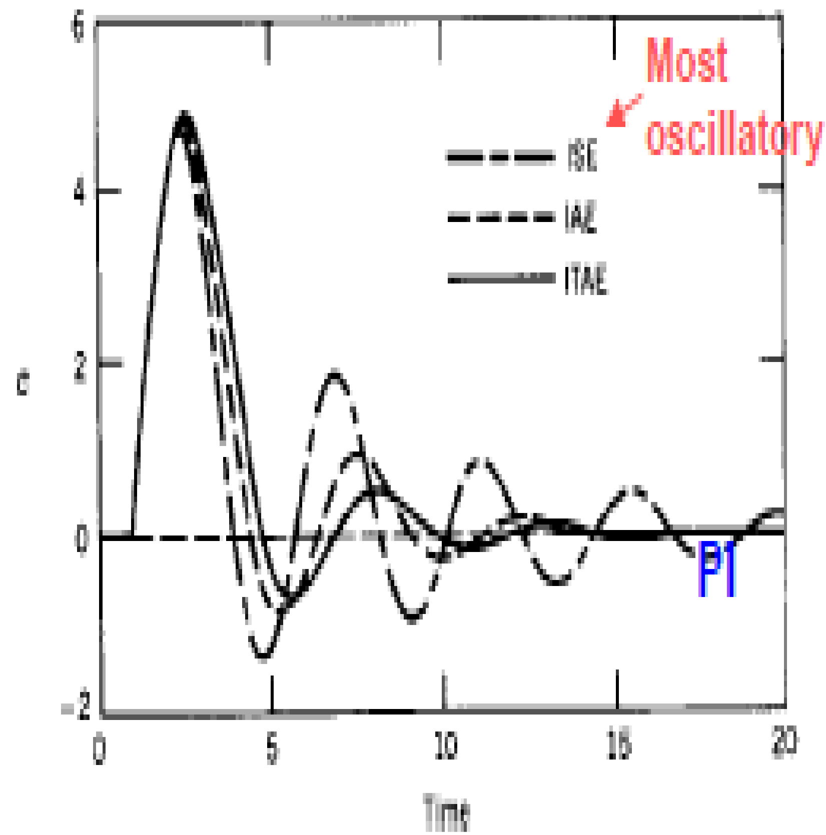
$$KK_c = (0.859)(1/2)^{-0.977} = 1.69$$

$$\Rightarrow K_c = 0.169$$

$$\tau/\tau_I = (0.674)(1/2)^{-0.680} = 1.08$$

$$\Rightarrow \tau_I = 1.85$$

Method	K_c	τ_I
IAE	0.195	2.02
ISE	0.245	2.44
ITAE	0.169	1.85

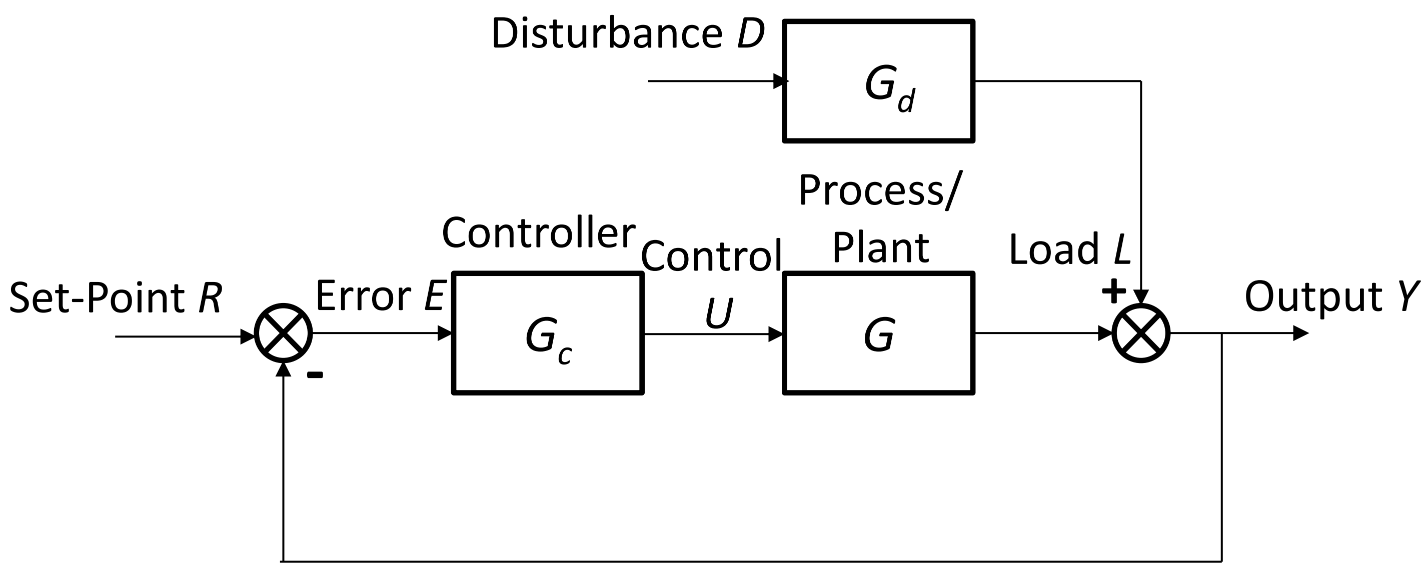


2.2 Direct Synthesis Method (DSM)

- In the Direct Synthesis Method (DSM), the controller design is based on a process model and a desired closed-loop transfer function
- The latter is usually specified for set-point changes ($T(s)$), but responses to disturbances ($S(s)$) can also be utilized
- Although these feedback controllers do not always have a PID structure, the DSM does produce PI or PID controllers for common process models

2.2 Feedback Control System

- Consider the following block diagram of a feedback control system



The closed-loop transfer function from set-point R to output Y is

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \quad (=T(s))$$

The closed-loop transfer function from disturbance D (load L) to output Y is

$$\frac{Y(s)}{D(s)} = \frac{G_d(s)}{1+G_c(s)G(s)} \quad \left(\frac{Y(s)}{L(s)} = \frac{1}{1+G_c(s)G(s)} = S(s)\right)$$

2.2 DSM Design Synthesis

$$\frac{Y}{R} = \frac{G_C G}{1 + G_C G}$$

1. Specify closed-loop response (transfer function)

$$\left(\frac{Y}{R}\right)_d$$

(= Complementary Sensitivity Function *T*(*s*))

2. Need process model, $\tilde{G}$ ($\approx$G)

3. Solve for G_c

$$G_C = \frac{1}{\tilde{G}} \left(\frac{\left(\frac{Y}{R}\right)_d}{1 - \left(\frac{Y}{R}\right)_d} \right) \qquad (12\text{-}3b)$$

Model Inversion

2.2 First Order Transfer Function

$$\left(\frac{Y}{R}\right)_d = \frac{e^{-\theta s}}{\tau_c s + 1} \qquad (12-6)$$

(first – order response, no offset)

$$\left(\tau_c = speed\ of\ response, \theta = process\ time\ delay\ in\ G\right)$$

But other variations of (12-6) can be used (e.g., replace time delay with polynomial approximation)

If $\theta = 0$, then (12 - 3b) yields $G_c = \frac{1}{\tilde{G}} \cdot \frac{1}{\tau_c s})$ (12 - 5)

$$\text{For } \tilde{G} = \frac{K}{\tau s + 1}, G_c = \frac{\tau s + 1}{K\tau_c s} = \frac{\tau}{K\tau_c} + \frac{1}{K\tau_c s} \qquad \text{(PI)}$$

2.2 First-Order-Plus-Time-Delay Model (FOPTD)

Consider the standard first-order-plus-time-delay model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \qquad (12\text{-}10)$$

Specify closed-loop response as FOPTD (12-6), but approximate using Taylor series approximation:
$e^{-\theta s} \approx 1 - \theta s$ in (12-3b).

Substituting and rearranging gives a PI controller,

$G_c = K_c\left(1 + 1/\tau_I s\right)$, with the following controller settings:

$$K_c = \frac{1}{K}\frac{\tau}{\theta + \tau_c}, \qquad \tau_I = \tau \qquad (12\text{-}11)$$

2.2 Summary : Plant without Delay

1. Perfect Control (K_c becomes infinite)

$$G(s)=\frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \text{ and } (Y/R)_d=1$$

$$G_c(s)=\frac{1}{G(s)}\left(\frac{1}{1-1}\right)=\frac{\infty}{G(s)} \text{ (infinite gain, unrealizable)}$$

2. Finite Settling Time for 1st-Order Process

$$G(s)=\frac{K}{(\tau s+1)} \text{ and } (Y/R)_d=\frac{1}{\tau_c s+1}$$

$$G_c(s)=\frac{1}{G(s)}\left(\frac{1/(\tau_c s+1)}{1-1/(\tau_c s+1)}\right)=\frac{\tau s+1}{K\tau_c s}=\frac{\tau}{\tau_c K}\left(1+\frac{1}{\tau s}\right) \text{ (PI)}$$

3. Finite Settling Time for 2nd-Order Process

$$G(s)=\frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \text{ and } (Y/R)_d=\frac{1}{\tau_c s+1}$$

$$G_c(s)=\frac{1}{G(s)}\left(\frac{1/(\tau_c s+1)}{1-1/(\tau_c s+1)}\right)=\frac{(\tau_1+\tau_2)}{K\tau_c}\left(1+\frac{1}{\underbrace{(\tau_1+\tau_2)}_{\tau_I}s}+\frac{\tau_1\tau_2}{\underbrace{(\tau_1+\tau_2)}_{\tau_D}}s\right) \text{ (PID)}$$

2.2 Summary Plants with Delay

- If there is a time delay, no physically realizable controller can overcome the time delay (Need time lead)
- A reasonable choice will be

$$(Y/R)_d = \frac{e^{-\theta_c s}}{\tau_c s+1}$$

1. FOPTD Without Approximation for Delay

$$G(s) = \frac{Ke^{-\theta s}}{\tau s+1} \text{ and } (Y/R)_d = \frac{e^{-\theta s}}{\tau_c s+1} \quad (\theta_c = \theta)$$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{e^{-\theta s}/(\tau_c s+1)}{1-e^{-\theta s}/(\tau_c s+1)}\right) = \frac{\tau s+1}{K}\left(\frac{1}{\tau_c s+1-e^{-\theta s}}\right) \text{ (Not realizable and Not PID)}$$

2. FOPTD with 1st Order Taylor Series Approximation for Delay $\left(e^{-\theta s} = 1-\theta s\right)$

$$G_c(s) = \frac{\tau s+1}{K}\left(\frac{1}{\tau_c s+1-(1-\theta s)}\right) = \frac{\tau}{K(\tau_c+\theta)}\left(1+\frac{1}{\tau s}\right) \quad \text{(PI)}$$

3. SOPTD with 1st Order Taylor Series Approximation for Delay $\left(e^{-\theta s} = 1-\theta s\right)$

$$G(s) = \frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \text{ and } (Y/R)_d = \frac{e^{-\theta s}}{\tau_c s+1} \quad (\theta_c = \theta)$$

$$G_c(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K}\frac{1}{(\tau_c+\theta)s} = \frac{(\tau_1+\tau_2)}{K(\tau_c+\theta)}\left(1+\frac{1}{\underbrace{(\tau_1+\tau_2)}_{\tau_I}s}+\underbrace{\frac{\tau_1\tau_2}{(\tau_1+\tau_2)}}_{\tau_D}s\right) \quad \text{(PID)}$$

2.2 DSM Design Example

Use the DS design method to calculate PID controller settings for the SOPTD process:

$$G(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

Consider 3 values of the desired closed-loop time constant: $\tau_c = 1, 3$ and 10.

Evaluate the controllers for unit step changes in

both the setpoint r and the disturbance d, assuming that $G_d(s) = G(s)$.

Perform the evaluation for 2 cases:

a. The process model is perfect $\left(\tilde{G}(s) = G(s)\right)$

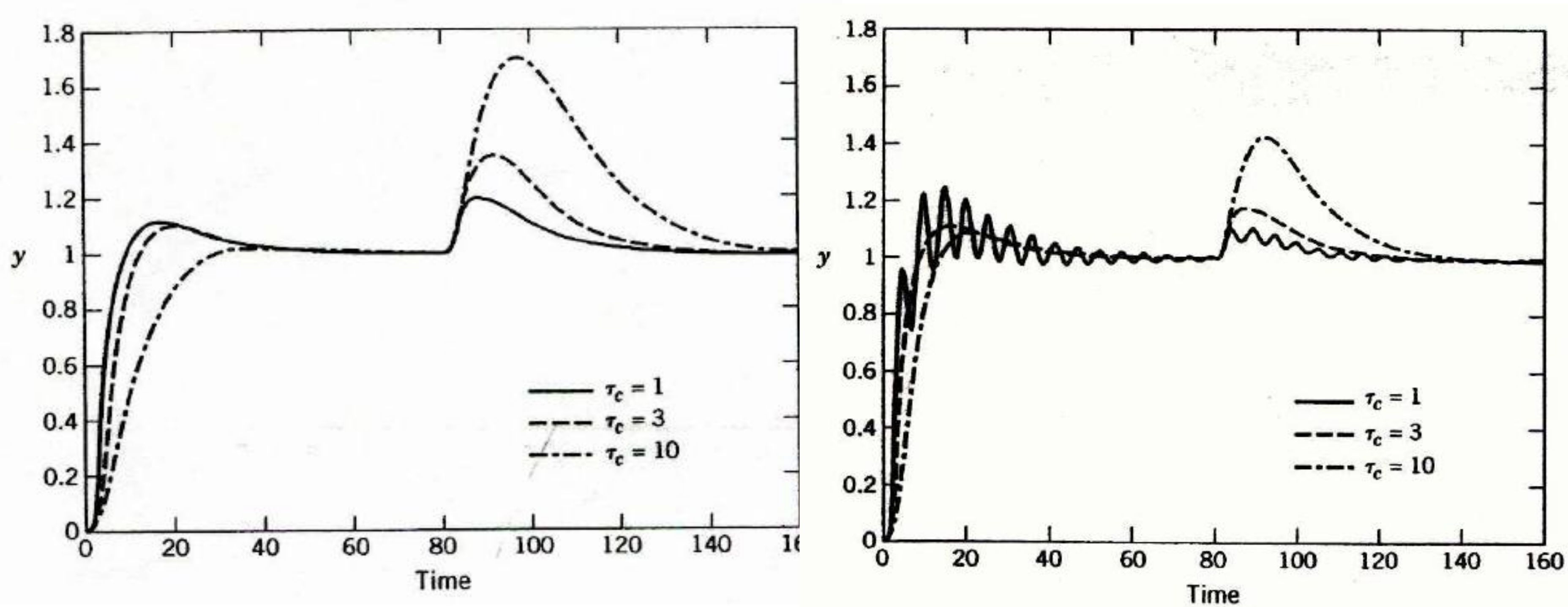
b. The model gain is incorrect.

$$\tilde{G}(s) = \frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

The controller settings for this example are:

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c\left(\tilde{K} = 2\right)$	3.75	1.88	0.682
$K_c\left(\tilde{K} = 0.9\right)$	8.33	4.17	1.51
τ_I	15	15	15
τ_D	3.33	3.33	3.33

2.2 DSM Design Example



(a) Correct Model (b) Incorrect Model (Robustness)

- Values of K_c decreases as τ_c increases. But values of τ_I and τ_D do not change.

2.2 Observations on DSM

- Resulting controllers could be quite complex and may not even be physically realizable
- PID parameters will be decided by a user-specified parameter (determined by closed-loop time constant τ_c)
- The shorter τ_c makes the action more aggressive (larger gain K_c)
- For limited cases, it results in PID structure:
  - 1^{st} order process without time delay: PI
  - FOPTD with 1^{st} order Taylor series approximation: PI
  - 2^{nd} order model without time delay: PID
  - SOPTD with 1^{st} order Taylor series approximation: PID
  - Delay modifies the K_c

$$\frac{\tau}{K\tau_c} \rightarrow \frac{\tau}{K\left(\tau_c+\theta\right)} \text{ (1}^{\text{st}} \text{ Order Process)} \qquad \frac{\left(\tau_1+\tau_2\right)}{K\tau_c} \rightarrow \frac{\left(\tau_1+\tau_2\right)}{K\left(\tau_c+\theta\right)} \text{ (2}^{\text{nd}} \text{ Order Process)}$$

- With time delay, the gain K_c will not become infinite even for perfect control, i.e. $(Y/R)_d = 1$

2.3 Internal Model Control

- Motivation
  - The resulting controller from DSM may not be physically realizable
  - If there is RHP in the process plant G(s), the resulting controller from DSM will be unstable
  - Unmeasured disturbance and modelling error are not considered in DSM
- Problem with Direct Synthesis Method
  -

$$G_c = \frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)$$

Resulting Controller may have higher order numerator than denominator

Direct inversion of process causes many problems

Process Plant is unknown

2.3 Internal Model Control (IMC)

- A more comprehensive model-based design method, *Internal Model Control* (*IMC*), was developed by Morari and coworkers (Garcia and Morari, 1982; Rivera et al., 1986).
- The IMC method, like the DS method, is based on an assumed process model and leads to analytical expressions for the controller settings.
- These two design methods are closely related and produce identical controllers if the design parameters are specified in a consistent manner.
- The IMC method is based on the simplified block diagram shown in Fig. 12.6b. A process model $\tilde{G}$ and the controller output P are used to calculate the model response, $\tilde{Y}$.

2.3 IMC

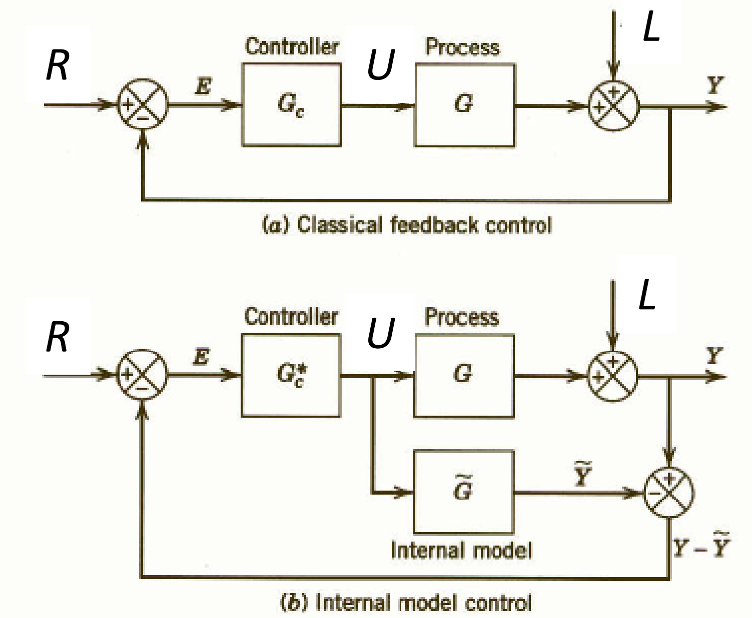


Figure 12.6. Feedback control strategies

- The model response is subtracted from the actual response Y, and the difference, $Y-\tilde{Y}$ is used as the input signal to the IMC controller, G_c^*.
- In general, $Y \neq \tilde{Y}$ due to modeling errors $\left(\tilde{G} \neq G\right)$ and unknown disturbances $\left(L \neq 0\right)$ that are not accounted for in the model.
- The block diagrams for conventional feedback control and IMC are compared in Fig. 12.6.

2.3 IMC

- It can be shown that the two block diagrams are identical if controllers G_c and G_c^* satisfy the relation

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} \qquad (12\text{-}16)$$

- Thus, any IMC controller G_c^* is equivalent to a standard feedback controller G_c, and vice versa.
- The following closed-loop relation for IMC can be derived from Fig. 12.6b using the block diagram algebra on next slide

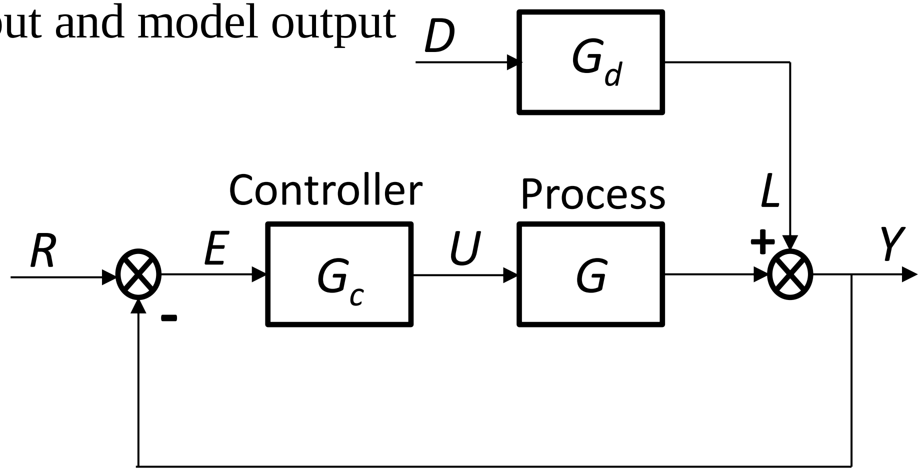
$$Y = \frac{G_c^* G}{1 + G_c^* \left(G - \tilde{G} \right)} R + \frac{1 - G_c^* \tilde{G}}{1 + G_c^* \left(G - \tilde{G} \right)} L \qquad (12\text{-}17)$$

2.3 IMC

Feedback the error between the process output and model output

Equivalent conventional controller:

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$$



Use block diagram algebra:

$$\tilde{Y} = \tilde{G}U, \quad E = R - \left(Y - \tilde{Y}\right) = R - Y + \tilde{G}U$$

$$U = G_c^* E = G_c^* \left(R - Y + \tilde{G}U\right) \Rightarrow U = G_c^* \left(R - Y\right) \Big/ \left(1 - G_c^* \tilde{G}\right)$$

$$Y = GU + L = G\,G_c^* \left(R - Y\right) \Big/ \left(1 - G_c^* \tilde{G}\right) + L$$

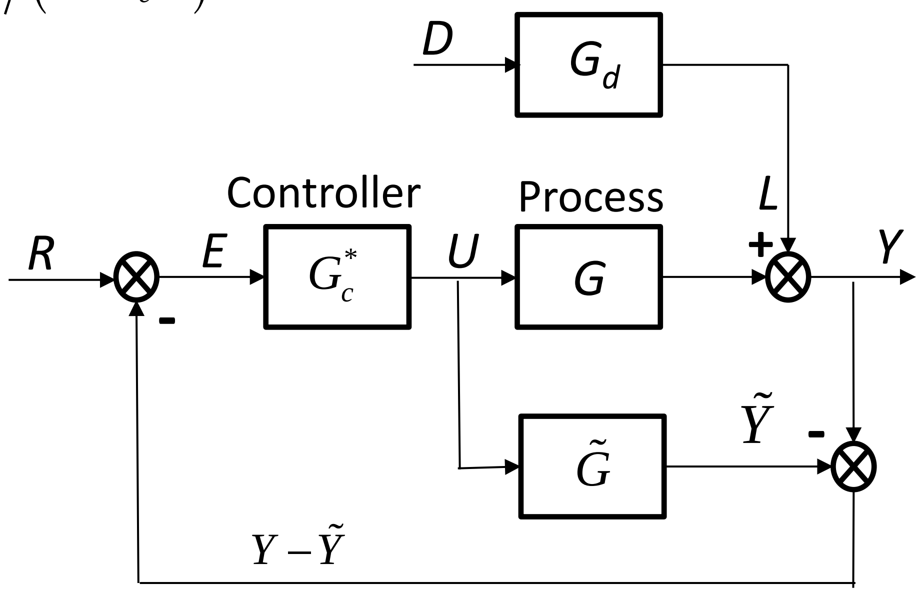
$$\Rightarrow \left(1 + GG_c^* - G_c^* \tilde{G}\right) Y = GG_c^* R + \left(1 - G_c^* \tilde{G}\right) L$$

Hence,

$$Y = \frac{G_c^* G}{1 + G_c^* \left(G - \tilde{G}\right)} R + \frac{\left(1 - G_c^* \tilde{G}\right)}{1 + G_c^* \left(G - \tilde{G}\right)} L$$

If $\tilde{G} = G$,

$$Y = G_c^* G R + \left(1 - G_c^* \tilde{G}\right) L$$



2.3 IMC Design Synthesis

For the special case of a perfect model, $\tilde{G} = G$, (12-17) reduces to

$$Y = G_c^* G\, R + \left(1 - G_c^* G\right) L \qquad (12\text{-}18)$$

The IMC controller is designed in two steps:

Step 1. The process model is factored as

$$\tilde{G} = \tilde{G}_+ \tilde{G}_- \qquad (12\text{-}19)$$

where $\tilde{G}_+$ contains any time delays and right-half plane zeros.

- In addition, $\tilde{G}_+$ is required to have a steady-state gain equal to one in order to ensure that the two factors in Eq. 12-19 are unique.

2.3 IMC Design Synthesis

Step 2. The controller is specified as

$$G_c^* = \frac{1}{\tilde{G}_-} f \qquad (12\text{-}20)$$

where f is a *low-pass filter* with a steady-state gain of one. It typically has the form:

$$f = \frac{1}{(\tau_c s + 1)^r} \qquad (12\text{-}21)$$

In analogy with the DS method, τ_c is the desired closed-loop time constant. Parameter r is a positive integer. The usual choice is $r = 1$.

2.3 IMC Design Synthesis

For the ideal situation where the process model is perfect $\left(\tilde{G} = G\right)$, substituting Eq. 12-20 into (12-18) gives the closed-loop expression

$$Y = \underbrace{\tilde{G}_+ f}_{T} R + \underbrace{\left(1 - f\tilde{G}_+\right)}_{\text{Sensitivity Function } S} L \qquad \text{(12-22)}$$

Recall: *S(s) + T(s) = I*

Thus, the closed-loop transfer function for set-point changes is

$$\frac{Y}{R} = \tilde{G}_+ f \qquad \text{(12-23)}$$

Selection of τ_c

- The choice of design parameter τ_c is a key decision in both the DS and IMC design methods.
- In general, increasing τ_c produces a more conservative controller because K_c decreases while τ_I increases.

2.3 IMC Design Example

FOPTD Process Model with First Order Pade Approximation $\left(e^{-\theta s} = \dfrac{1-\theta s/2}{1+\theta s/2} \right)$

$$\tilde{G} = \frac{K\left(1-\theta s/2\right)}{\left(\tau s+1\right)\left(1+\theta s/2\right)}$$

$$\tilde{G}_{+} = 1-\theta s/2 \qquad \tilde{G}_{-} = \frac{K}{\left(\tau s+1\right)\left(1+\theta s/2\right)}$$

$$G_c^{*} = \frac{1}{\tilde{G}_{-}} f = \frac{\left(\tau s+1\right)\left(1+\theta s/2\right)}{K} \frac{1}{\left(\tau_c s+1\right)}$$

$$G_c = \frac{G_c^{*}}{1-G_c^{*}\tilde{G}} = \frac{\left(\tau s+1\right)\left(1+\theta s/2\right)}{K\left(\tau_c+\theta/2\right)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K} \frac{\left(\tau+\theta/2\right)}{\left(\tau_c+\theta/2\right)} \qquad \tau_I = \tau+\theta/2 \qquad \tau_D = \frac{\tau\theta/2}{\tau+\theta/2}$$

2.3 IMC Tuning

- Several IMC guidelines for τ_c have been published for the model in Eq. 12-10: FOPTD $\dfrac{Ke^{-\theta s}}{s\tau+1}$

1. $\tau_c/\theta > 0.8$ and $\tau_c > 0.1\tau$ (Rivera et al., 1986)
2. $\tau > \tau_c > \theta$ (Chien and Fruehauf, 1990)
3. $\tau_c = \theta$ (Skogestad, 2003)

Controller Tuning Relations

In the last section, we have seen that model-based design methods such as DS and IMC produce PI or PID controllers for certain classes of process models.

IMC Tuning Relations

The IMC method can be used to derive PID controller settings for a variety of transfer function models.

2.3 IMC-Based PID Controller Setting

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4][a]

Case	*Model*	$K_c K$	τ_I	τ_D
A	$\dfrac{K}{\tau s + 1}$	$\dfrac{\tau}{\tau_c}$	τ	—
B	$\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\dfrac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
D	$\dfrac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\dfrac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
E	$\dfrac{K}{s}$	$\dfrac{1}{\tau_c}$	—	—
F	$\dfrac{K}{s(\tau s + 1)}$	$\dfrac{1}{\tau_c}$	—	τ

[a]Based on Eq. 12-30 with $r = 1$.

2.3 IMC-Based PID Controller Setting

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\frac{K}{\tau^2s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s+1)}{\tau^2s^2+2\zeta\tau s+1},\ \beta>0$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s+1)}$	$\frac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\frac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau}{\tau_c+\theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$
I	$\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$	$2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

2.3 IMC Tuning for Lag Dominant Models

- First- or second-order models with relatively small time delays $(\theta / \tau \ll 1)$ are referred to as *lag-dominant models*.
- The IMC and DS methods provide satisfactory set-point responses, but very slow disturbance responses, because the value of τ_I is very large.
- Fortunately, this problem can be solved in three different ways.

Method 1: Integrator Approximation

$$\text{Approximate } \tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \text{ by } \tilde{G}(s) = \frac{K^* e^{-\theta s}}{s}$$

where $K^* \triangleq K / \tau$.

- Then can use the IMC tuning rules (Rule M or N) to specify the controller settings.

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2.3 IMC Tuning for Lag Dominant Models

Method 2. Limit the Value of τ_I

- For lag-dominant models, the standard IMC controllers for first-order and second-order models provide sluggish disturbance responses because τ_I is very large.

- For example, controller G in Table 12.1 has $\tau_I = \tau$ where τ is very large.

- As a remedy, Skogestad (2003) has proposed limiting the value of τ_I:

$$\tau_I = \min\left\{\tau_1, 4\left(\tau_c + \theta\right)\right\} \qquad (12\text{-}34)$$

where τ_1 is the largest time constant (if there are two).

Method 3. Design the Controller for Disturbances, Rather Set-point Changes

- The desired CLTF is expressed in terms of $(Y/L)_{des}$ rather than $(Y/R)_{des}$
- *Reference*: Chen & Seborg (2002)

Example 2.2 IMC for Lag-Dominant Model

Consider a lag-dominant model with $\theta / \tau = 0.01$:

$$\tilde{G}(s) = \frac{100}{100s+1} e^{-s}$$

Design four PI controllers:

a) IMC $(\tau_c = 1)$

b) IMC $(\tau_c = 2)$ based on the integrator approximation

c) IMC $(\tau_c = 1)$ with Skogestad's modification (Eq. 12-34)

d) Direct Synthesis method for disturbance rejection (Chen and Seborg, 2002): The controller settings are $K_c = 0.551$ and $\tau_I = 4.91$.

Example 2.2 IMC for Lag-Dominant Model

Evaluate the four controllers by comparing their performance for unit step changes in both set point and disturbance. Assume that the model is perfect and that $G_d(s) = G(s)$.

Solution

The PI controller settings are:

Controller	K_c	τ_I
(a) IMC	0.5	100
(b) Integrator approximation	0.556	5
(c) Skogestad	0.5	8
(d) DS-d	0.551	4.91

Example 2.2 IMC for Lag-Dominant Model

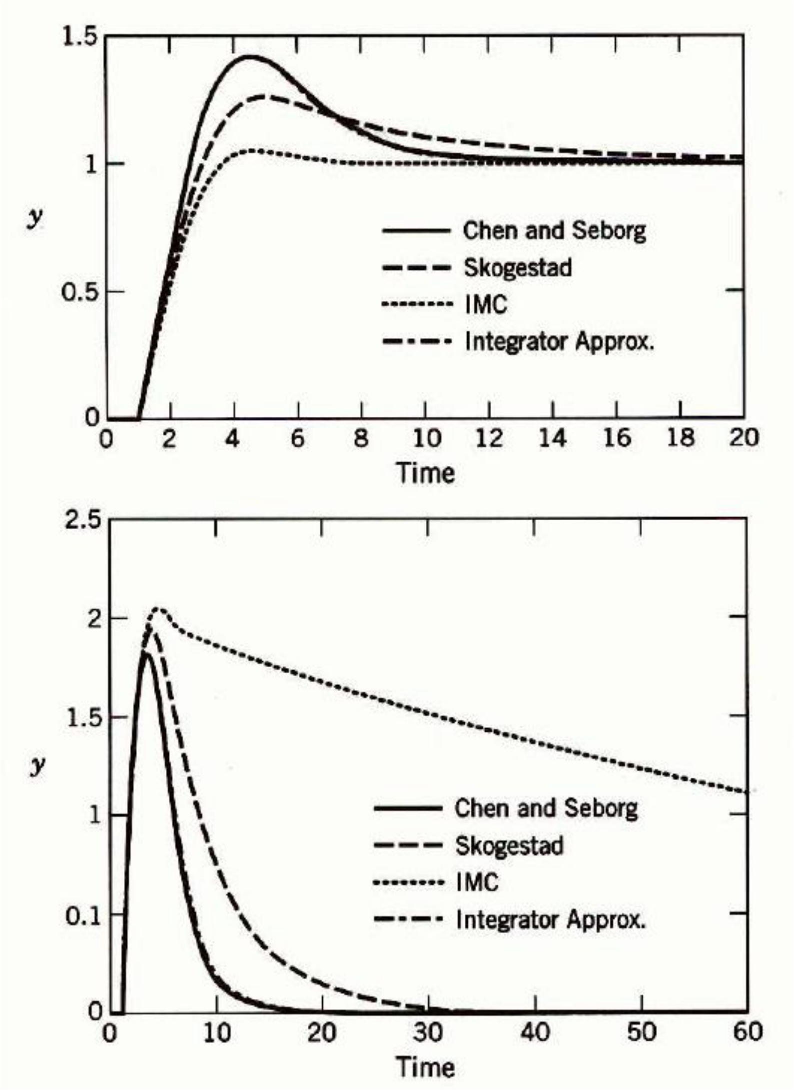


Figure 12.8. Comparison of set-point responses (top) and disturbance responses (bottom) for Example 12.4. The responses for the Chen and Seborg and integrator approximation methods are essentially identical.

2.3 Time Delay Compensation

- **Model-based feedback controller that improves closed-loop performance when time delays are present and substantial (i.e. $\theta/(\theta+\tau) > 0.3$)**
- **Effect of added time delay on PI controller performance for a second order process ($\tau_1 = 3$, $\tau_2 = 5$) shown below**

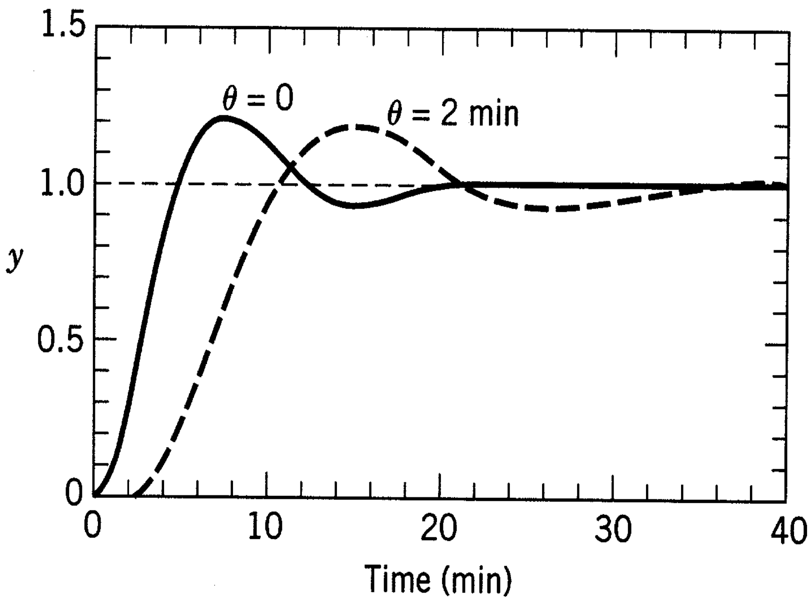


Figure 16.7 A comparison of closed-loop set-point changes.

2.3 Smith Predictor

- **If the process plant model G(s) is available and is asymptotically stable, we can use it to predict the delayed output**

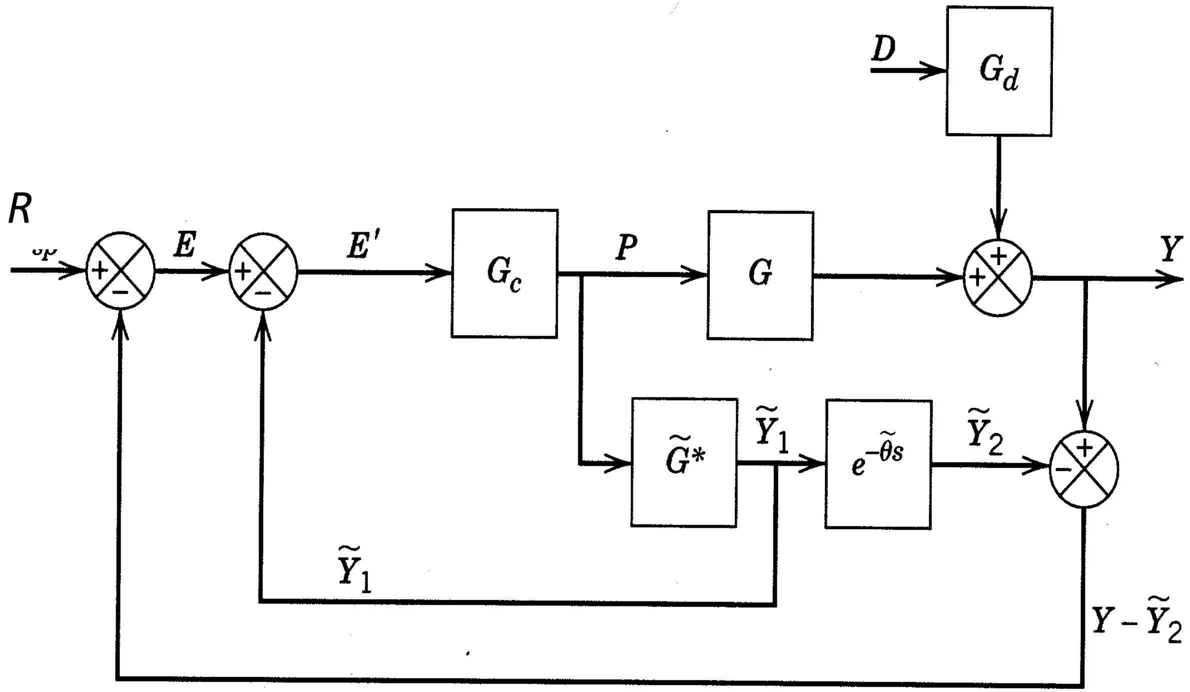


Figure 16.8 Block diagram of the Smith predictor.

2.3 Smith Predictor

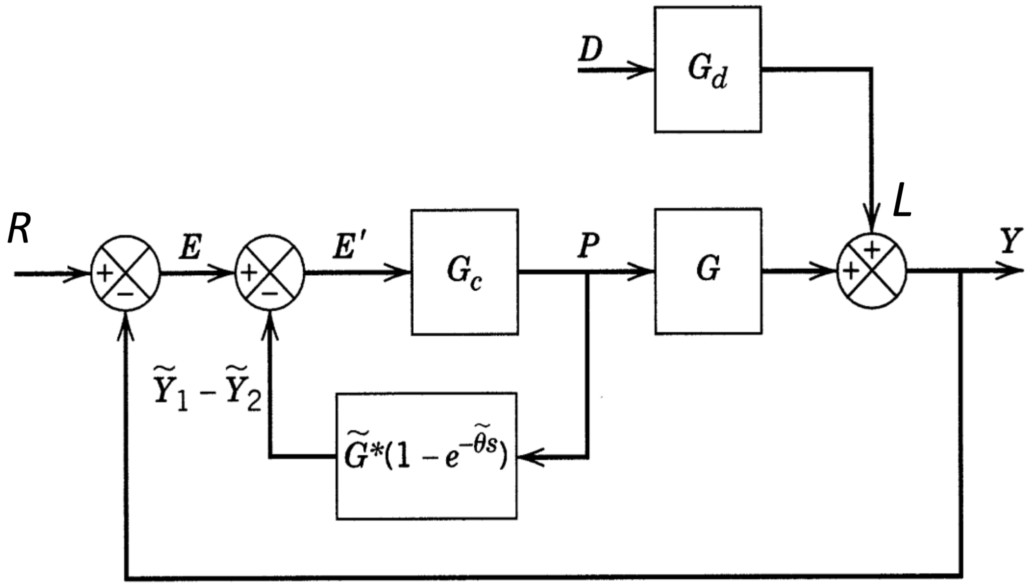


Figure 16.9 An alternative block diagram of a Smith predictor.

No model error: $\tilde{G} = G = G^* e^{-\theta s}$, so $\tilde{G}^* = G^*, \tilde{\theta} = \theta$

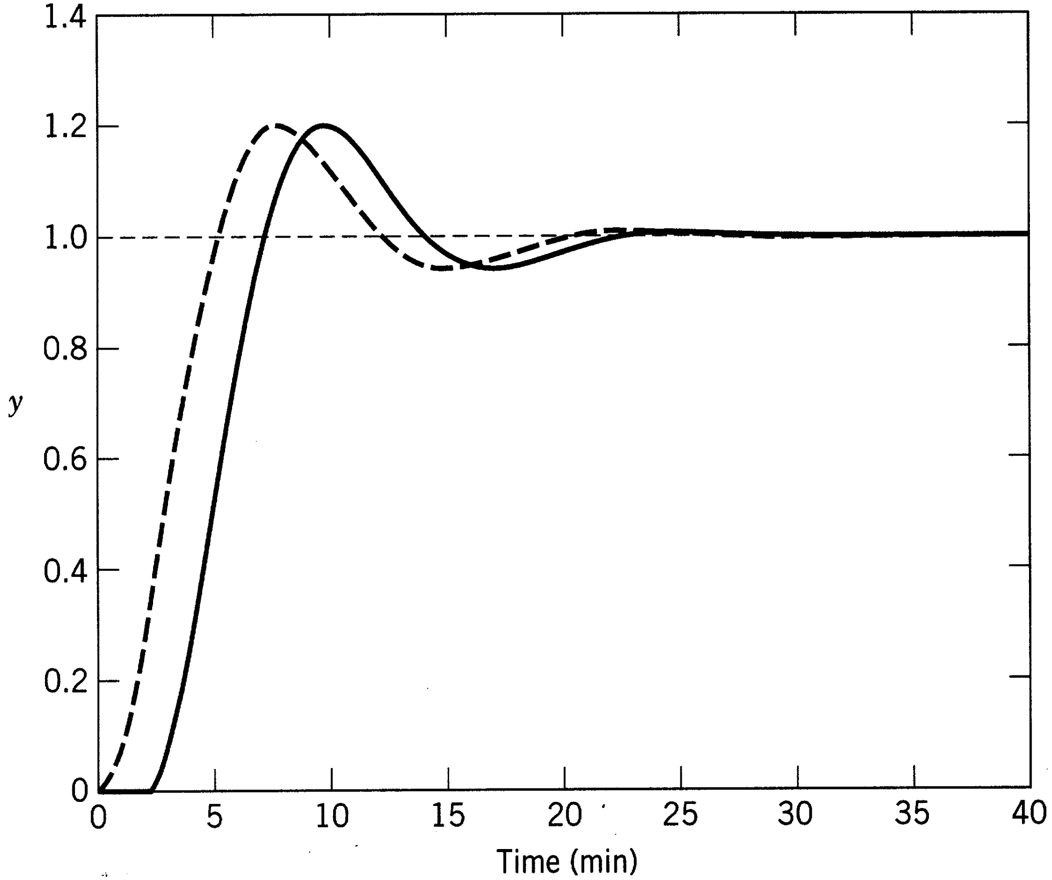
$$G_C' = \frac{G_C}{1 + G_C G^*\left(1 - e^{-\theta s}\right)} = \frac{1}{\dfrac{1}{G_c} + G^* - G^* e^{-\theta s}} \quad \text{and} \quad \frac{Y}{R} = \frac{G_C' G}{1 + G_C' G}$$

No need for approximating time delay θ!

$$\therefore, \quad \frac{Y}{R} = \frac{G_C G}{1 + G_C G^*} = \frac{G_C G^* e^{-\theta s}}{1 + G_C G^*} = Q(s) e^{-\theta s} \Rightarrow G_c = \frac{Q}{G^*(1 - Q)}$$

(sensitive to model errors > +/- 20%)

Example 2.3 Smith Predictor



- The delay cannot be completely eliminated
- Advantages of the prediction are:
  - ✓ Very fast
  - ✓ Same Robustness.
- Disadvantages of the prediction are:
  - ✓ Very difficult to implement.
  - ✓ Very difficult to analyze.
  - ✓ Problems if there are model errors.

Figure 16.10 Closed-loop set-point change (solid line) for Smith predictor with $\theta = 2$. The dashed line is the response for $\theta = 0$ from Fig. 16.7.

Exercise 1: DSM

Consider a second order plant with time delay,

$$G(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

(a) What is the controller G_c to achieve the below desired closed loop transfer function

$$\left(\frac{Y}{R}\right)_d = \frac{e^{-\theta s}}{\tau_c s + 1}$$

(b) Using the first order Taylor series approximation $e^{-\theta s} = 1 - \theta s$, express the controller G_c in the following PID form:

$$G_c(s) = K_p\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Derive K_p, τ_I and τ_D.

Exercise 2: IMC

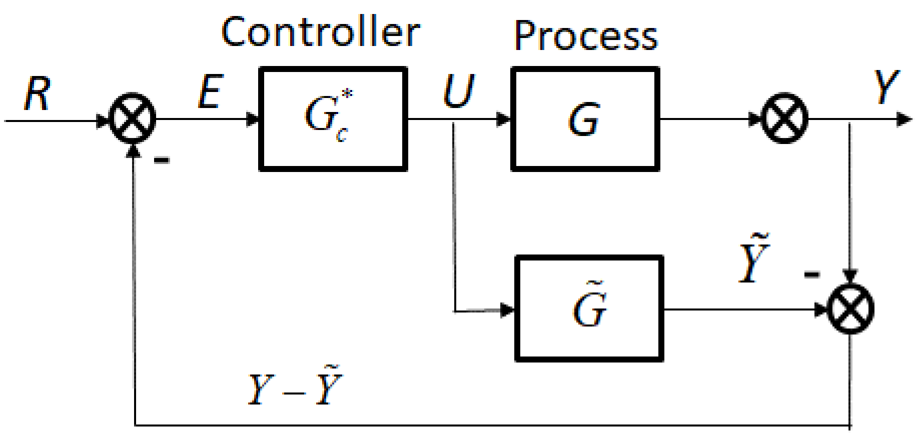
1) Consider the following first-order process with time delay

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

We shall use the below model where we increase the time constant to approximate the time delay θ

$$\tilde{G}(s) = \frac{K}{(\tau + \theta)s + 1}$$

a) Find the IMC controller $G_c^* = \frac{1}{\tilde{G}_-} f$ where $f = \frac{1}{\tau_c s + 1}$ is a low pass filter.



Exercise 2: IMC

b) Find the equivalent standard feedback controller $G_c = \dfrac{G_c^*}{1 - G_c^* \tilde{G}}$

c) Determine the corresponding parameters of a PID controller

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + s\tau_D\right).$$

Exercise 3: DSM for Smith Predictor

Refer to Figure 16.9. Choose

$$\frac{Y}{R} = \frac{G_C' G}{1 + G_C' G} = Q(s) e^{-\theta s}$$

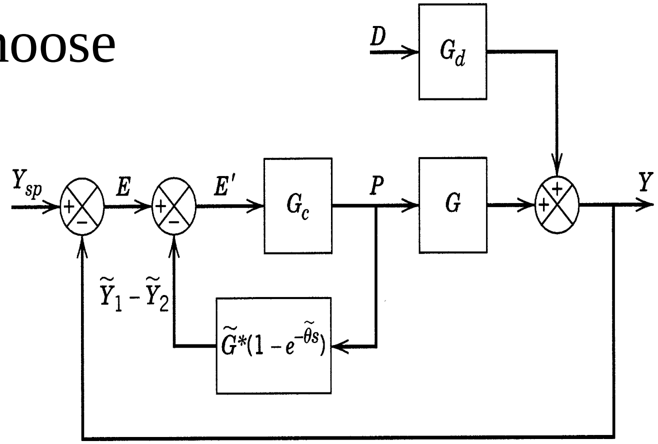


Figure 16.9 An alternative block diagram of a Smith predictor.

a) If $G = G^* e^{-\theta s}$

$$\text{show } G_C' = \frac{Y/R}{1 - Y/R} \cdot \frac{1}{G} = \frac{1}{\dfrac{G^*}{Q} - G^* e^{-\theta s}} \qquad (1)$$

b) From Figure 16.9, show $G_C' = \dfrac{1}{\dfrac{1}{G_C} + G^* - G^* e^{-\theta s}}$ (2)

c) From 2) and 3), show that $G_C = \dfrac{Q}{G^*(1 - Q)}$