

1. Given two polynomial matrices

$$N(s) = \begin{bmatrix} s+2 & (s+2)^2 \\ (s+2)(s+3) & (s+4)(s^2-4) \end{bmatrix} \text{ and } D(s) = \begin{bmatrix} (s+2)^2 & s+2 \\ s+2 & s^3+8 \end{bmatrix}$$

(1) Determine whether $N(s)$ and $D(s)$ are coprime.

(2) Calculate a unimodular matrix $U(s)$ that can reduce the matrix $\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}$ to a row Hermite form, and determine the greatest common right divisor of $N(s)$ and $D(s)$.

(3) Let $\hat{N}(s)$ and $\hat{D}(s)$ be coprime matrices derived from (2). Determine whether $\hat{D}(s)$ is column reduced. If not, find a unimodular matrix $U(s)$ to make it column reduced.

2. Given a matrix transfer function $G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{2}{s+4} \\ \frac{s+2}{(s+1)^2} & \frac{s+2}{(s+4)(s+1)} \\ \frac{s+2}{(s+1)(s+4)} & \frac{s+2}{(s+4)^2} \end{bmatrix}$ of a MIMO system,

(1) Find one coprime right polynomial fraction description $G(s) = N(s)D(s)^{-1}$.

(2) Determine all poles and transmission zeros of $G(s)$.

(3) Determine whether $D(s)$ is column reduced. If not, make it column reduced.

(4) Determine a minimal realization of $G(s)$ by using integrator coefficient matrices.

3. Given a matrix transfer function $G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{2}{s+4} \\ \frac{s+2}{(s+1)^2} & \frac{s+2}{(s+4)(s+1)} \end{bmatrix}$, determine a minimal realization of $G(s)$

by using integrator coefficient metrices. Design a state feedback law $u=r-kx$, that assigns all system poles to $s=-3$ by using the Lyapunov method.

4. Given an SISO system shown below, where $A = \begin{bmatrix} -2 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $c = [1 \ 1]$, design feedback $k = [k \ k_a]$ to reject a constant disturbance w and asymptotically track a step reference r .

