EE4341/EE6341
Advanced Analog Circuits

Tutorial 7

Dr See Kye Yak
Associate Professor
School of EEE
Office: S2-B2C-112
Email: ekysee@ntu.edu.sg

NANYANG TECHNOLOGICAL UNIVERSITY

Q1. Fig. 1 shows a Class AB power amplifier biased by a constant current source I_{bias} = 5 mA. The load R_L = 100 Ω and the transistor and the diode parameters are: $I_S = 10^{-13}$ A and V_T = 26 mV. Current gains β_n = 100 and β_p = 20 for npn and pnp transistors, respectively.

(a) Determine V_{BB}, I_C and V_{BE} for each transistor when $v_o = 0$.

(b) Repeat part (a) when v_o = +10 V.

(c) What is the instantaneous powers delivered to the load and dissipated in each transistor when v_o = + 10 V?

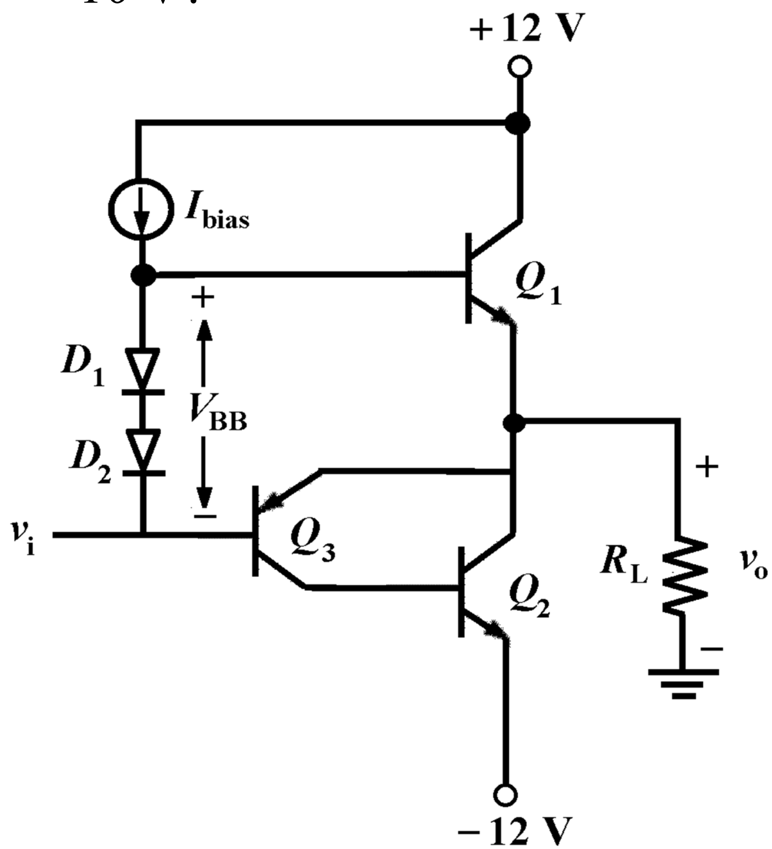


Figure 1

NANYANG TECHNOLOGICAL UNIVERSITY

(a) When $v_o = 0$ V, $i_L = 0$ mA.

$$i_{E1} = i_{E3} + i_{C2}$$

$$\left(\frac{\beta_n + 1}{\beta_n}\right) i_{C1} = \left(\frac{\beta_p + 1}{\beta_p}\right) i_{C3} + \beta_n i_{C3}$$

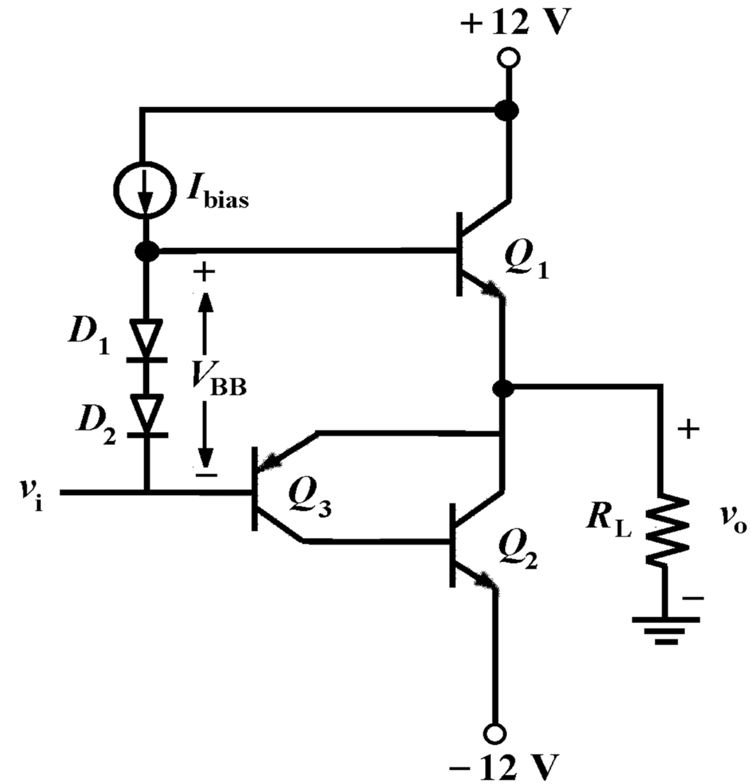
$$\left(\frac{101}{100}\right) i_{C1} = \left(\frac{21}{20}\right) i_{C3} + 100 i_{C3}$$

$$1.01 i_{C1} = 1.05 i_{C3} + 100 i_{C3}$$

$$i_{C1} = \left(\frac{1.05 + 100}{1.01}\right) i_{C3} = 100.05 i_{C3} \quad \text{-----}(1)$$

$$V_{BB} = v_{BE1} + v_{EB3} = 2V_D$$

$$V_T \ln \frac{i_{C1}}{I_S} + V_T \ln \frac{i_{C3}}{I_S} = 2V_T \ln \frac{i_D}{I_S}$$



$$V_T \ln \frac{i_{C1} i_{C3}}{I_S{}^2} = V_T \ln \frac{i_D{}^2}{I_S{}^2}$$

$$i_{C1} i_{C3} = i_D{}^2 \quad \text{------}(2)$$

NANYANG TECHNOLOGICAL UNIVERSITY

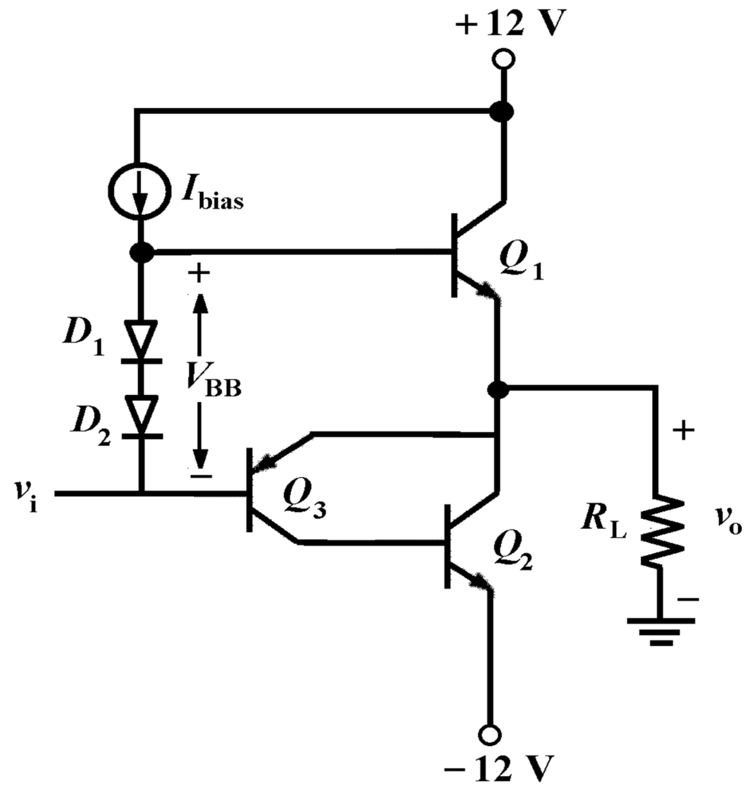
Substitute (1) into (2):

$$i_{C1}\left(\frac{i_{C1}}{100.05}\right)=i_D{}^2=\left(I_{Bias}-i_{B1}\right)^2$$

$$\frac{i_{C1}{}^2}{100.05}=\left(5-\frac{i_{C1}}{100}\right)^2$$

$$i_{C1}{}^2=100.05\left(25-0.1i_{C1}+\frac{i_{C1}{}^2}{100^2}\right)$$

$$0.99i_{C1}{}^2+10.005i_{C1}-2501.25=0$$



$$i_{C1}=\frac{-10.005\pm\sqrt{10.005^2+4(0.99)(2501.25)}}{2(0.99)}$$

$$i_{C1}=45.465\text{ mA (take }+\text{ve value)}$$

$$i_{B1} = \frac{i_{C1}}{100} = 454.65\ \mu\text{A}$$

$$i_D = 5\ \text{mA} - 454.65\ \mu\text{A} = 4.54535\ \text{mA}$$

$$V_{BB} = 2V_T \ln\frac{i_D}{I_S} = 2\left(26\times10^{-3}\right)\ln\frac{4.54535\times10^{-3}}{10^{-13}} = 1.2761\ \text{V}$$

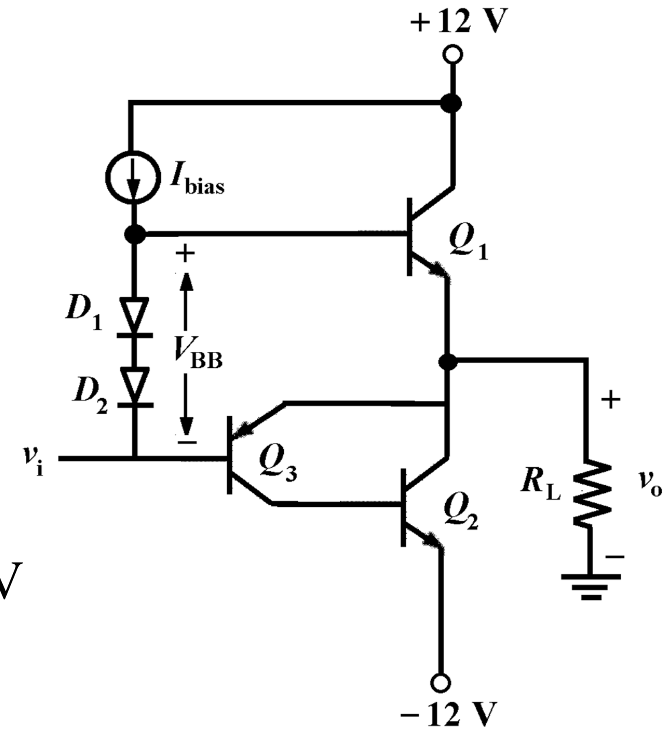
$$v_{BE1} = V_T\, ln\frac{i_{C1}}{I_S} = \left(26\times10^{-3}\right) ln\frac{45.465\times10^{-3}}{10^{-13}} = 0.6979\ \text{V}$$

$$v_{EB3} = V_{BB} - v_{BE1} = 1.2761 - 0.6979 = 0.5782\ \text{V}$$

$$i_{C3} = I_S e^{\frac{v_{EB3}}{V_T}} = 10^{-13} e^{\frac{0.5782}{26\times10^{-3}}} = 0.455\ \text{mA}$$

$$i_{C2} = 100 i_{C3} = 45.5\ \text{mA}$$

$$v_{BE2} = V_T \ln\frac{i_{C2}}{I_S} = \left(26\times10^{-3}\right)\ln\frac{45.5\times10^{-3}}{10^{-13}} = 0.6979\ \text{V}$$



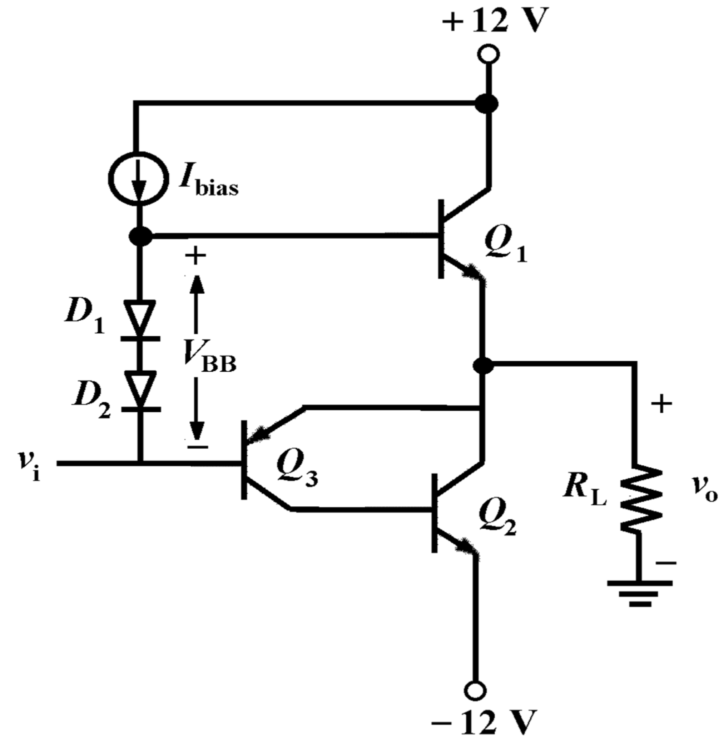
NANYANG TECHNOLOGICAL UNIVERSITY

(b) When $v_o = 10$ V, $i_L = v_o/R_L = 100$ mA.

$$i_{E1} = i_{E3} + i_{C2} + i_L$$

$$1.01 i_{C1} = 1.05 i_{C3} + 100 i_{C3} + 100$$

$$i_{C3} = \frac{i_{C1} - 99.01}{100.05} \quad \text{-------}(3)$$



Substitute (3) into (2):

$$i_{C1}\left(\frac{i_{C1} - 99.01}{100.05}\right) = i_D{}^2 = \left(5 - \frac{i_{C1}}{100}\right)^2$$

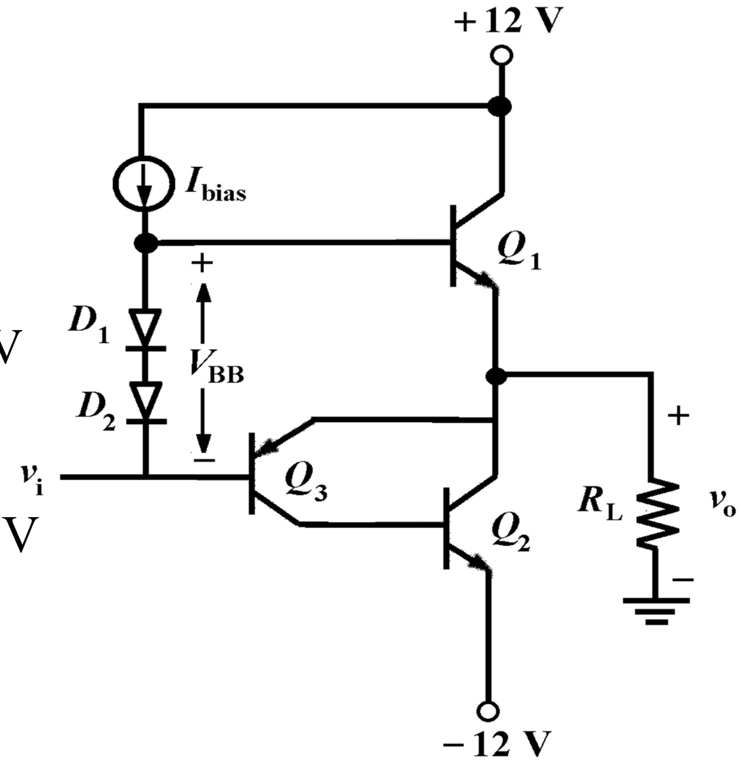
$$0.0099 i_{C1}{}^2 - 0.89 i_{C1} - 25 = 0$$

$$i_{C1} = \frac{0.89 \pm \sqrt{0.89^2 + 4(0.0099)(25)}}{2(0.0099)}$$

$$i_{C1} = 112.37 \text{ mA (take } +\text{ve value)}$$

$$i_{B1} = \frac{i_{C1}}{100} = 1.1237\,\text{mA}$$

$$i_D = 5\text{ mA} - 1.1237\text{ mA} = 3.8763\text{ mA}$$



$$V_{BB} = 2V_T \ln\frac{i_D}{I_S} = 2\left(26\times10^{-3}\right)\ln\frac{3.8763\times10^{-3}}{10^{-13}} = 1.2678\text{ V}$$

$$v_{BE1} = V_T\, ln\frac{i_{C1}}{I_S} = \left(26\times10^{-3}\right) ln\frac{112.37\times10^{-3}}{10^{-13}} = 0.7214\text{ V}$$

$$v_{EB3} = V_{BB} - v_{BE1} = 1.2678 - 0.7214 = 0.5464\text{ V}$$

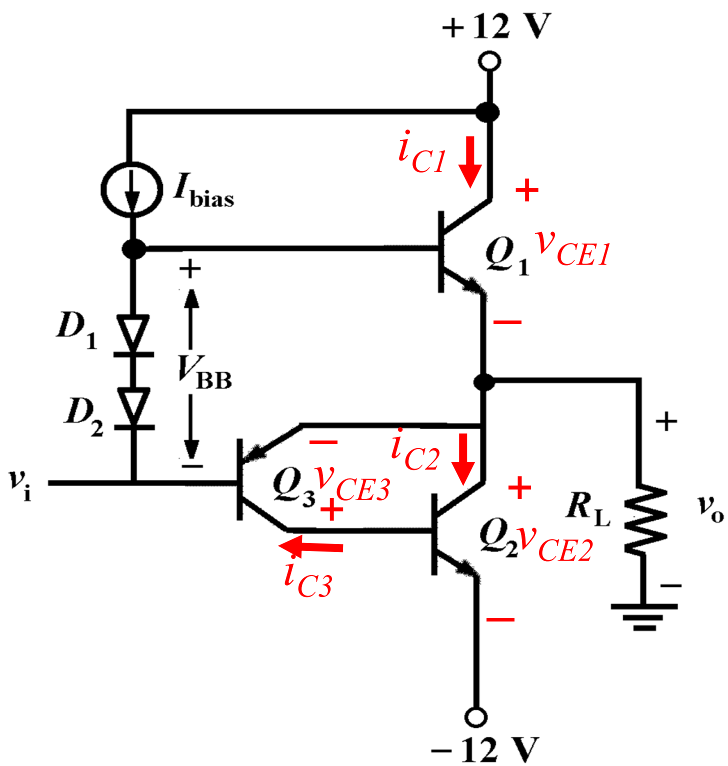
$$i_{C3} = I_S e^{\frac{v_{EB3}}{V_T}} = 10^{-13} e^{\frac{0.5464}{26\times10^{-3}}} = 133.9\ \mu\text{A}$$

$$i_{C2} = 100 i_{C3} = 13.39\text{ mA}$$

$$v_{BE2} = V_T\, ln\frac{i_{C2}}{I_S} = \left(26\times10^{-3}\right) ln\frac{13.39\times10^{-3}}{10^{-13}} = 0.6661\text{ V}$$

NANYANG TECHNOLOGICAL UNIVERSITY

The instantaneous powers to the load and the power dissipation in the transistors:



$$
\begin{aligned}
p_{Q1} &= i_{C1} v_{CE1} \\
&= 112.37\ \text{mA} \times (12 - 10)\,\text{V} \\
&= 225\ \text{mW}
\end{aligned}
$$

$$
\begin{aligned}
p_{Q2} &= i_{C2} v_{CE2} \\
&= 13.39\ \text{mA} \times [10 - (-12)]\,\text{V} \\
&= 295\ \text{mW}
\end{aligned}
$$

$$
\begin{aligned}
p_{Q3} &= i_{C3} v_{EC3} \\
&= 133.9\ \mu\text{A} \times \left[10 - (-12 + 0.6661)\right]\text{V} \\
&= 2.86\ \text{mW}
\end{aligned}
$$

$$p_L = \frac{v_o{}^2}{R_L} = \frac{10^2}{100} = 1\ \text{W}$$

2. Fig. 2 shows a Class D power amplifier to drive a load $R_L = 8\ \Omega$. Assume $V_{CE(sat)} = 0.3$ V and the LC low pass filter is lossless.

(a) What is the maximum power that can be delivered to the load?

(b) If each switching transition of the transistor is about 5% of the period of switching frequency, what is the conversion efficiency at maximum output power?

(c) Repeat part (b) if faster transistors are used with the switching transition reduced to 2.5%.

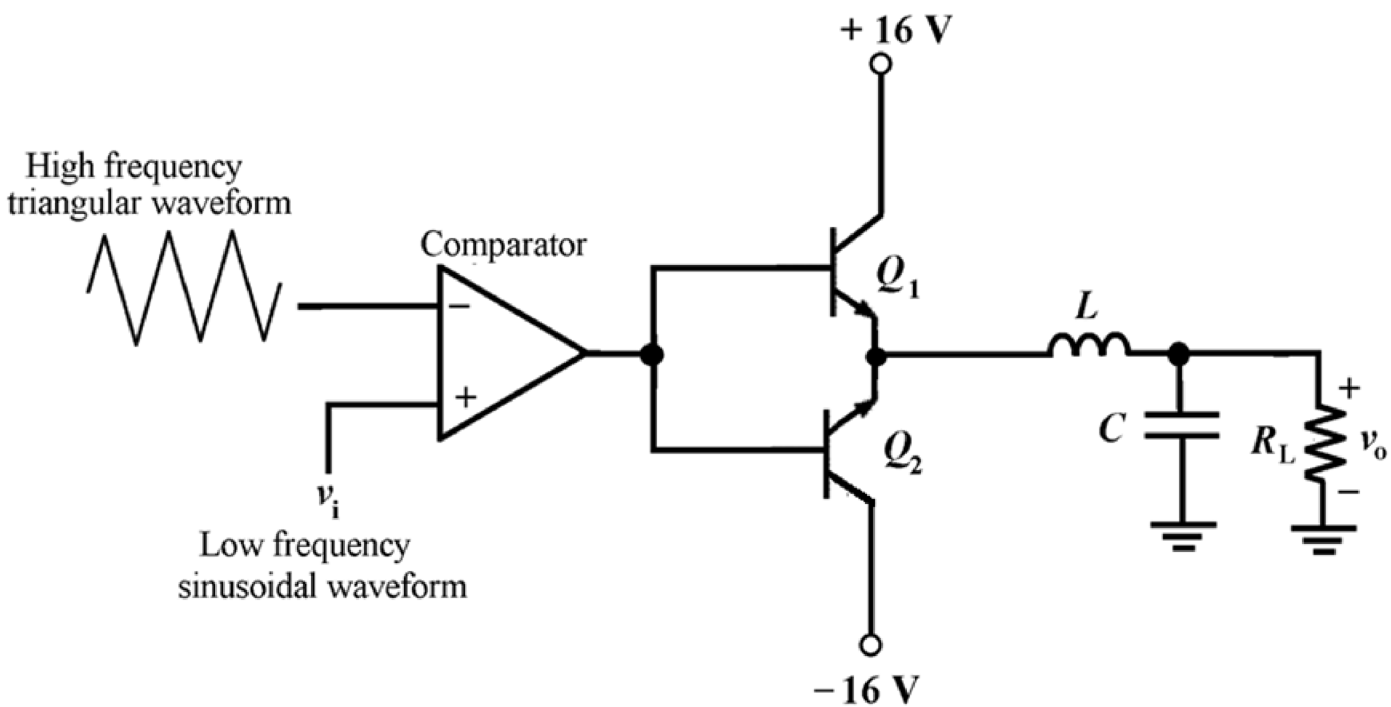
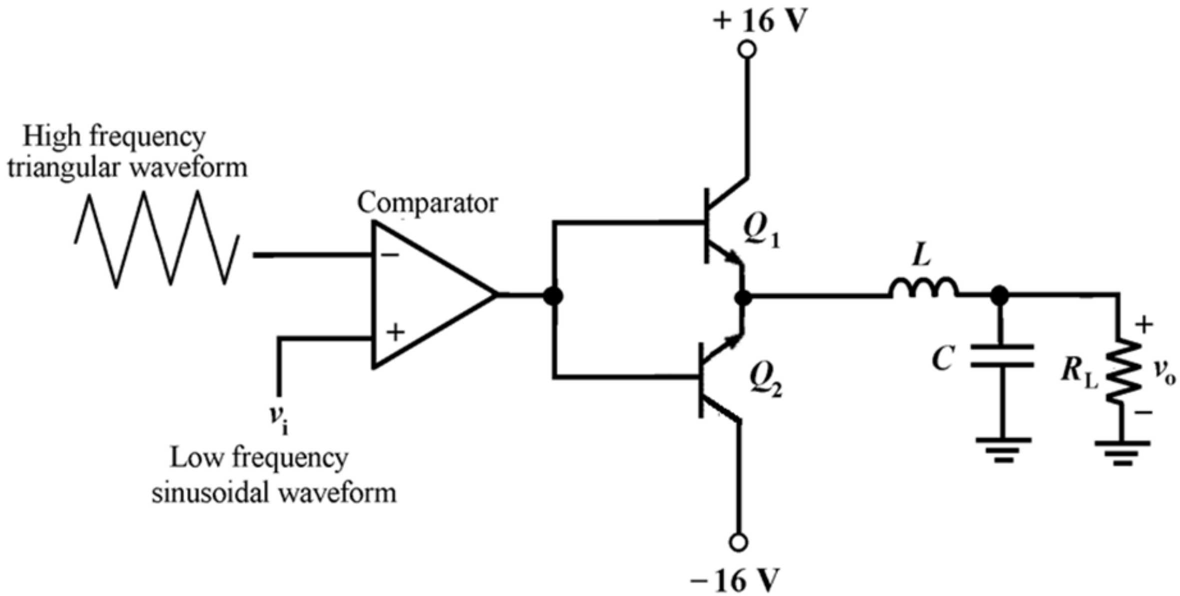


Figure 2

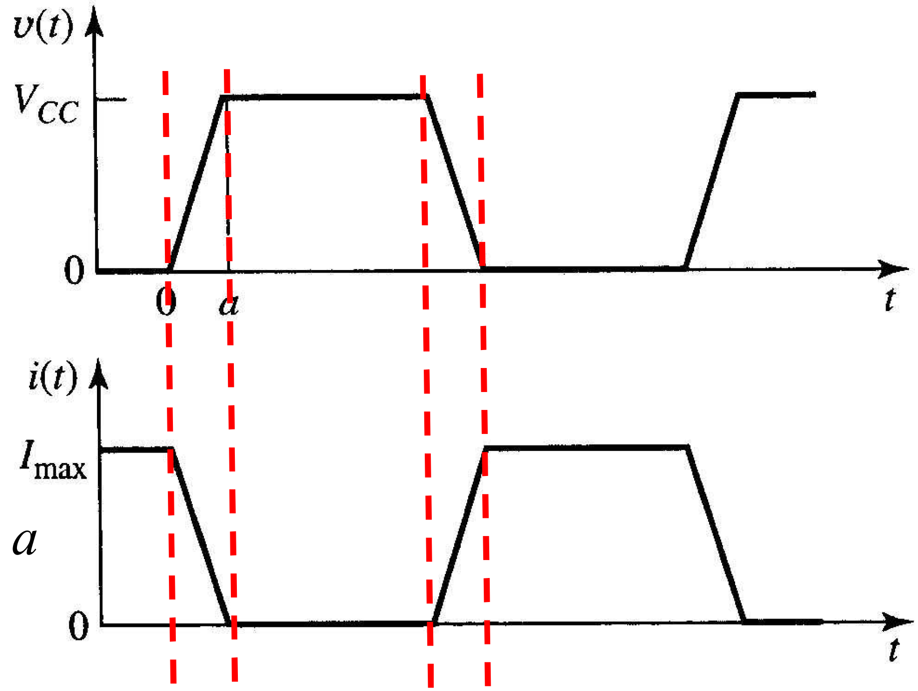


(a) Maximum power that can be delivered to the load:

$$V_{o,\max} = V_{CC} - V_{CE(sat)} = 16 - 0.3 = 15.7 \text{ V}$$

$$P_{L,\max} = \frac{V_{o,\max}{}^2}{2R_L} = \frac{(15.7)^2}{2\times 8} = 15.4 \text{ W}$$

(b) Each transistor has two switching transitions and the power dissipated in each transistor:



$$v(t) = \frac{t}{a} V_{CC} \quad \text{for} \quad 0 \le t \le a$$

$$i(t) = I_{max} - \frac{t}{a} I_{max} = \frac{V_{CC}}{R_L}\left(1 - \frac{t}{a}\right) \quad \text{for } 0 \le t \le a$$

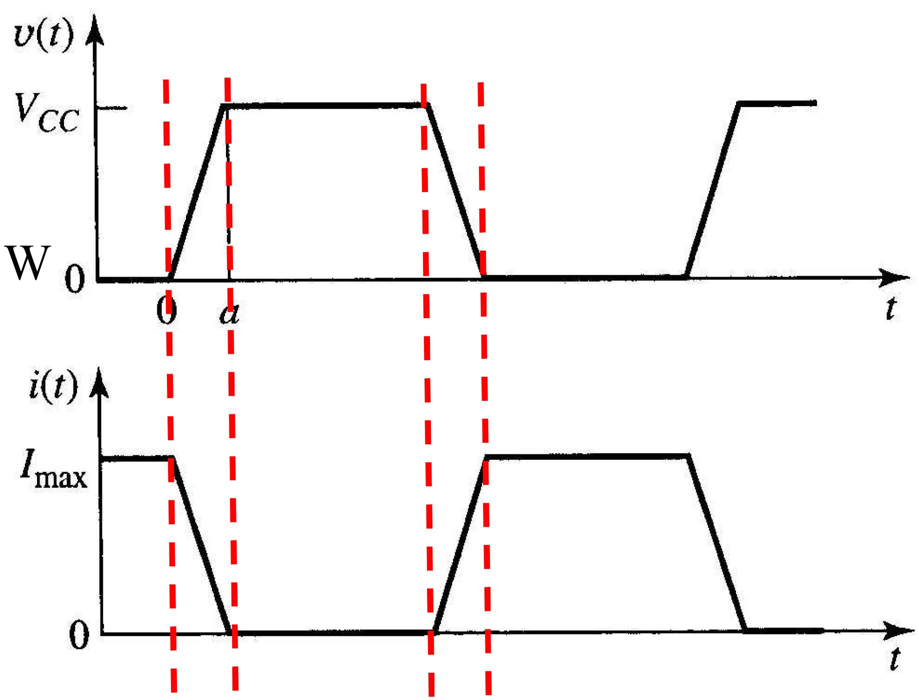
$$p(t) = v(t) i(t) = \frac{V_{CC}^{\;2}}{R_L}\left(\frac{t}{a} - \frac{t^2}{a^2}\right)$$

$$P_T = 2 \times \frac{1}{T}\int_0^a \frac{V_{CC}^{\;2}}{R_L}\left(\frac{t}{a} - \frac{t^2}{a^2}\right) dt = \frac{2}{T}\left(\frac{V_{CC}^{\;2}}{R_L}\right)\left[\frac{t^2}{2a} - \frac{t^3}{3a^2}\right]_0^a = \frac{a}{T}\left(\frac{V_{CC}^{\;2}}{3R_L}\right)$$

NANYANG TECHNOLOGICAL UNIVERSITY

$$P_T = \frac{a}{T}\left(\frac{V_{CC}{}^2}{3R_L}\right)$$

$$2P_T = \frac{a}{T}\left(\frac{2V_{o,max}{}^2}{3R_L}\right) = (0.05)\left(\frac{2\times 15.7^2}{3\times 8}\right) = 1.027 \text{ W}$$



$$I_{max} = \frac{V_{max}}{R_L} = \frac{15.7}{8} = 1.96 \text{ A}$$

$$\eta = \frac{P_{L,\max}}{P_{L,\max} + 2P_T + 0.9 I_{\max} V_{CE(sat)}} = \frac{15.4}{15.4 + 1.027 + 0.9\times 1.96\times 0.3} = 90.8\%$$

NANYANG TECHNOLOGICAL UNIVERSITY

(c)

$$2P_T = \frac{a}{T}\left(\frac{2V_{o,\max}{}^2}{3R_L}\right) = (0.025)\left(\frac{2\times 15.7^2}{3\times 8}\right) = 0.514\ \text{W}$$

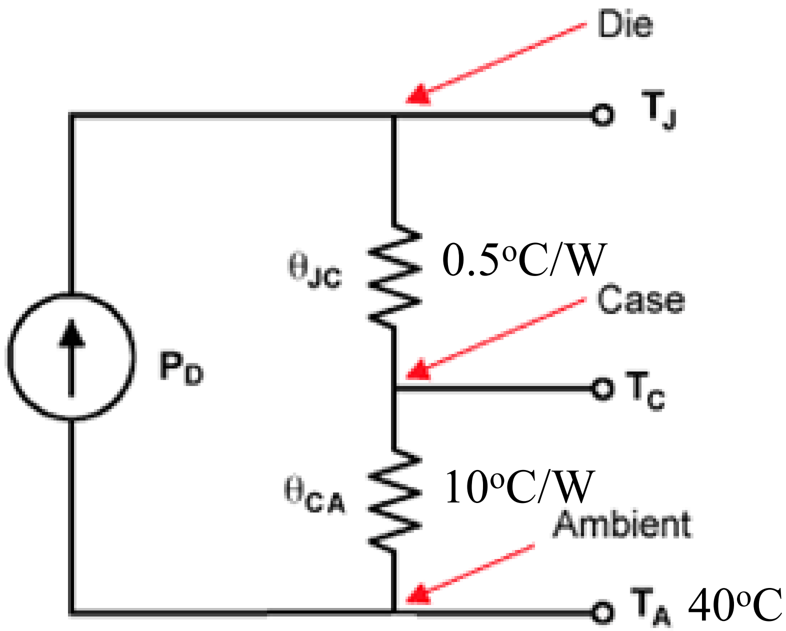
$$I_{\max} = 1.96\ \text{A}$$

$$\eta = \frac{P_{L,\max}}{P_{L,\max} + 2P_T + 0.95 I_{\max} V_{CE(sat)}} = \frac{15.4}{15.4 + 0.514 + 0.95\times 1.96\times 0.3} = 93.5\%$$

NANYANG TECHNOLOGICAL UNIVERSITY

3. The maximum permissible junction temperature of a power transistor is 150^{o}C. It is desired to operate the transistor with a power dissipation of 15 W in an ambient temperature of 40^{o}C. The thermal resistances of the transistor:
θ_{JC} = 0.5^{o}C/W (junction to case) and θ_{CA} = 10^{o}C/W (case to ambient).
(a) Determine whether a heat sink is required for this application.
(b) If a heat sink is needed, determine the required thermal resistance. To mount the heat sink, there is a mica washer between the transistor case and the heat sink. The thermal resistance of the mica washer θ_{W} = 0.5^{o}C/W.

(a)



$$\theta_T = \theta_{JC} + \theta_{CA} = 0.5^{\circ}\mathrm{C/W} + 10^{\circ}\mathrm{C/W} = 10.5\ ^{\circ}\mathrm{C/W}$$

$$T_J - T_A = \theta_T P_D = 10.5\ ^{\circ}\mathrm{C/W} \times 15\ \mathrm{W} = 157.5^{\circ}\mathrm{C}$$

$$T_J = 157.5^{\circ}\mathrm{C} + T_A = 157.5^{\circ}\mathrm{C} + 40^{\circ}\mathrm{C} = 197.5^{\circ}\mathrm{C}$$

The junction temperature has exceeded the maximum permissible junction temperature (150°C). Hence, a heat sink is necessary to reduce the total thermal resistance.

(b) Let set the T_J to its maximum permissible value of 150°C, then we can solve for the required total thermal resistance.

$$T_J - T_A = \theta_T P$$

$$150 - 40 = \theta_T \times 15$$

$$\theta_T = \frac{110}{15} = 7.33^{\circ}\,\mathrm{C/W}$$

$$\theta_T = \theta_{JC} + \theta_W + \theta_{SA} = 7.33^{\circ}\,\mathrm{C/W}$$

$$\theta_{SA} = \theta_T - (\theta_{JC} + \theta_W)$$

$$\theta_{SA} = 7.33 - (0.5 + 0.5)$$

$$\theta_{SA} = 6.33\ ^{\circ}\mathrm{C/W}$$