Model Predictive Control — Lecture 6
Feedforward in MPC

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Measured Disturbance and Feedforward, 1/3

- It is frequently the case that the effects of some disturbances can be anticipated and approximately cancelled out by suitable control actions.
- When this is done it is called feedforward control. It can be more effective than feedback control, because the latter has to wait until the effect of the disturbance becomes apparent before taking corrective action.
- Feedforward is easily incorporated into predictive control. All that has to be done is to include the effects of the measured disturbances in the predictions of future outputs. This can be achieved by modifying the plant model to include the measured disturbance vector $d_m(k)$:

$$x(k+1) = Ax(k) + Bu(k) + B_d d_m(k), \quad y(k) = Cx(k)$$

Measured Disturbance and Feedforward, 2/3

Then the prediction equation becomes

$$\hat{\mathbf{Y}} = \Phi x(k) + G\hat{\mathbf{U}} + \Upsilon u(k-1) + G_d \hat{\mathbf{D}}_\mathbf{m}$$

where $\hat{\mathbf{D}}_\mathbf{m}$ is a vector of future measured disturbance:

$$\hat{\mathbf{D}}_\mathbf{m} = \begin{bmatrix} d_m(k) \\ \hat{d}_m(k+1|k) \\ \vdots \\ \hat{d}_m(k+N-1|k) \end{bmatrix}, \text{ and } G_d = \ldots$$

The output prediction will clearly be influenced by what one assumes about the future behaviour of the measured disturbance. A common practice is to assume that it will remain constant at the last measured value, i.e. $\hat{d}_m(k+i|k) = d_m(k), i = 1, \ldots, N-1$.

Measured Disturbance and Feedforward, 3/3

The optimal solution in the unconstrained case becomes

$$\hat{\mathbf{U}} = (G^T G + \lambda I)^{-1} G^T (\hat{\mathbf{W}} - \Phi x(k) - \Upsilon u(k-1) - G_d \hat{\mathbf{D}}_{\mathbf{m}})$$

The receding-horizon unconstrained MPC control law then becomes

$$\Delta u(k) = K_1 w(k) + K_2 x(k) + K_3 u(k-1) + K_4 d_m(k)$$

which has a feedforward control structure.

Example 5.1[a]

[a] See JMM(2001), pp.147

Suppose that plant model can be described by

$$y(k) = P_2(z)[P_1(z)u(k) + P_d(z)d_m(k)]$$

and the transfer functions $P_1(z)$, $P_2(z)$ and $P_d(z)$ have state space realisations (A_1, B_1, C_1, D_1), (A_2, B_2, C_2, D_2) and $(A_d, B_d, C_d, 0)$, with corresponding state vectors x_1, x_2, and x_d. Assume that $D_2 D_1 = 0$ so that there is no direct feed-through from $u(k)$ to $y(k)$. Then a complete state space model is given by

$$\begin{aligned}
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_d(k+1) \end{bmatrix} &= \begin{bmatrix} A_1 & 0 & 0 \\ B_2C_1 & A_2 & B_2C_d \\ 0 & 0 & A_d \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_d(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2D_1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ B_d \end{bmatrix} d_m(k) \\
y(k) &= \begin{bmatrix} D_2C_1 & C_2 & D_2C_d \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_d(k) \end{bmatrix}
\end{aligned}$$