NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# Wideband Amplifiers

## EE4341: Advanced Analog Circuits

Professor Zheng Yuanjin

School of Electrical and Electronic Engineering

Email: YJZHENG@ntu.edu.sg

# Learning Objectives

By the end of this section, you should be able to:

- Exactly know how to draw the frequency response and the relationship between unity gain frequency and -3 dB frequency of an amplifier.
- Fully master how to analyse a C-E amplifier frequency response using the miller capacitor splitting technique.
- Understand how to analyse a CE-CB amplifier frequency response.
- Understand the concept of feedback technique extending the amplifier bandwidth, and calculating cascaded system's bandwidth.

# Wideband Amplifiers

## Topic 1: Amplifier Frequency Response

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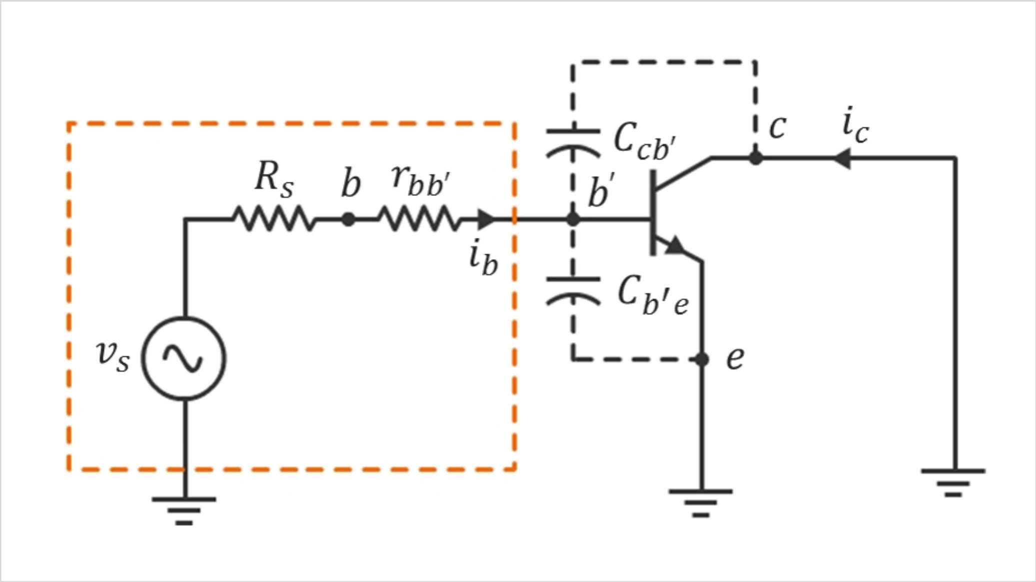
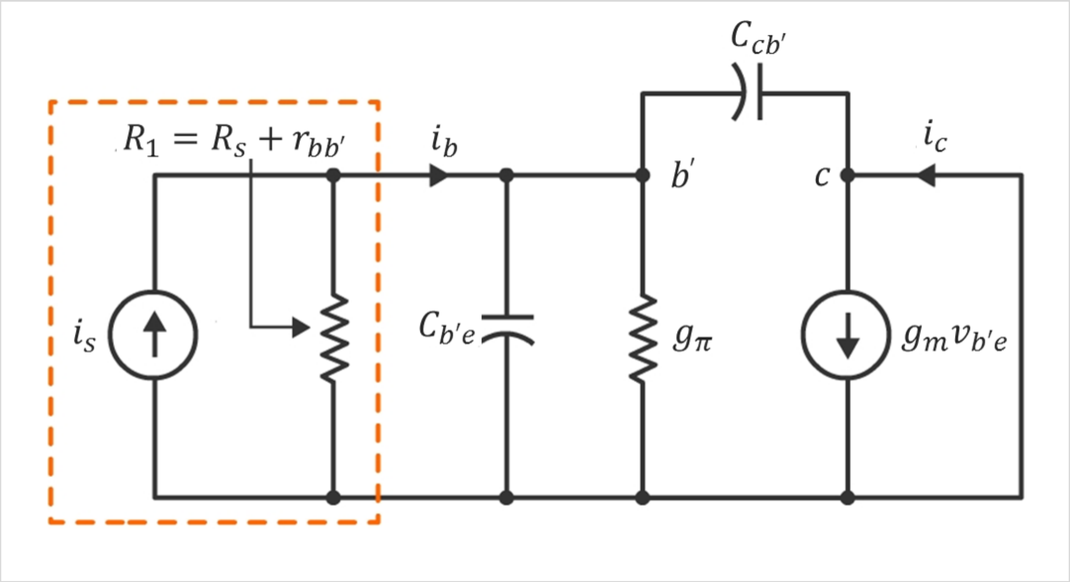
# Unity Gain Frequency of BJT

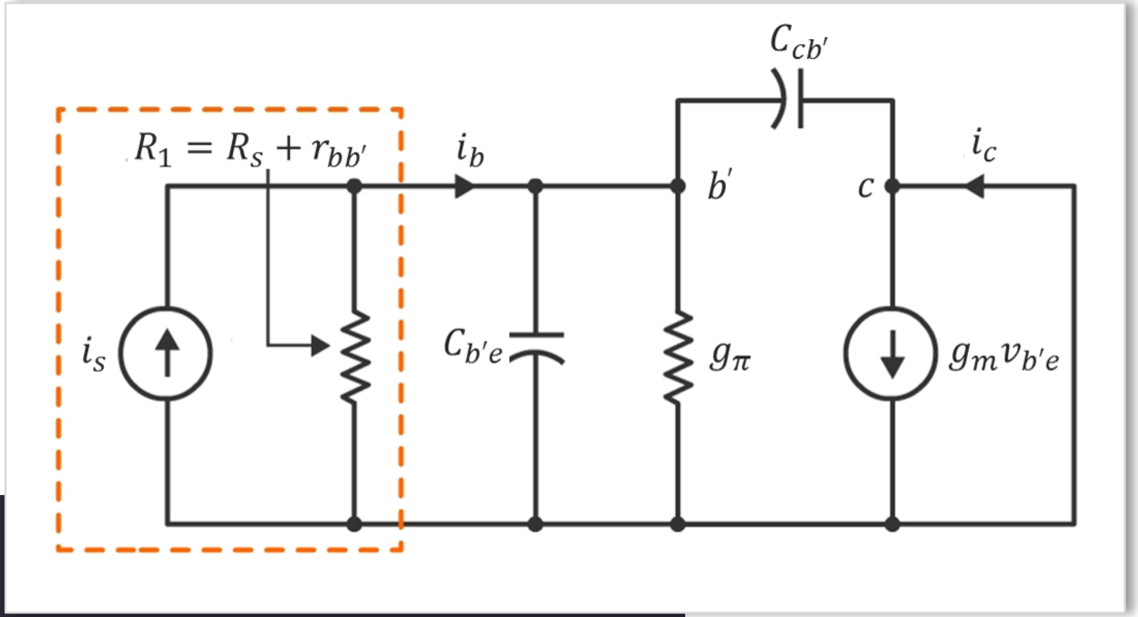
The short-circuit current gain is determined with output shorted.




$$i_s = \frac{v_s}{R_s + r_{bb'}}$$

# Unity Gain Frequency of BJT




$i_c = g_m v_{b'e}$

Apply KCL at node $b'$ :

$$i_b = g_\pi v_{b'e} + j\omega C_{b'e} v_{b'e} + j\omega C_{cb'}(v_{b'e} - v_{ce})$$

$$= g_\pi v_{b'e} + j\omega C_{b'e} v_{b'e} + j\omega C_{cb'} v_{b'e}$$

$$= [g_\pi + j\omega(C_{b'e} + C_{cb'})]v_{b'e}$$

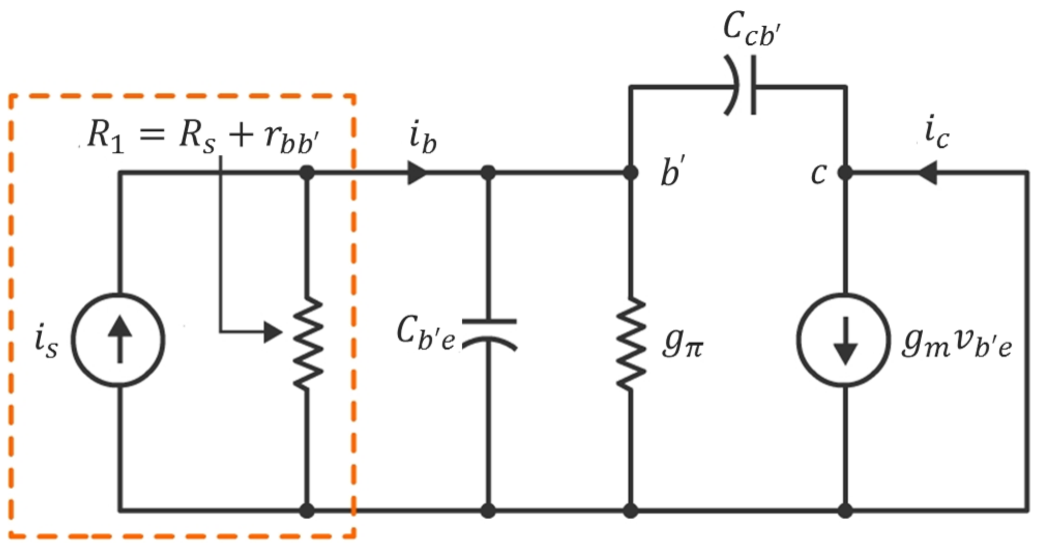
$$g_\pi = \frac{1}{r_\pi}$$

$$A_{i(sc)} = \frac{i_c}{i_b} = \frac{g_m}{g_\pi + j\omega(C_{b'e} + C_{cb'})}$$

$$= \left(\frac{g_m}{g_\pi}\right)\left[\frac{1}{1 + j\omega\,(C_{b'e} + C_{cb'})/g_\pi}\right]$$

$$\because \frac{g_m}{g_\pi} = \frac{I_C}{V_T} \times \frac{V_T}{I_B} = \beta$$

# Unity Gain Frequency of BJT




$$\therefore A_{i(sc)} = \frac{\beta}{1 + j\omega(C_{b'e} + C_{cb'})\,\beta/g_m}$$

By definition, unity gain frequency $f_T$ occurs when:

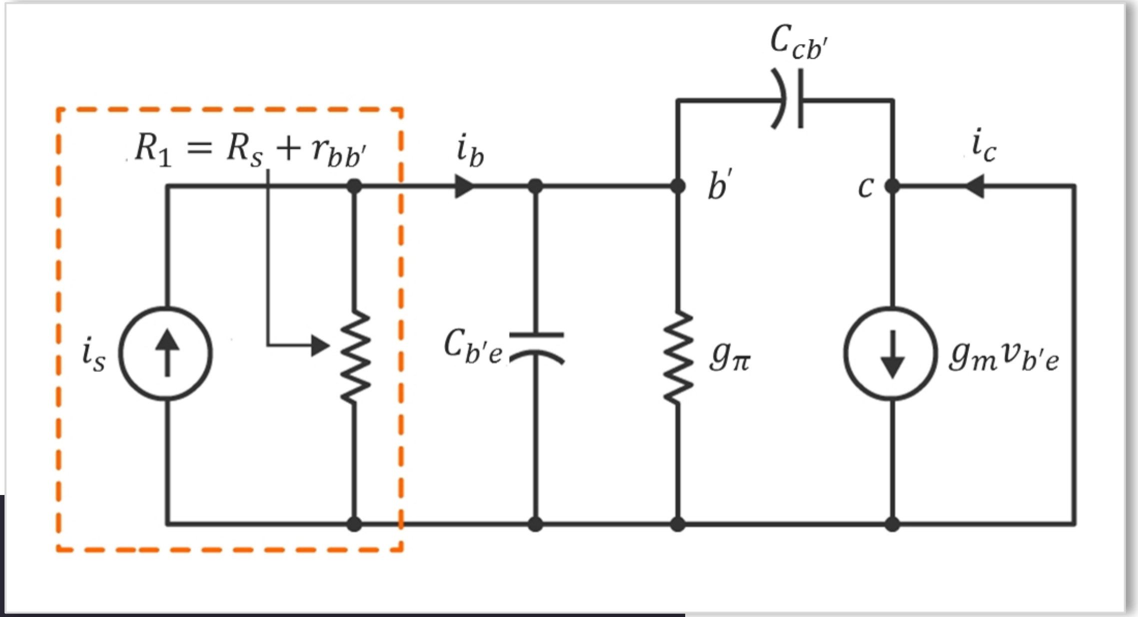
$$|A_{i(sc)}| = \left|\frac{\beta}{1 + j\omega_T(C_{b'e} + C_{cb'})\,\beta/g_m}\right| = 1$$

$$\because \omega_T(C_{b'e} + C_{cb'})\,\beta/g_m >> 1$$

$$\therefore \frac{\beta}{\frac{\omega_T(C_{b'e} + C_{cb'})\beta}{g_m}} \approx 1$$

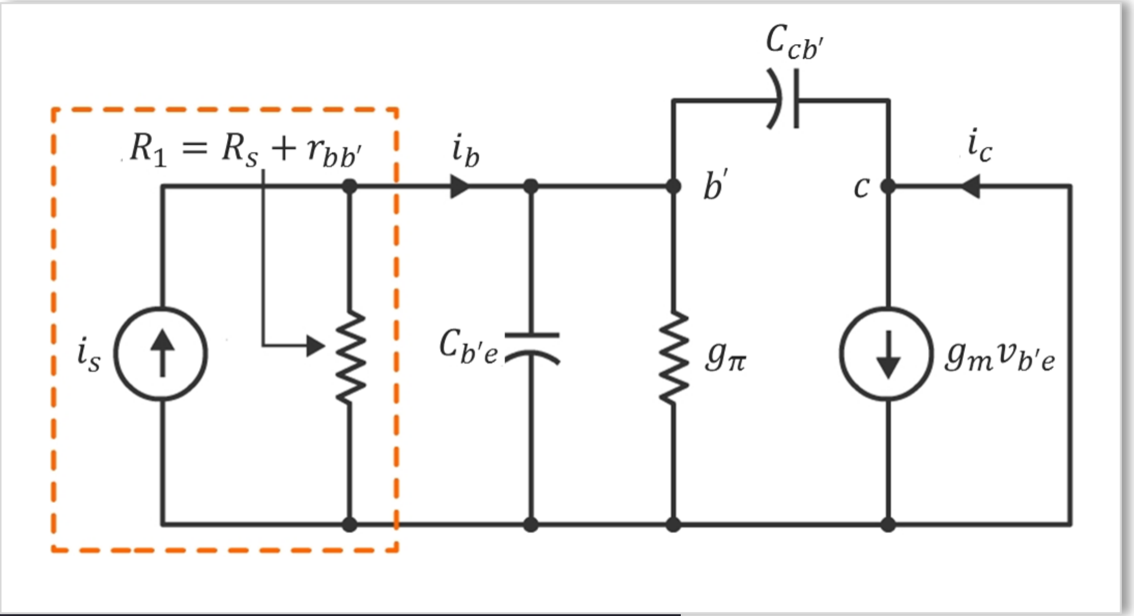
$$\omega_T \approx \frac{g_m}{C_{b'e} + C_{cb'}} \Rightarrow f_T = \frac{g_m}{2\pi(C_{b'e} + C_{cb'})}$$

# Unity Gain Frequency of BJT




**Note:** The above expression shows that $f_T$ is independent of $\beta$ but is dependent on the biasing point (that determines $g_m$) and the inherent parasitic capacitances of the device. As a general rule, to design a wideband amplifier with a -3 dB bandwidth of $\mathrm{BW}_{3\mathrm{dB}}$, the BJT must have a $f_T > 5$ to 10 times $\mathrm{BW}_{3\mathrm{dB}}$.

# Unity Gain Frequency of BJT




$$A_{i(sc)}(j\omega) = \frac{\beta}{1 + j\omega(C_{b'e} + C_{cb'})\,\beta/g_m}$$

$$\because\ \omega_T = \frac{g_m}{C_{b'e} + C_{cb'}}$$

$$\therefore\ A_{i(sc)}(j\omega) = \frac{\beta}{1 + j\left(\frac{\beta\omega}{\omega_T}\right)}$$

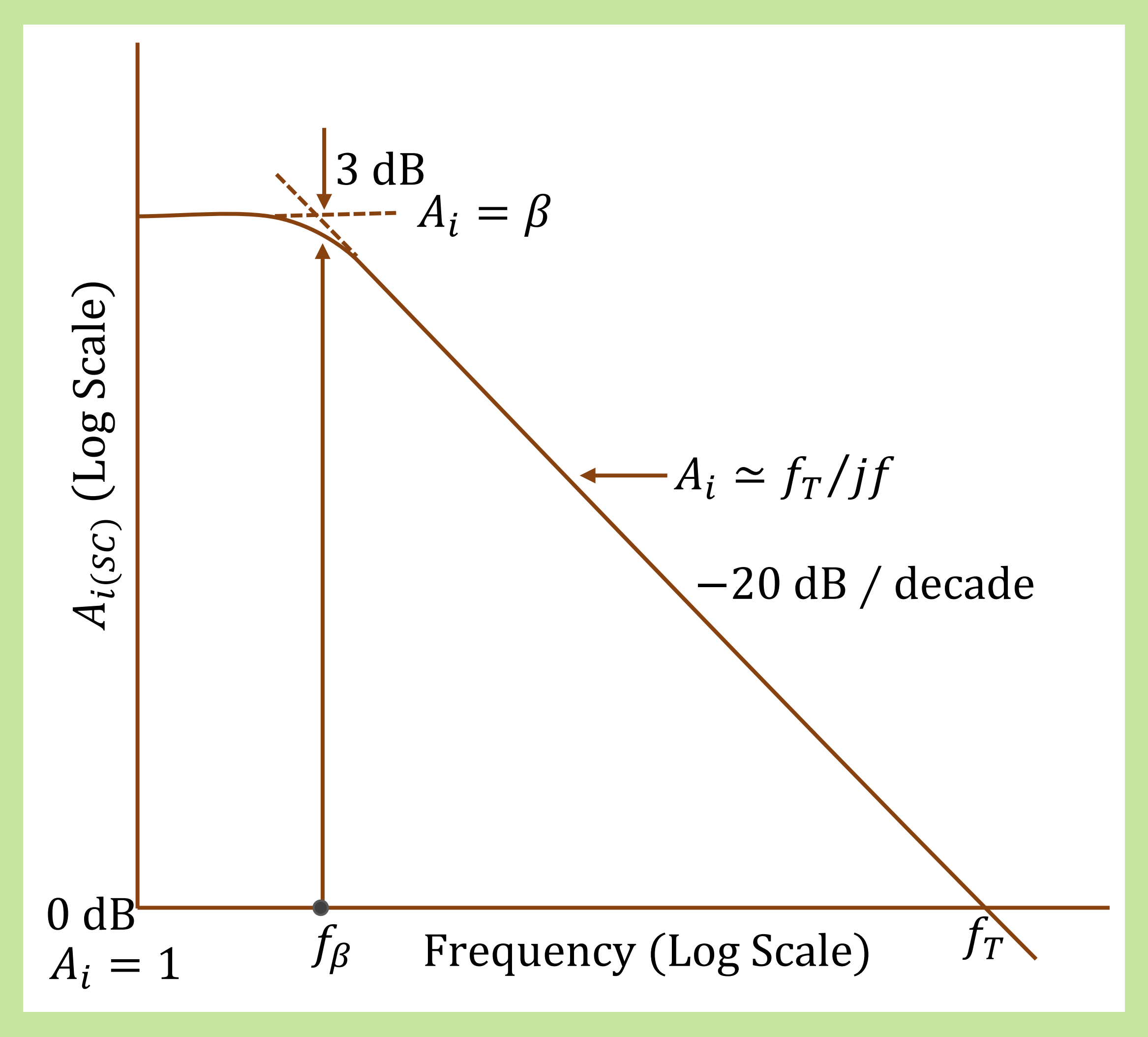
$$A_{i(sc)}(jf) = \frac{\beta}{1 + j\left(\frac{\beta f}{f_T}\right)}$$

# Unity Gain Frequency of BJT

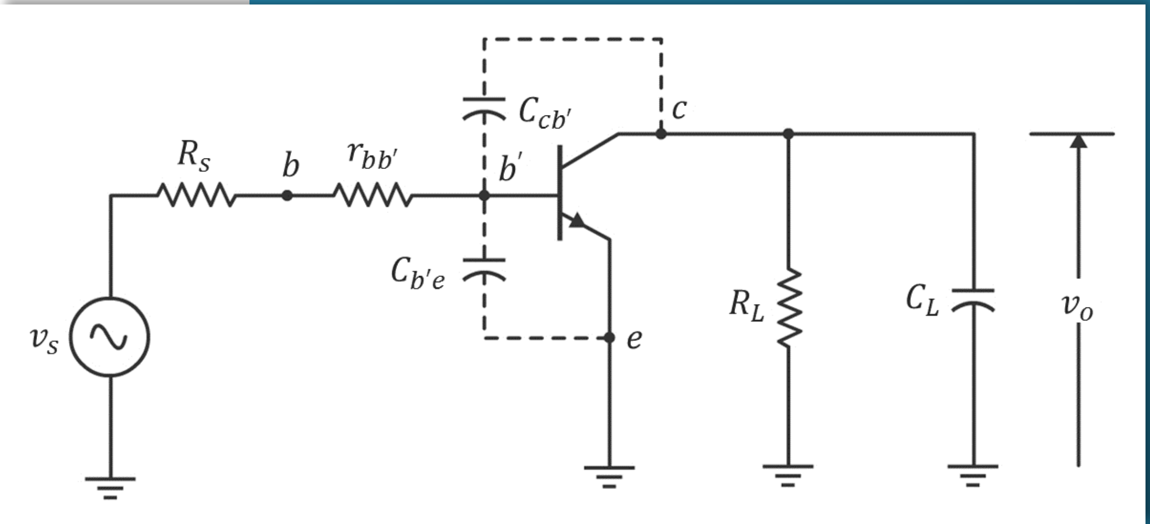
$$A_{i(sc)} = \frac{\beta}{1 + j\left(\frac{\beta f}{f_T}\right)} = \frac{\beta}{1 + j\left(\frac{f}{f_\beta}\right)}$$

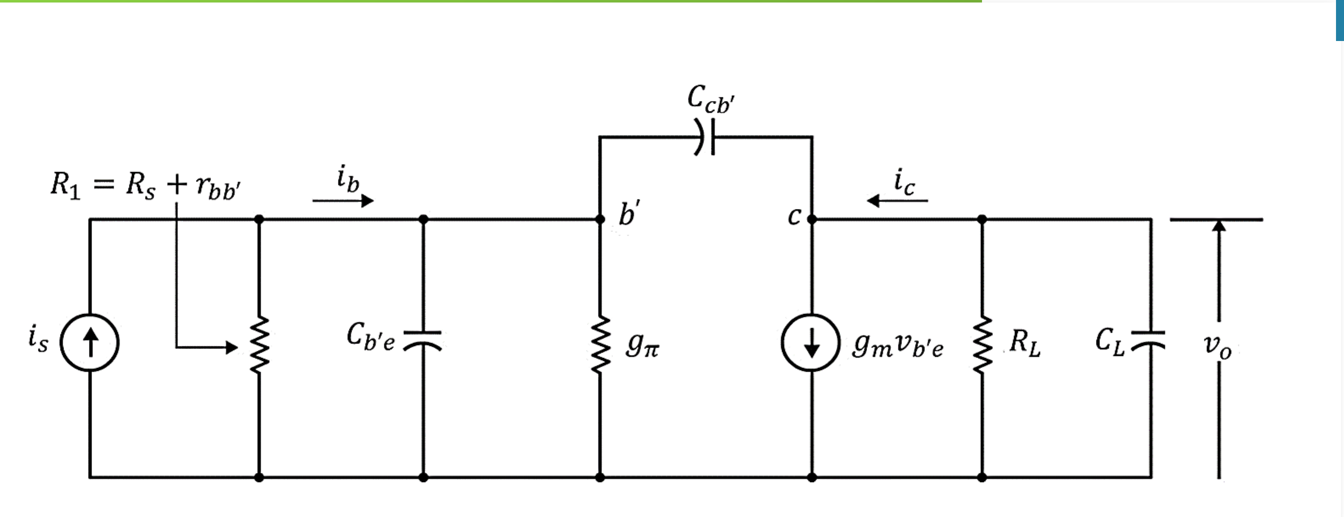
Where, $f_\beta = \frac{f_T}{\beta}$ is the -3 dB frequency.

$$A_{i(sc)} \approx \frac{\beta}{j\left(\frac{f}{f_\beta}\right)} = \frac{\beta f_\beta}{jf} = \frac{f_T}{jf} \text{ for } f > 3f_\beta$$

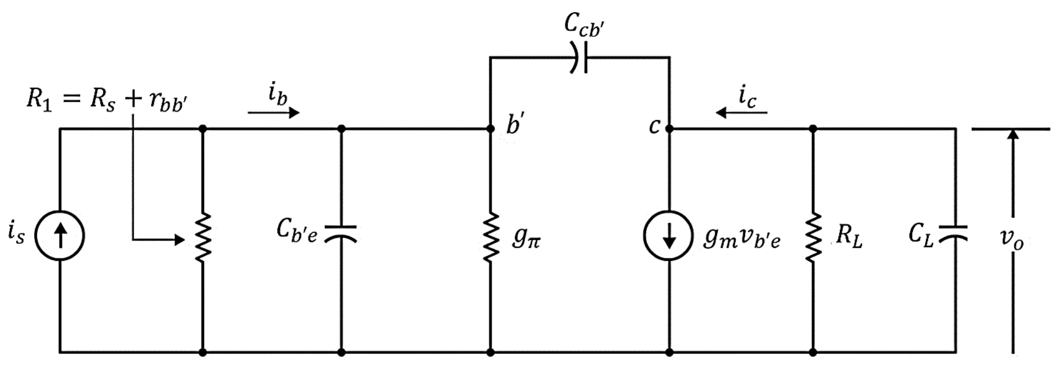


# Frequency Response of CE Amplifier



# Frequency Response of CE Amplifier




$$G_1 = \frac{1}{R_S + r_{bb'}} \qquad G_L = \frac{1}{R_L}$$

Apply KCL at node $b'$:

$$v_{b'e}(G_1 + g_\pi + j\omega C_{b'e}) = i_s + (v_o - v_{b'e})j\omega C_{cb'}$$

$$v_{b'e}[G_1 + g_\pi + j\omega(C_{b'e} + C_{cb'})] - j\omega C_{cb'} v_o = i_s \text{ ------- } (1)$$

Apply KCL at node c:

$$v_o(G_L + j\omega C_L) + g_m v_{b'e} = (v_{b'e} - v_o)j\omega C_{cb'}$$

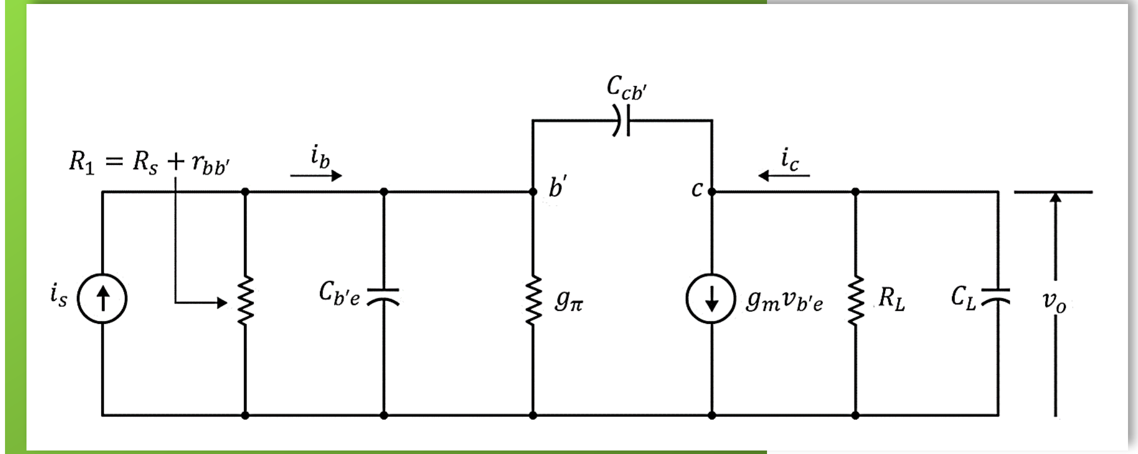
$$v_o[G_L + j\omega(C_L + C_{cb'})] + g_m v_{b'e} = j\omega C_{cb'} v_{b'e}$$

$$v_o = \frac{-(g_m - j\omega C_{cb'})v_{b'e}}{G_L + j\omega(C_L + C_{cb'})} \text{ -------- } (2)$$

Substitute (2) into (1):

$$v_{b'e}\left[G_1 + g_\pi + j\omega(C_{b'e} + C_{cb'}) + \frac{j\omega C_{cb'}(g_m - j\omega C_{cb'})}{G_L + j\omega(C_{cb'} + C_L)}\right] = i_s$$

# Frequency Response of CE Amplifier



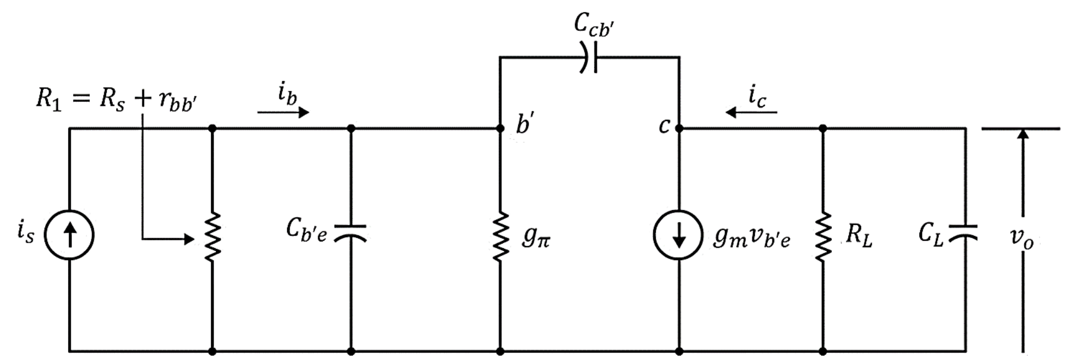

$$v_{b'e} = \frac{i_s}{G_1 + g_\pi + j\omega(C_{b'e} + C_{cb'}) + \dfrac{j\omega C_{cb'}(g_m - j\omega C_{cb'})}{G_L + j\omega(C_{cb'} + C_L)}} \quad ----(3)$$

Substitute (3) into (2):

$$\therefore v_o = \left[\frac{-(g_m - j\omega C_{cb'})}{G_L + j\omega(C_{cb'} + C_L)}\right] \frac{i_s}{G_1 + g_\pi + j\omega(C_{b'e} + C_{cb'}) + \dfrac{j\omega C_{cb'}(gm - j\omega C_{cb'})}{G_L + j\omega(C_{cb'} + C_L)}}$$

$$i_s = \frac{v_s}{R_S + r_{bb'}} = v_s G_1$$

# Frequency Response of CE Amplifier




$$A_v = \frac{v_o}{v_s} = \frac{-(g_m - j\omega C_{cb'})G_1}{[G_L + j\omega(C_{cb'} + C_L)]\left[G_1 + g_\pi + j\omega(C_{b'e} + C_{cb'}) + \frac{j\omega C_{cb'}(gm - j\omega C_{cb'})}{G_L + j\omega(C_{cb'} + C_L)}\right]}$$

In the frequency range of interest ($f$ in the MHz range):

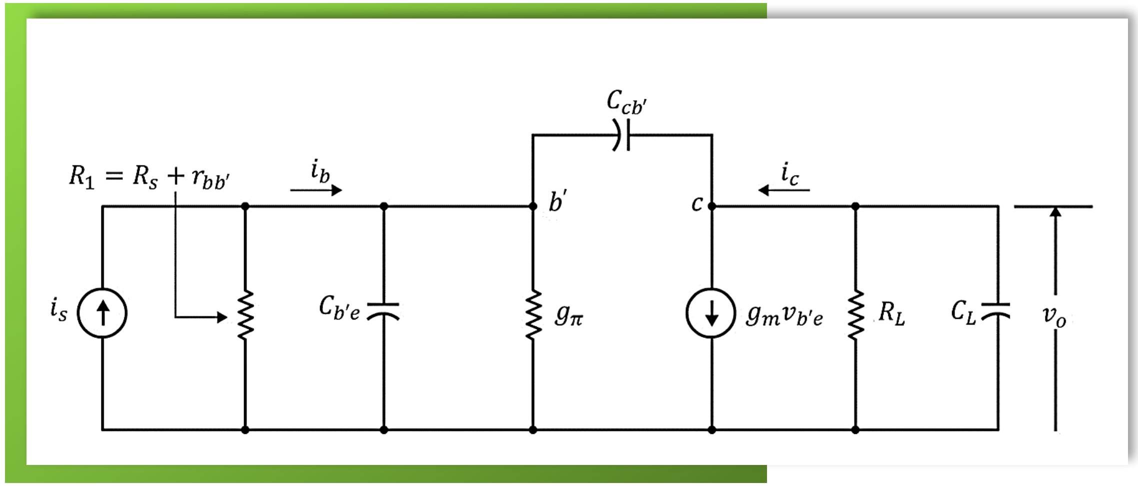
$g_m$(in the range of $10^{-3}$) >> $\omega C_{cb'}$(in the range of $10^{6} \times 10^{-12} \approx 10^{-6}$)

$\therefore (g_m - j\omega C_{cb'})G_1 \approx g_m G_1$

$G_1$(in the range of $10^{-2}$) >> $g_\pi$(in the range of $10^{-4}$)

$\therefore G_1 + g_\pi \approx G_1$

# Frequency Response of CE Amplifier

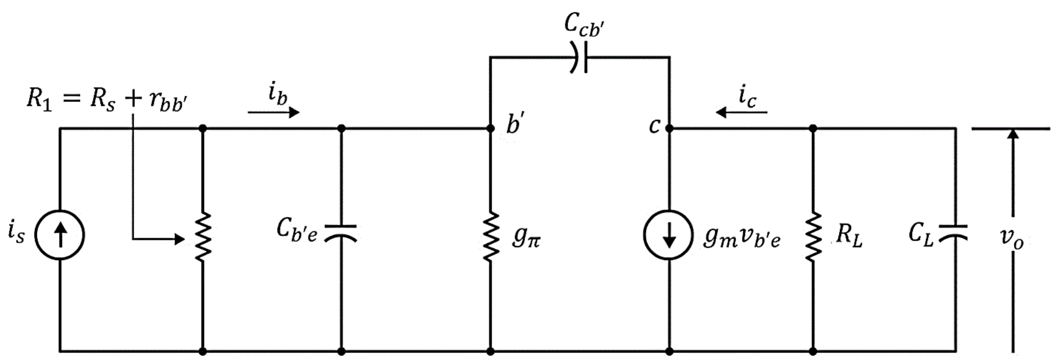



$G_L$(in the range of $10^{-3}$) >> $\omega(C_{cb'} + C_L)$(in the range of $10^6 \times 10^{-12} \approx 10^{-6}$)

$$\therefore G_L + j\omega(C_{cb'} + C_L) \approx G_L$$

$$A_v \approx \frac{-g_m G_1}{[G_L + j\omega(C_{cb'} + C_L)]\left[G_1 + j\omega(C_{b'e} + C_{cb'}) + \frac{j\omega C_{cb'} g_m}{G_L}\right]}$$

# Frequency Response of CE Amplifier



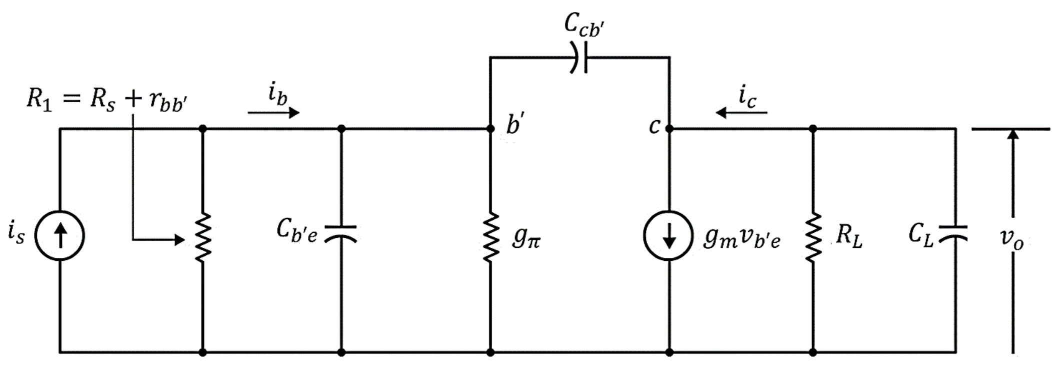

$$A_v = \frac{-g_m G_1}{[G_L + j\omega(C_{cb'} + C_L)][G_1 + j\omega(C_{b'e} + C_{cb'} + g_m R_L C_{cb'})]}$$

$$= \frac{-g_m G_1}{G_L G_1 \left[1 + \frac{j\omega(C_{cb'} + C_L)}{G_L}\right]\left[1 + \frac{j\omega(C_{b'e} + C_{cb'} + g_m R_L C_{cb'})}{G_1}\right]}$$

$$= \frac{-g_m R_L}{\left[1 + \frac{j\omega(C_{b'e} + C_{cb'} + g_m R_L C_{cb'})}{G_1}\right]\left[1 + \frac{j\omega(C_{cb'} + C_L)}{G_L}\right]}$$

$$= \frac{-g_m R_L}{(1 + j\,\omega/\omega_1)(1 + j\,\omega/\omega_2)}$$

# Frequency Response of CE Amplifier




The two break frequencies:

$$\omega_1 = \frac{G_1}{C_{b'e} + C_{cb'}(1 + g_m R_L)} = \frac{1}{(R_S + r_{bb'})[C_{b'e} + C_{cb'}(1 + g_m R_L)]}$$

$$\therefore f_1 = \frac{1}{2\pi(R_S + r_{bb'})C_i}$$ where, $C_i = C_{b'e} + (1 + g_m R_L)C_{cb'}$

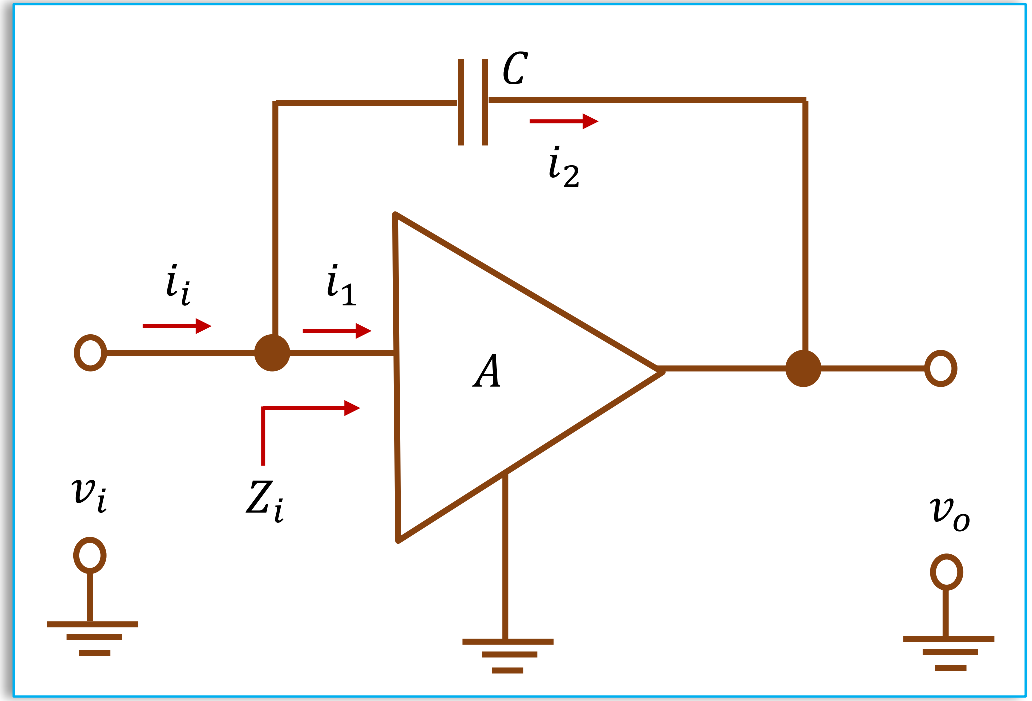
$$\omega_2 = \frac{G_L}{C_{cb'} + C_L} = \frac{1}{R_L(C_{cb'} + C_L)}$$

$$\therefore f_2 = \frac{1}{2\pi R_L(C_{cb'} + C_L)}$$

**Note:** The effective input capacitance is equal to $C_{cb'}$ multiplied by $(1 + g_m R_L)$.

This is known as the Miller effect.

# Miller Effect




$$i_i = i_1 + i_2$$

$$i_i = \frac{v_i}{Z_i} + \frac{v_i - v_o}{(1/j\omega C)}$$

$$i_i = \frac{v_i}{Z_i} + j\omega C(v_i - v_o)$$

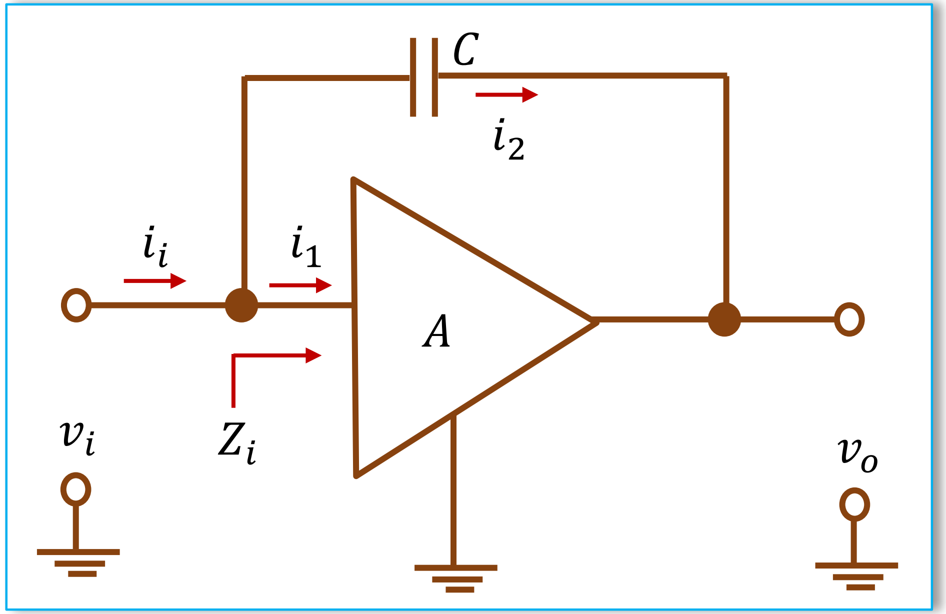
$$\because A = \frac{v_o}{v_i}$$

$$\therefore i_i = \frac{v_i}{Z_i} + j\omega C(v_i - Av_i)$$

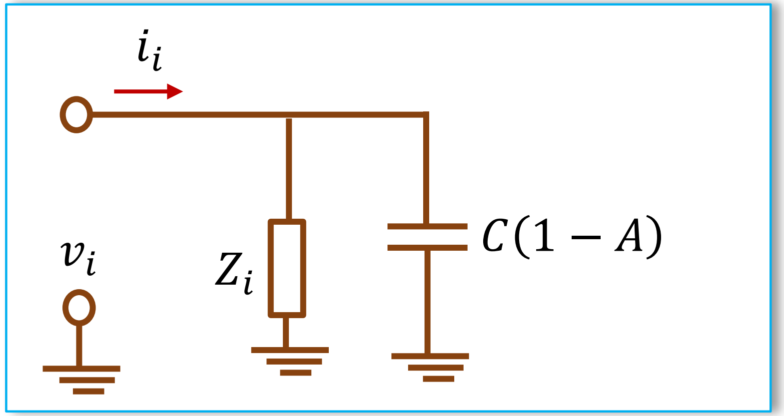
# Miller Effect

$$i_i = \frac{v_i}{Z_i} + j\omega C(v_i - Av_i)$$

$$i_i = v_i\left[\frac{1}{Z_i} + j\omega C(1 - A)\right]$$




- The above expression shows that the effective input impedance is basically $Z_i$ in parallel with a capacitance $C(1\text{-}A)$.
- The magnification of the capacitance $C$ at the input of an amplifier is called the 'Miller Effect'.



# Miller Effect: Example

The BJT device parameters are:

$r_{bb'} = 60\ \Omega$

$R_s = 40\ \Omega$

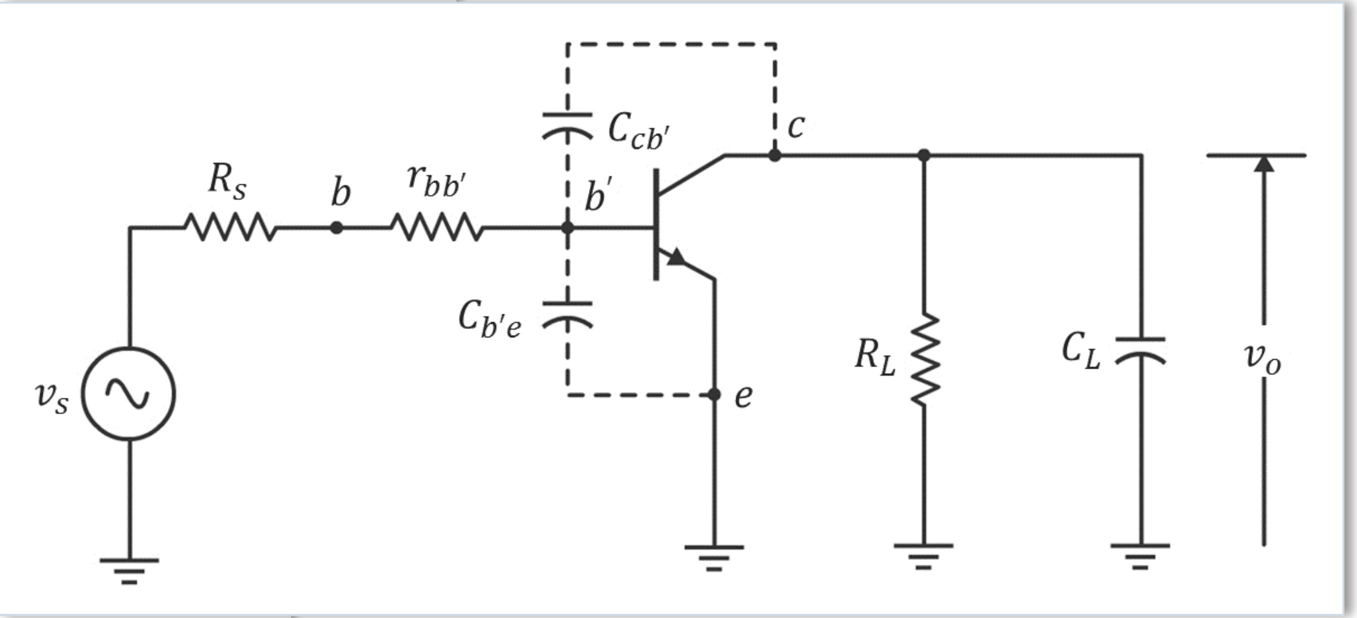
$C_{cb'} = 1.5\ \mathrm{pF}$

$f_T = 1.6\ \mathrm{GHz}$

Load capacitance: $C_L = 1\ \mathrm{pF}$

The BJT is biased at $2.5\ \mathrm{mA}$.

Determine the voltage gain frequency response for different values of $R_L$ varying from $30\ \Omega$ to $10\ \mathrm{k}\Omega$.



# Miller Effect: Example

$$\omega_T = 2\pi\times1.6\times10^9 = 1.005\times10^{10}\ \text{rad/s}$$

$$g_m = \frac{I_C}{V_T} = \frac{2.5\ \text{mA}}{26\ \text{mV}} = 0.096\ \text{S}$$

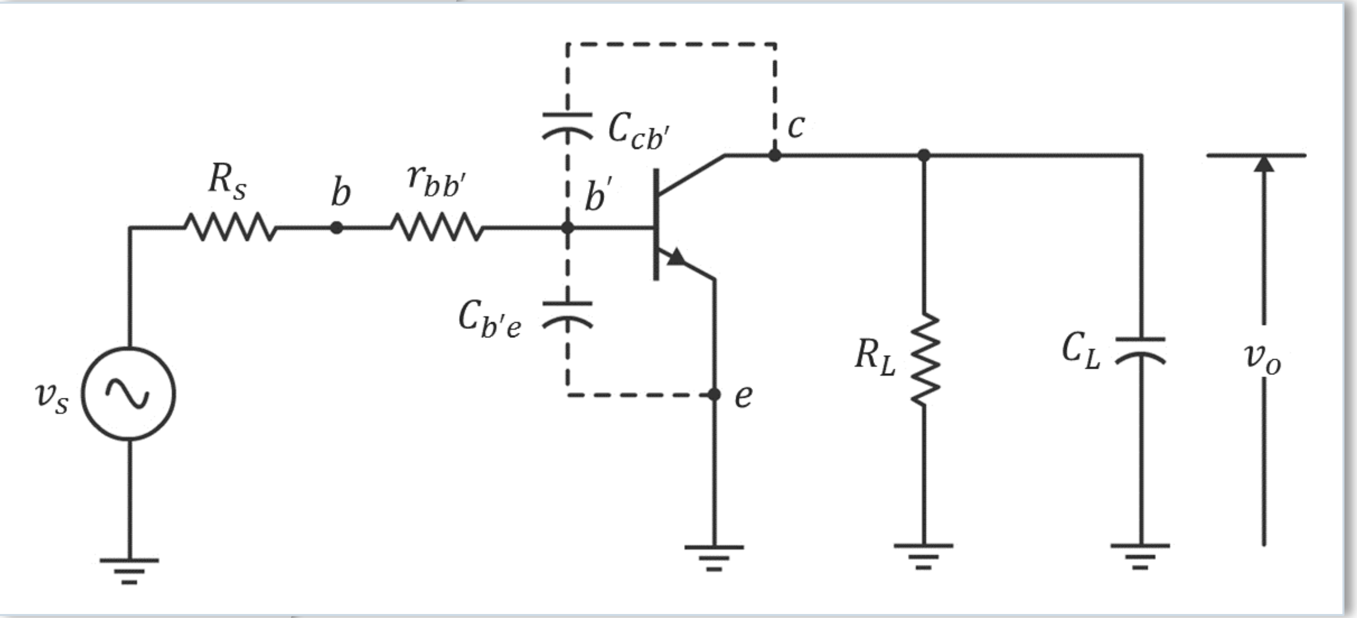
$$C_{b'e} + C_{cb'} = \frac{g_m}{\omega_T} = \frac{0.096}{1.005\times10^{10}} = 9.6\ \text{pF}$$

$$|A_{v(MID)}| = g_m R_L = (0.096)R_L$$

$$f_1 = \frac{1}{2\pi(R_s + r_{bb'})(C_{b'e} + C_{cb'} + g_m R_L C_{cb'})}$$

$$= \frac{1}{2\pi(100)(9.6 + 0.096\times R_L\times1.5)\times10^{-12}}$$

$$= \frac{1}{6.28\times10^{-10}(11.1 + 0.144R_L)}$$



# Miller Effect: Example

If $R_L = 1\ \mathrm{k}\Omega$, $f_1 = 10.26\ \mathrm{MHz}$

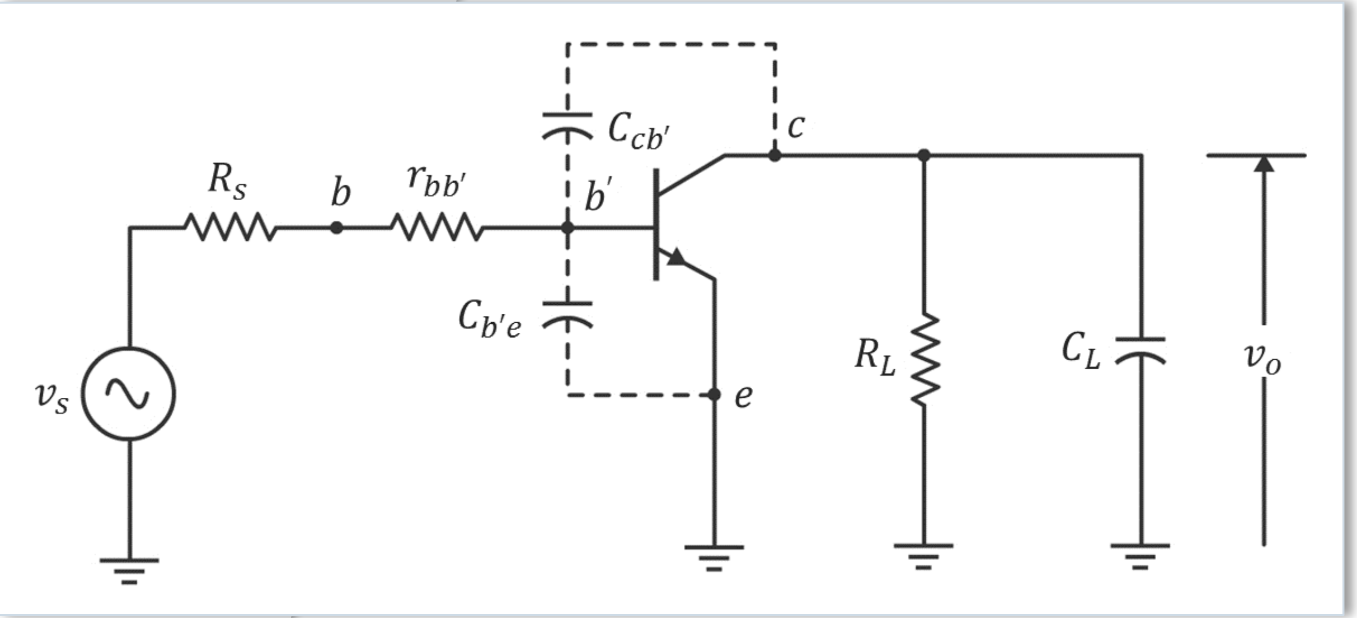
$$f_2 = \frac{1}{2\pi \times R_L (C_{cb'} + C_L)}$$

$$= \frac{1}{2\pi \times R_L (2.5\times 10^{-12})}$$

$$= \frac{1}{1.57\times 10^{-11} \times R_L}$$

If $R_L = 1\ \mathrm{k}\Omega$, $f_2 = 63.7\ \mathrm{MHz}$

Both $f_1$ and $f_2$ can be calculated for different load $R_L$.



# Miller Effect: Example

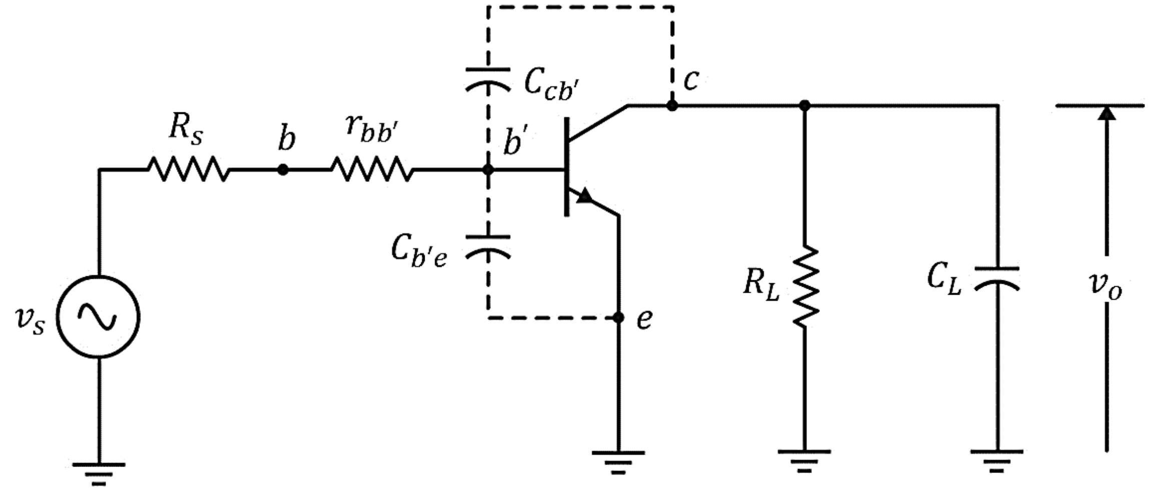
From the calculations, we have:

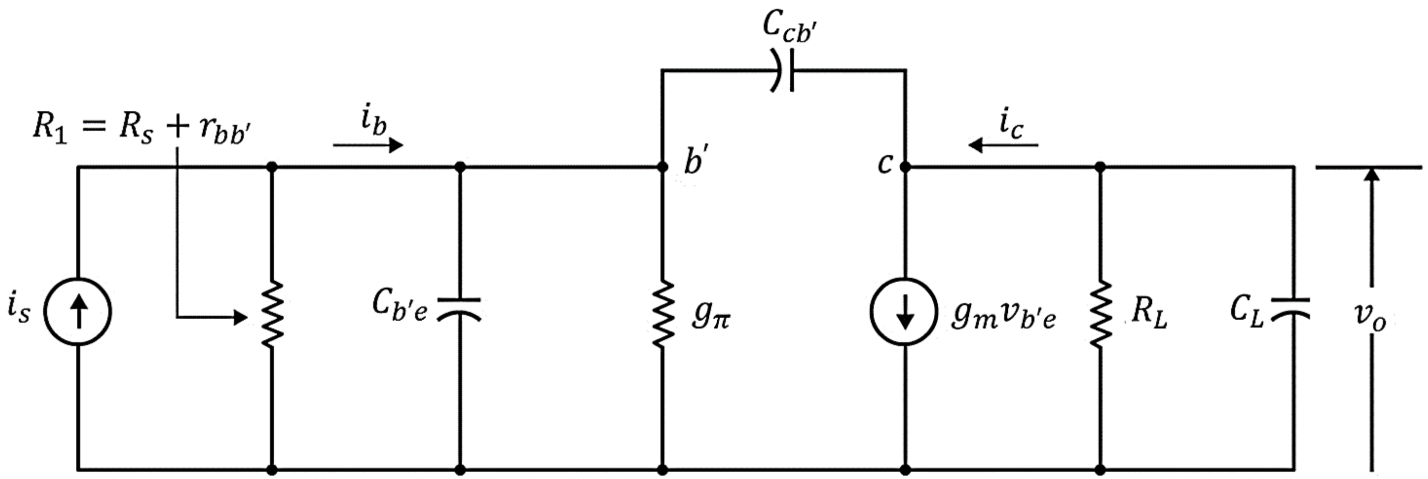
| $R_L$ (Ω) | $f_1$ (MHz) | $f_2$ (MHz) | $BW$ (MHz) | $A_{V(\text{MID})}$ | $A_{V(\text{MID})} \times BW$ (MHz) |
|---|---|---|---|---|---|
| 30 | 110 | 2,122 | 110 | 3 | 330 |
| 100 | 64 | 637 | 64 | 10 | 640 |
| 300 | 29 | 212 | 28 | 30 | 868 |
| 1,000 | 10 | 64 | 10 | 100 | 1,000 |
| 3,000 | 3.5 | 21 | 3.5 | 300 | 1,038 |
| 10,000 | 1.05 | 6.4 | 1.04 | 1,000 | 1,040 |

$f_1 << f_2$ for all load resistance values. Therefore, $f_1$ is the primary factor that determines the -3dB BW of the amplifier.

Larger $R_L$ gives larger mid-band gain but at the expense of reduction in BW.

# Summary: C-E Amplifier Frequency Analysis using Miller Effect



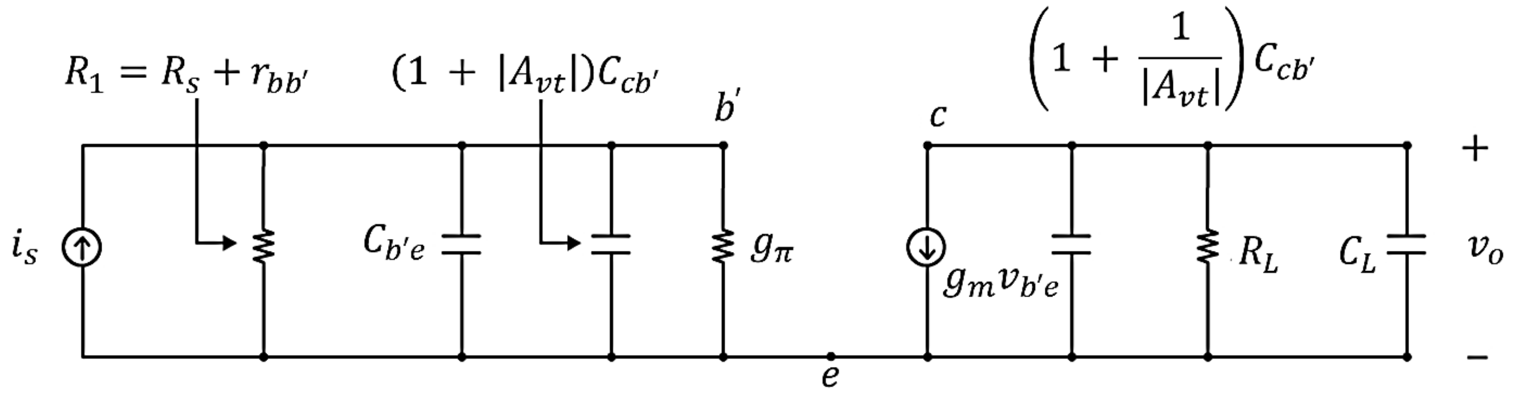




Miller capacitor splitting

$A_{vt}$ : Terminal voltage gain

$A_{vt} = -g_m R_L$

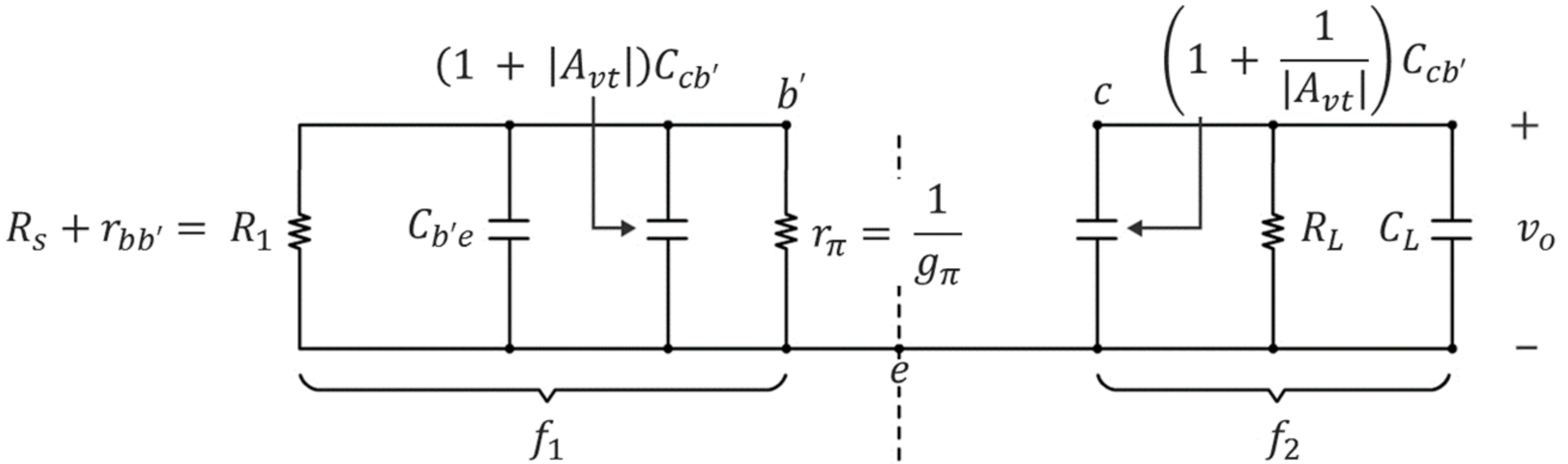
# Summary: C-E Amplifier Frequency Analysis using Miller Effect




Open current source: $i_s$

$\therefore v_{b'e} = 0$

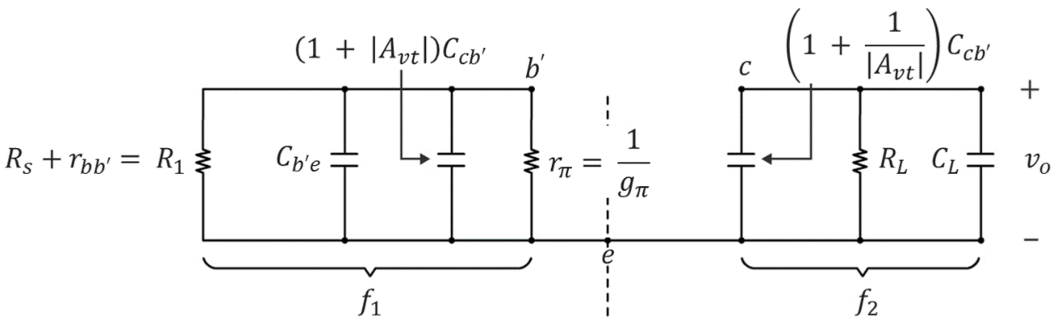
$g_m v_{b'e}$= 0, open it.




$r_\pi \gg R_s + r_{bb'}$

$R_s + r_{bb'} // r_\pi$
$\cong R_s + r_{bb'}$

# Summary: C-E Amplifier Frequency Analysis using Miller Effect




Calculate break frequency $f_1$ and $f_2$.

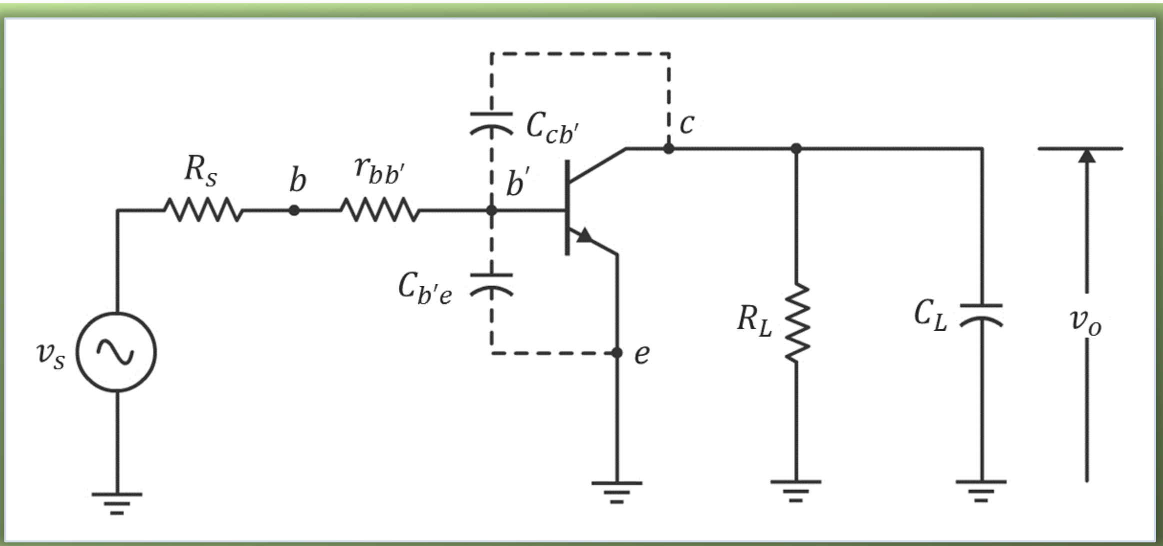
$$f_1 = \frac{1}{2\pi[(R_s + r_{bb'})\ //\ r_\pi][C_{b'e} + (1 + g_m R_L)C_{cb'}]}$$

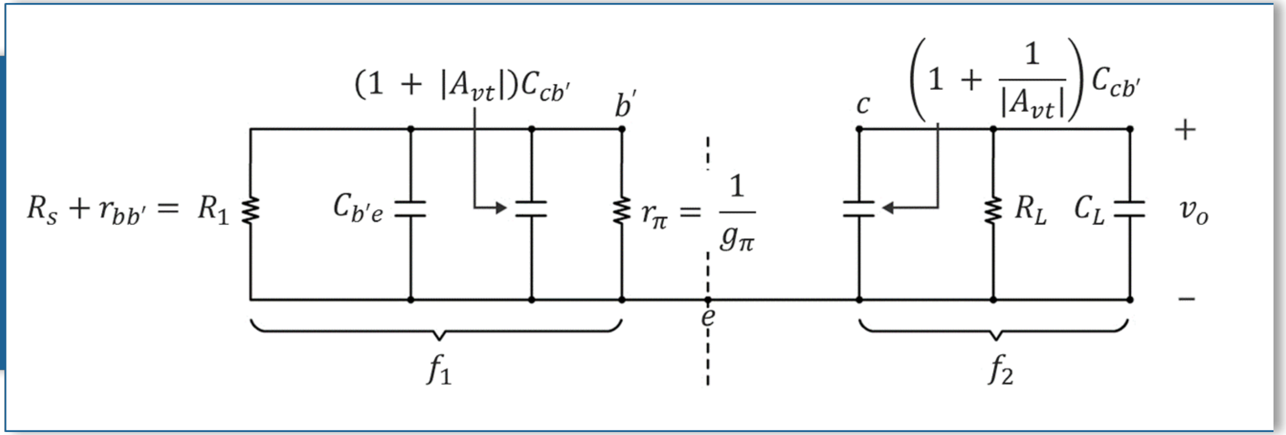
$$\cong \frac{1}{2\pi[(R_s + r_{bb'})][C_{b'e} + (1 + g_m R_L)C_{cb'}]}$$

$$f_2 = \frac{1}{2\pi R_L\left[\left(1 + \frac{1}{g_m R_L}\right)C_{cb'} + C_L\right]}$$

$$\cong \frac{1}{2\pi R_L(C_{cb'} + C_L)}$$

# Limitation of CE Stage for Wideband Application




$$|A_{v(MID)}| = g_m R_L$$

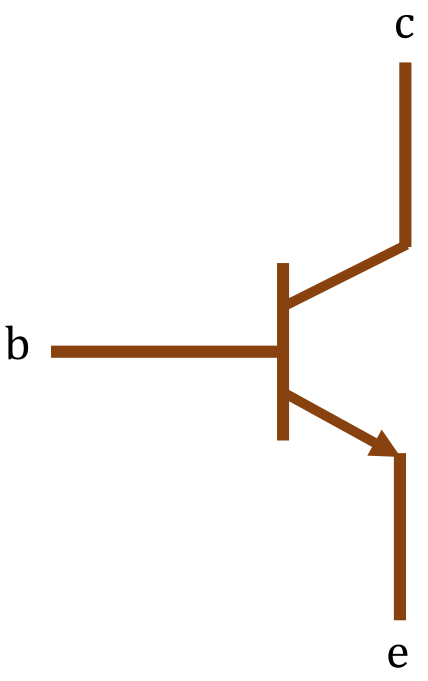
$$f_1 = \frac{1}{2\pi(R_s + r_{bb'})C_i}$$

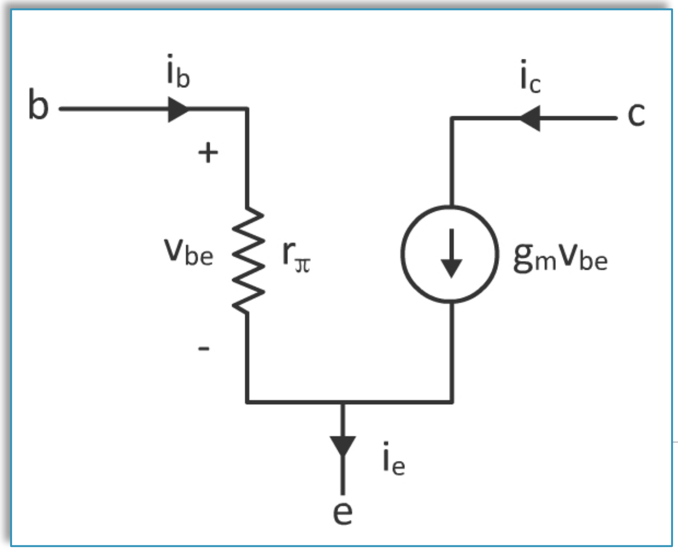
Where, $C_i = C_{b'e} + \left(1 + |A_{v(MID)}|\right)C_{cb'}$

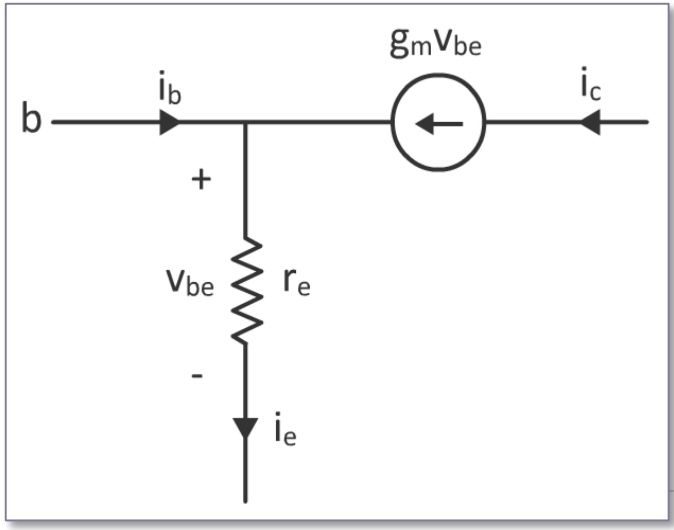
$$f_2 = \frac{1}{2\pi R_L(C_{cb'} + C_L)}$$

The BW is determined by $f_1$ and $C_i$ is large due to **Miller effect**. Hence, CE amplifier alone is not suitable for wideband application.

# BJT Models




$$\boxed{r_\pi = \frac{V_T}{I_B}}$$

$$i_c = g_m v_{be}$$

$$v_{be} = i_b r_\pi$$

$$i_e = i_b + i_c$$

$$\boxed{r_e = \frac{r_\pi}{(\beta+1)} \approx \frac{r_\pi}{g_m r_\pi} = \frac{1}{g_m}}$$

$$\boxed{r_e = \frac{V_T}{I_E}}$$

$$v_{be} = i_e r_e = i_b(\beta + 1) r_e$$

$$r_\pi = (\beta + 1) r_e$$

$$i_e = i_b + i_c$$

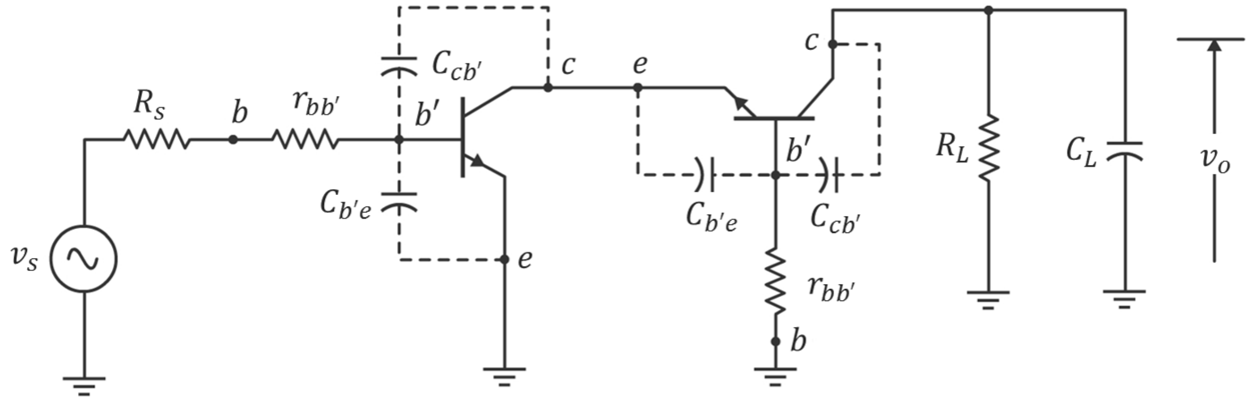
# CE-CB Configuration (Cascode Amplifier)

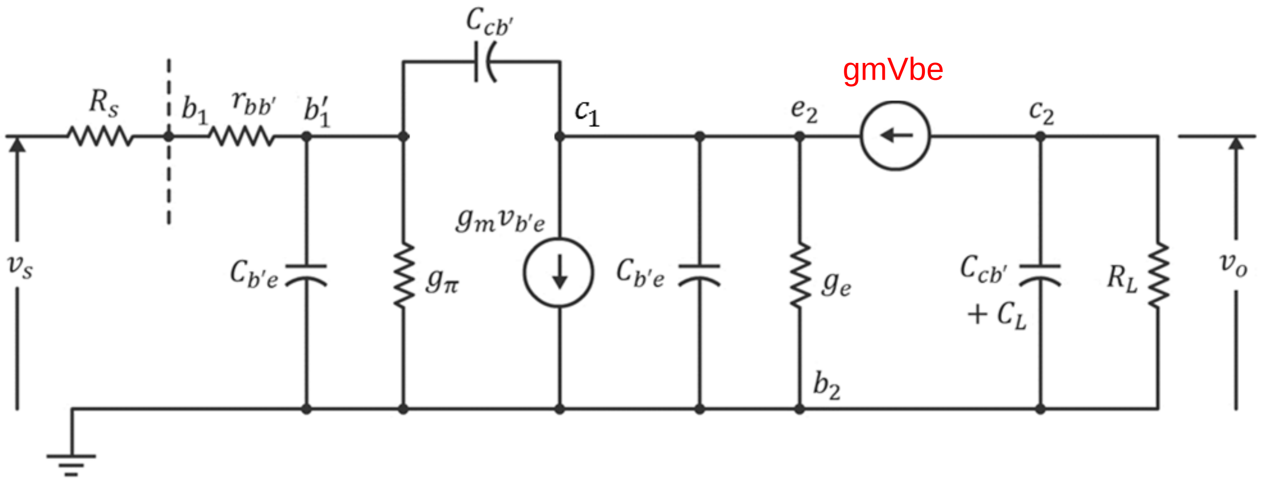
The term **cascode** means **<u>cas</u>cading tri<u>ode</u>** (it has been used since 1939).

$$r_e = \frac{(r_\pi + r_{bb'})}{(\beta + 1)} \approx \frac{r_\pi}{(\beta + 1)} \because r_\pi >> r_{bb'}$$

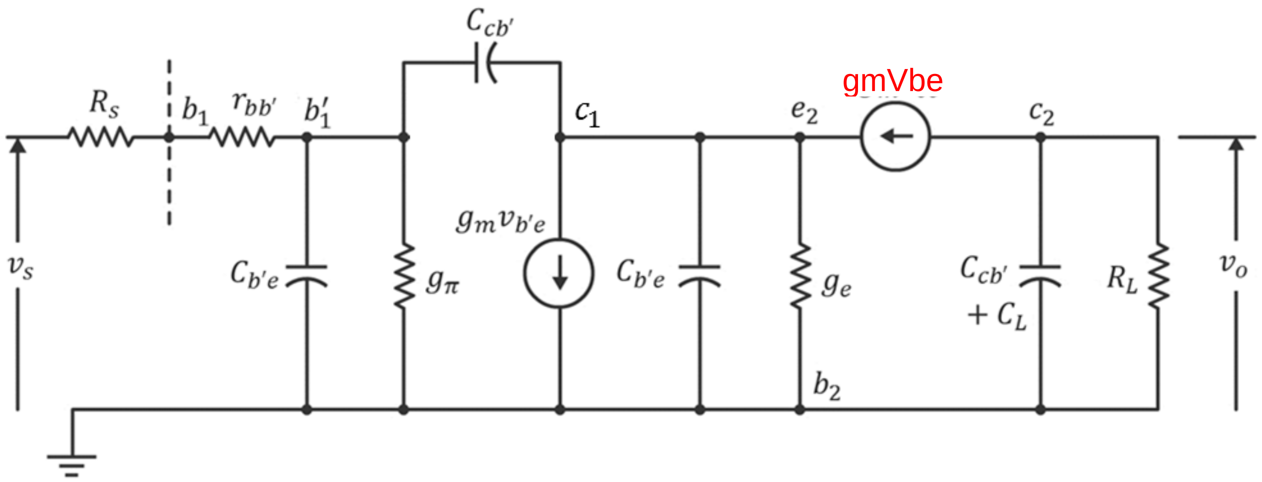
$$\approx \frac{r_\pi}{\beta} = \frac{r_\pi}{g_m r_\pi} = \frac{1}{g_m}$$

For CB amplifier, the effect of $r_{bb'}$ can be neglected.



# CE-CB Configuration (Cascode Amplifier)




$$Y_{L(CE)} \approx g_e + j\omega(C_{b'e} + C_{cb'})$$

$$= \frac{I_Q}{V_T} + j\omega(C_{b'e} + C_{cb'})$$

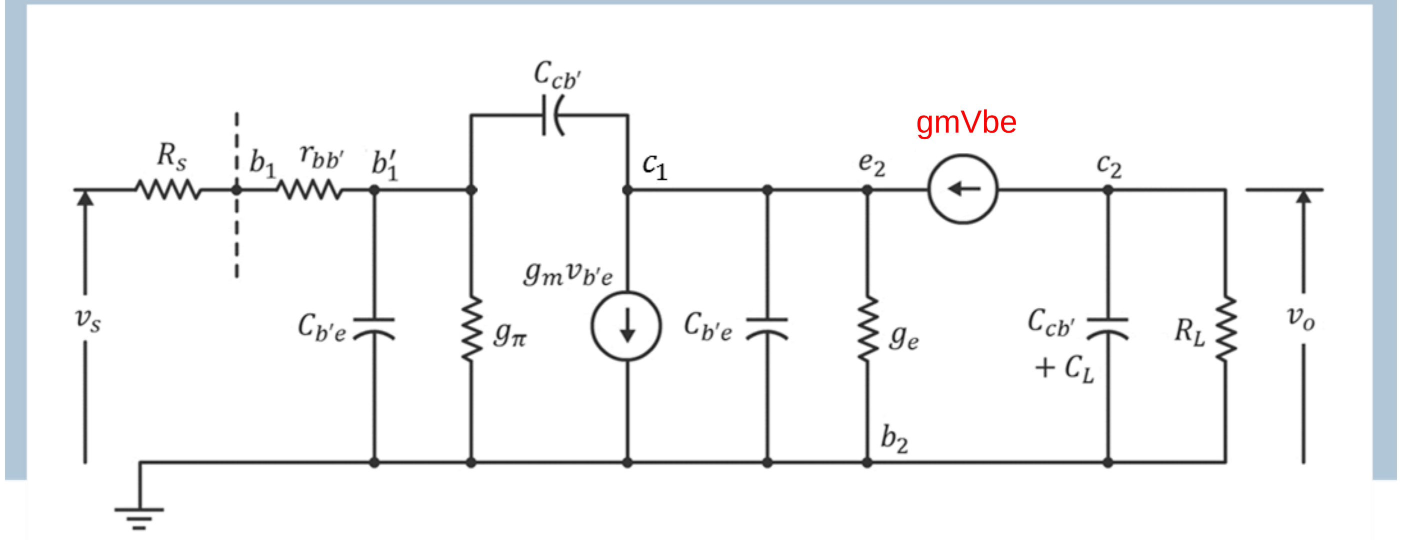
Where, $g_e = \dfrac{1}{r_{e2}} = \dfrac{I_{E2}}{V_T}$, $I_Q = I_{E2} = I_{C1}, g_e = g_m$

$$Y_{L(CE)} = g_m + j\omega(C_{b'e} + C_{cb'}) = g_m\left[1 + \frac{j\omega(C_{b'e} + C_{cb'})}{g_m}\right] = g_m\left(1 + \frac{j\omega}{\omega_T}\right)$$

$$Y_{L(CE)} = g_m\left(1 + \frac{jf}{f_T}\right) \quad \text{For } f << f_T,\ Y_{L(CE)} \approx g_m$$

The voltage gain for CE stage is: $A_{v(CE)} = -g_m\left(\dfrac{1}{Y_{L(CE)}}\right) = -\dfrac{g_m}{g_m} = -1$

# CE-CB Configuration (Cascode Amplifier)




$$\omega_1 = \frac{1}{(R_S + r_{bb'})\left[C_{b'e} + C_{cb'}\left(1 + |A_{v(CE)}|\right)\right]}$$

$$= \frac{1}{(R_S + r_{bb'})(C_{b'e} + 2C_{cb'})}$$

$\because C_{b'e} >> C_{cb'}$

$$\therefore C_{b'e} + 2C_{cb'} \approx C_{b'e} + C_{cb'} \approx \frac{g_m}{\omega_T}$$

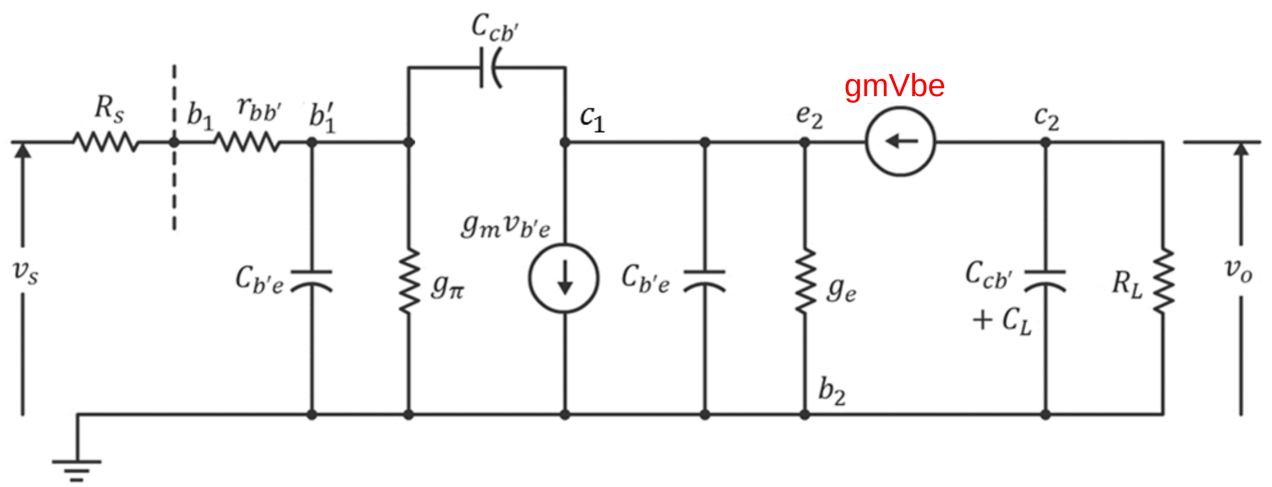
$$\omega_1 = \frac{\omega_T}{g_m(R_s + r_{bb'})} \Rightarrow f_1 = \frac{f_T}{g_m(R_s + r_{bb'})}$$

**Note:** Now, $f_1$ is independent of $R_L$.

The second break frequency $\omega_2$ is: $$\omega_2 = \frac{g_e}{C_{b'e} + C_{cb'}\left(1 + \frac{1}{|A_{v(CE)}|}\right)} \approx \frac{g_m}{C_{b'e} + 2C_{cb'}} \approx \omega_T \Rightarrow f_2 \approx f_T$$

$\omega_2$ will not have any significant effect in finding the overall BW.

# CE-CB Configuration (Cascode Amplifier)




The third break frequency $\omega_3$ is:

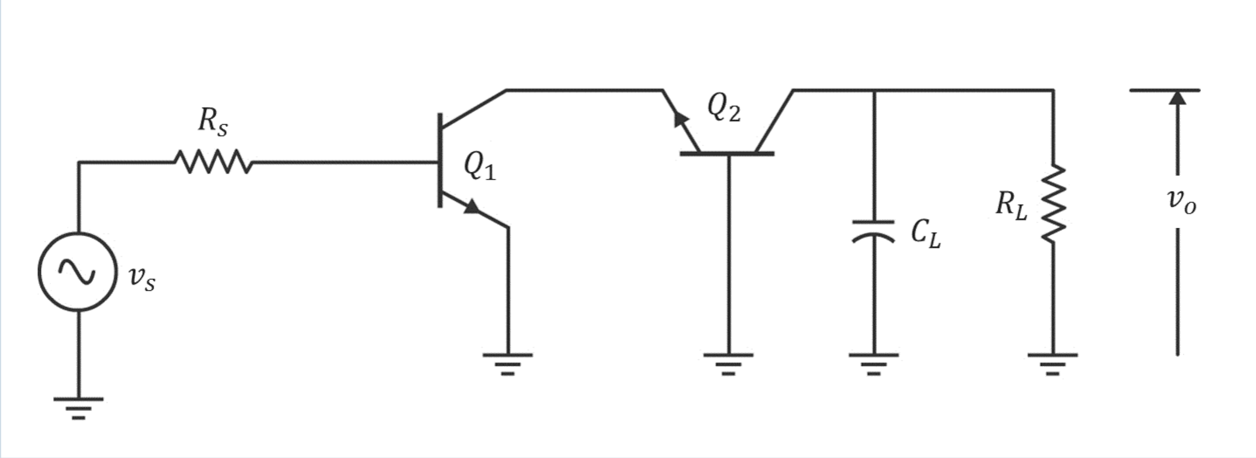
$$\omega_3 = \frac{1}{R_L(C_{cb'} + C_L)} \Rightarrow f_3 = \frac{1}{2\pi R_L(C_{cb'} + C_L)}$$

$A_{v(CB)} = \dfrac{\alpha R_L}{r_e} \approx g_m R_L$ **Note:** $\alpha \approx 1$ and $r_e \approx \dfrac{1}{g_m}$

The overall mid-band gain:

$$A_{V(MID)} = A_{v(CE)} A_{v(CB)} = (-1)(g_m R_L) = -g_m R_L$$

# CE-CB Configuration (Cascode Amplifier): Example




Two identical transistors $Q_1$ and $Q_2$ are configured as a Cascode Amplifier (CE-CB):

The BJT device parameters are:

$r_{bb'} = 60\ \Omega$

$R_s = 40\ \Omega$

$C_{cb'} = 1.5\ \text{pF}$

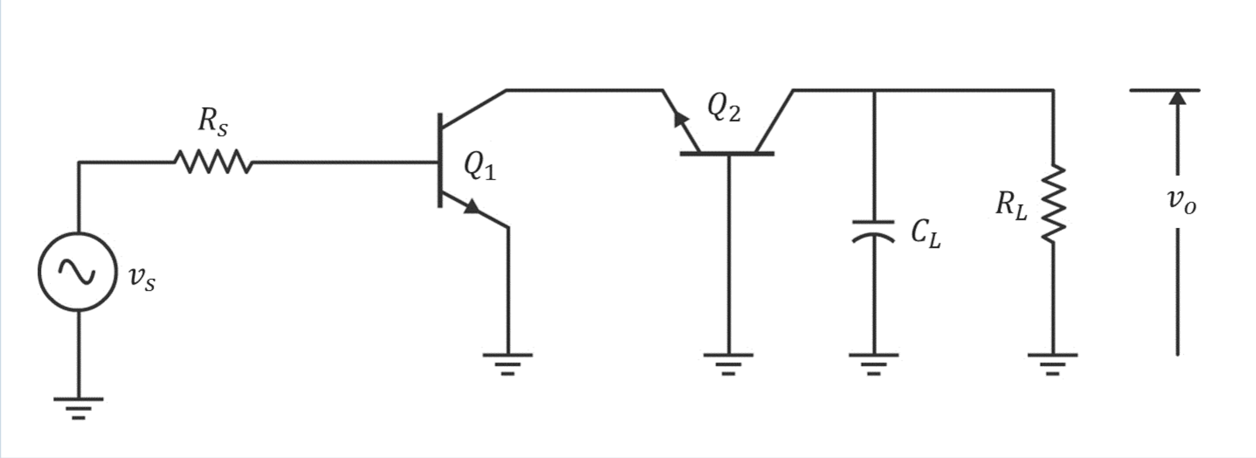
$f_T = 1.6\ \text{GHz}$

Load capacitance: $C_L = 1\ \text{pF}$

The BJT is biased at $2.5\ \text{mA}$.

Determine the overall voltage gain frequency response for different values of $R_L$ varying from $30\ \Omega$ to $10\ \text{k}\Omega$.

# CE-CB Configuration (Cascode Amplifier): Example




$$A_{v(MID)} = -g_m R_L = -0.096\, R_L$$

$$f_1 = \frac{f_T}{g_m(R_s + r_{bb'})} = \frac{1.6\times 10^9}{0.096(100)} \approx 160\text{ MHz}$$

$$f_2 \approx f_T \approx 1.6\text{ GHz}$$

$$f_3 = \frac{1}{2\pi R_L(C_{cb'} + C_L)} = \frac{1}{2\pi R_L(2.5\times 10^{-12})}$$

# CE-CB Configuration (Cascode Amplifier): Example

From the calculations, we have:

| $R_L$ (Ω) | $f_1$ (MHz) | $f_2$ (MHz) | $f_3$ (MHz) | $BW$ (MHz) | $A_{V(\mathrm{MID})}$ | $A_{V(\mathrm{MID})} \times BW$ (MHz) |
|---|---|---|---|---|---|---|
| 30 | 160 | 1,600 | 2,122 | 160 | 3 | 480 |
| 100 | 160 | 1,600 | 637 | 160 | 10 | 1,600 |
| 300 | 160 | 1,600 | 212 | 128 | 30 | 3,831 |
| 1,000 | 160 | 1,600 | 64 | 59 | 100 | 5,900 |
| 3,000 | 160 | 1,600 | 21 | 21 | 300 | 6,305 |
| 10,000 | 160 | 1,600 | 6.4 | 6.4 | 1,000 | 6,400 |

# CE and Cascode Comparison

CE Stage

| $R_L$ (Ω) | $f_1$ (MHz) | $f_2$ (MHz) | $BW$ (MHz) | $A_{V(\mathrm{MID})}$ | $A_{V(\mathrm{MID})}$ X $BW$ (MHz) |
|---|---|---|---|---|---|
| 30 | 110 | 2,122 | 110 | 3 | 330 |
| 100 | 64 | 637 | 64 | 10 | 640 |
| 300 | 29 | 212 | 28 | 30 | 868 |
| 1,000 | 10 | 64 | 10 | 100 | 1,000 |
| 3,000 | 3.5 | 21 | 3.5 | 300 | 1,038 |
| 10,000 | 1.05 | 6.4 | 1.04 | 1,000 | 1,040 |

Cascode

| $R_L$ (Ω) | $f_1$ (MHz) | $f_2$ (MHz) | $f_3$ (MHz) | $BW$ (MHz) | $A_{V(\mathrm{MID})}$ | $A_{V(\mathrm{MID})} \times BW$ (MHz) |
|---|---|---|---|---|---|---|
| 30 | 160 | 1,600 | 2,122 | 160 | 3 | 480 |
| 100 | 160 | 1,600 | 637 | 160 | 10 | 1,600 |
| 300 | 160 | 1,600 | 212 | 128 | 30 | 3,831 |
| 1,000 | 160 | 1,600 | 64 | 59 | 100 | 5,900 |
| 3,000 | 160 | 1,600 | 21 | 21 | 300 | 6,305 |
| 10,000 | 160 | 1,600 | 6.4 | 6.4 | 1,000 | 6,400 |

**For $R_L = 1\ \mathrm{k\Omega}$, mid-band gain is 100 in both the cases but $\mathrm{BW} = 10$ MHz for CE and $\mathrm{BW} \approx 60$ MHz for the cascode stage which is nearly six times wider in BW.**

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# Wideband Amplifiers

## Topic 2: Amplifier Feedback Analysis

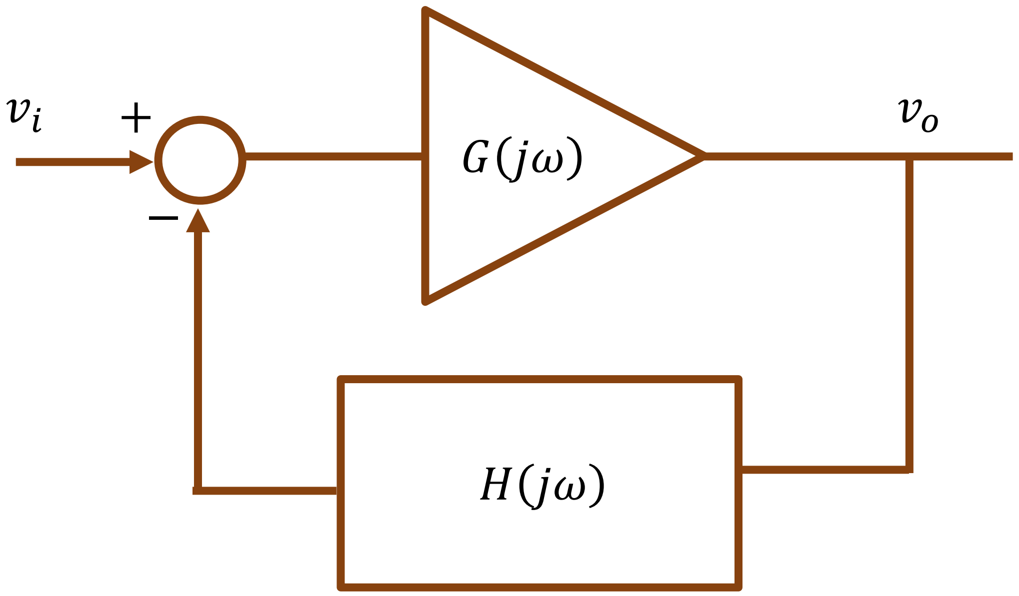
## EE4341: Advanced Analog Circuits

A/P Zheng Yuanjin

School of Electrical and Electronic Engineering

Email: YJZHENG@ntu.edu.sg

# Applying Feedback to Broaden BW

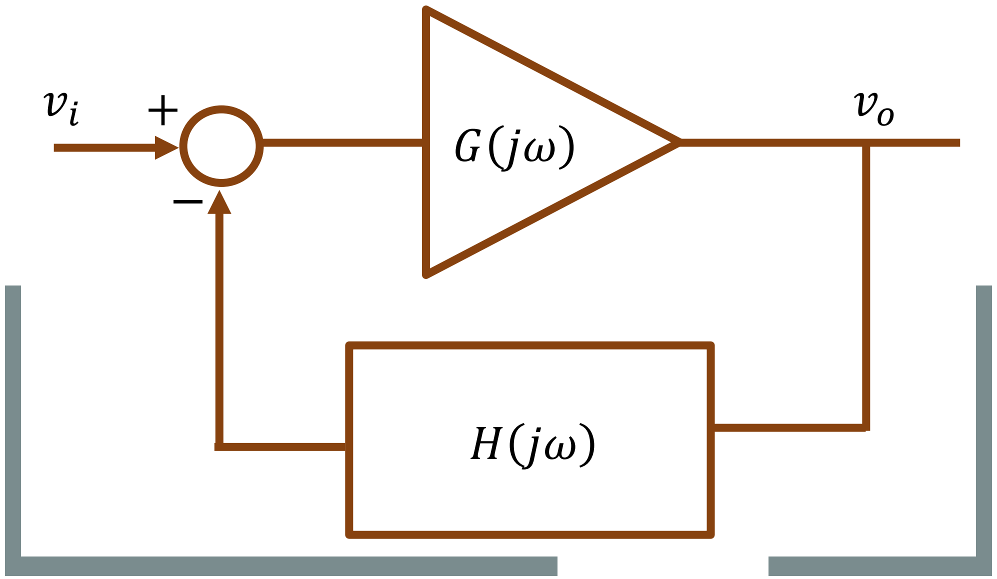



$$G(j\omega) = \frac{A_o}{1 + j\,\omega/\omega_o}$$

$$A_v(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)}$$

As the bandwidth of a transistor is restricted by its device parameters, negative feedback technique can be employed to broaden the amplifier's bandwidth. It would be at the expense of lower gain.

# Applying Feedback to Broaden BW




$G\,(j\omega)$ is the voltage transfer function of the amplifier and $H\,(j\omega)$ is the negative feedback network.

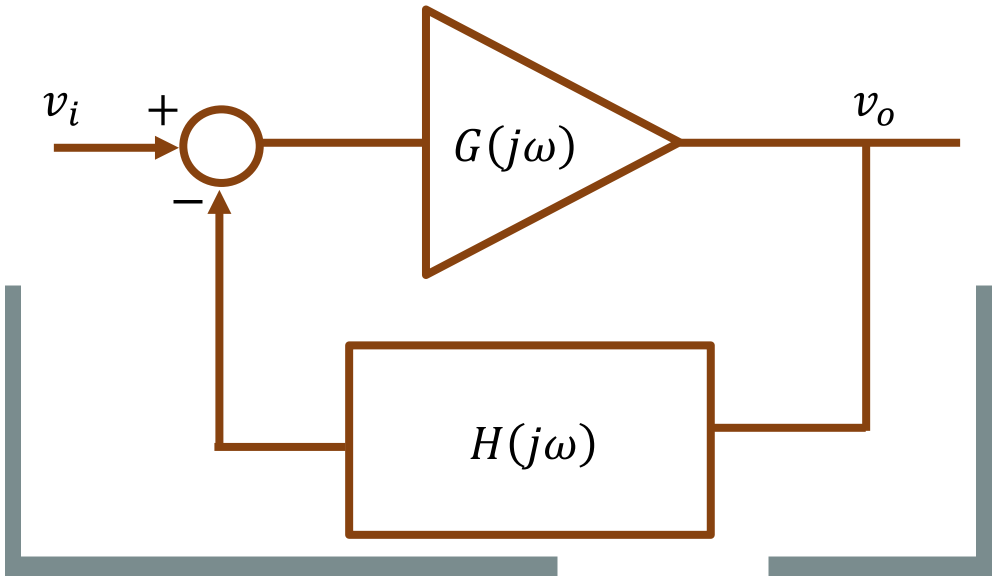
If $H\,(j\omega)$ is frequency-independent in the frequency of interest that is $H(j\omega) = H$.

$$A_v(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H}$$

$$A_v(j\omega) = \frac{\dfrac{A_o}{1 + j\,\omega/\omega_o}}{1 + \dfrac{A_oH}{1 + j\,\omega/\omega_o}} = \frac{A_o}{1 + j\,\omega/\omega_o + A_oH}$$

$$= \frac{A_o}{1 + A_oH}\left(\frac{1}{1 + \dfrac{j\omega}{\omega_o(1 + A_oH)}}\right) = \frac{A_o}{1 + A_oH}\left(\frac{1}{1 + j\,\omega/\omega_L}\right)$$

# Applying Feedback to Broaden BW




$$A_v(j\omega) = \frac{A_o}{1 + A_o H}\left(\frac{1}{1 + \frac{j\omega}{\omega_o(1 + A_o H)}}\right) = A_{v(MID)}\left(\frac{1}{1 + j\,\omega/\omega_L}\right)$$

The mid-band gain of the amplifier with feedback is:

$$A_{v(MID)} = \frac{A_o}{1 + A_o H} \qquad \because A_o H >> 1 \;\therefore A_{v(MID)} \approx \frac{1}{H}$$

$\therefore$ The mid-band gain is controlled by the feedback network.

Now, the bandwidth of the amplifier with feedback has been broadened by a factor of $(1 + A_o H)$:

$$\omega_L = \omega_o(1 + A_o H)$$

# Summary of Feedback Configurations

| Feedback Configuration | Input | Output | Transfer Function |
|---|---|---|---|
| Shunt-Shunt | Current | Voltage | Transresistance Amplifier |
| Shunt-Series | Current | Current | Current Amplifier |
| Series-Shunt | Voltage | Voltage | Voltage Amplifier |
| Series-Series | Voltage | Current | Transconductance Amplifier |

To sample the output voltage, measure the voltage with a voltmeter. Hence, voltage sampling at output is a ‘shunt’ connection.

To sample the output current, measure the current with an amp-meter. Hence, current sampling at output is a ‘series’ connection.

To feedback a voltage at the input, it needs to be in ‘series’ with the voltage signal source (think of the Thevenin equivalent model).

To feedback a current at the input, it needs to be in ‘shunt’ with the current signal source (think of the Norton equivalent model).

# Feedback Configuration

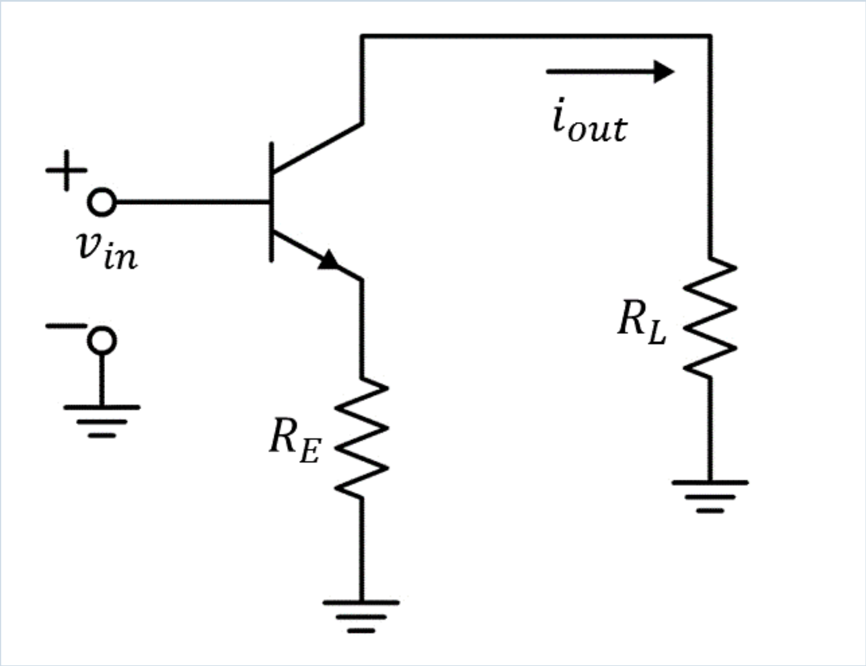
$$i_{out} = -i_c \approx -i_e$$

$$v_{in} = v_{be} + i_e R_E$$

$$\approx -i_{out}(r_e + R_E) \approx -i_{out} R_E$$

$$\because R_E >> r_e$$

$$\therefore \frac{i_{out}}{v_{in}} = -\frac{1}{R_E}$$

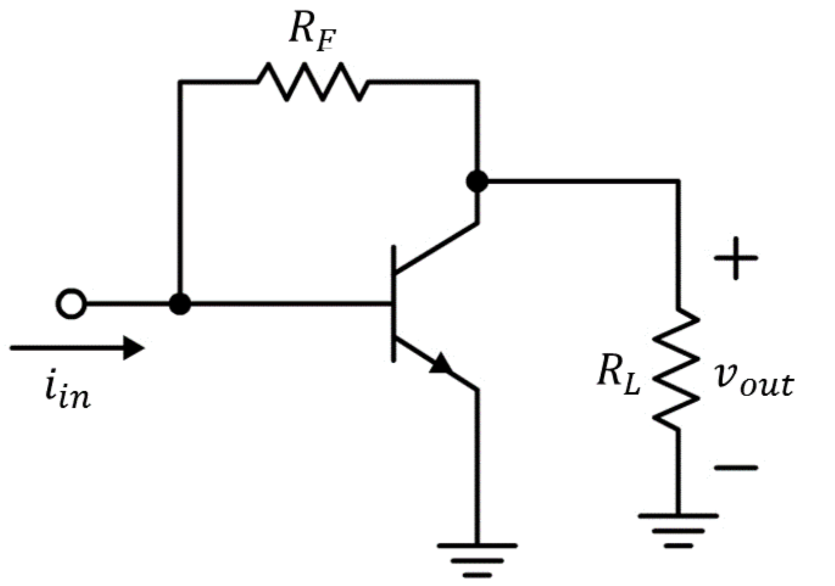



$R_E$ samples output current in 'series' and feedback a signal voltage in 'series' with the input voltage.

It is a trans-conductance amplifier with a gain of $-1/R_E$.

# Feedback Configuration

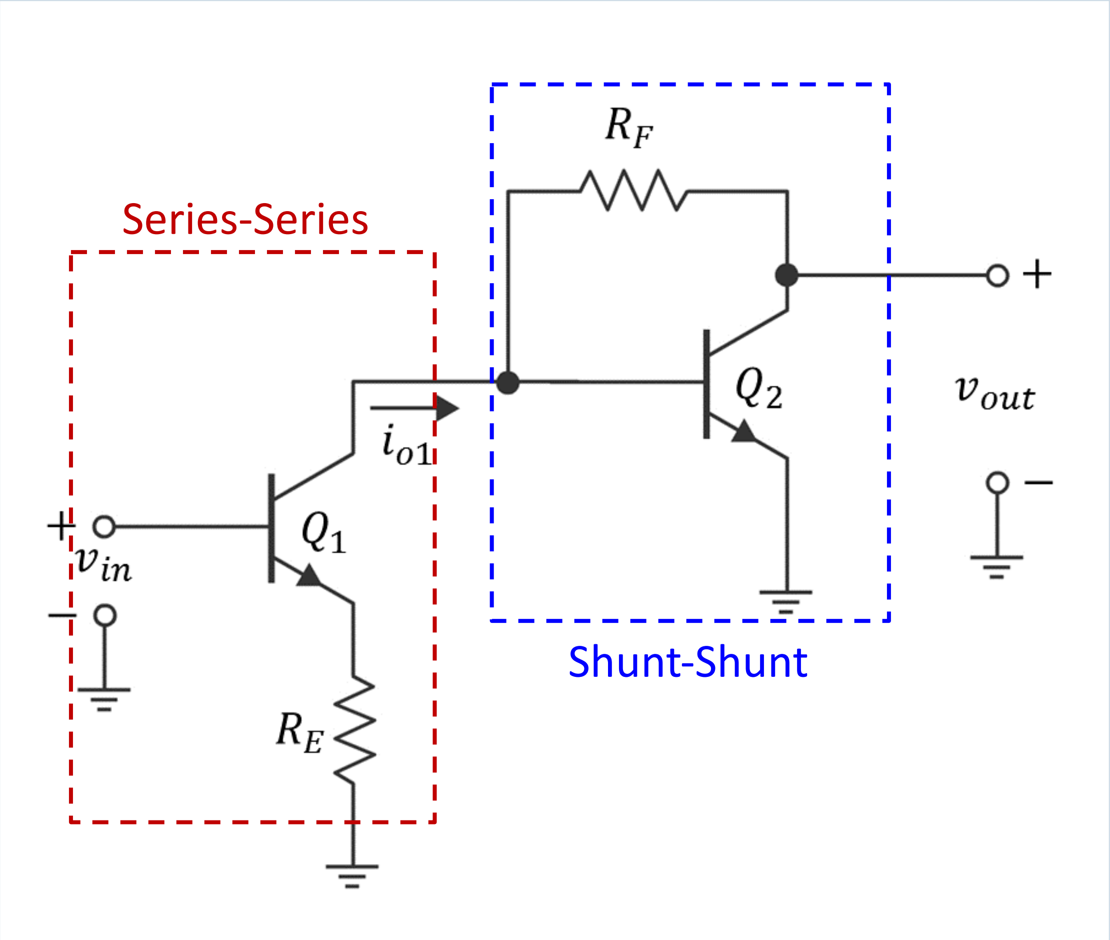
$$v_{out} = v_{be} - (i_{in} - i_b)R_F$$
$$= i_b r_\pi - i_{in}R_F + i_b R_F$$
$$= i_b(r_\pi + R_F) - i_{in}R_F$$
$$\because i_{in} >> i_b, \therefore v_{out} \approx -i_{in}R_F$$
$$\frac{v_{out}}{i_{in}} = -R_F$$




$R_F$ samples output voltage in 'shunt' and feedback a signal current in 'shunt' with the input current.

It is a trans-impedance amplifier with a gain of $-R_F$.

# Feedback Configuration



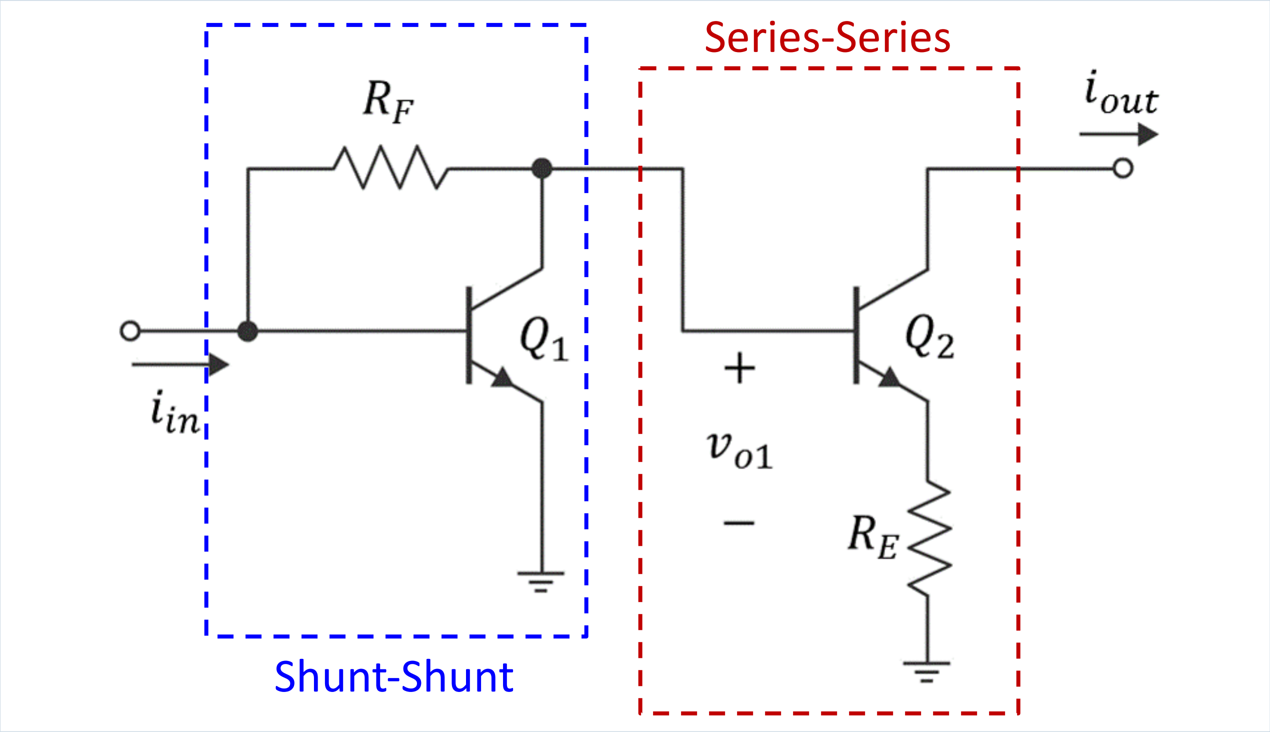

It is a voltage amplifier with series-shunt feedback configuration.

$$\frac{v_{out}}{v_{in}} = \left(-\frac{1}{R_E}\right)(-R_F) = \frac{R_F}{R_E}$$

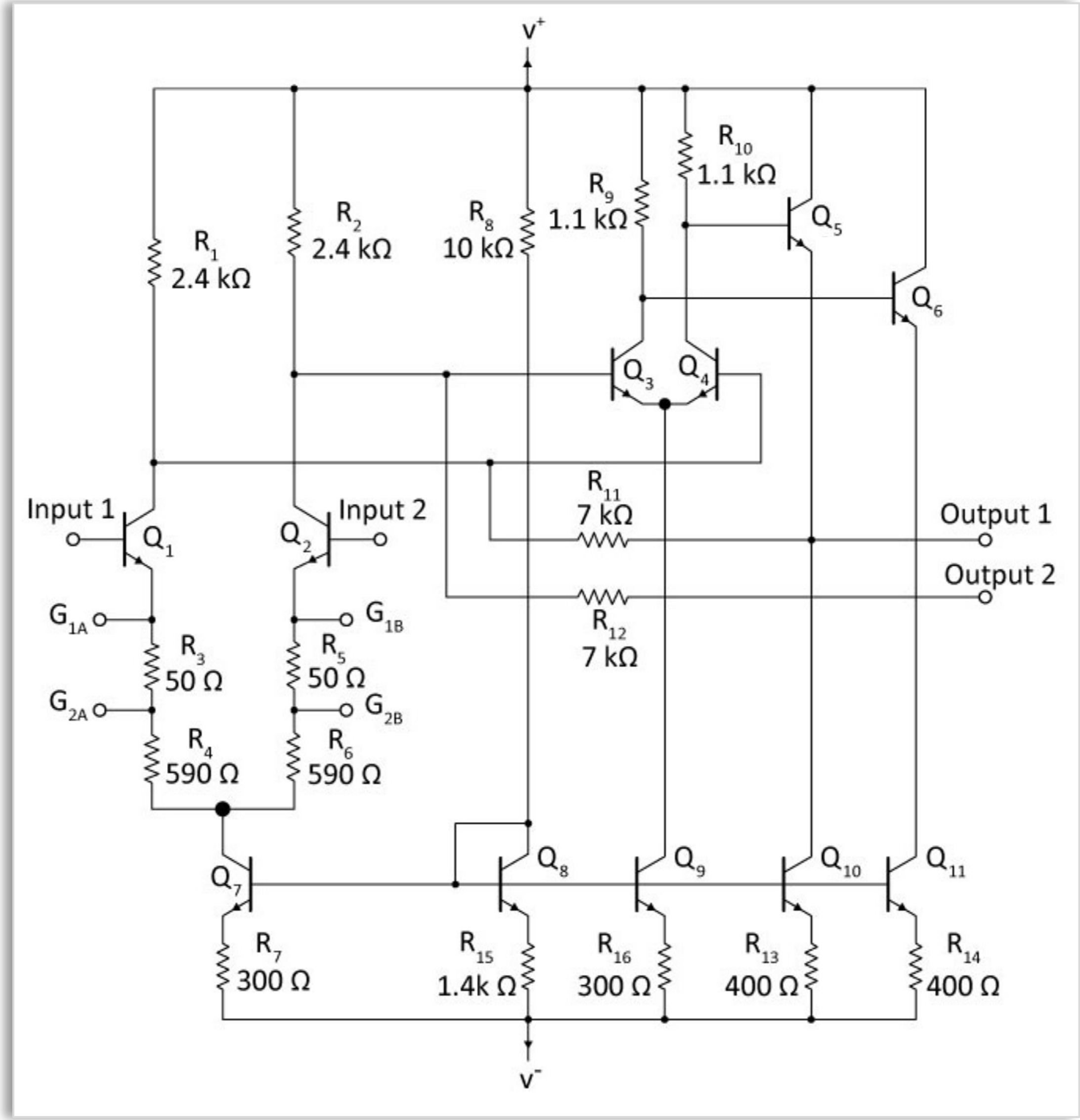
# Feedback Configuration

It is a current amplifier with shunt-series feedback configuration.

$$\frac{i_{out}}{i_{in}} = (-R_F)\left(-\frac{1}{R_E}\right) = \frac{R_F}{R_E}$$



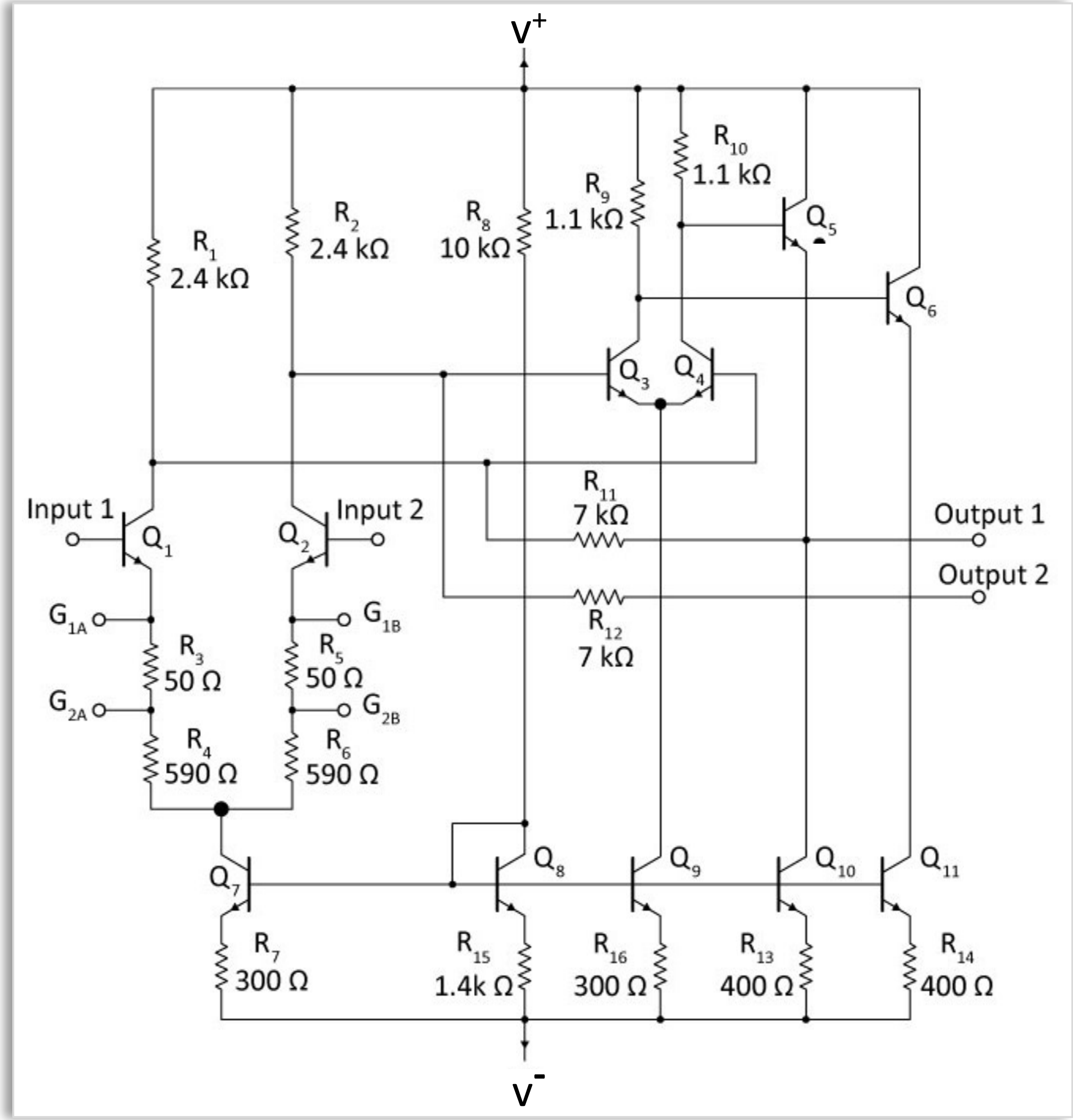
# Differential Video Amplifier




Circuit diagram of $\mu A$ 733 wideband differential video amplifier that uses the series-shunt cascade feedback topology.

The circuit has a good CMRR (Common-mode Rejection Ratio).

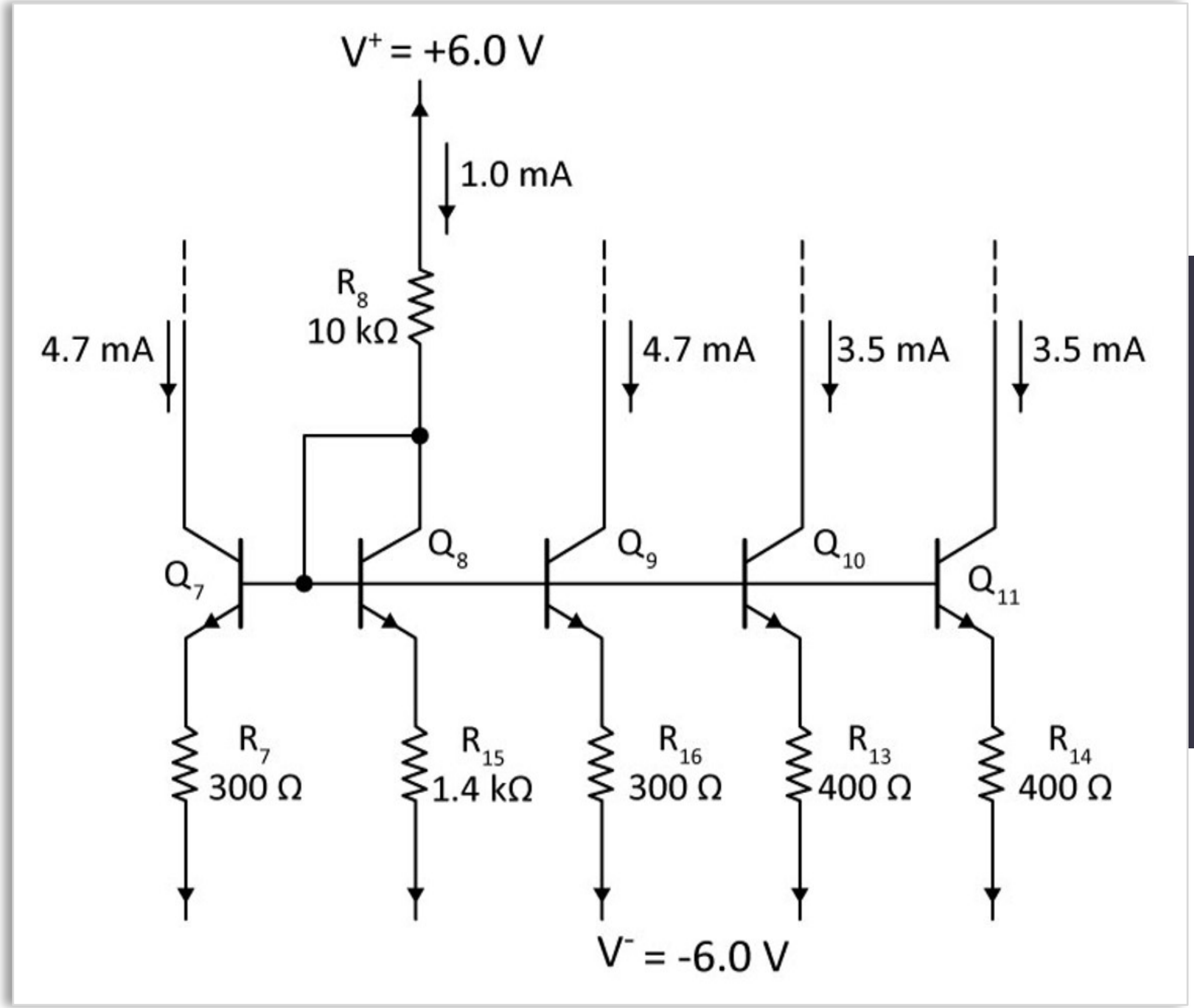
# Differential Video Amplifier




Series-shunt feedback

Current mirrors for DC biasing

# DC Biasing Analysis




$$I_{C8} = \frac{(V^+ - V^-) - V_{BE8}}{R_8 + R_{15}}$$

$$= \frac{12 - 0.6}{(10 + 1.4)\times 10^3}$$

$$= 1 \text{ mA}$$

$$V_{R15} = 1 \text{ mA} \times 1.4 \text{ k}\Omega = 1.4 \text{ V}$$

$$I_{C7} = I_{C9} = \frac{1.4 \text{ V}}{300\ \Omega} = 4.7 \text{ mA}$$

$$I_{C10} = I_{C11} = \frac{1.4 \text{ V}}{400\ \Omega} = 3.5 \text{ mA}$$

# Voltage Gain Analysis

$$g_{f1} = \frac{i_{c1}}{v_{b1}} = \frac{i_{c1}}{i_{e1}(r_{e1} + R_E)} \approx \frac{i_{c1}}{i_{c1}(r_{e1} + R_E)} = \frac{1}{r_{e1} + R_E}$$

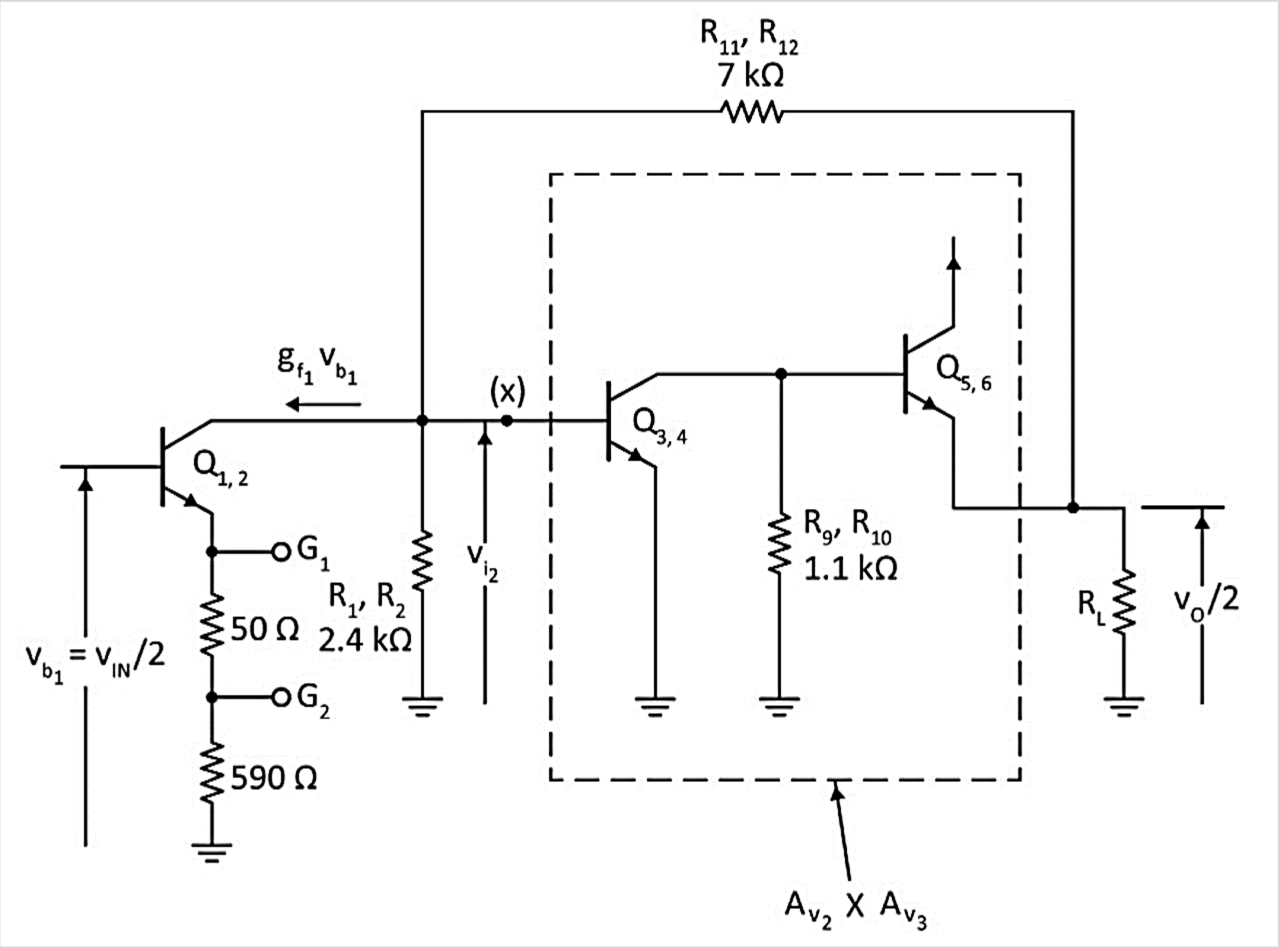
$$r_{e1} \approx \frac{V_T}{I_{C1}}$$

$$i_{c1} = g_{f1} v_{b1} = \frac{g_{f1} v_{in}}{2}$$

The voltage gain for second stage ($Q_3$ or $Q_4$) is:

$$A_{v2} = \frac{v_{c3}}{v_{i2}} = -\frac{g_{f3} v_{i2} R_9}{v_{i2}} = -\left(\frac{I_{C3}}{V_T}\right) R_9$$

$$= -\frac{4.7\text{ mA}/2}{26\text{ mV}} \times 1.1\text{ k}\Omega = -103$$



# Voltage Gain Analysis

The voltage gain for third stage ($Q_5$ or $Q_6$) is:

$$A_{v3} = \frac{v_o/2}{v_{c3}} = \frac{i_{e5}R_L}{i_{e5}(R_L + r_{e5})} = \frac{R_L}{R_L + r_{e5}} \approx 1$$

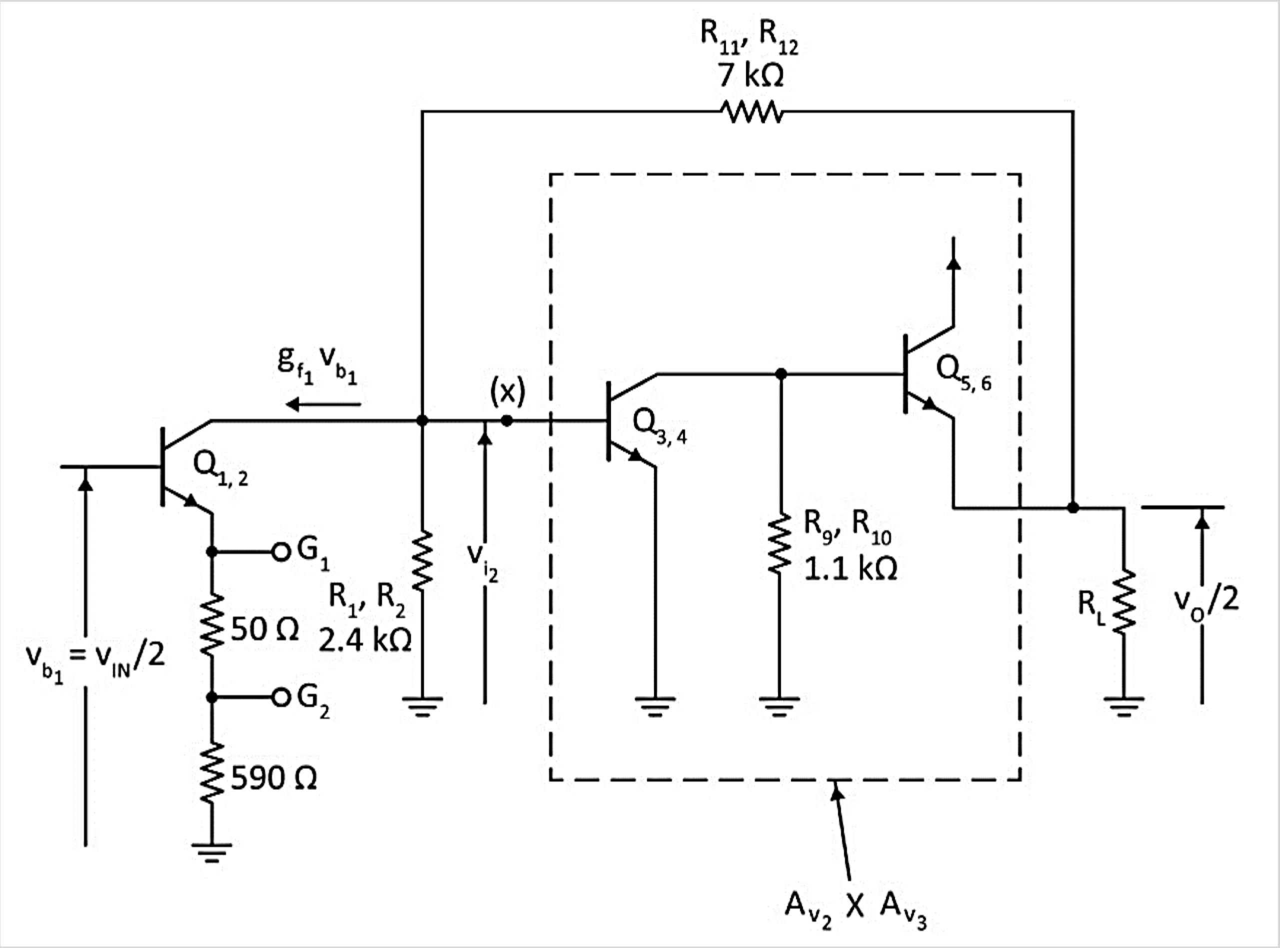
$\because R_L >> r_{e5}$

$\therefore A_{v2} \times A_{v3} \approx -100$

Applying KCL at node "x":

$$\frac{g_{f1}v_{in}}{2} + \frac{v_x}{R_1} = \left(\frac{v_o}{2} - v_x\right)\left(\frac{1}{R_{11}}\right) \Rightarrow$$

$$\frac{g_{f1}v_{in}}{2} = \frac{v_o}{2R_{11}} - v_x\left(\frac{1}{R_1} + \frac{1}{R_{11}}\right) ----(1)$$



# Voltage Gain Analysis

Note: $\dfrac{v_o}{2} = A_{v2}A_{v3}v_x \Rightarrow v_x = -\dfrac{v_o}{200} ----(2)$

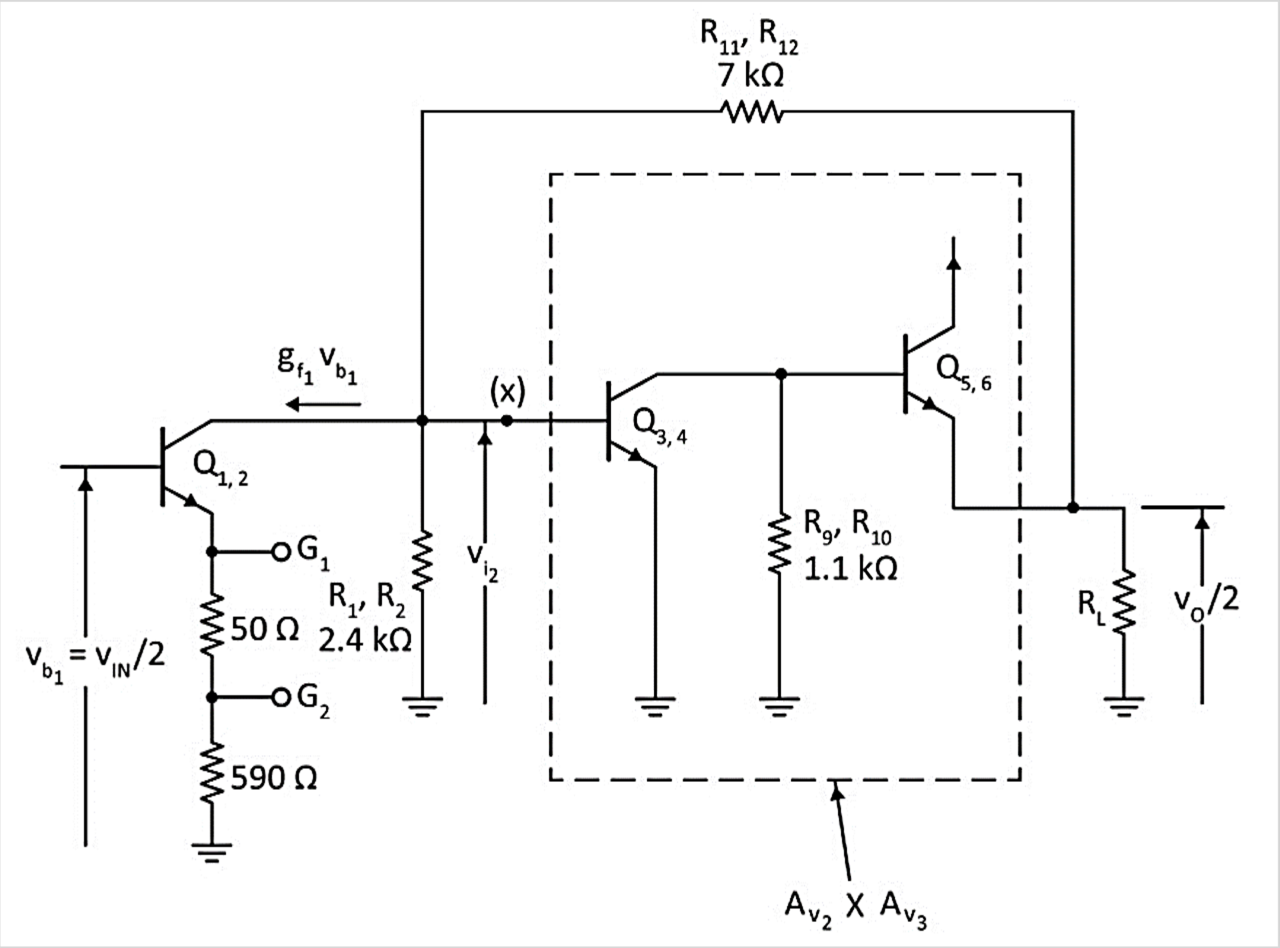
Substituting (2) into (1):

$$\frac{g_{f1}v_{in}}{2} = \frac{v_o}{2R_{11}} + \left(\frac{v_o}{200}\right)\left(\frac{1}{R_1} + \frac{1}{R_{11}}\right)$$

$$A_v = \frac{v_o}{v_{in}} = \frac{g_{f1}}{1/R_{11} + (1/100)(1/R_1 + 1/R_{11})}$$

$$= \frac{g_{f1}}{1/7\,\mathrm{k} + (1/100)(1/7\,\mathrm{k} + 1/2.4\,\mathrm{k})} \approx g_{f1}(7\,\mathrm{k})$$

$$A_v = g_{f1}(7\,\mathrm{k}) = \frac{7\,\mathrm{k}}{r_{e1} + R_E} = \frac{7\times10^3}{11 + R_E}$$



# Voltage Gain Analysis

The gain for $R_E = 0$, 50 $\Omega$ and 640 $\Omega$ are calculated as follows:

$$R_E = 0: A_v = \frac{7\times10^3}{11} = 640$$

$$R_E = 50\ \Omega: A_v = \frac{7\times10^3}{11+50} = 115$$

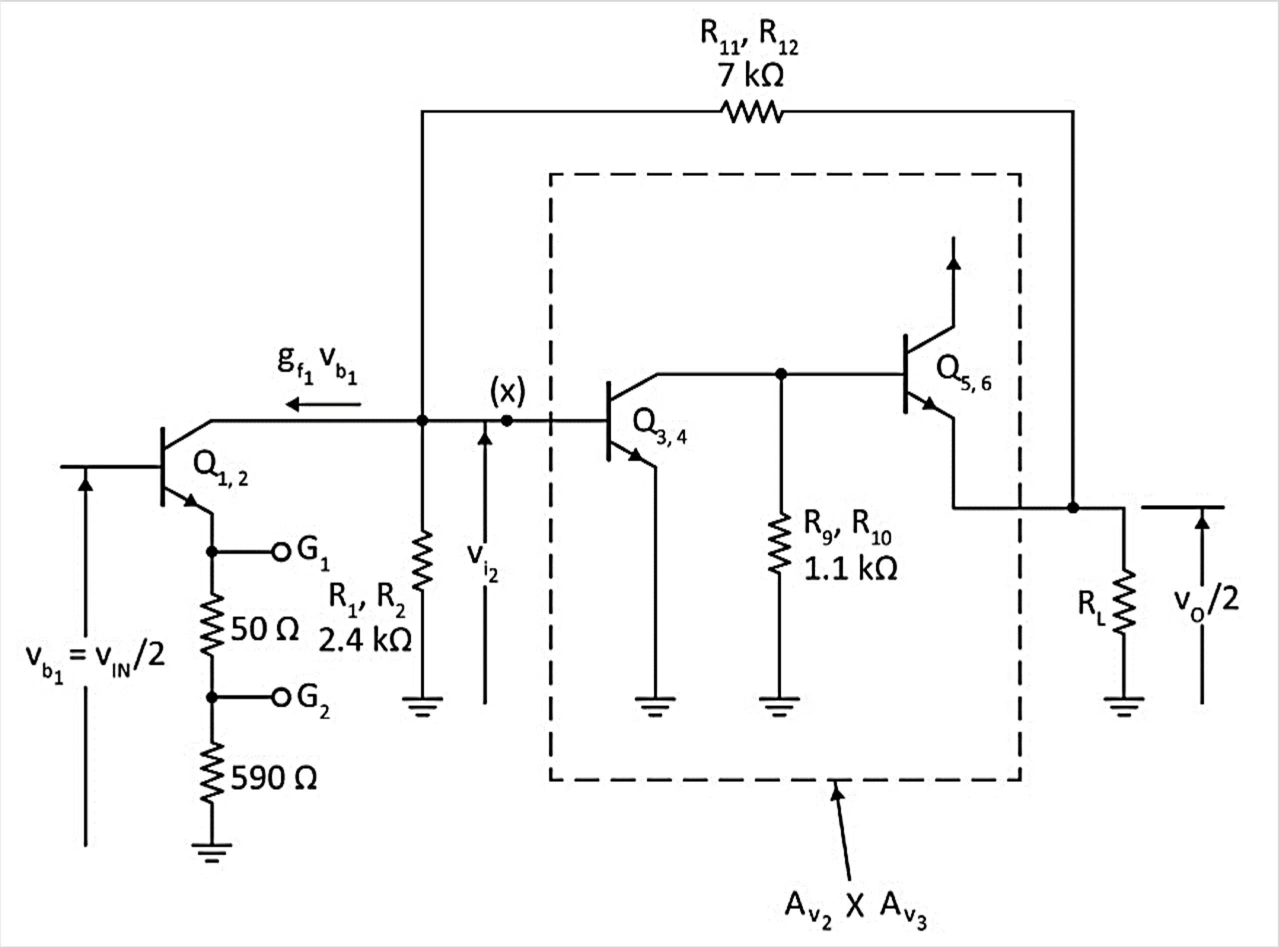
$$R_E = 640\ \Omega: A_v = \frac{7\times10^3}{11+640} = 10.8$$

The 3 dB bandwidth obtained from the data sheet:

$$R_E = 0: \mathrm{BW} = 50\ \mathrm{MHz\ (Typical)}$$

$$R_E = 50\ \Omega: \mathrm{BW} = 90\ \mathrm{MHz\ (Typical)}$$

$$R_E = 640\ \Omega: \mathrm{BW} = 200\ \mathrm{MHz\ (Typical)}$$



# Cascading Identical Stages

If a large voltage gain is required, it is convenient to cascade several identical amplifier stages.

The voltage transfer function of each stage is:

$$A_v = \frac{A}{1 + j\,\omega/\omega_p}$$

The overall voltage transfer function of $n$ stage is:

$$A_T = \frac{A^n}{\left(1 + j\,\omega/\omega_p\right)^n}$$

The overall bandwidth $\omega_1$ can be found by:

$$|A_T| = \frac{A^n}{|1 + j\,\omega_1/\omega_p\,|^n} = \frac{A^n}{\sqrt{2}} \Rightarrow \left[1 + \left(\omega_1/\omega_p\right)^2\right]^{n/2} = 2^{1/2}$$

$$\therefore\ \omega_1 = \omega_p\left(2^{1/n} - 1\right)^{1/2} \Rightarrow f_1 = f_p\sqrt{2^{1/n} - 1}$$

# Cascading Identical Stages: Example

Three identical amplifiers, each with a voltage gain of 10 and a bandwidth of 10 MHz, are cascaded. What is the overall gain and bandwidth?

The overall voltage gain:

$$A^n = 10^3 = 1{,}000$$

The overall bandwidth:

$$f_1 = f_p\sqrt{2^{1/n} - 1}$$

$$= 10\times10^6\sqrt{2^{1/3} - 1}$$

$$= 5.1\ \mathrm{MHz}$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

# Wideband Amplifiers

## Summary

## EE4341: Advanced Analog Circuits

A/P Zheng Yuanjin

School of Electrical and Electronic Engineering

Email: YJZHENG@ntu.edu.sg

# Summary

- BJT Frequency Response
- Amplifier Frequency Response for CE Amplifiers
- CE-CB Amplifier Analysis
- Feedback Circuit Analysis for Wideband Amplifiers
- Cascaded Integrated Circuit Analysis