EE4341 Advanced Analog Circuits

Wideband Amplifier Design

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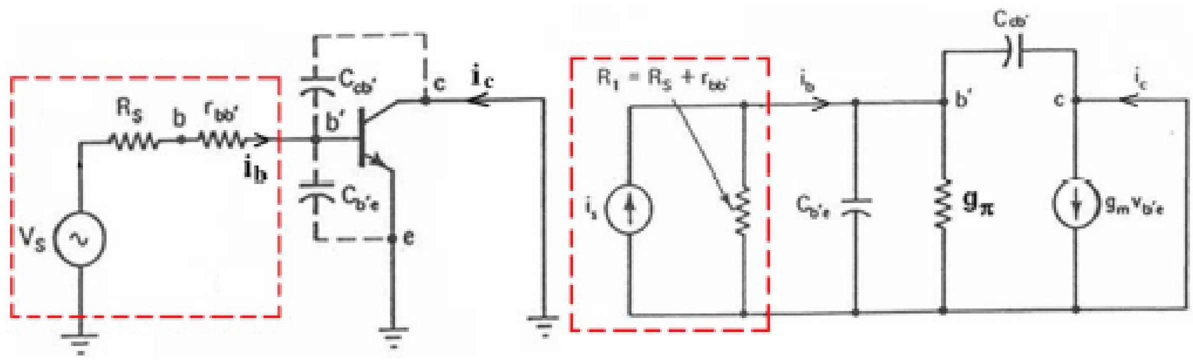
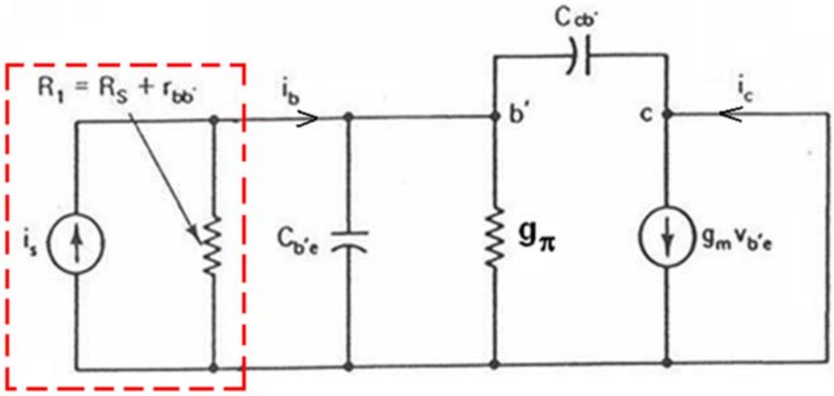
Unity Gain Frequency of BJT

The short-circuit current gain is determined with output shorted.



$$i_s = \frac{v_s}{R_s + r_{bb'}}$$

Unity Gain Frequency of BJT



$$g_\pi = \frac{1}{r_\pi}$$

$$i_c = g_m v_{b'e}$$

Apply KCL at node b':

$$\begin{aligned} i_b &= g_\pi v_{b'e} + j\omega C_{b'e} v_{b'e} + j\omega C_{cb'}\left(v_{b'e} - v_{ce}\right) \\ &= g_\pi v_{b'e} + j\omega C_{b'e} v_{b'e} + j\omega C_{cb'} v_{b'e} \\ &= \left[g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right)\right] v_{b'e} \end{aligned}$$

$$i_c = g_m v_{b'e} \qquad\qquad i_b = \left[g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right)\right] v_{b'e}$$

$$A_{i(sc)} = \frac{i_c}{i_b} = \frac{g_m}{g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right)} = \left(\frac{g_m}{g_\pi}\right)\left[\frac{1}{1 + j\omega\left(C_{b'e} + C_{cb'}\right)/\, g_\pi}\right]$$

$$\because \frac{g_m}{g_\pi} = \frac{I_C}{V_T} \times \frac{V_T}{I_B} = \beta$$

$$\therefore A_{i(sc)} = \frac{\beta}{1 + j\omega\left(C_{b'e} + C_{cb'}\right)\beta / g_m}$$

By definition, unity gain frequency f_T occurs when:

$$\left|A_{i(sc)}\right| = \left|\frac{\beta}{1 + j\omega_T\left(C_{b'e} + C_{cb'}\right)\beta / g_m}\right| = 1$$

$$\left| \frac{\beta}{1 + j\omega_T (C_{b'e} + C_{cb'})\beta / g_m} \right| = 1$$

$$\because \omega_T (C_{b'e} + C_{cb'})\beta / g_m >> 1 \qquad \therefore \frac{\beta}{\dfrac{\omega_T (C_{b'e} + C_{cb'})\beta}{g_m}} \approx 1$$

$$\omega_T \approx \frac{g_m}{C_{b'e} + C_{cb'}} \Rightarrow f_T = \frac{g_m}{2\pi(C_{b'e} + C_{cb'})}$$

Note: The above expression shows that f_T is independent of β but is dependent on the biasing point (that determines g_m) and the inherent parasitic capacitances of the device. As a general rule, to design a wideband amplifier with a -3dB bandwidth of BW_{3bB}, the BJT must have a f_T > 5 to 10 times BW_{3bB}.

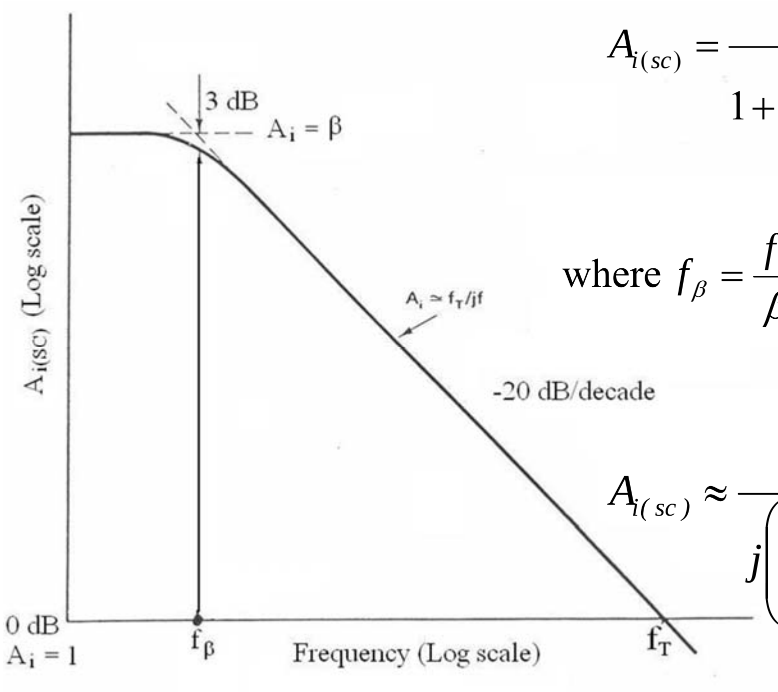
Unity Gain Frequency of BJT

$$A_{i(sc)}(j\omega) = \frac{\beta}{1 + j\omega(C_{b'e} + C_{cb'})\beta / g_m}$$

$$\because \omega_T = \frac{g_m}{C_{b'e} + C_{cb'}}$$

$$\therefore A_{i(sc)}(j\omega) = \frac{\beta}{1 + j\left(\dfrac{\beta\omega}{\omega_T}\right)} \qquad A_{i(sc)}(jf) = \frac{\beta}{1 + j\left(\dfrac{\beta f}{f_T}\right)}$$

Unity Gain Frequency of BJT

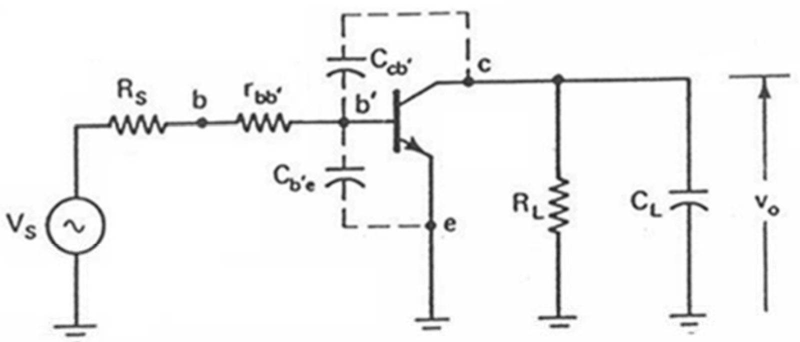


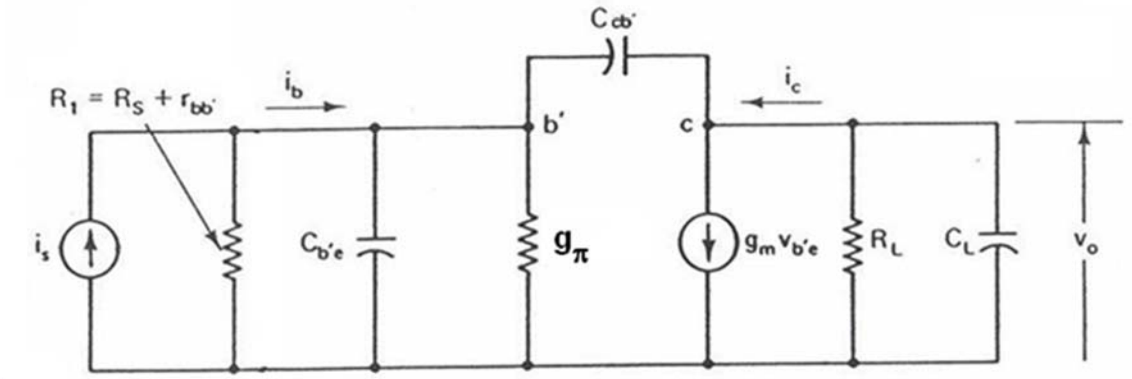
$$A_{i(sc)} = \frac{\beta}{1 + j\left(\frac{\beta f}{f_T}\right)} = \frac{\beta}{1 + j\left(\frac{f}{f_\beta}\right)}$$

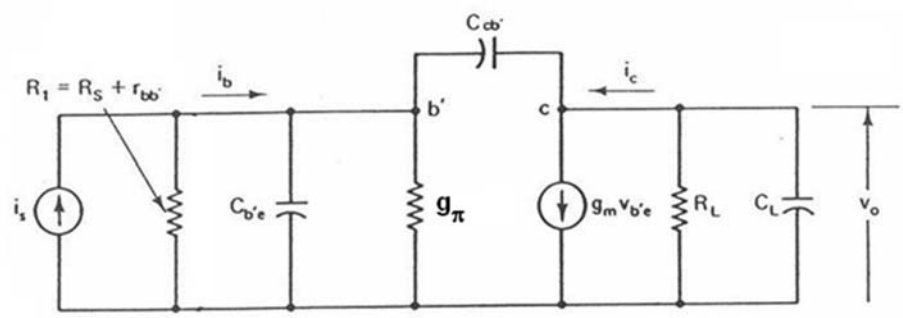
where $f_\beta = \frac{f_T}{\beta}$ is the - 3dB frquency of β

$$A_{i(sc)} \approx \frac{\beta}{j\left(\frac{f}{f_\beta}\right)} = \frac{\beta f_\beta}{jf} = \frac{f_T}{jf} \text{ for } f > 3 f_\beta$$

Frequency Response of CE Amplifier







$$G_1 = \frac{1}{R_S + r_{bb'}}$$

$$G_L = \frac{1}{R_L}$$

Apply KCL at node b':

$$v_{b'e}\left(G_1 + g_\pi + j\omega C_{b'e}\right) = i_s + \left(v_o - v_{b'e}\right) j\omega C_{cb'}$$

$$v_{b'e}\left[G_1 + g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right)\right] - j\omega C_{cb'} v_o = i_s \text{ ------}(1)$$

Apply KCL at node c:

$$v_o\left(G_L + j\omega C_L\right) + g_m v_{b'e} = \left(v_{b'e} - v_o\right) j\omega C_{cb'}$$

$$v_o\left[G_L + j\omega\left(C_L + C_{cb'}\right)\right] + g_m v_{b'e} = j\omega C_{cb'} v_{b'e}$$

$$v_o = \frac{-\left(g_m - j\omega C_{cb'}\right) v_{b'e}}{G_L + j\omega\left(C_L + C_{cb'}\right)} \text{ -------}(2)$$

Substitute (2) into (1):

$$v_{b'e}\left[G_1 + g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right) + \frac{j\omega C_{cb'}\left(g_m - j\omega C_{cb'}\right)}{G_L + j\omega\left(C_{cb'} + C_L\right)}\right] = i_s$$

$$v_{b'e} = \frac{i_s}{G_1 + g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right) + \dfrac{j\omega C_{cb'}\left(g_m - j\omega C_{cb'}\right)}{G_L + j\omega\left(C_{cb'} + C_L\right)}} \text{ ----}(3)$$

Substitute (3) into (2):

$$\therefore v_o = \left[\frac{-\left(g_m - j\omega C_{cb'}\right)}{G_L + j\omega\left(C_{cb'} + C_L\right)}\right] \frac{i_s}{G_1 + g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right) + \dfrac{j\omega C_{cb'}\left(g_m - j\omega C_{cb'}\right)}{G_L + j\omega\left(C_{cb'} + C_L\right)}}$$

$$i_s = \frac{v_s}{R_S + r_{bb'}} = v_s G_1$$

$$A_v = \frac{v_o}{v_s} = \frac{-\left(g_m - j\omega C_{cb'}\right)G_1}{\left[G_L + j\omega\left(C_{cb'} + C_L\right)\right]\left[G_1 + g_\pi + j\omega\left(C_{b'e} + C_{cb'}\right) + \dfrac{j\omega C_{cb'}\left(g_m - j\omega C_{cb'}\right)}{G_L + j\omega\left(C_{cb'} + C_L\right)}\right]}$$

In the frequency range of interest (*f* in the MHz range):

$$g_m\left(\text{in the range of } 10^{-3}\right) >> \omega C_{cb'}\left(\text{in the range of } 10^6 \times 10^{-12} \approx 10^{-6}\right)$$

$$\therefore \left(g_m - j\omega C_{cb'}\right)G_1 \approx g_m G_1$$

$$G_1\left(\text{in the range of } 10^{-2}\right) >> g_\pi \left(\text{in the range of } 10^{-4}\right)$$

$$\therefore G_1 + g_\pi \approx G_1$$

$$G_L\left(\text{in the range of } 10^{-3}\right) >> \omega\left(C_{cb'} + C_L\right)\left(\text{in the range of } 10^6 \times 10^{-12} \approx 10^{-6}\right)$$

$$\therefore G_L + j\omega\left(C_{cb'} + C_L\right) \approx G_L$$

$$A_v \approx \frac{-g_m G_1}{\left[G_L + j\omega\left(C_{cb'} + C_L\right)\right]\left[G_1 + j\omega\left(C_{b'e} + C_{cb'}\right) + \dfrac{j\omega C_{cb'} g_m}{G_L}\right]}$$

$$A_v = \frac{-g_m G_1}{\left[G_L + j\omega\left(C_{cb'} + C_L\right)\right]\left[G_1 + j\omega\left(C_{b'e} + C_{cb'} + g_m R_L C_{cb'}\right)\right]}$$

$$= \frac{-g_m G_1}{G_L G_1\left[1 + \dfrac{j\omega\left(C_{cb'} + C_L\right)}{G_L}\right]\left[1 + \dfrac{j\omega\left(C_{b'e} + C_{cb'} + g_m R_L C_{cb'}\right)}{G_1}\right]}$$

$$= \frac{-g_m R_L}{\left[1 + \dfrac{j\omega\left(C_{b'e} + C_{cb'} + g_m R_L C_{cb'}\right)}{G_1}\right]\left[1 + \dfrac{j\omega\left(C_{cb'} + C_L\right)}{G_L}\right]}$$

$$= \frac{-g_m R_L}{\left(1 + j\omega/\omega_1\right)\left(1 + j\omega/\omega_2\right)}$$

The two break frequencies:

$$\omega_1 = \frac{G_1}{C_{b'e} + C_{cb'}\left(1 + g_m R_L\right)} = \frac{1}{\left(R_S + r_{bb'}\right)\left[C_{b'e} + C_{cb'}\left(1 + g_m R_L\right)\right]}$$

$$\therefore f_1 = \frac{1}{2\pi\left(R_S + r_{bb'}\right)C_i} \qquad \text{where } C_i = C_{b'e} + \left(1 + g_m R_L\right)C_{cb'}$$

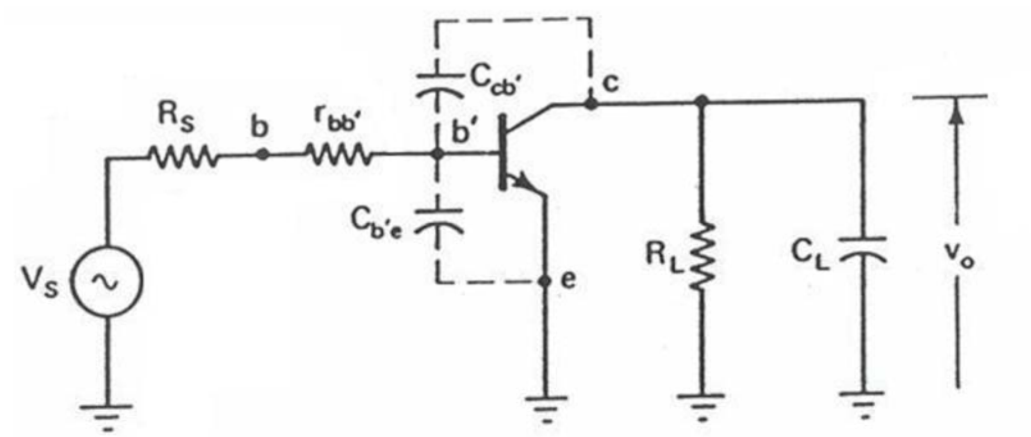
Note: The effective input capacitance is equal to $C_{cb'}$ multiplied by $(1 + g_m R_L)$. This is known as the Miller effect.

$$\omega_2 = \frac{G_L}{C_{cb'} + C_L} = \frac{1}{R_L\left(C_{cb'} + C_L\right)}$$

$$\therefore f_2 = \frac{1}{2\pi R_L\left(C_{cb'} + C_L\right)}$$

Example

BJT Device Parameters: $r_{bb'} = 60\ \Omega$, $R_s = 40\ \Omega$, $C_{cb'} = 1.5$ pF and $f_T = 1.6$ GHz. Load capacitance: $C_L = 1$ pF. The BJT is biased at 2.5 mA. Determine voltage gain frequency response for different values of R_L (varying from 30 Ω to 10 kΩ.



$$\omega_T = 2\pi \times 1.6 \times 10^9 = 1.005 \times 10^{10} \text{ rad/s} \quad g_m = \frac{I_C}{V_T} = \frac{2.5\,\text{mA}}{26\,\text{mV}} = 0.096\,\text{S}$$

$$C_{b'e} + C_{cb'} = \frac{g_m}{\omega_T} = \frac{0.096}{1.005 \times 10^{10}} = 9.6\,\text{pF}$$

$$\left|A_{v(MID)}\right| = g_m R_L = (0.096) R_L$$

$$f_1 = \frac{1}{2\pi (R_s + r_{bb'})(C_{b'e} + C_{cb'} + g_m R_L C_{cb'})}$$

$$= \frac{1}{2\pi (100)(9.6 + 0.096 \times R_L \times 1.5) \times 10^{-12}}$$

$$= \frac{1}{6.28 \times 10^{-10} (11.1 + 0.144 R_L)}$$

If $R_L = 1\ \text{k}\Omega$, $f_1 = 10.26$ MHz.

$$f_2 = \frac{1}{2\pi \times R_L (C_{cb'} + C_L)}$$

$$= \frac{1}{2\pi \times R_L (2.5 \times 10^{-12})}$$

$$= \frac{1}{1.57 \times 10^{-11} \times R_L}$$

If $R_L = 1\ \text{k}\Omega$, $f_2 = 63.7$ MHz.

We could calculate both f_1 and f_2 for different load R_L.

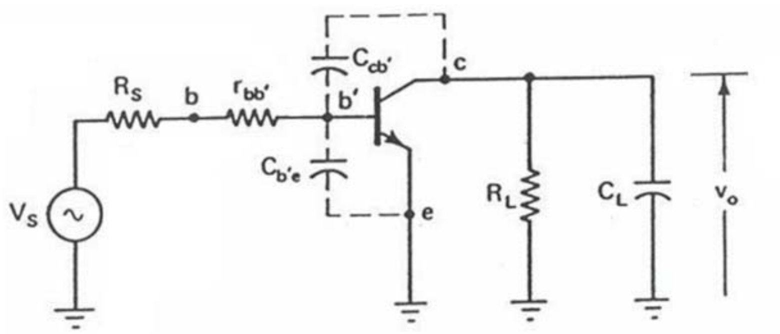
From the calculations, we have:

R_L (Ω)	f_1 (MHz)	f_2 (MHz)	BW (MHz)	$A_{V(MID)}$	$A_{V(MID)} \times BW$ (MHz)
30	110	2,122	110	3	330
100	64	637	64	10	640
300	29	212	29	30	868
1.000	10	64	10	100	1,000
3,000	3.5	21	3.5	300	1,038
10,000	1.05	6.4	1.04	1,000	1,040

$f_1 << f_2$ for all load resistance values. Therefore, f_1 is the primary factor that determines the -3dB BW of the amplifier.

Larger R_L gives larger mid-band gain but at the expense of reduction in BW.

Limitation of CE Stage for Wideband Application



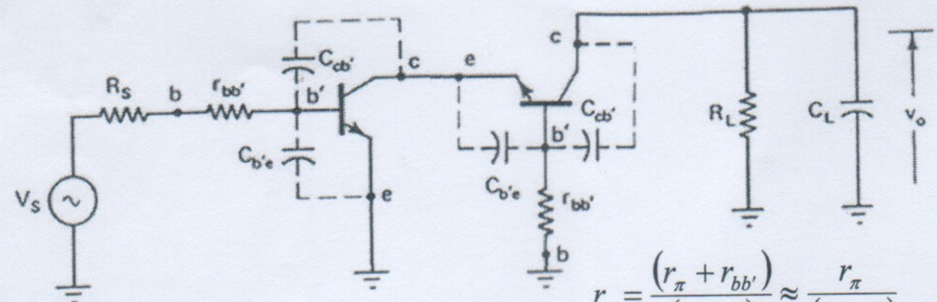
$$\left|A_{v(MID)}\right| = g_m R_L$$

$$f_1 = \frac{1}{2\pi(R_s + r_{bb'})C_i} \quad \text{where } C_i = C_{b'e} + \left(1 + \left|A_{v(MID)}\right|\right)C_{cb'}$$

$$f_2 = \frac{1}{2\pi R_L (C_{cb'} + C_L)}$$

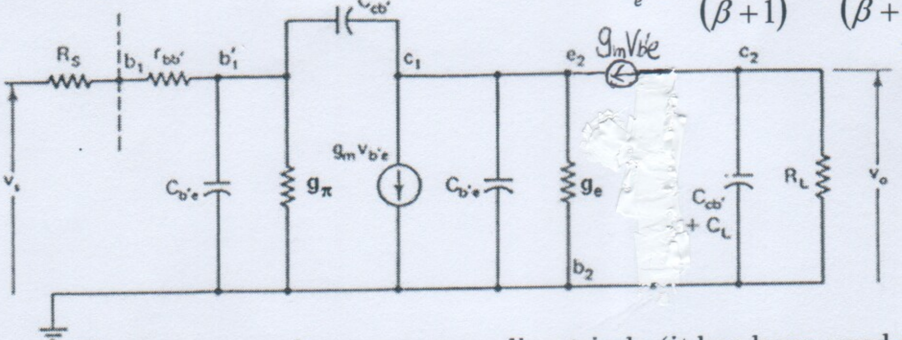
The BW is determined by f_1 and C_i is large due to Miller effect. Hence, CE amplifier alone is not suitable for wideband application.

CE-CB Configuration (Cascode Amplifier)



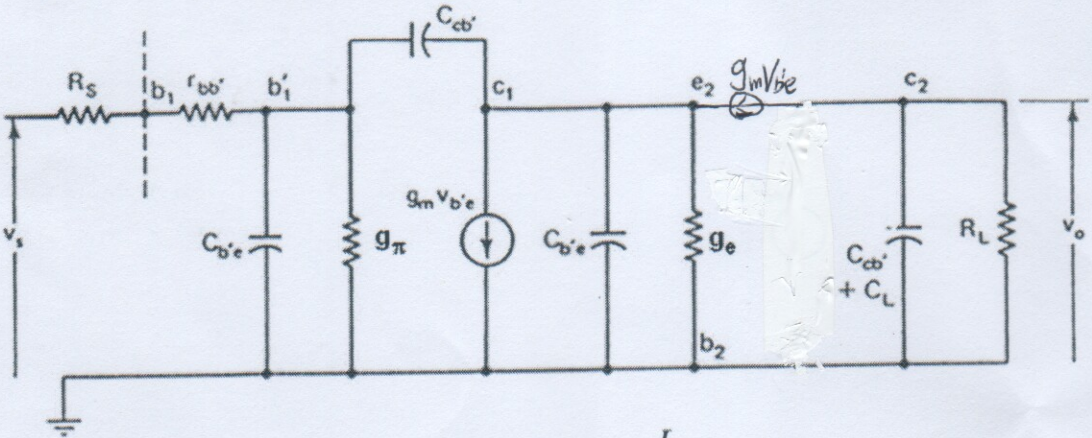
$$r_e = \frac{(r_\pi + r_{bb'})}{(\beta+1)} \approx \frac{r_\pi}{(\beta+1)} \quad \because r_\pi >> r_{bb'}$$

$$\approx \frac{r_\pi}{\beta} = \frac{1}{g_m}$$



For CB amplifier, the effect of $r_{bb'}$ can be neglected.

The term cascode means <u>cas</u>cading tri<u>ode</u> (it has been used since 1939)



$$Y_{L(CE)} \approx g_e + j\omega(C_{b'e} + C_{cb'}) = \frac{I_Q}{V_T} + j\omega(C_{b'e} + C_{cb'})$$

where $g_e = \dfrac{1}{r_{e2}}$, $I_Q = I_{E2} = I_{C1}$, $g_e = g_m = \dfrac{I_Q}{V_T}$,

$$Y_{L(CE)} = g_m + j\omega(C_{b'e} + C_{cb'}) = g_m\left[1 + \frac{j\omega(C_{b'e} + C_{cb'})}{g_m}\right] = g_m\left(1 + \frac{j\omega}{\omega_T}\right)$$

$$Y_{L(CE)} = g_m\left(1+\frac{jf}{f_T}\right) \quad \text{For } f << f_T,\ Y_{L(CE)} \approx g_m$$

The voltage gain for CE stage is:

$$A_{V(CE)} = -g_m\left(\frac{1}{Y_{L(CE)}}\right) = -\frac{g_m}{g_m} = -1$$

$$\omega_1 = \frac{1}{(R_S + r_{bb'})\left[C_{b'e} + C_{cb'}\left(1+\left|A_{v(CE)}\right|\right)\right]} = \frac{1}{(R_S + r_{bb'})(C_{b'e} + 2C_{cb'})}$$

$$\because C_{b'e} >> C_{cb'} \quad \therefore C_{b'e} + 2C_{cb'} \approx C_{b'e} + C_{cb'} \approx \frac{g_m}{\omega_T}$$

$$\omega_1 = \frac{\omega_T}{g_m(R_s + r_{bb'})} \Rightarrow f_1 = \frac{f_T}{g_m(R_s + r_{bb'})}$$

Note: now f_1 is independent of R_L.

The second break frequency ω_2 is:

$$\omega_2 = \frac{g_e}{C_{b'e} + C_{cb'}\left(1+\frac{1}{\left|A_{v(CE)}\right|}\right)} \approx \frac{g_m}{C_{b'e} + 2C_{cb'}} \approx \omega_T \Rightarrow f_2 \approx f_T$$

ω_2 will not have any significant effect in finding the overall BW.

The third break frequency ω_3 is:

$$\omega_3 = \frac{1}{R_L(C_{cb'} + C_L)} \Rightarrow f_3 = \frac{1}{2\pi R_L(C_{cb'} + C_L)}$$

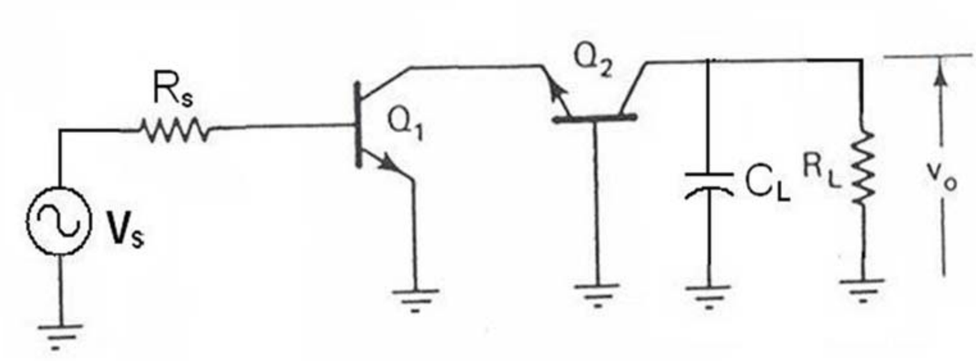
$$A_{V(CB)} = \frac{\alpha R_L}{r_e} \approx g_m R_L \quad \text{Note}: \alpha \approx 1 \text{ and } r_e \approx \frac{1}{g_m}$$

The overall mid-band gain:

$$A_{V(MID)} = A_{V(CE)} A_{V(CB)} = (-1)(g_m R_L) = -g_m R_L$$

Example

Two identical transistors Q_1 and Q_2 are configured as a Cascode Amplifier (CE-CB): BJT Device Parameters: $r_{bb'} = 60\ \Omega$, $R_s = 40\ \Omega$, $C_{cb'} = 1.5$ pF and $f_T = 1.6$ GHz. Load capacitance: $C_L = 1$ pF. The BJT is biased at 2.5 mA. Determine overall voltage gain frequency response for different values of R_L (varying from 30 Ω to 10 kΩ.



$$A_{V(MID)} = -g_m R_L = -0.096\, R_L$$

$$f_1 = \frac{f_T}{g_m\left(R_s + r_{bb'}\right)} = \frac{1.6\times10^9}{0.096(100)} \approx 160\ \text{MHz}$$

$$f_2 \approx f_T \approx 1.6\ \text{GHz}$$

$$f_3 = \frac{1}{2\pi R_L\left(C_{cb'} + C_L\right)} = \frac{1}{2\pi R_L\left(2.5\times10^{-12}\right)}$$

R_L (Ω)	f_1 (MHz)	f_2 (MHz)	f_3 (MHz)	BW (MHz)	$A_{V(MID)}$	$A_{V(MID)} \times BW$ (MHz)
30	160	1,600	2,122	160	3	480
100	160	1,600	637	160	10	1,600
300	160	1,600	212	128	30	3,831
1,000	160	1,600	64	59	100	5,900
3,000	160	1,600	21	21	300	6,305
10,000	160	1,600	6.4	6.4	1,000	6,400

Comparison

R_L (Ω)	f_1 (MHz)	f_2 (MHz)	BW (MHz)	$A_{V(MID)}$	$A_{V(MID)} \times BW$ (MHz)
30	110	2,122	110	3	330
100	64	637	64	10	640
300	29	212	29	30	868
1,000	10	64	10	100	1,000
3,000	3.5	21	3.5	300	1,038
10,000	1.05	6.4	1.04	1,000	1,040

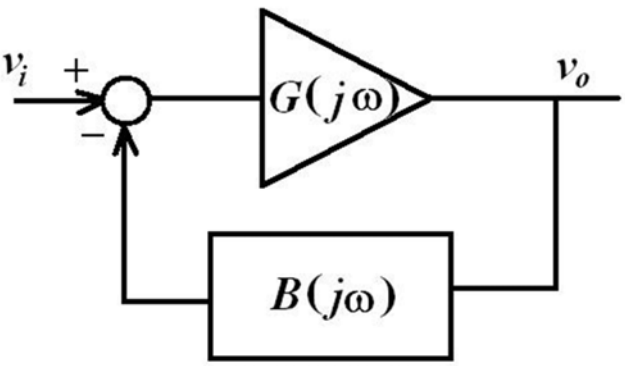
CE Stage

R_L (Ω)	f_1 (MHz)	f_2 (MHz)	f_3 (MHz)	BW (MHz)	$A_{V(MID)}$	$A_{V(MID)} \times BW$ (MHz)
30	160	1,600	2,122	160	3	480
100	160	1,600	637	160	10	1,600
300	160	1,600	212	128	30	3,831
1,000	160	1,600	64	59	100	5,900
3,000	160	1,600	21	21	300	6,305
10,000	160	1,600	6.4	6.4	1,000	6,400

Cascode

For R_L = 1kΩ , mid-band gain is 100 in both cases but BW = 10 MHz for CE and BW ≈ 60 MHz for the cascode stage. Nearly 6 times wider in BW.

Applying Feedback to Broaden BW



$$G(j\omega) = \frac{A_o}{1 + j\omega/\omega_o}$$

$$\frac{v_o}{v_i} = A_v(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)}$$

As the bandwidth of the transistor is restricted by its device parameters, negative feedback technique can be employed to broaden the amplifier's bandwidth. Of course, at the expense of lower gain.

$G(j\omega)$ is the voltage transfer function of the amplifier and $H(j\omega)$ is the negative feedback network.

If $H(j\omega)$ is frequency-independent in the frequency of interest, i.e. $H(j\omega) = H$:

$$A_v(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H}$$

$$A_v(j\omega) = \frac{\dfrac{A_o}{1 + j\omega/\omega_o}}{1 + \dfrac{A_o H}{1 + j\omega/\omega_o}} = \frac{A_o}{1 + j\omega/\omega_o + A_o H}$$

$$= \frac{A_o}{1 + A_o H}\left(\frac{1}{1 + \dfrac{j\omega}{\omega_o(1 + A_o H)}}\right) = \frac{A_o}{1 + A_o H}\left(\frac{1}{1 + j\omega/\omega_L}\right)$$

$$A_v(j\omega) = \frac{A_o}{1 + A_o H}\left(\frac{1}{1 + j\omega/\omega_L}\right)$$

The mid-band gain of the amplifier with feedback is:

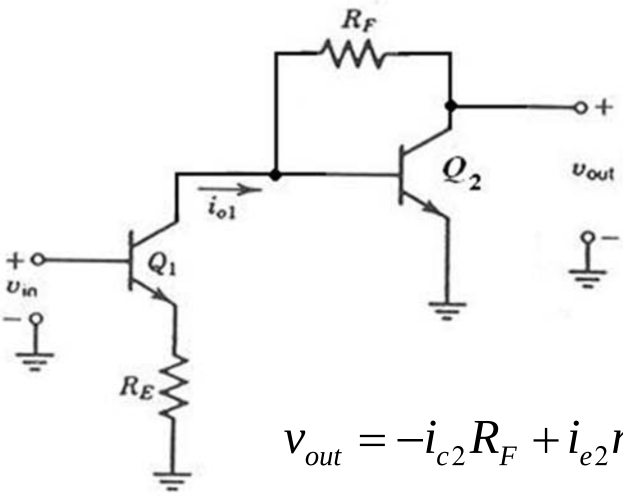
$$A_{v(MID)} = \frac{A_o}{1 + A_o H} \qquad \because A_o H >> 1 \;\therefore A_{v(MID)} \approx \frac{1}{H}$$

$\therefore$ The mid-band gain is controlled by the feedback network.

Now, the bandwidth has been broaden by a factor of $(1+A_oH)$:

$$\omega_L = \omega_o(1 + A_o H)$$

Amplifier with Series-Shunt Cascade



$$i_{o1} = -i_{c1} \approx -i_{e1}$$

$$i_{e1} = \frac{v_{in}}{r_e + R_E} \approx \frac{v_{in}}{R_E} \quad \because R_E >> r_e$$

$$\therefore i_{o1} \approx -\frac{v_{in}}{R_E} \Rightarrow v_{in} = -i_{o1} R_E$$

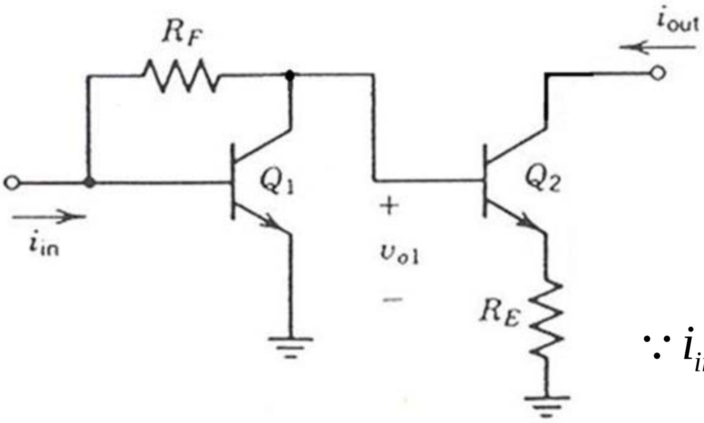
$$v_{out} = -i_{c2} R_F + i_{e2} r_{e2} \approx -i_{c2}\left(R_F - r_{e2}\right) \approx -i_{c2} R_F$$

$$\because i_{o1} = i_{c2} + i_{b2} \approx i_{c2} \quad \therefore v_{out} = -i_{o1} R_F$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_F}{R_E}$$

The amplifier with series-shunt feedback cascade is a wideband voltage amplifier.

Amplifier with Shunt-Series Cascade



$$v_{o1} = v_{c1} = -i_{c1} R_F + i_{e1} r_{e1}$$
$$\approx -i_{c1}\left(R_F - r_{e1}\right) = -i_{c1} R_F$$

$$\because i_{in} = i_{c1} + i_{b1} \approx i_{c1} \quad \therefore v_{o1} = -i_{in} R_F$$

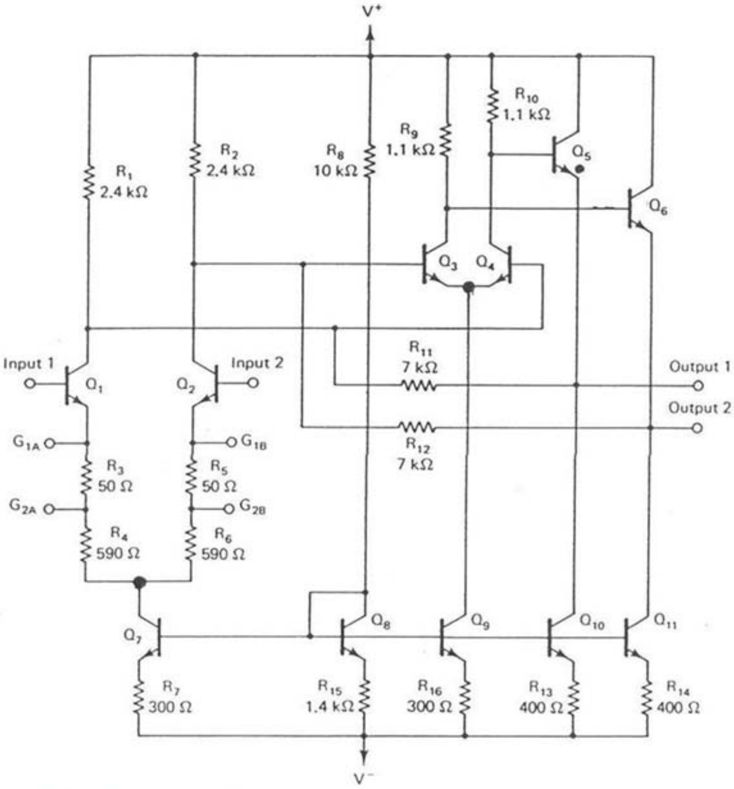
$$v_{o1} = i_{e2}\left(R_E + r_{e2}\right) \approx i_{e2} R_E$$

$$\because i_{out} = i_{c2} \approx i_{e2} \quad \therefore v_{o1} = i_{out} R_E$$

$$A_i = \frac{i_{out}}{i_{in}} = -\frac{R_F}{R_E}$$

The amplifier with shunt-series feedback cascade is a wideband current amplifier.

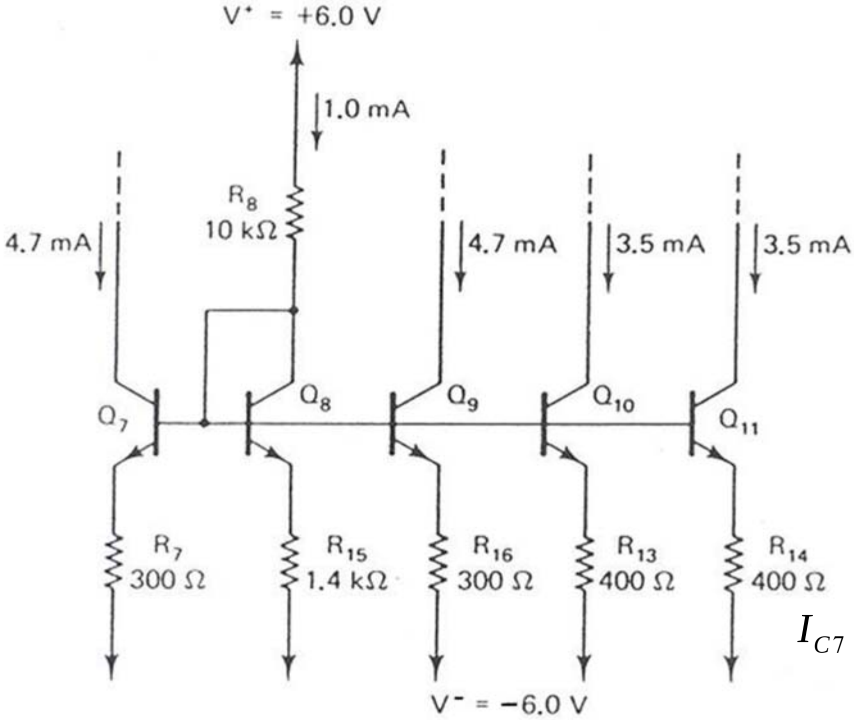
Differential wideband amplifier



Circuit diagram of μA 733 wideband differential amplifier that uses the series-shunt cascade feedback topology.

The circuit has very good CMRR (Common-mode rejection ratio)

DC Biasing Analysis



$$I_{C8} = \frac{(V^+ - V^-) - V_{BE8}}{R_8 + R_{15}} = \frac{12 - 0.6}{(10 + 1.4) \times 10^3} = 1\,\text{mA}$$

$$V_{R15} = 1\,\text{mA} \times 1.4\text{k}\Omega = 1.4\,\text{V}$$

$$I_{C7} = I_{C9} = \frac{1.4\,\text{V}}{300\,\Omega} = 4.7\,\text{mA}$$

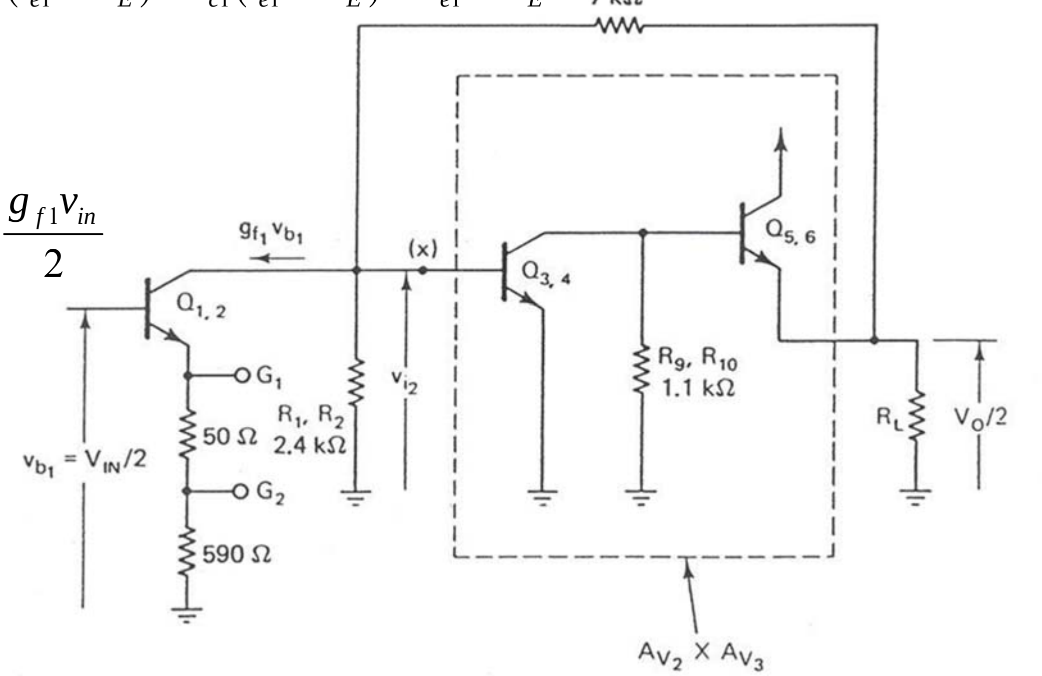
$$I_{C10} = I_{C11} = \frac{1.4\,\text{V}}{400\,\Omega} = 3.5\,\text{mA}$$

AC Signal Analysis

$$g_{f1} = \frac{i_{c1}}{v_{b1}} = \frac{i_{c1}}{i_{e1}(r_{e1} + R_E)} \approx \frac{i_{c1}}{i_{c1}(r_{e1} + R_E)} = \frac{1}{r_{e1} + R_E}$$

$$r_{e1} \approx \frac{V_T}{I_{C1}}$$

$$i_{c1} = g_{f1} v_{b1} = \frac{g_{f1} v_{in}}{2}$$



The voltage gain for second stage (Q_3 or Q_4) is:

$$A_{V2} = \frac{v_{c3}}{v_{i2}} = -\frac{g_{f2} v_{i2} R_9}{v_{i2}} = -\left(\frac{I_{C3}}{V_T}\right) R_9 = -\frac{4.7\text{mA}/2}{26\text{mV}} \times 1.1\,\text{k}\Omega = -103$$

The voltage gain for third stage (Q_5 or Q_6) is:

$$A_{V3} = \frac{v_o/2}{v_{c3}} = \frac{i_{e5} R_L}{i_{e5}(R_L + r_{e5})} = \frac{R_L}{R_L + r_{e5}} \approx 1 \quad \because R_L >> r_{e5}$$

$$\therefore A_{V2} \times A_{V3} \approx -100$$

Applying KCL at node "x":

$$\frac{g_{f1} v_{in}}{2} + \frac{v_x}{R_1} = \left(\frac{v_o}{2} - v_x\right)\left(\frac{1}{R_{11}}\right) \Rightarrow \frac{g_{f1} v_{in}}{2} = \frac{v_o}{2R_{11}} - v_x\left(\frac{1}{R_1} + \frac{1}{R_{11}}\right) \quad \text{----(1)}$$

Note that: $\dfrac{v_o}{2} = A_{V1}A_{V2}v_x \Rightarrow v_x = -\dfrac{v_o}{200}$ ----(2)

Substituting (2) into (1):

$$\frac{g_{f1}v_{in}}{2} = \frac{v_o}{2R_{11}} + \left(\frac{v_o}{200}\right)\left(\frac{1}{R_1} + \frac{1}{R_{11}}\right)$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_{f1}}{1/R_{11} + (1/100)(1/R_1 + 1/R_{11})}$$

$$= \frac{g_{f1}}{1/7\text{k} + (1/100)(1/7\text{k} + 1/2.4\text{k})} \approx g_{f1}(7\text{k})$$

$$A_v = g_{f1}(7\text{k}) = \frac{7\text{k}}{r_{e1} + R_E} = \frac{7\times10^3}{11 + R_E}$$

The gain for R_E = 0, 50 Ω and 640 Ω are calculated as follows:

$$R_E = 0 : A_v = \frac{7\times10^3}{11} = 640$$

$$R_E = 50\,\Omega : A_v = \frac{7\times10^3}{11+50} = 115$$

$$R_E = 640\,\Omega : A_v = \frac{7\times10^3}{11+640} = 10.8$$

The 3dB bandwidth obtained from the data sheet:

$$R_E = 0 : BW = 40\text{ MHz (Typical)}$$

$$R_E = 50\,\Omega : BW = 90\text{ MHz (Typical)}$$

$$R_E = 640\,\Omega : BW = 120\text{ MHz (Typical)}$$

Cascading Identical Stages

If a large voltage gain is required, it is convenient to cascade several identical amplifier stages.

The voltage transfer function of each stage is: $A_v = \dfrac{A}{1 + j\omega/\omega_p}$

The overall voltage transfer function of *n* stage is:

$$A_T = \frac{A^n}{\left(1 + j\omega/\omega_p\right)^n}$$

The overall bandwidth ω_1 can be found by:

$$|A_T| = \frac{A^n}{\left|1 + j\omega_1/\omega_p\right|^n} = \frac{A^n}{\sqrt{2}} \Rightarrow \left[1 + (\omega_1/\omega_p)^2\right]^{n/2} = 2^{1/2}$$

$$\therefore \omega_1 = \omega_p\left(2^{1/n} - 1\right)^{1/2} \Rightarrow f_1 = f_p\sqrt{2^{1/n} - 1}$$

Example

Three identical amplifiers, each with a voltage gain of 10 and a bandwidth of 10 MHz, are cascaded. What are the overall gain and bandwidth?

The overall voltage gain:

$$A^n = 10^3 = 1{,}000$$

The overall bandwidth:

$$f_1 = f_p\sqrt{2^{1/n} - 1} = 10 \times 10^6\sqrt{2^{1/3} - 1} = 5.1 \text{ MHz}$$