Kirchoff's Voltage Law (KVL)

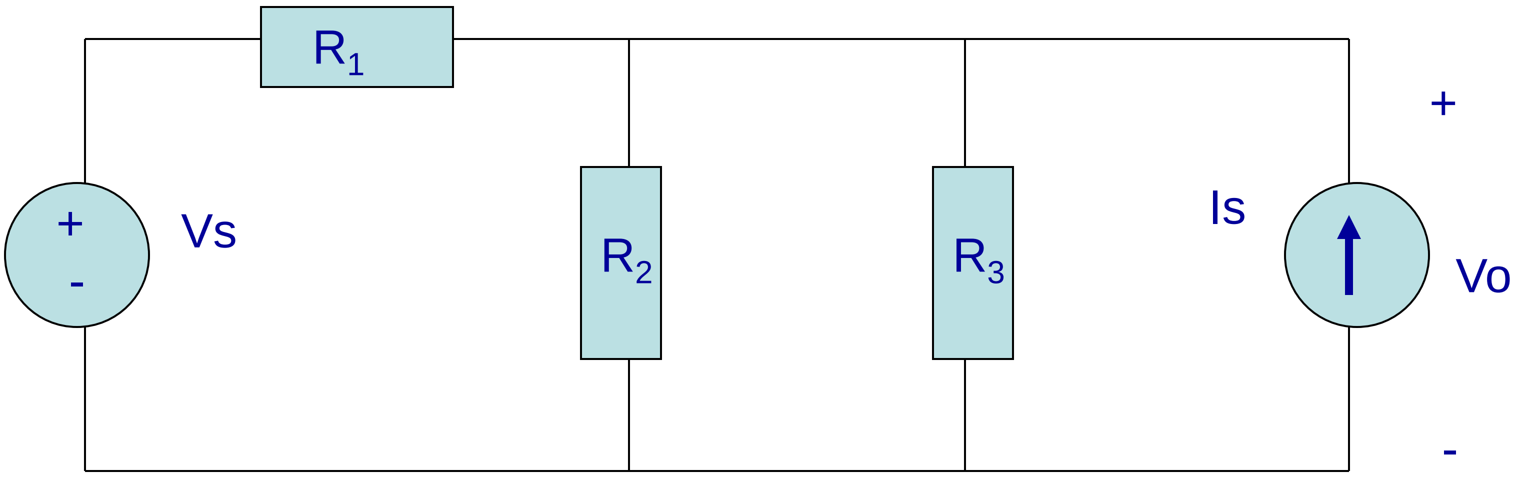
Kirchoff's Voltage Law (KCL)

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

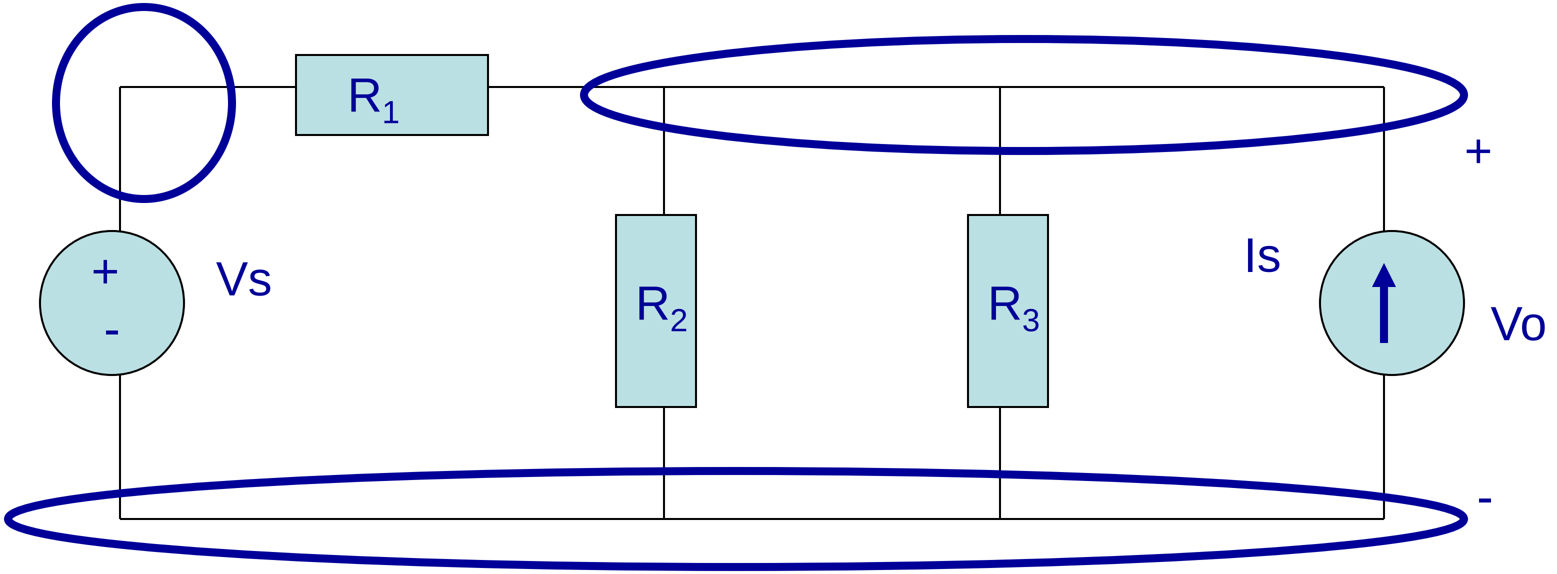
Example

- How many nodes, branches & loops?



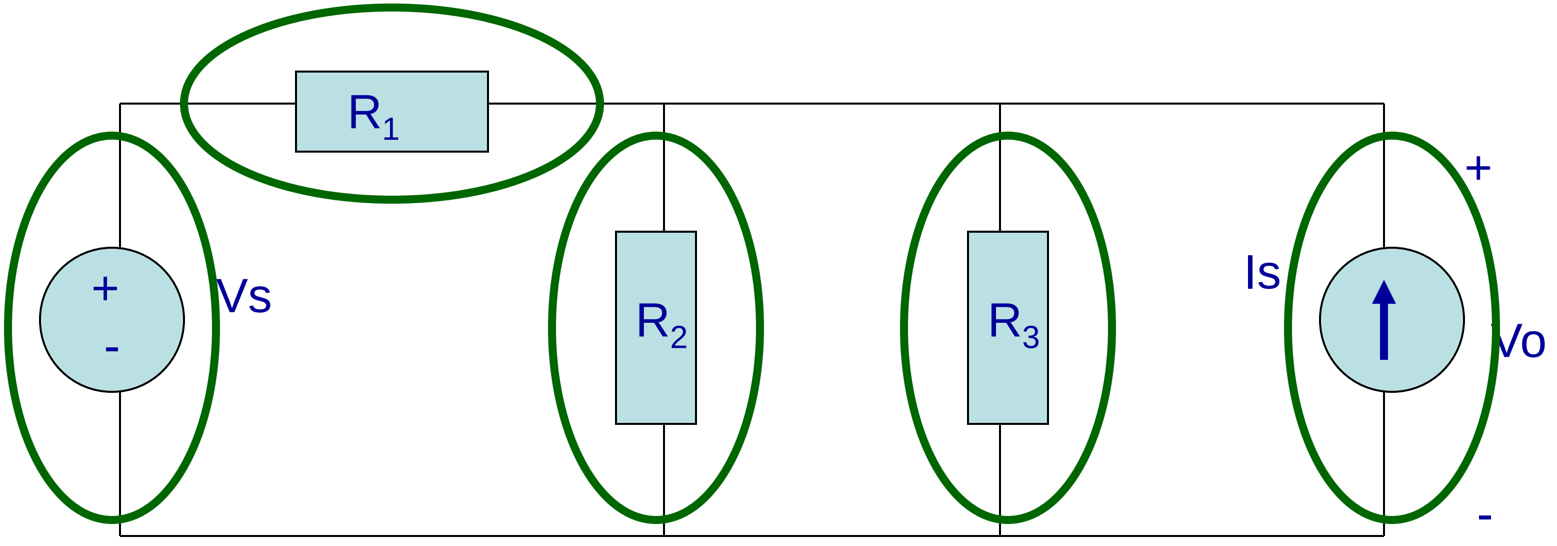
Example

- Three nodes



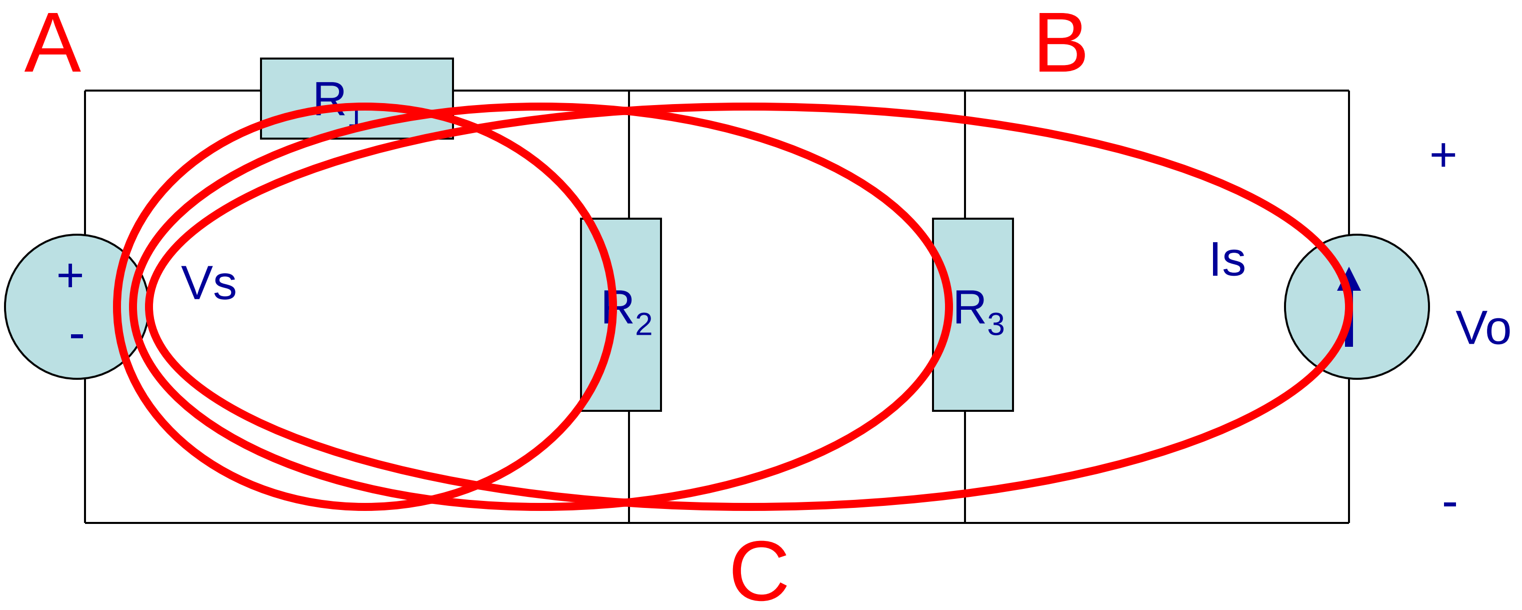
Example

- 5 Branches



Example

- Three Loops, if starting at node A

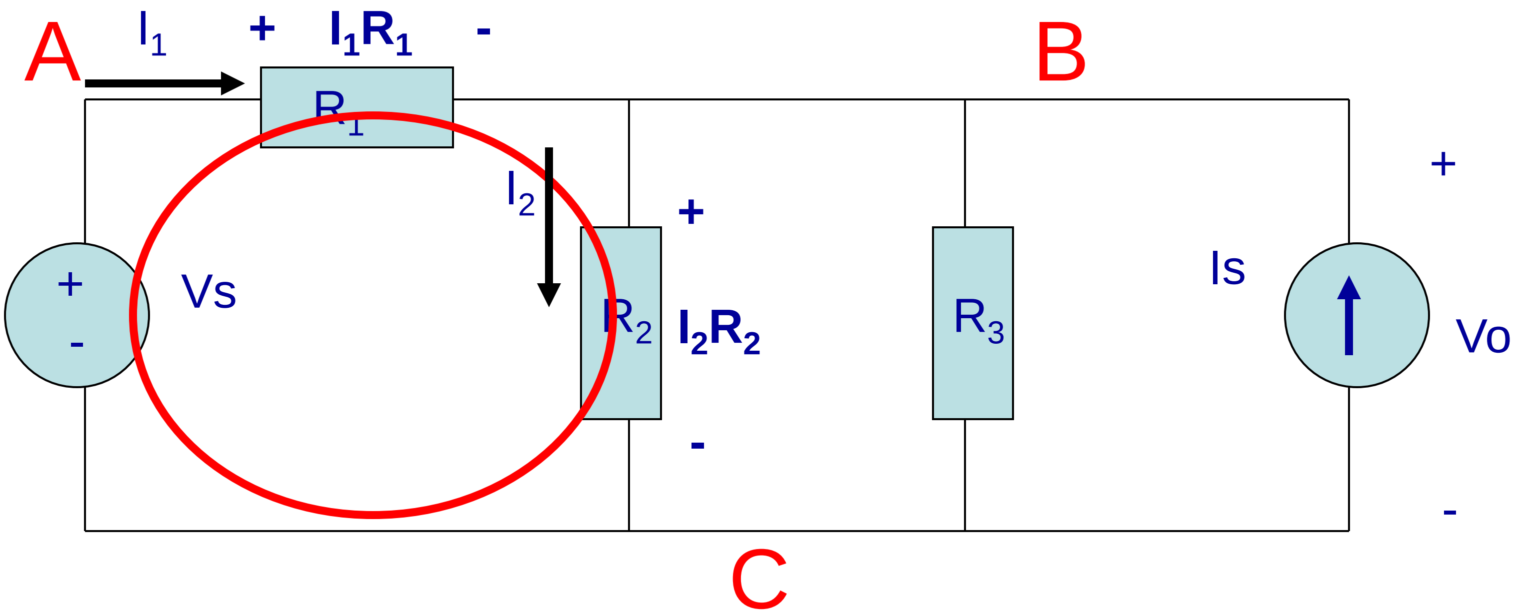


Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1^{st} Loop

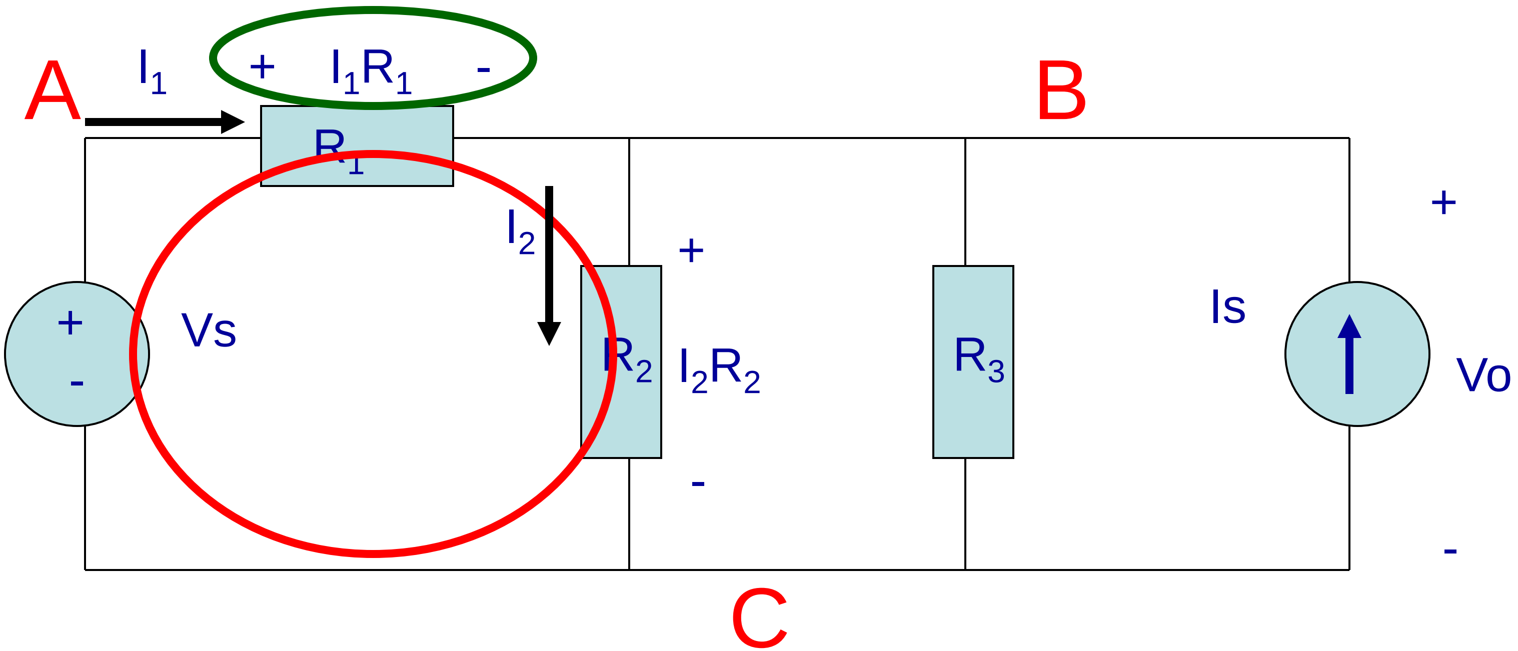


Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

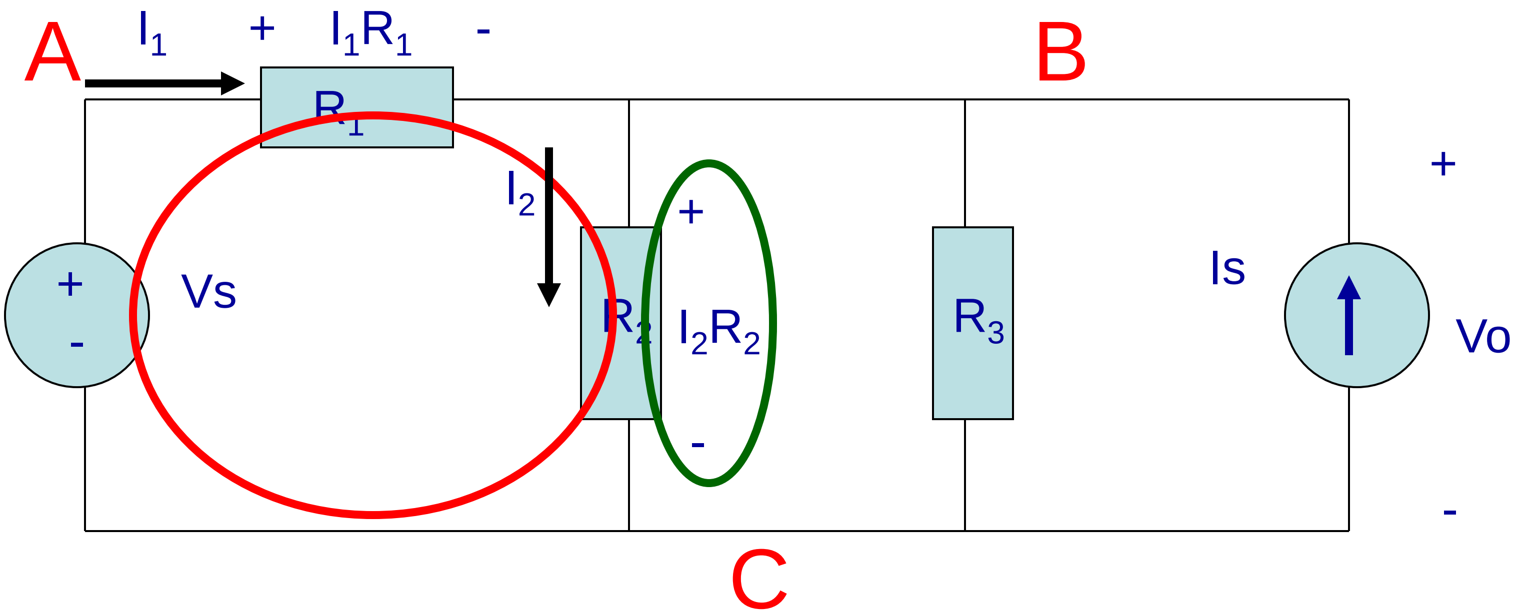
- Kirchoff's Voltage Law around 1st Loop



Starting at node A, add the 1st voltage drop: + I_1R_1

Example

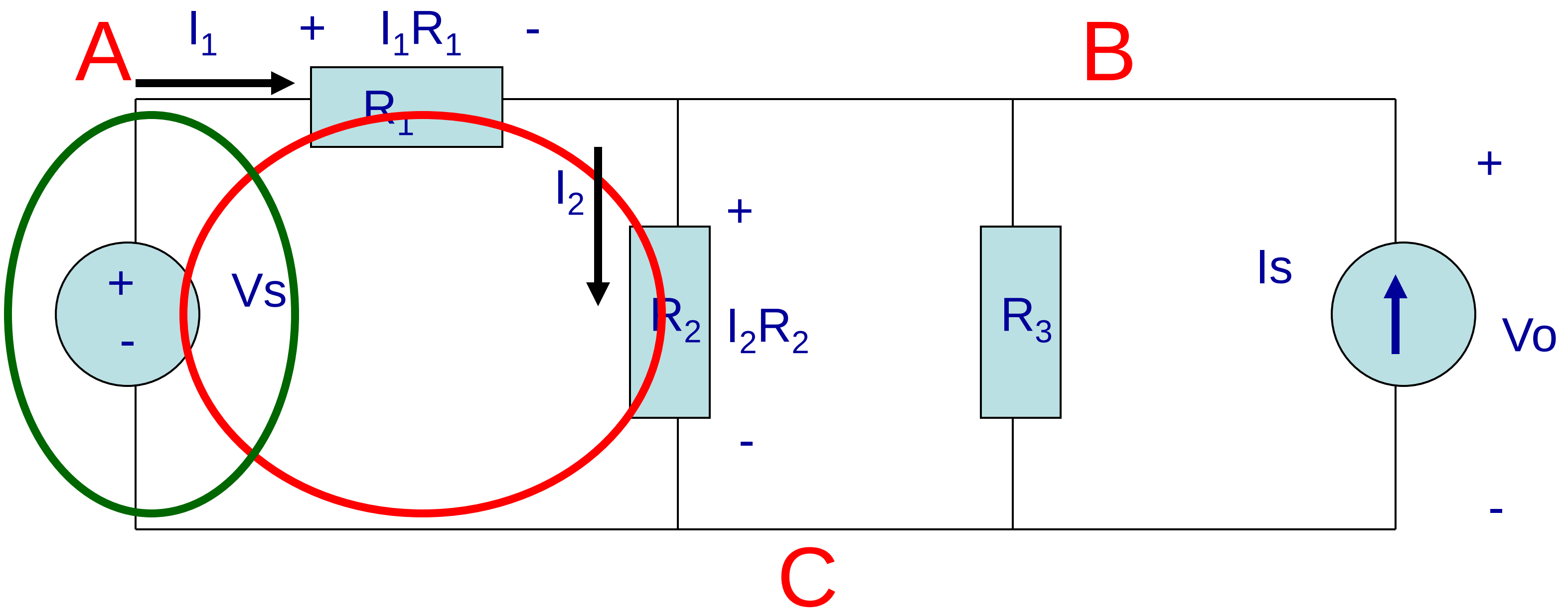
- Kirchoff's Voltage Law around 1st Loop



Add the voltage drop from B to C through R_2: + I_1R_1 + I_2R_2

Example

- Kirchoff's Voltage Law around 1st Loop

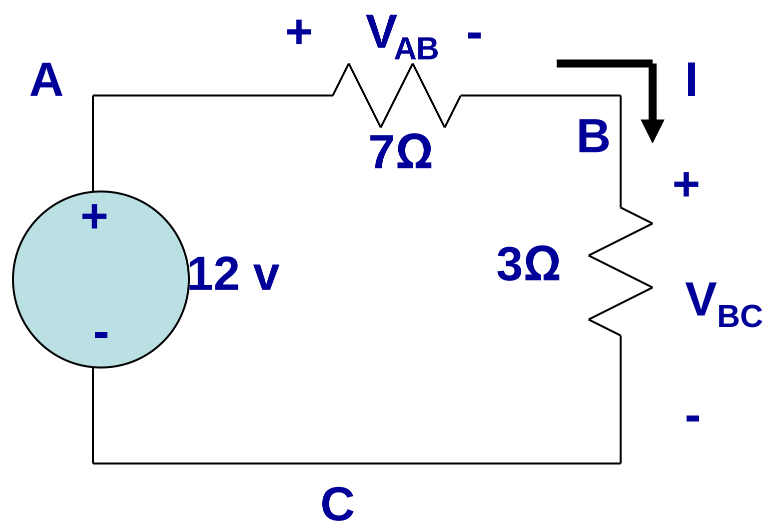


Subtract the voltage rise from C to A through Vs: **$+ I_1R_1 + I_2R_2 - Vs = 0$**

Notice that the sign of each term matches the polarity encountered 1st

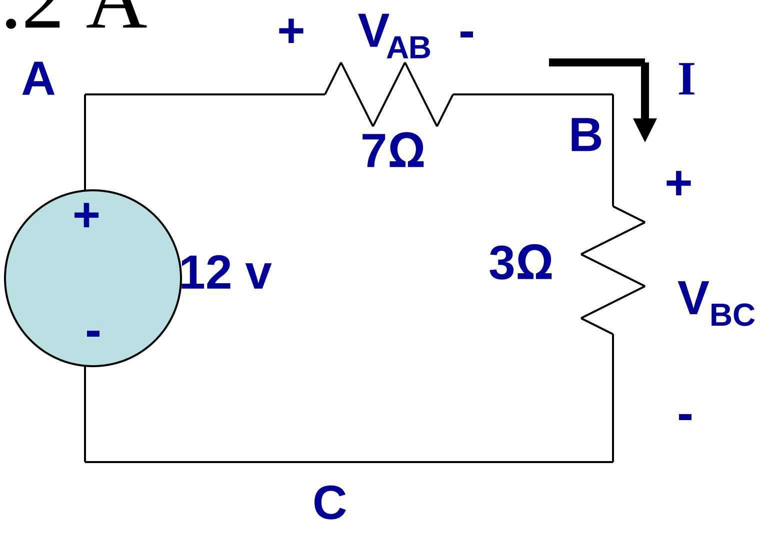
Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff's two laws and Ohm's law to determine all branch voltages and currents



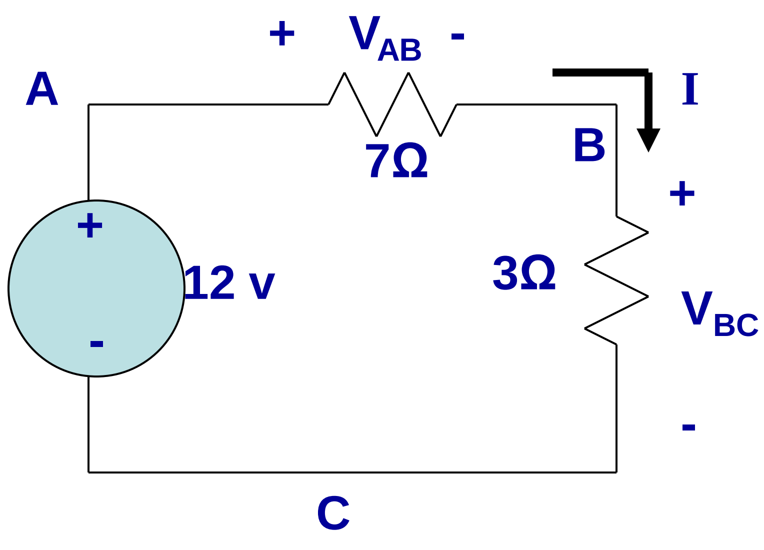
Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$
- Solving: $I = 1.2\text{ A}$



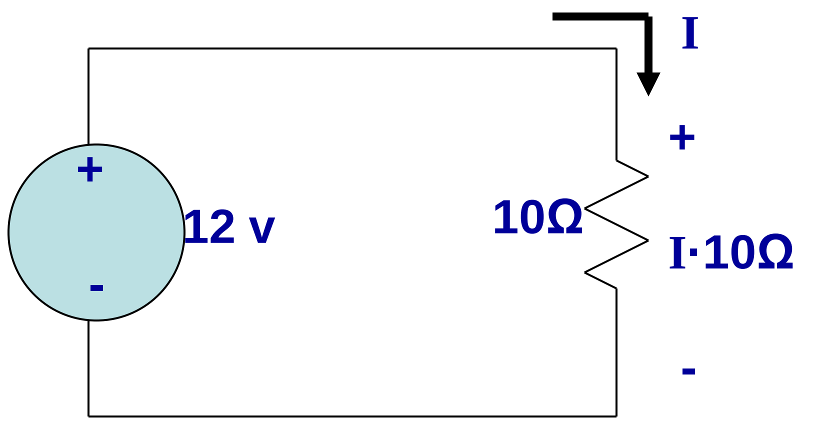
Circuit Analysis

- Since $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- And $I = 1.2$ A
- So $V_{AB} = 8.4$ v and $V_{BC} = 3.6$ v



Series Resistors

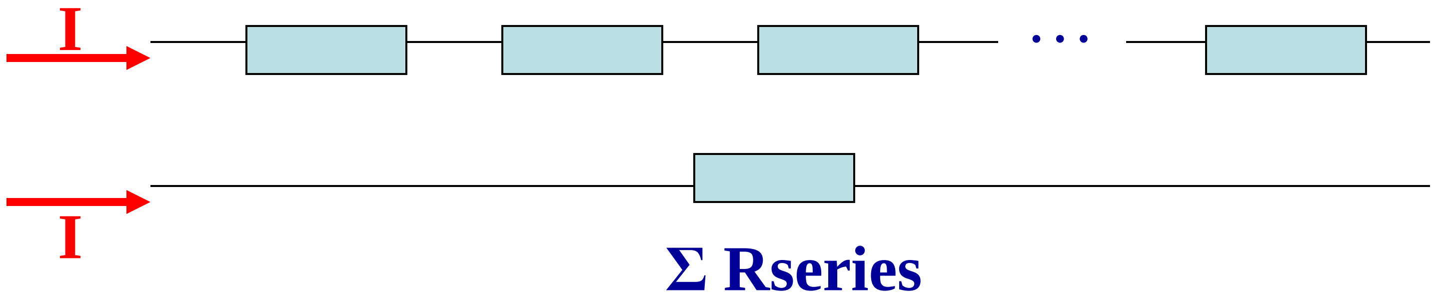
- KVL: $+I \cdot 10\Omega - 12\text{ v} = 0$, So $I = 1.2$ A
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



Series Resistors

- To the rest of the circuit, series resistors can be replaced by an equivalent resistance equal to the sum of all resistors

Series resistors (same current through all)

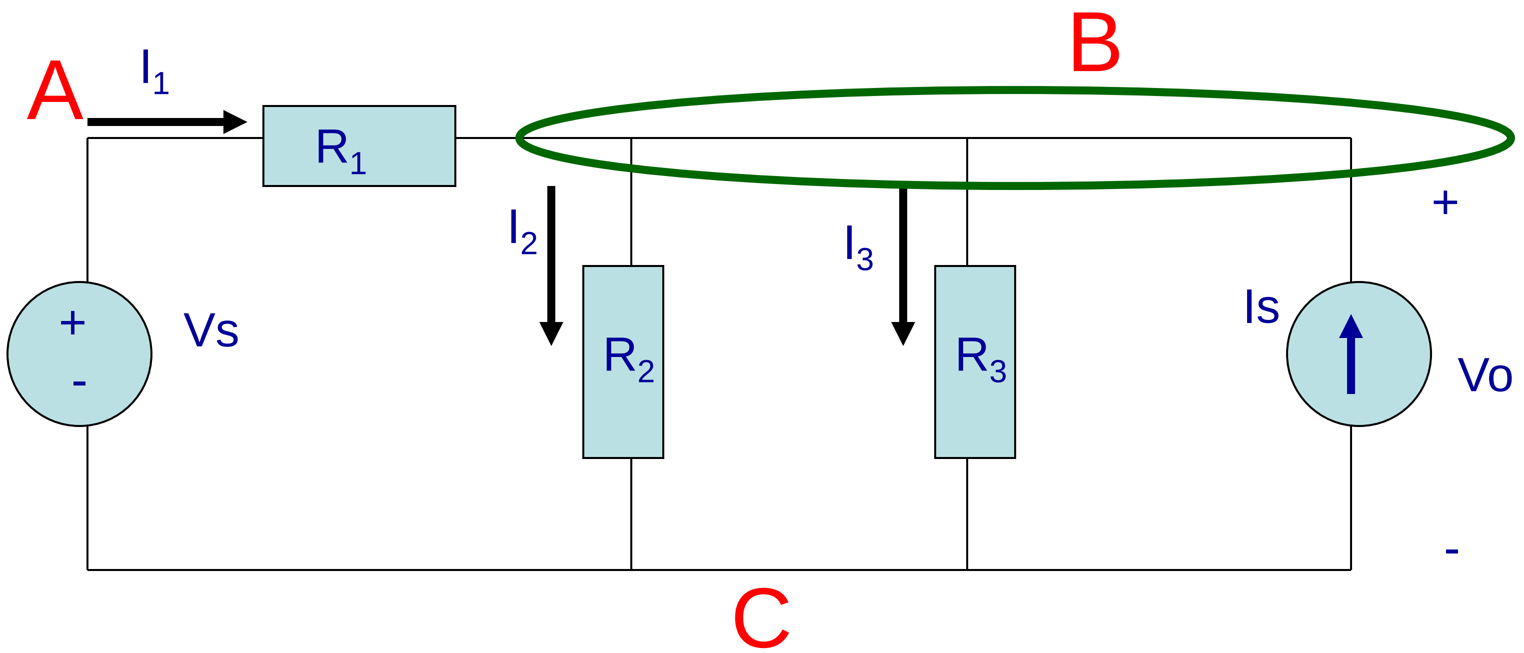


Kirchoff’s Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

Example

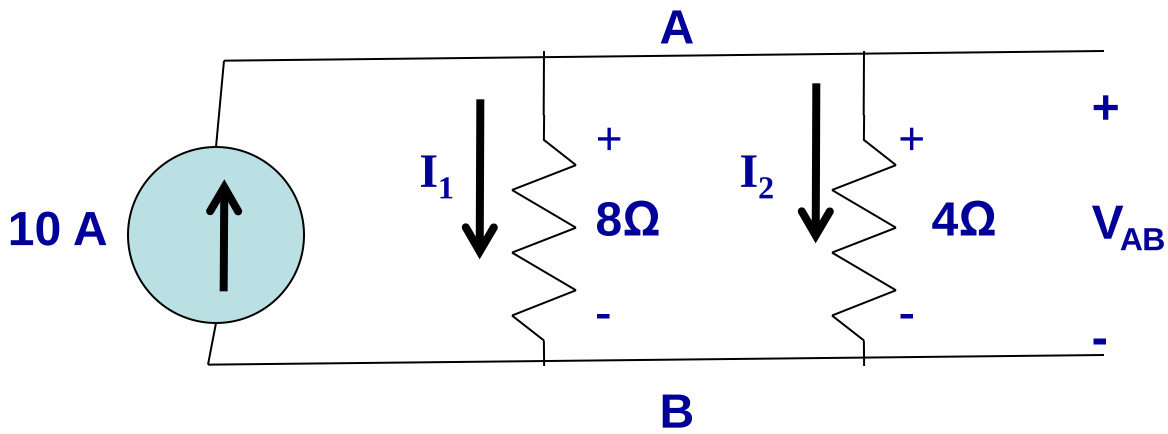
- Kirchoff's Current Law at B



Assign current variables and directions

Add currents in, subtract currents out: $I_1 - I_2 - I_3 + Is = 0$

Circuit Analysis



By KVL: $-I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$

Solving: $I_2 = 2 \cdot I_1$

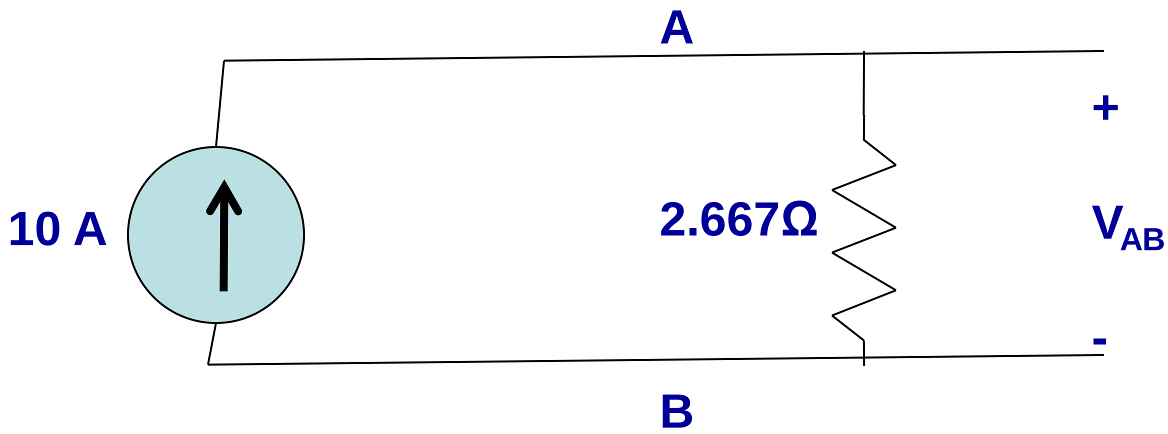
By KCL: $10A = I_1 + I_2$

Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$

So $I_1 = 3.33$ A and $I_2 = 6.67$ A

And $V_{AB} = 26.33$ volts

Circuit Analysis



By Ohm's Law: V_{AB} = 10 A · 2.667 Ω

So V_{AB} = 26.67 volts

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

Parallel Resistors

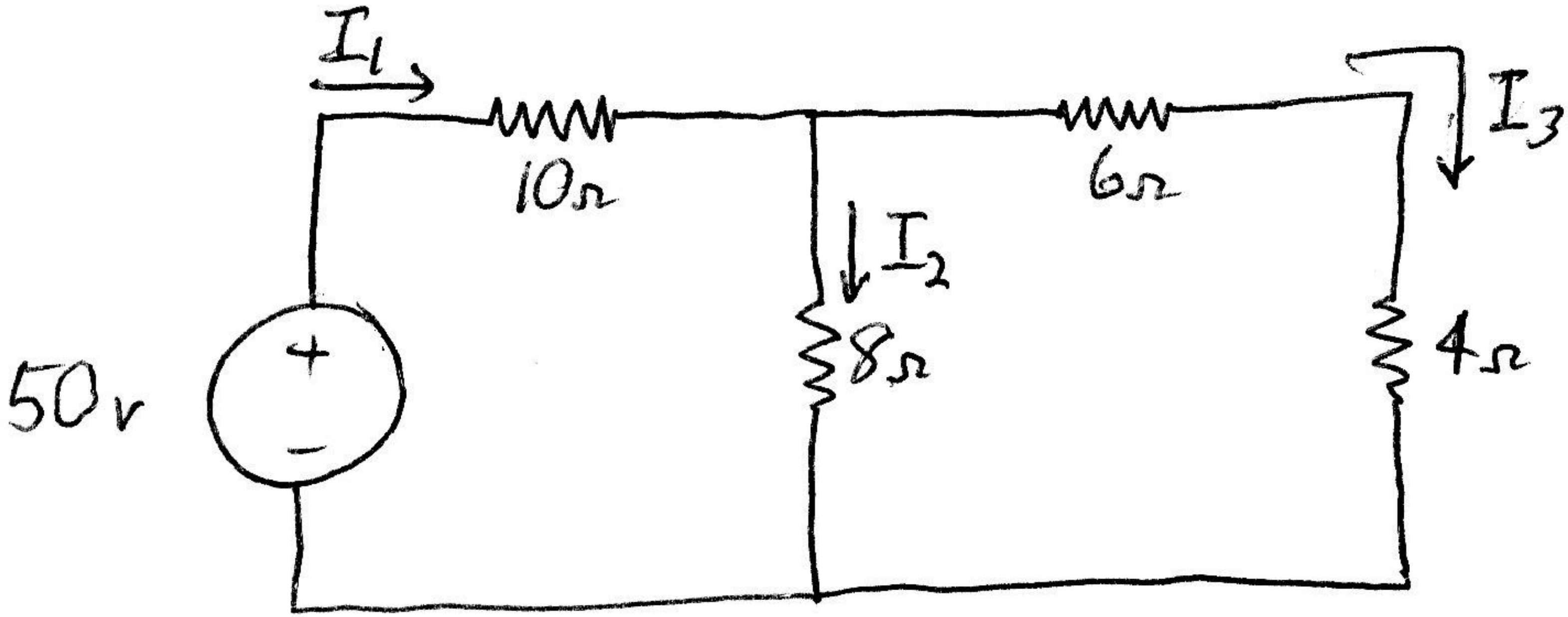
- The equivalent resistance for any number of resistors in parallel (i.e. they have the same voltage across each resistor):

$$Req = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}$$

- For two parallel resistors:

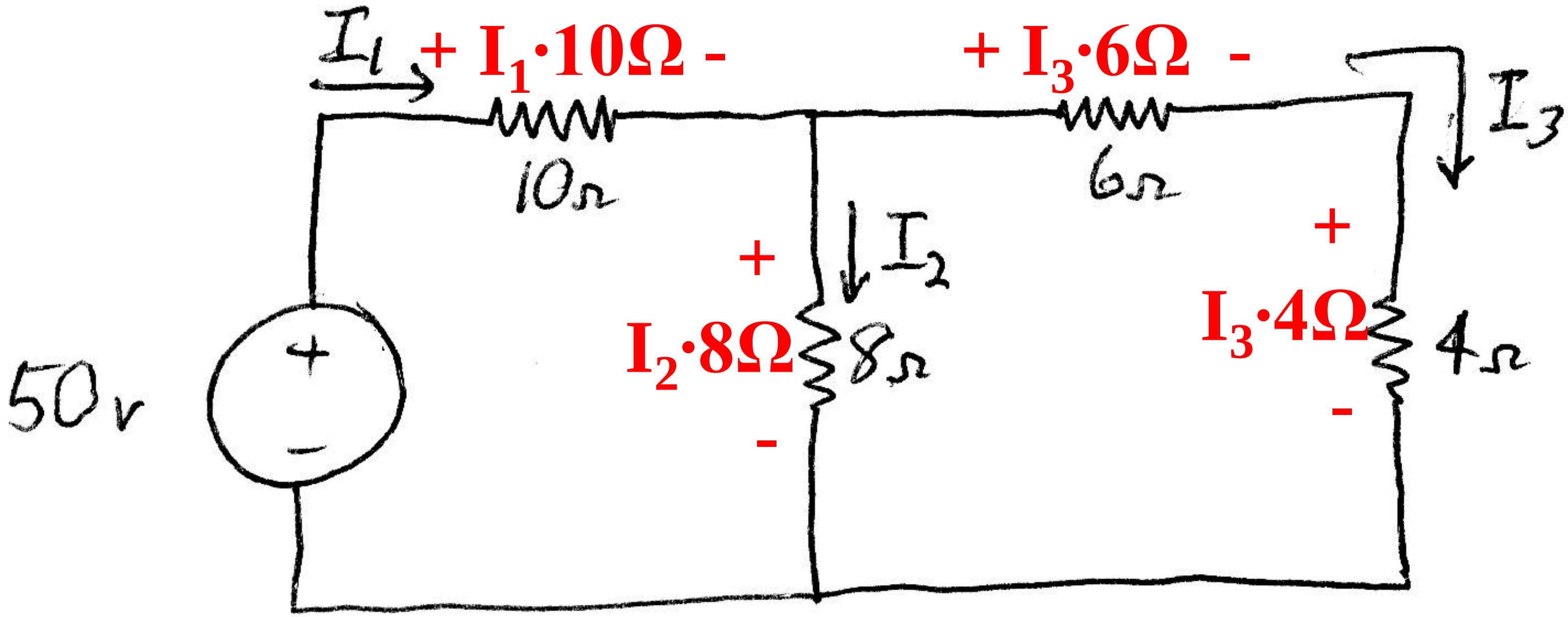
$$Req = R_1 \cdot R_2/(R_1+R_2)$$

Example Circuit



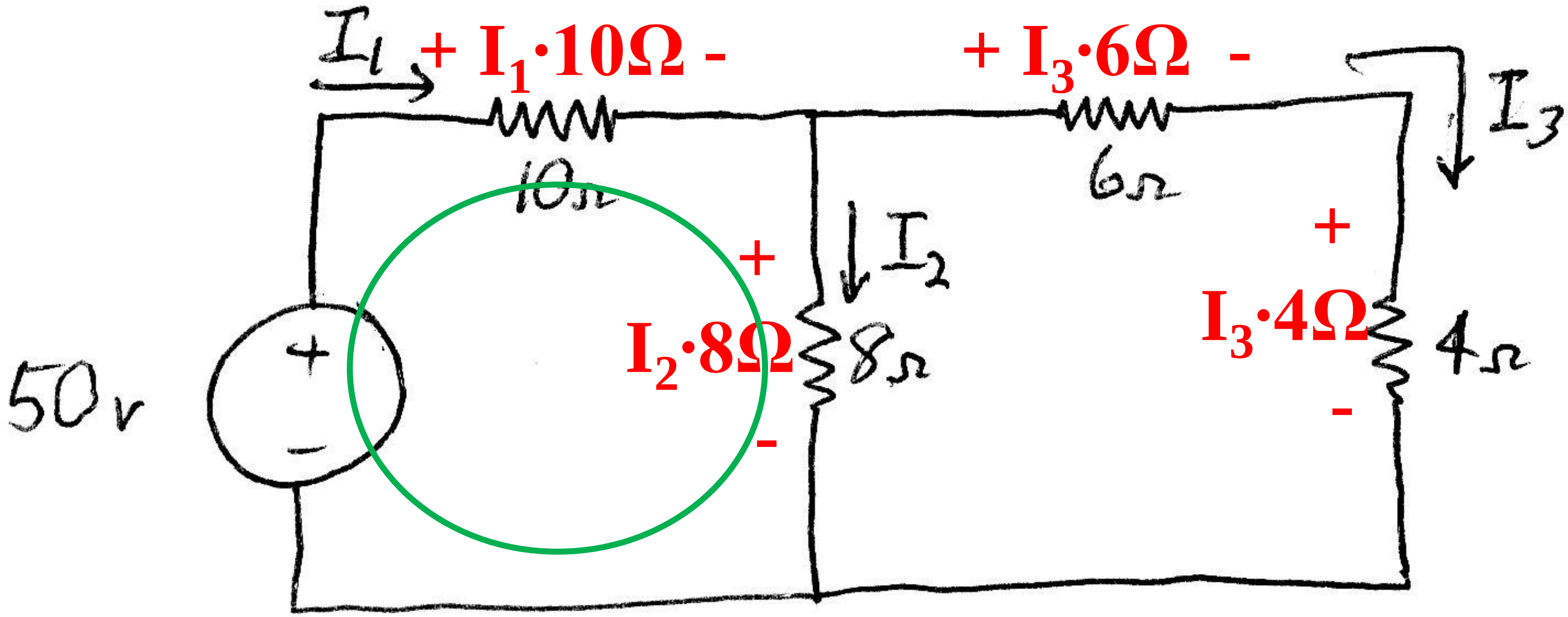
Solve for the currents through each resistor
And the voltages across each resistor

Example Circuit



Using Ohm's law, add polarities and expressions for each resistor voltage

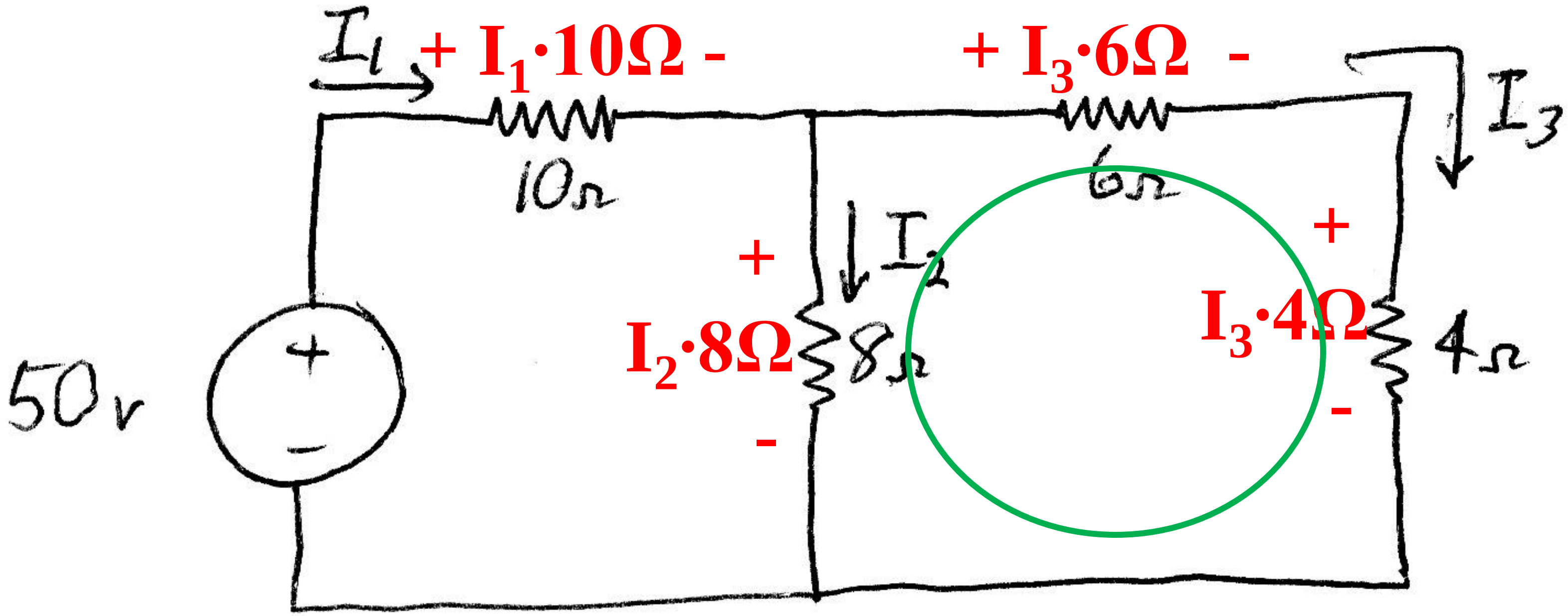
Example Circuit



Write 1st Kirchoff's voltage law equation

$$-50\text{ v} + I_1 \cdot 10\Omega + I_2 \cdot 8\Omega = 0$$

Example Circuit

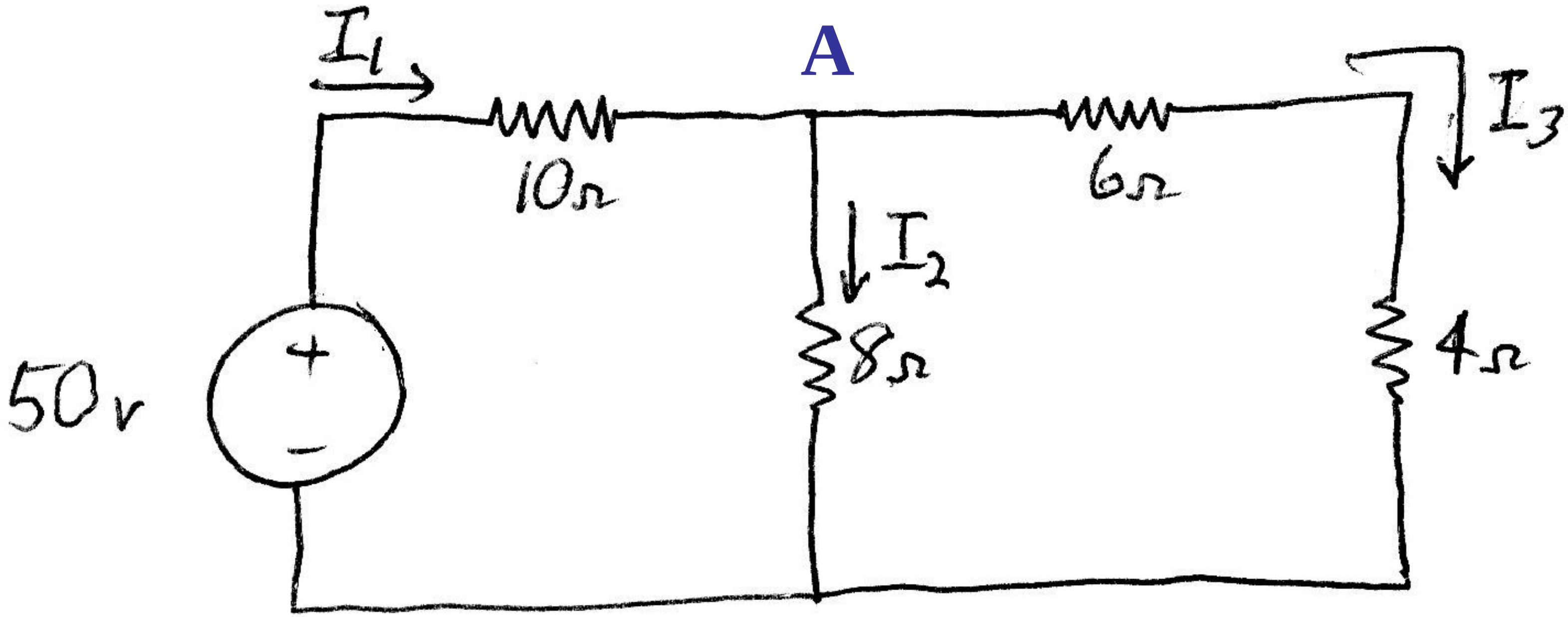


Write 2nd Kirchoff's voltage law equation

$$-I_2 \cdot 8\Omega + I_3 \cdot 6\Omega + I_3 \cdot 4\Omega = 0$$

$$\text{or } I_2 = I_3 \cdot (6+4)/8 = 1.25 \cdot I_3$$

Example Circuit



Write Kirchoff's current law equation at A

$$+I_1 - I_2 - I_3 = 0$$

Thevenin’s and Norton’s Equivalent

THEVENIN & NORTON

THEVENIN'S THEOREM:

Consider the following:

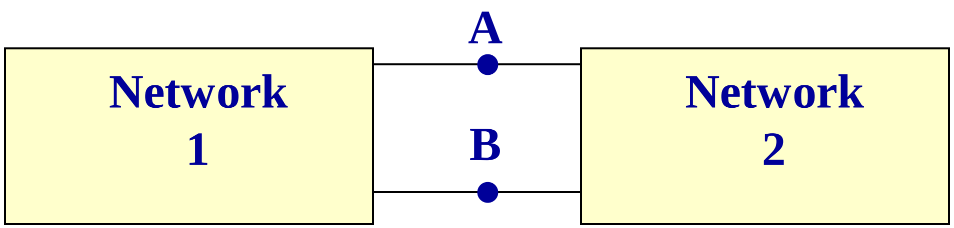


Figure 10.1: Coupled networks.

For purposes of discussion, at this point, we consider that both networks are composed of resistors and independent voltage and current sources

THEVENIN & NORTON

THEVENIN'S THEOREM:

Suppose Network 2 is detached from Network 1 and we focus temporarily only on Network 1.

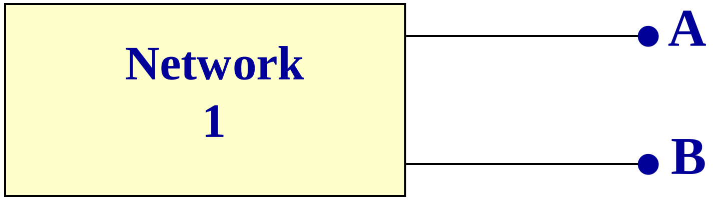
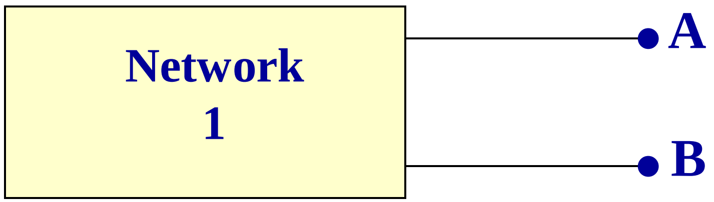


Figure 10.2: Network 1, open-circuited.

Network 1 can be as complicated in structure as one can imagine. Maybe 45 meshes, 387 resistors, 91 voltage sources and 39 current sources.

THEVENIN & NORTON

THEVENIN'S THEOREM:



Now place a voltmeter across terminals A-B and read the voltage. We call this the open-circuit voltage.

No matter how complicated Network 1 is, we read one voltage. It is either positive at A, (with respect to B) or negative at A.

We call this voltage V_{os} and we also call it $V_{THEVENIN} = V_{TH}$

THEVENIN & NORTON

THEVENIN’S THEOREM:

- We now deactivate all sources of Network 1.
- To deactivate a voltage source, we remove the source and replace it with a short circuit.
- To deactivate a current source, we remove the source.

THEVENIN & NORTON

THEVENIN'S THEOREM:

Consider the following circuit.

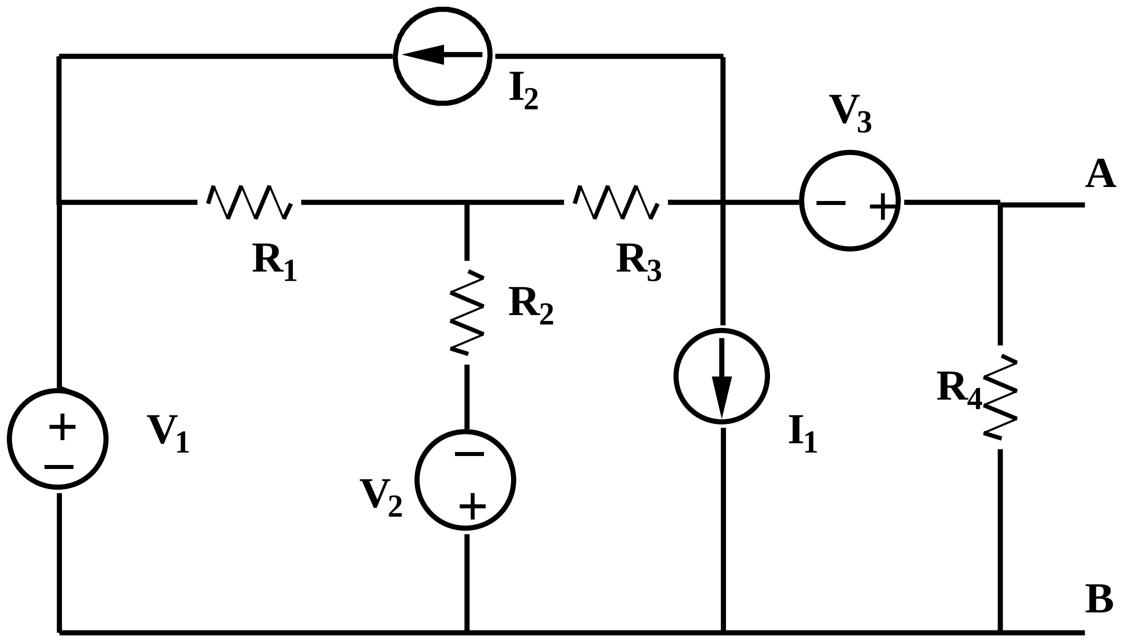


Figure 10.3: A typical circuit with independent sources

How do we deactivate the sources of this circuit?

THEVENIN & NORTON

THEVENIN'S THEOREM:

When the sources are deactivated the circuit appears as in Figure 10.4.

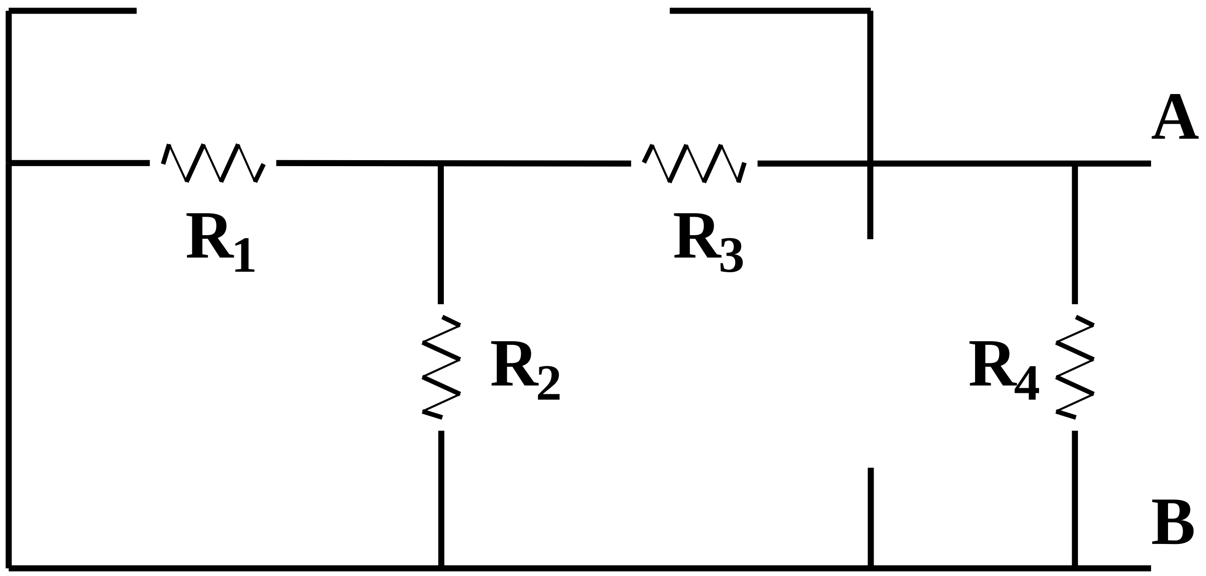


Figure 10.4: Circuit of Figure 10.3 with sources deactivated

Now place an ohmmeter across A-B and read the resistance.
If $R_1 = R_2 = R_4 = 20\ \Omega$ and $R_3 = 10\ \Omega$ then the meter reads $10\ \Omega$.

THEVENIN & NORTON

THEVENIN'S THEOREM:

We call the ohmmeter reading, under these conditions, $R_{THEVENIN}$ and shorten this to R_{TH}. Therefore, the important results are that we can replace Network 1 with the following network.

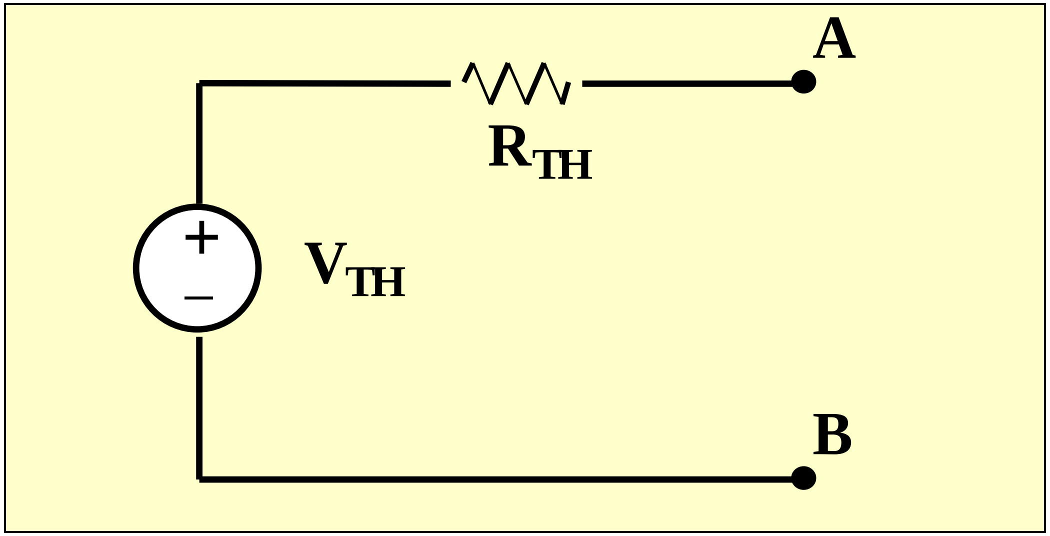


Figure 10.5: The Thevenin equivalent structure.

THEVENIN & NORTON

THEVENIN'S THEOREM:

We can now tie (reconnect) Network 2 back to terminals A-B.

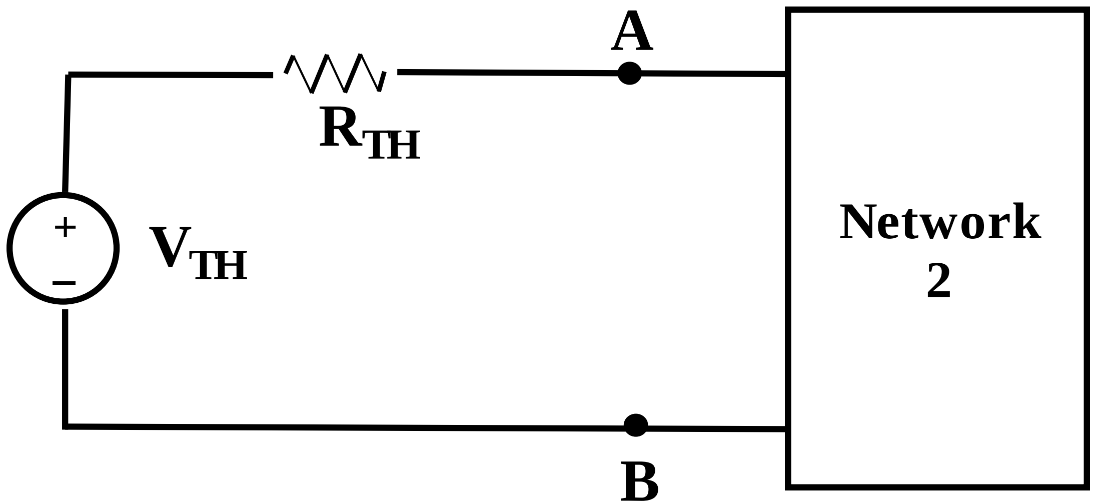


Figure 10.6: System of Figure 10.1 with Network 1 replaced by the Thevenin equivalent circuit.

We can now make any calculations we desire within Network 2 and they will give the same results as if we still had Network 1 connected.

THEVENIN & NORTON

THEVENIN'S THEOREM:

It follows that we could also replace Network 2 with a Thevenin voltage and Thevenin resistance. The results would be as shown in Figure 10.7.

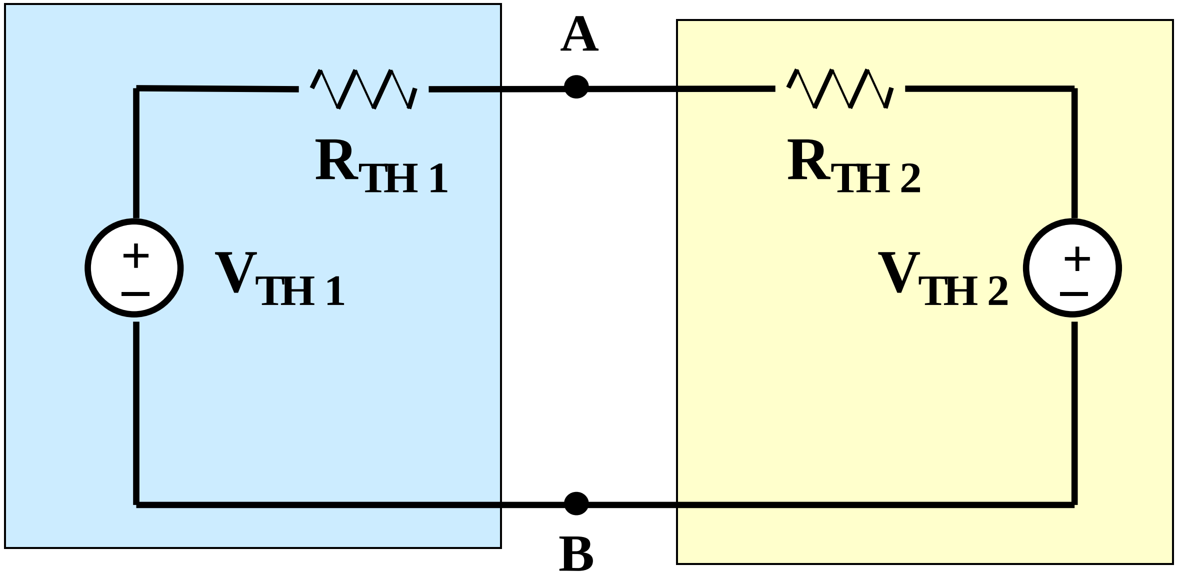


Figure 10.7: The network system of Figure 10.1 replaced by Thevenin voltages and resistances.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.1.

Find V_X by first finding V_{TH} and R_{TH} to the left of A-B.

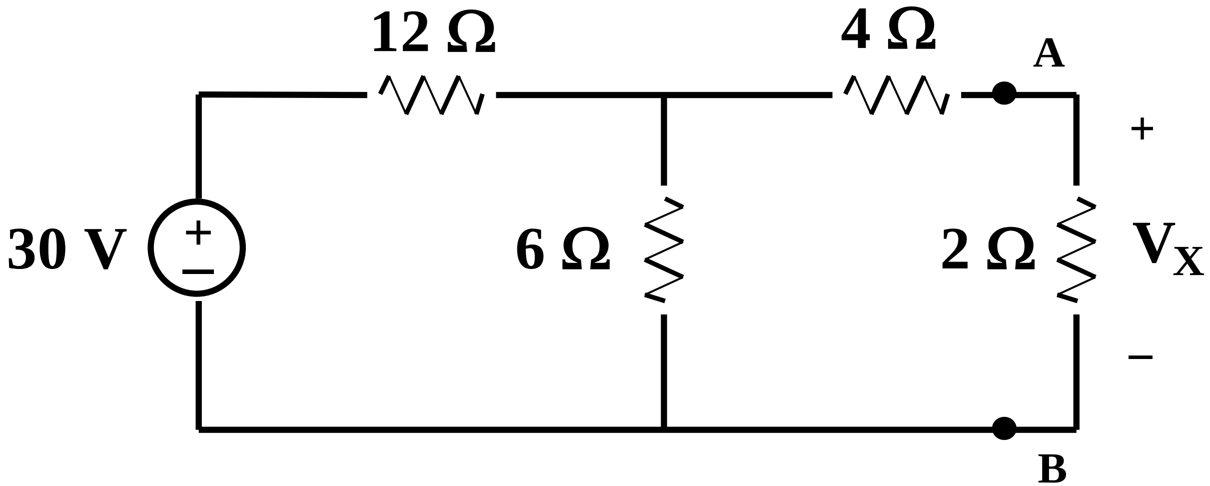


Figure 10.8: Circuit for Example 10.1.

First remove everything to the right of A-B.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.1. continued

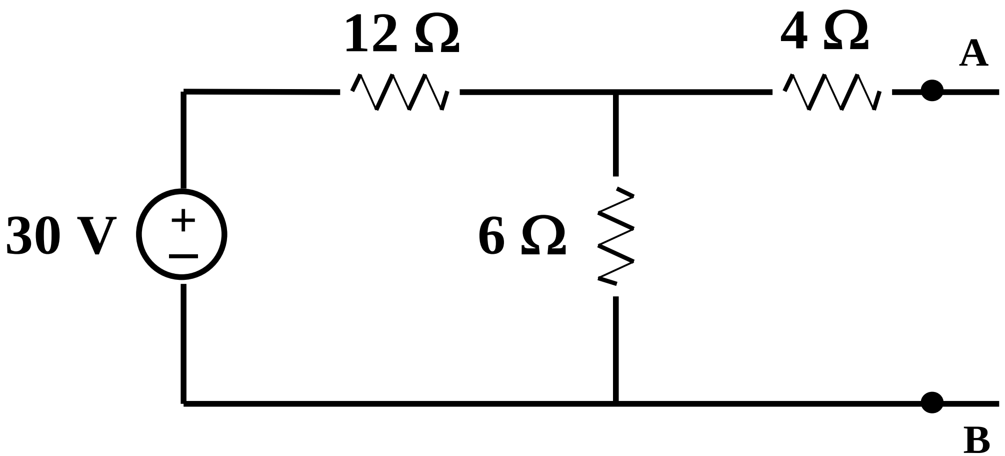


Figure 10.9: Circuit for finding V_{TH} for Example 10.1.

$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

Notice that there is no current flowing in the 4 Ω resistor (A-B) is open. Thus there can be no voltage across the resistor.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.1. continued

We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.

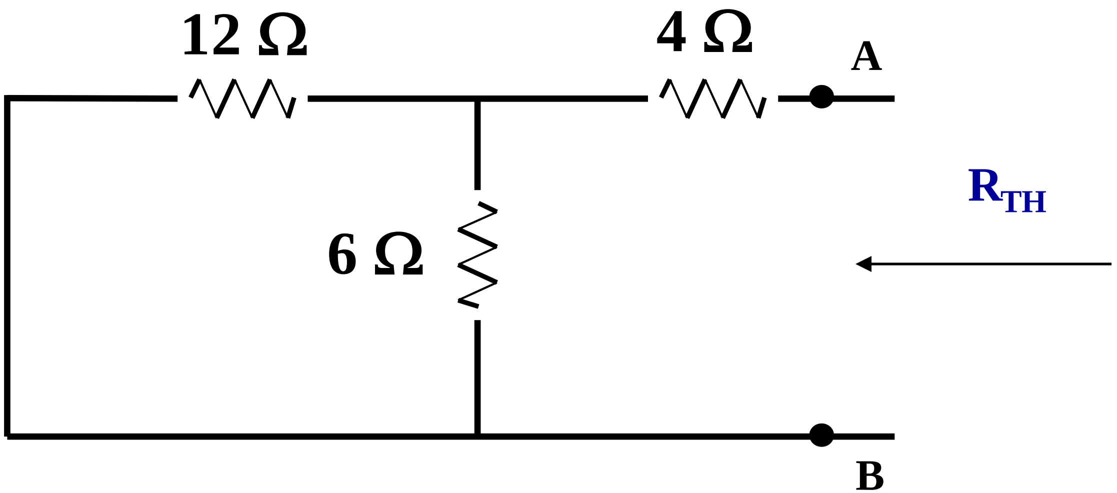


Figure 10.10: Circuit for find R_{TH} for Example 10.10.

We see,

$$R_{TH} = 12||6 + 4 = 8\ \Omega$$

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.1. continued

After having found the Thevenin circuit, we connect this to the load in order to find V_X.

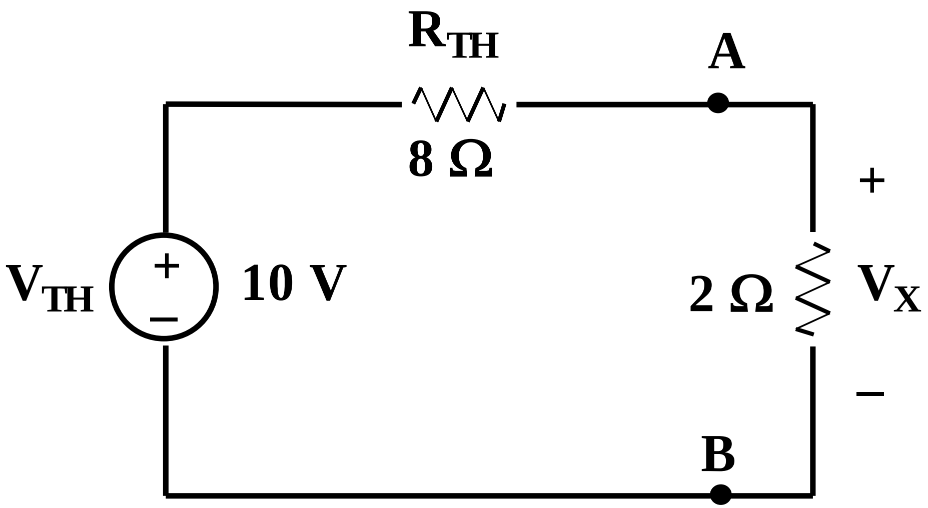


Figure 10.11: Circuit of Ex 10.1 after connecting Thevenin circuit.

$$V_X = \frac{(10)(2)}{2+8} = 2V$$

THEVENIN & NORTON

THEVENIN'S THEOREM:

In some cases it may become tedious to find R_{TH} by reducing the resistive network with the sources deactivated. Consider the following:

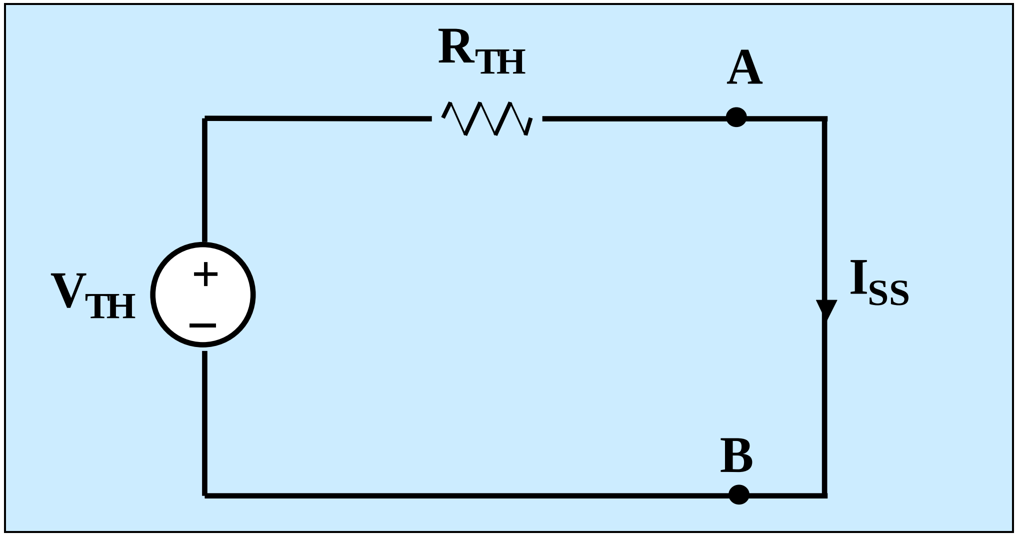


Figure 10.12: A Thevenin circuit with the output shorted.

We see;

$$R_{TH} = \frac{V_{TH}}{I_{SS}}$$

Eq 10.1

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.2.

For the circuit in Figure 10.13, find R_{TH} by using Eq 10.1.

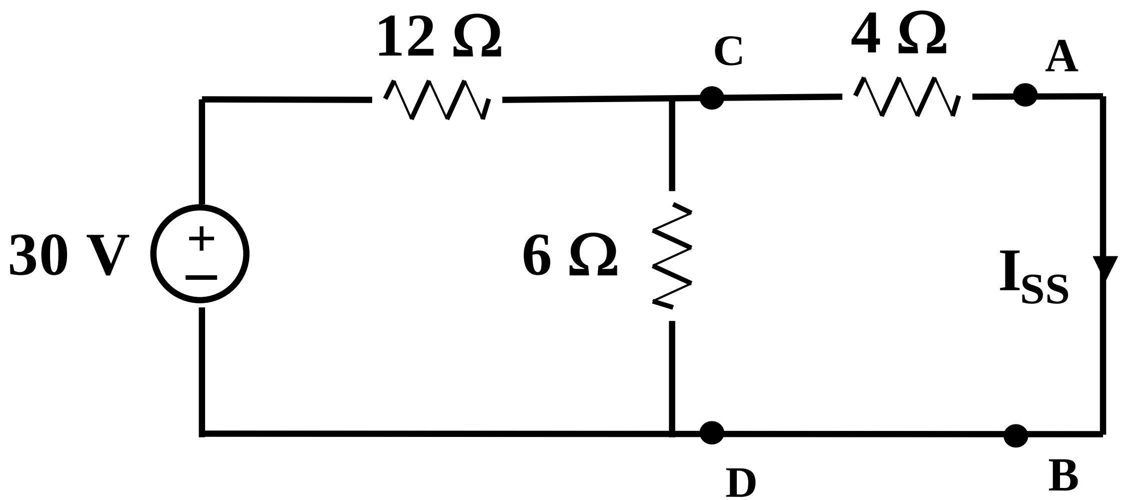


Figure 10.13: Given circuit with load shorted

The task now is to find I_{SS}. One way to do this is to replace the circuit to the left of C-D with a Thevenin voltage and Thevenin resistance.

THEVENIN & NORTON

THEVENIN'S THEOREM: Example 10.2. continued

Applying Thevenin's theorem to the left of terminals C-D and reconnecting to the load gives,

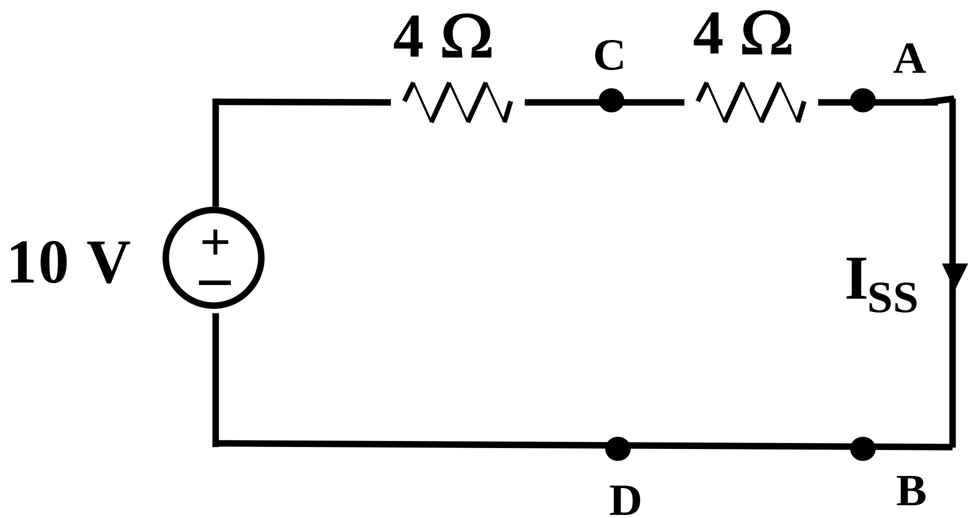


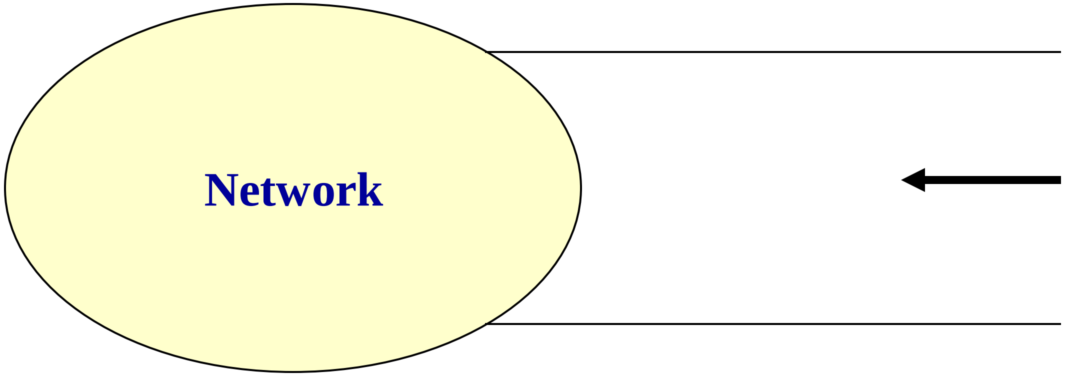
Figure 10.14: Thevenin reduction for Example 10.2.

$$R_{TH} = \frac{V_{TH}}{I_{SS}} = \frac{10}{10/8} = 8\Omega$$

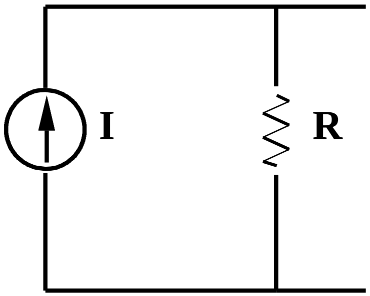
THEVENIN & NORTON

NORTON'S THEOREM:

Assume that the network enclosed below is composed of independent sources and resistors.



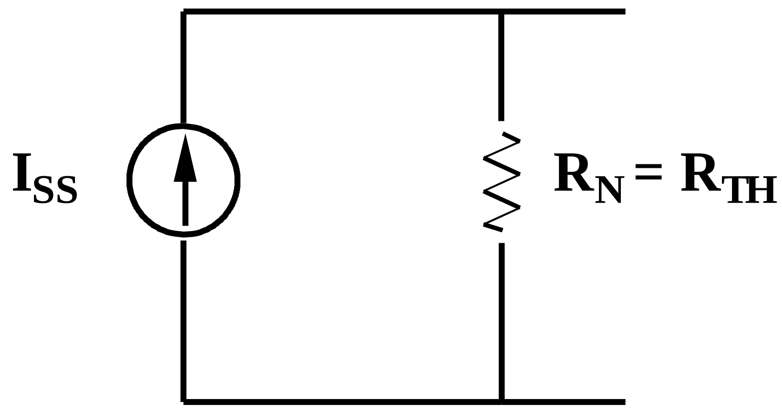
Norton's Theorem states that this network can be replaced by a current source shunted by a resistance R.



THEVENIN & NORTON

NORTON'S THEOREM:

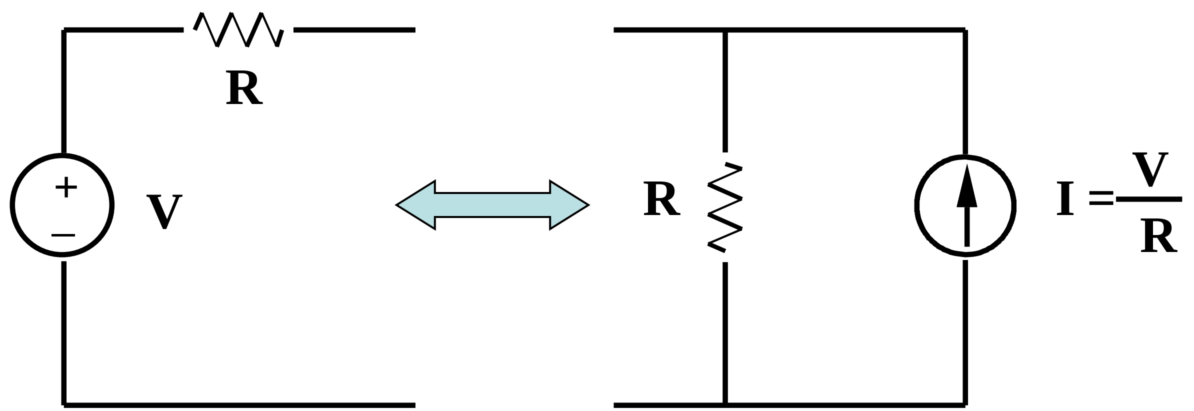
In the Norton circuit, the current source is the short circuit current of the network, that is, the current obtained by shorting the output of the network. The resistance is the resistance seen looking into the network with all sources deactivated. This is the same as R_{TH}.



THEVENIN & NORTON

NORTON'S THEOREM:

We recall the following from source transformations.



In view of the above, if we have the Thevenin equivalent circuit of a network, we can obtain the Norton equivalent by using source transformation.

However, this is not how we normally go about finding the Norton equivalent circuit.