Small-Signal BJT Amplifiers

E2002 Analog Electronics
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Module Goals

Understanding of concepts related to:

- Biasing of Transistors (BJT and MOSFET)
- dc and ac equivalent circuits for small-signal amplifier
- Small-signal models of BJT and MOSFET
- Amplifier characteristics such as voltage gain, input and output resistances
- Analysis of three broad classes of single-stage amplifiers

  - Inverting amplifiers – common-emitter and common-source configurations

  - Followers – common-collector and common-drain configurations

  - Noninverting amplifiers – common-base and common-gate configurations

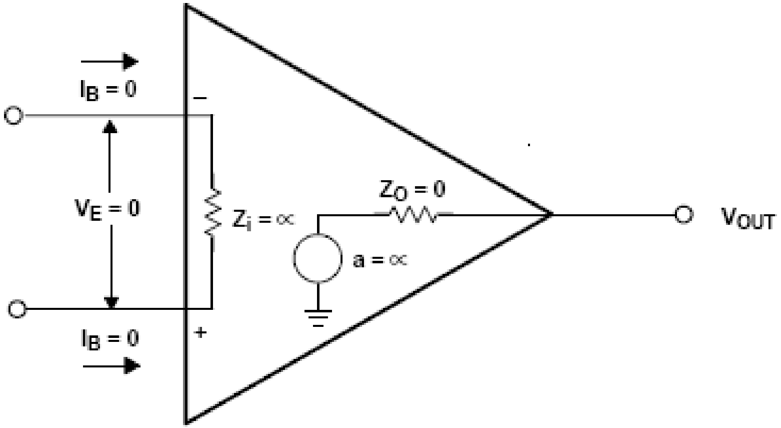
References

Text Book

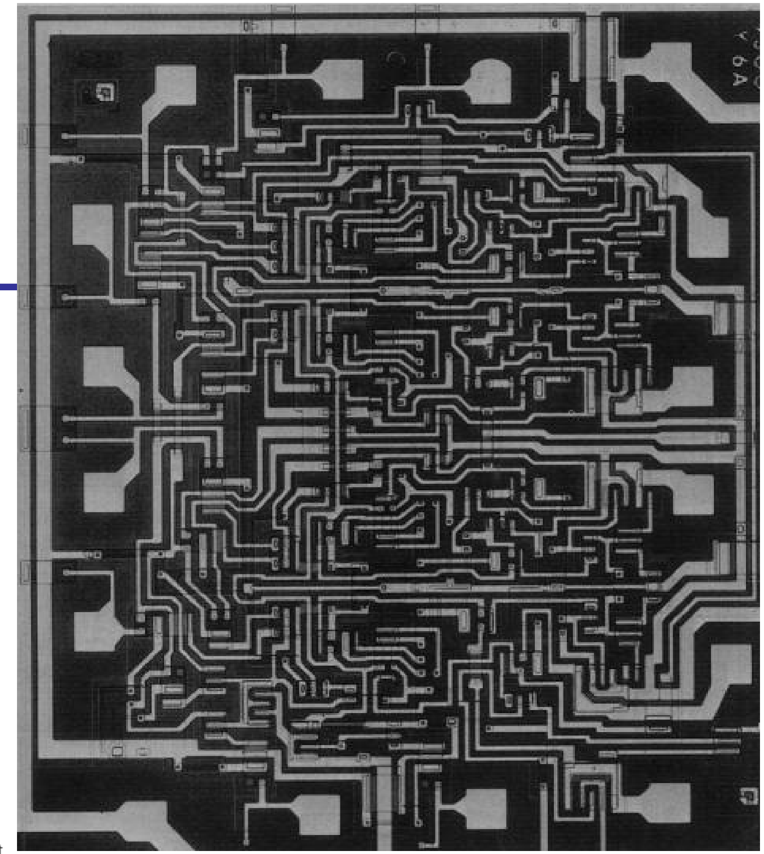
1. Richard C. Jaeger and Travis N. Blalock, "Microelectronic Circuit Design", 4th Edition, McGraw Hill, 2011, Chapters 4, 5, 13 and 14.

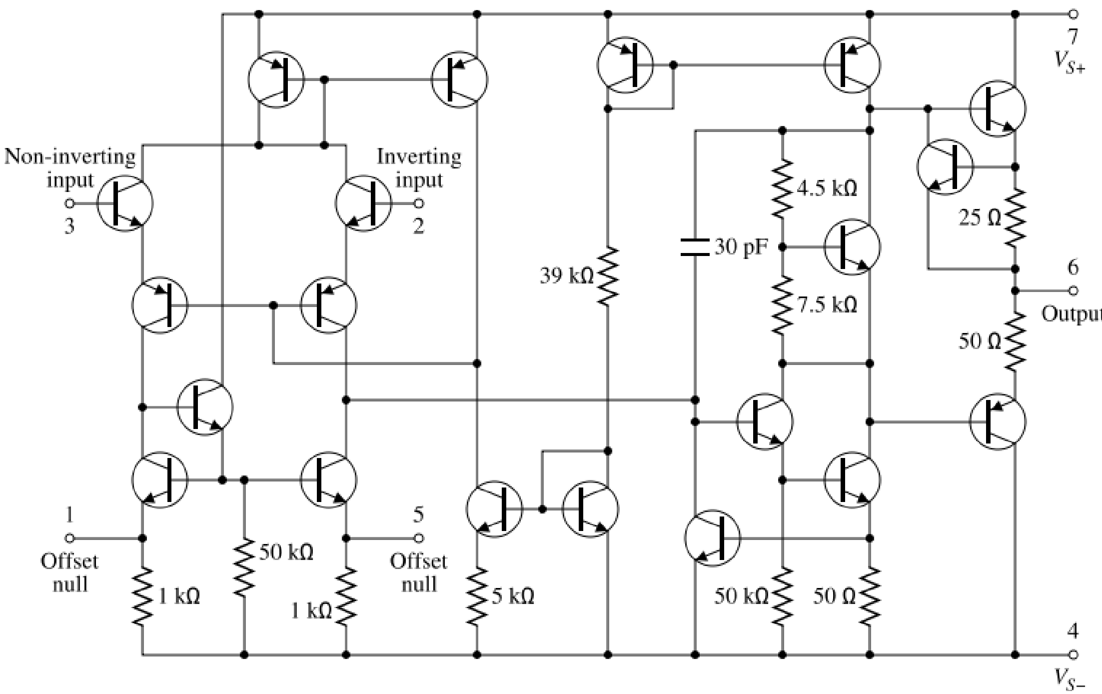
References

1. Allan R. Hambley, "Electronics", 2^{nd} Edition, Prentice Hall, 2000
2. Donald A. Neamen, "Electronic Circuit Analysis and Design", 2^{nd} Edition, McGraw-Hill, 2002

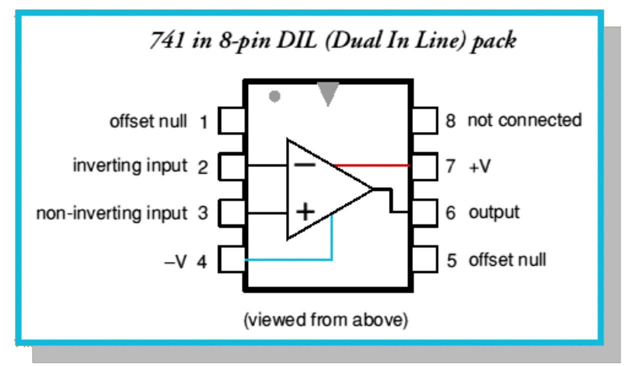


IB = 0
VE = 0
IB = 0
Zi = ∝
ZO = 0
a = ∝
VOUT





Non-inverting input
3
Inverting input
2
39 kΩ
4.5 kΩ
30 pF
7.5 kΩ
25 Ω
50 Ω
7
VS+
6
Output
1
Offset null
5
Offset null
1 kΩ
50 kΩ
1 kΩ
5 kΩ
50 kΩ
50 Ω
4
VS−

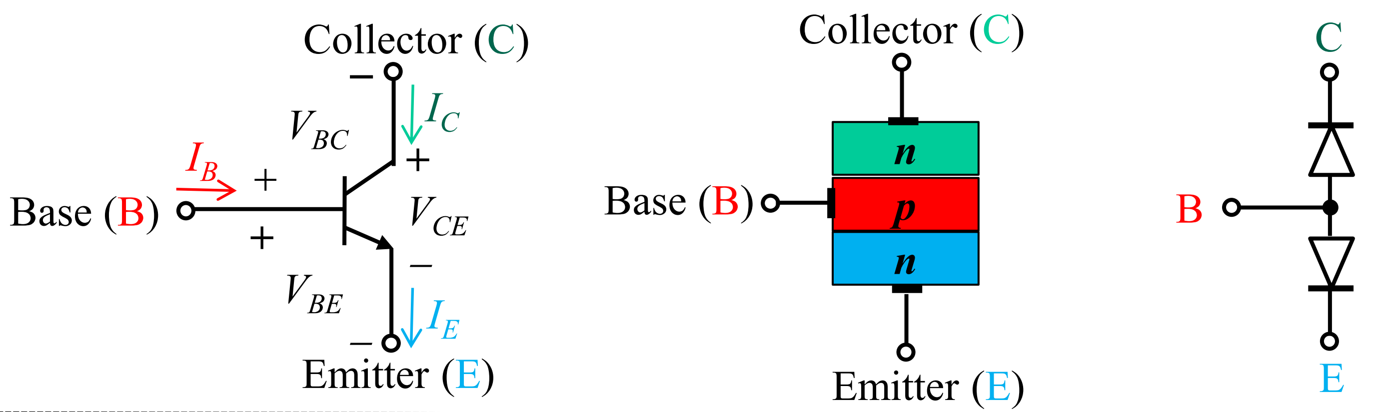


741 in 8-pin DIL (Dual In Line) pack
offset null 1
inverting input 2
non-inverting input 3
−V 4
8 not connected
7 +V
6 output
5 offset null
(viewed from above)

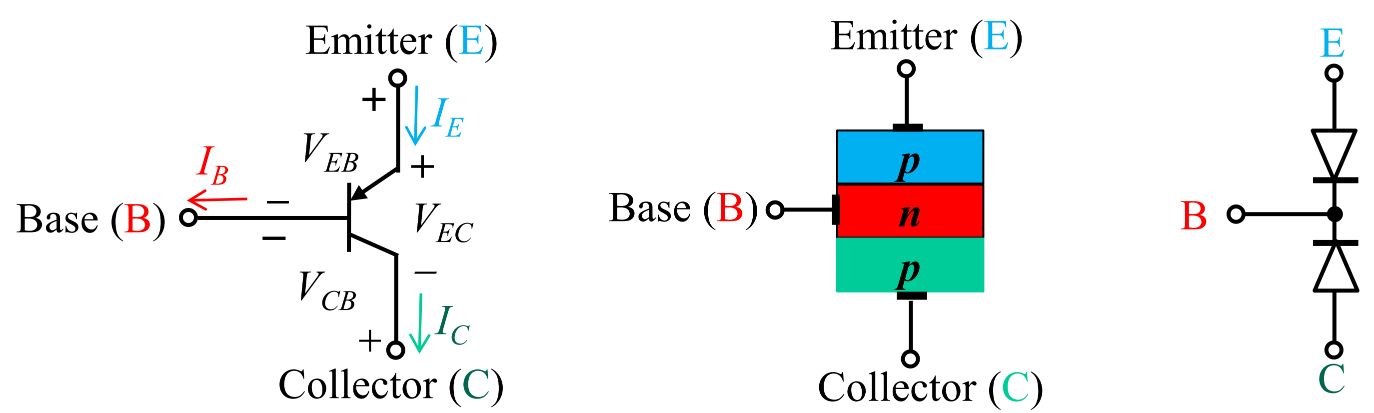
Bipolar Junction Transistors

Bipolar transistor can be thought of as a sandwich of three doped Si regions. The outer two regions are doped with the same polarity, while the middle region is doped with opposite polarity.

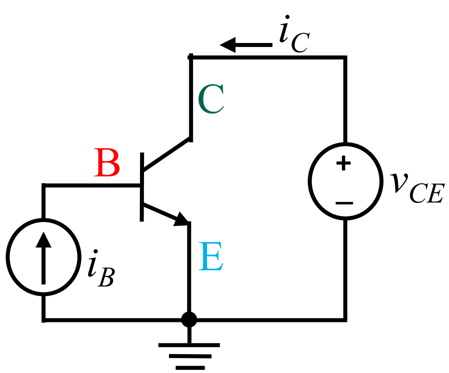
npn

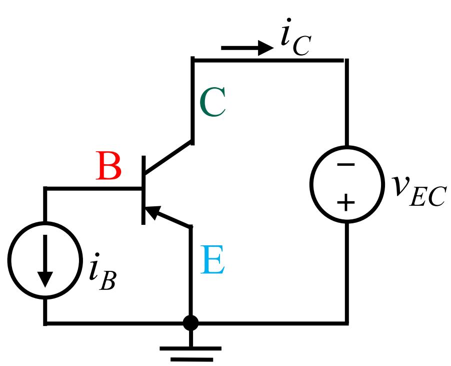


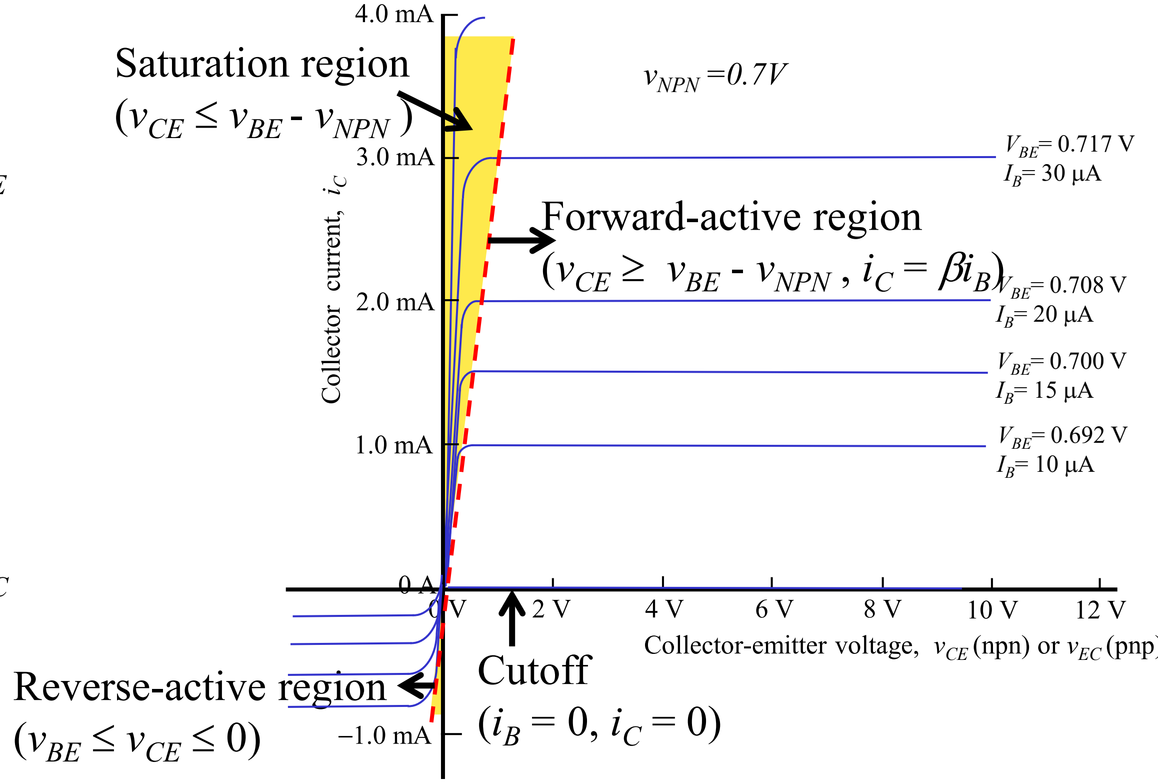
pnp



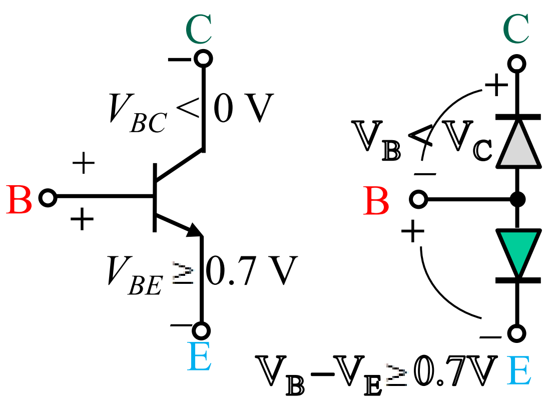
Output Characteristics of BJT







Operation Regions of BJT



Forward-active region

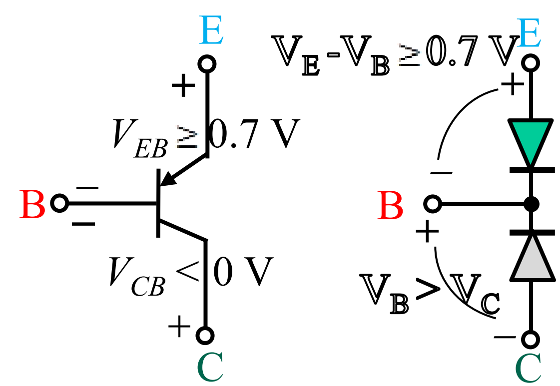
BEJ (npn) forward biased

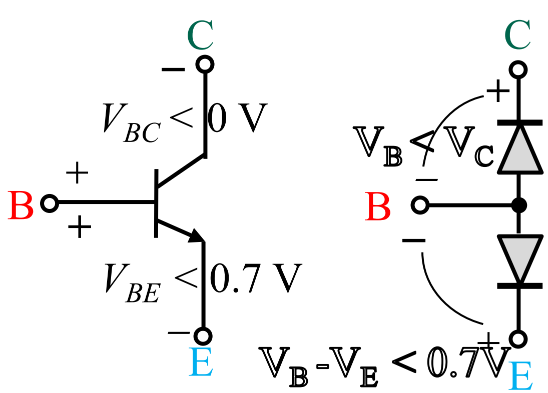
BCJ (npn) reversed biased

$V_{BE} \geq 0.7$ V

$I_C = \beta I_B = \alpha I_E$

=> Good amplifier





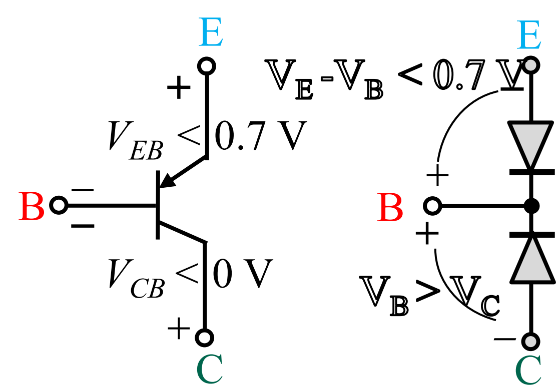
Cutoff region

BEJ (npn) reverse biased

BCJ (npn) reverse biased

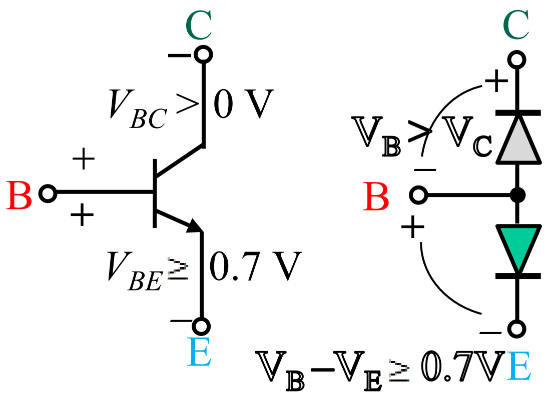
$I_C = 0$

=> Open Switch



Note: the junctions refer to EBJ and CBJ for pnp transistor.

Operation Regions of BJT (Cont.)



Saturation region

BEJ (npn) forward biased

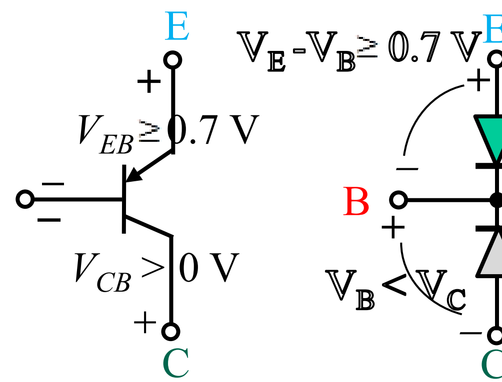
BCJ (npn) forward biased

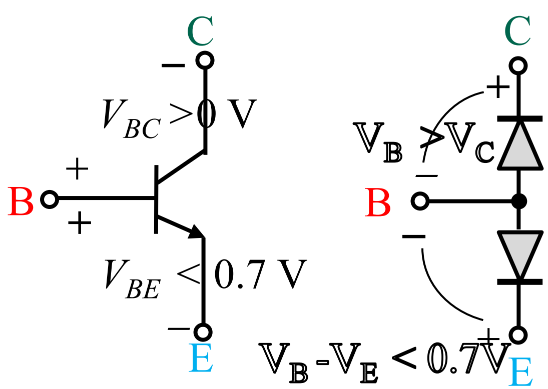
⇒ Closed switch

⇒ $V_{BE} \geq 0.7$ V

$V_{BC} = 0.4\sim 0.5$ V

⇒ $V_{CE(SAT)} = 0.2\sim 0.3$ V





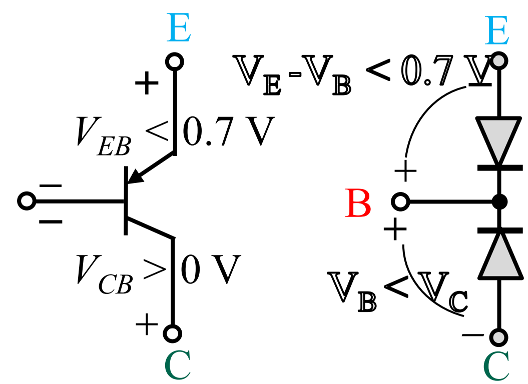
Reverse-active region

BEJ (npn) reverse biased

BCJ (npn) forward biased

⇒ Weak amplifier

⇒ Normally not use



Note: the junctions refer to EBJ and CBJ for pnp transistor.

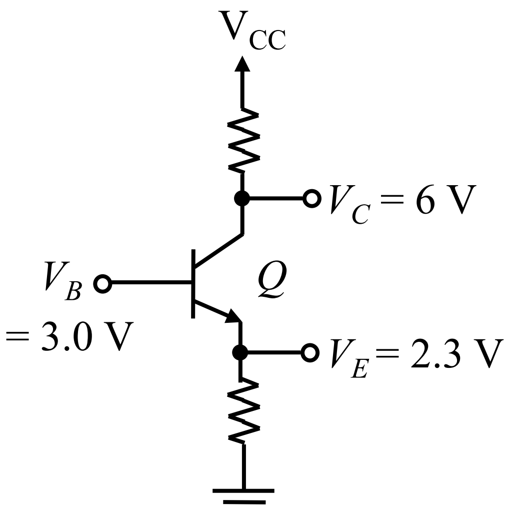
BJT Biasing for Different Regions of Operation

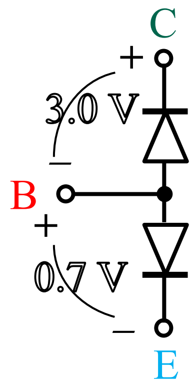
Region	NPN	PNP
Forward-active	$V_{BE} \geq 0.7$ V $V_{BC} < 0$ V	$V_{EB} \geq 0.7$ V $V_{CB} < 0$ V
Saturation	$V_{BE} \geq 0.7$ V $V_{BC} > 0$ V	$V_{EB} \geq 0.7$ V $V_{CB} > 0$ V
Cutoff	$V_{BE} < 0.7$ V $V_{BC} < 0$ V	$V_{EB} < 0.7$ V $V_{CB} < 0$ V
Reverse-active	$V_{BE} < 0.7$ V $V_{BC} > 0$ V	$V_{EB} < 0.7$ V $V_{CB} > 0$ V

BJT Bias Analysis: Active Mode

$$V_{BE} = V_B - V_E = 3.0 - 2.3 = 0.7 \text{ V}$$

$$V_{BC} = V_B - V_C = 3.0 - 6 \text{ V} = -3.0 \text{ V}$$



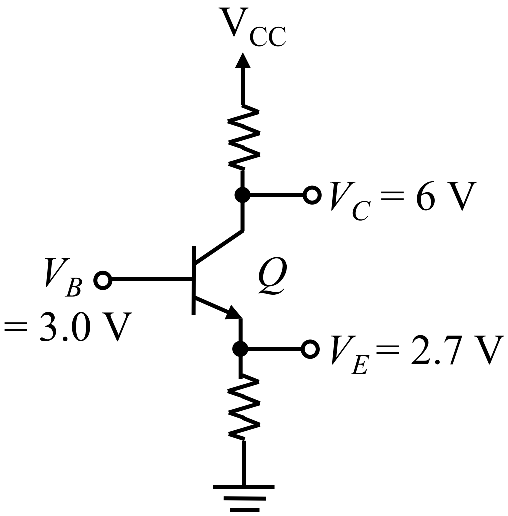


BCJ is reversed biased by 3.0 V

BEJ is forward biased by 0.7 V

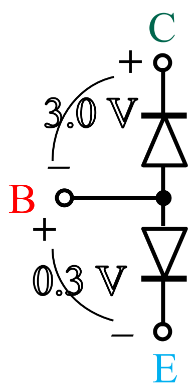
⇒ Q in active mode and can be used as linear amplifier.

BJT Bias Analysis: Cut-off Mode



$$V_{BE} = V_B - V_E = 3.0 - 2.7 = 0.3 \text{ V}$$

$$V_{BC} = V_B - V_C = 3.0 - 6 \text{ V} = -3.0 \text{ V}$$



BCJ is reversed biased by 3.0 V

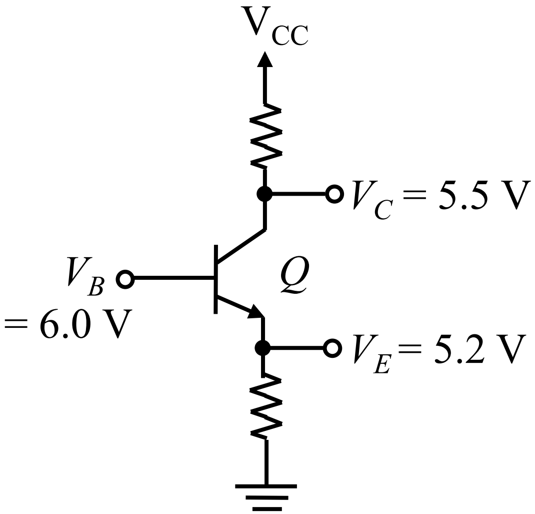
BEJ is forward biased by 0.3 V

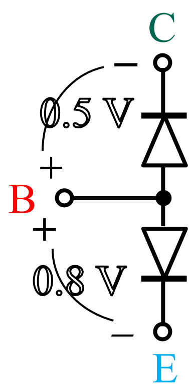
$\Rightarrow$ *Q* is cutoff.

BJT Bias Analysis: Saturation mode

$$V_{BE} = V_B - V_E = 6.0 - 5.2 = 0.8 \text{ V}$$

$$V_{BC} = V_B - V_C = 6.0 - 5.5 \text{ V} = 0.5 \text{ V}$$





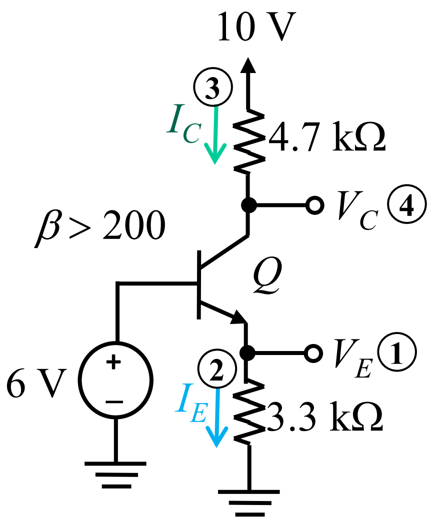
BCJ is forward biased by 0.5 V

BEJ is forward biased by 0.8 V

Q is in Saturation!

$$V_{CE} = V_{CB} + V_{BE} = -0.5 + 0.8 = 0.3 \text{ V}$$

BJT Bias Analysis: Determine DC node voltages and branch currents



Assume active-mode operation,

$V_E = V_B - V_{BE} = 6 - 0.7 = 5.3$ V.

$I_E = 5.3/3.3 = 1.6$ mA

Since β is very large, $\alpha \approx 1 \Rightarrow I_C \approx I_E = 1.6$ mA

$V_C = 10 - 1.6 \times 4.7 = 2.48$ V

$V_{BC} = V_B - V_C = 6 - 2.48 = 3.52$ V

=> Wrong assumption! Q is in saturation mode.

In saturation region, $V_{CE} \approx 0.2$ to 0.3 V. Assume $V_{CE(SAT)} = 0.2$ V,

$V_E = 6 - 0.7 = 5.3$ V and $I_E = 5.3/3.3 = 1.6$ mA.

$V_C = V_E + V_{CE(SAT)} = 5.3 + 0.2 = 5.5$ V

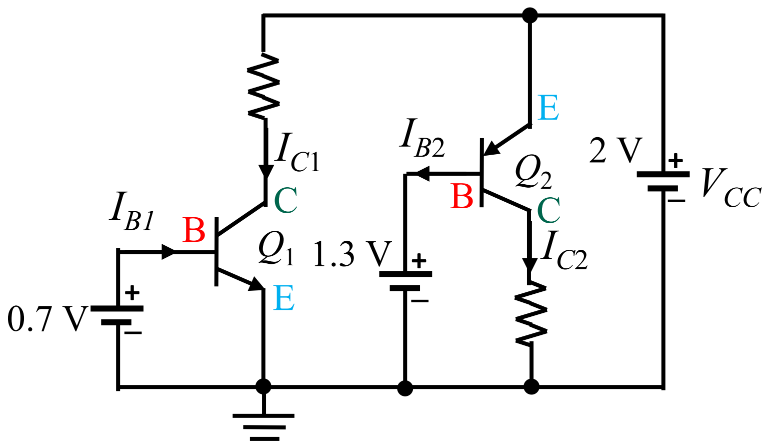
$I_C = (10 - 5.5)/4.7 = 0.96$ mA. $I_B = I_E - I_C = 1.6 - 0.96 = 0.64$ mA

$\beta_{\text{forced}} = I_C/I_B = 0.96/0.64 = 1.5$

Introduction to Amplifiers

- BJT is an excellent amplifier when biased in forward-active region
- MOSFET can be used as amplifier when biased in saturation region.
- In these regions, transistors can provide high voltage, current and power gains.
- Bias refers to setting the ‘quiescent’ (idle) current when there is no signal presence. It sets the transistor in the desired operation region.
- Q-point (determined by DC analysis) also determines
  - Small-signal parameters of transistor
  - Voltage gain, input resistance, output resistance
  - Maximum input and output signal amplitudes
  - Power consumption
  - Efficiency (o/p signal power vs DC i/p power)

Biasing BJT for linear amplification



BJT is forward biased for small-signal amplifier.

All the principles that applied to npn's also apply to pnp's with the exception that emitter is at a higher potential than base and base at a higher potential than collector

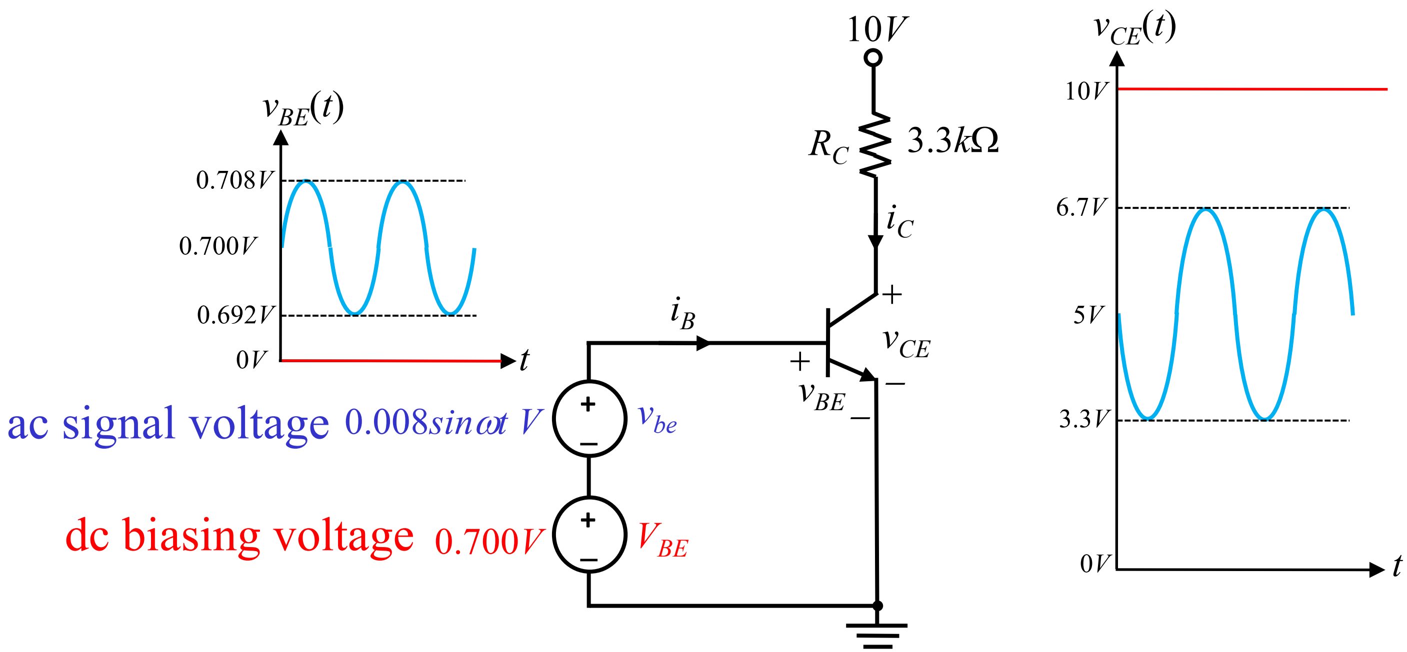
npn	**pnp**
$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$ with Early effect: $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$	$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right)$ with Early effect: $I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right)\left(1+\frac{V_{EC}}{V_A}\right)$

$$I_C = \beta I_B;\ I_E = \frac{I_C}{\alpha};\ \alpha = \frac{\beta}{\beta+1}$$

$$V_T = \frac{kT}{q} \approx 25\ mV\ @\ 25^\circ C$$

V_T : Thermal voltage in V.
k : Boltzmann's constant, 8.62×10^{-5} eV/K.
T : absolute temperature in K.
q : charge, 1.602×10^{-19} C.

BJT Amplifier

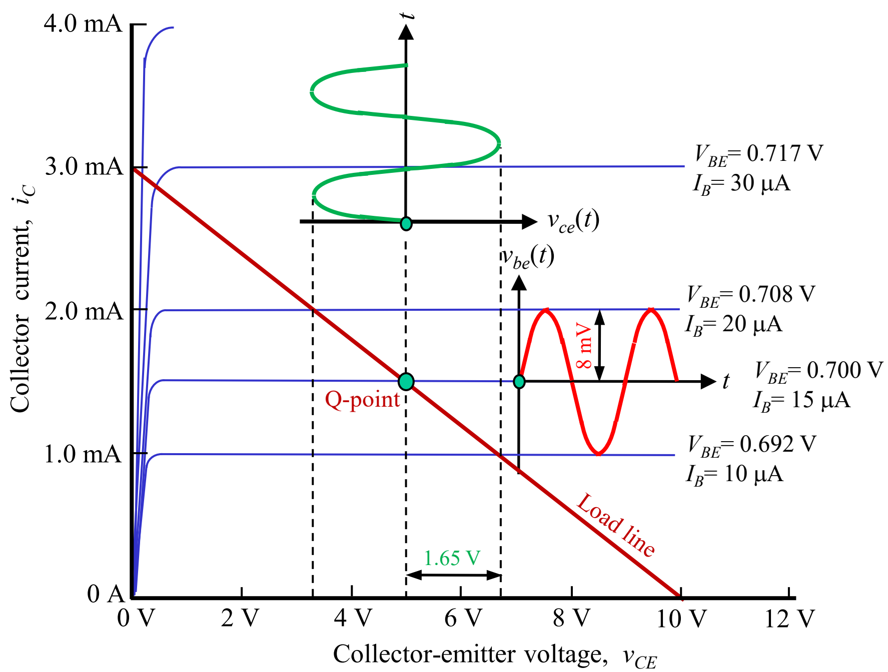


Q-point is set at (I_C, V_{CE}) = (1.5 mA, 5 V) with I_B = 15 μA.

Total base-emitter voltage is: $v_{BE} = V_{BE} + v_{be}$

Collector-emitter voltage is: $v_{CE} = V_{CC} - i_C R_C \Rightarrow$ load line

BJT Amplifier (contd.)



180° phase shift between input and output signals.

8 mV peak change in v_{BE}

$\Rightarrow$ 5 µA change in i_B

$\Rightarrow$ 0.5 mA change in i_C

$\Rightarrow$1.65 V change in v_{CE} .

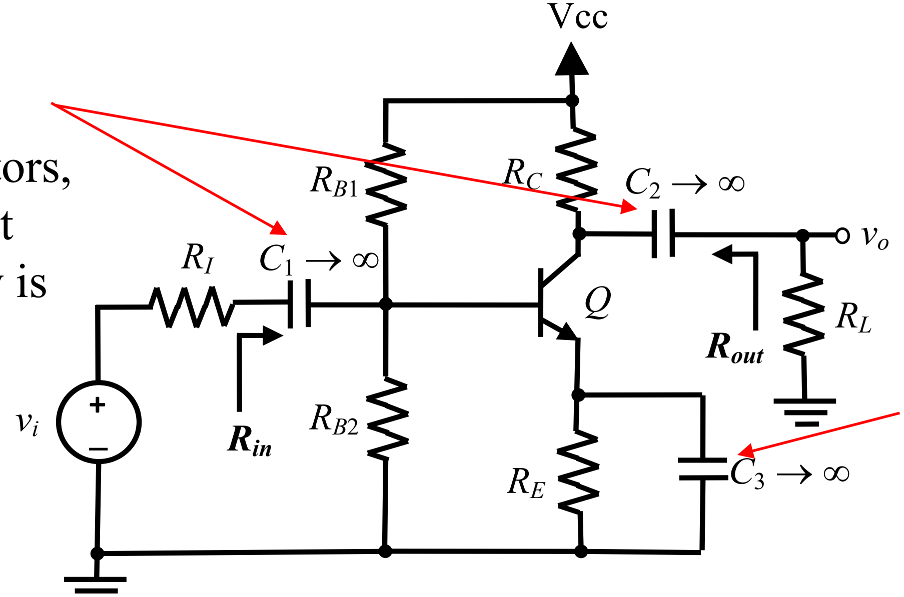
If changes in v_{BE} are small enough, then i_C and v_{CE} waveforms are undistorted replicas of input signal.

$$A_v = \frac{v_{ce}}{v_{be}} = \frac{1.65\angle 180°}{0.008\angle 0°}$$

$$= 206\angle 180° = -206$$

Coupling and Bypass Capacitors

large coupling capacitors or dc blocking capacitors, their reactance at signal frequency is negligible.



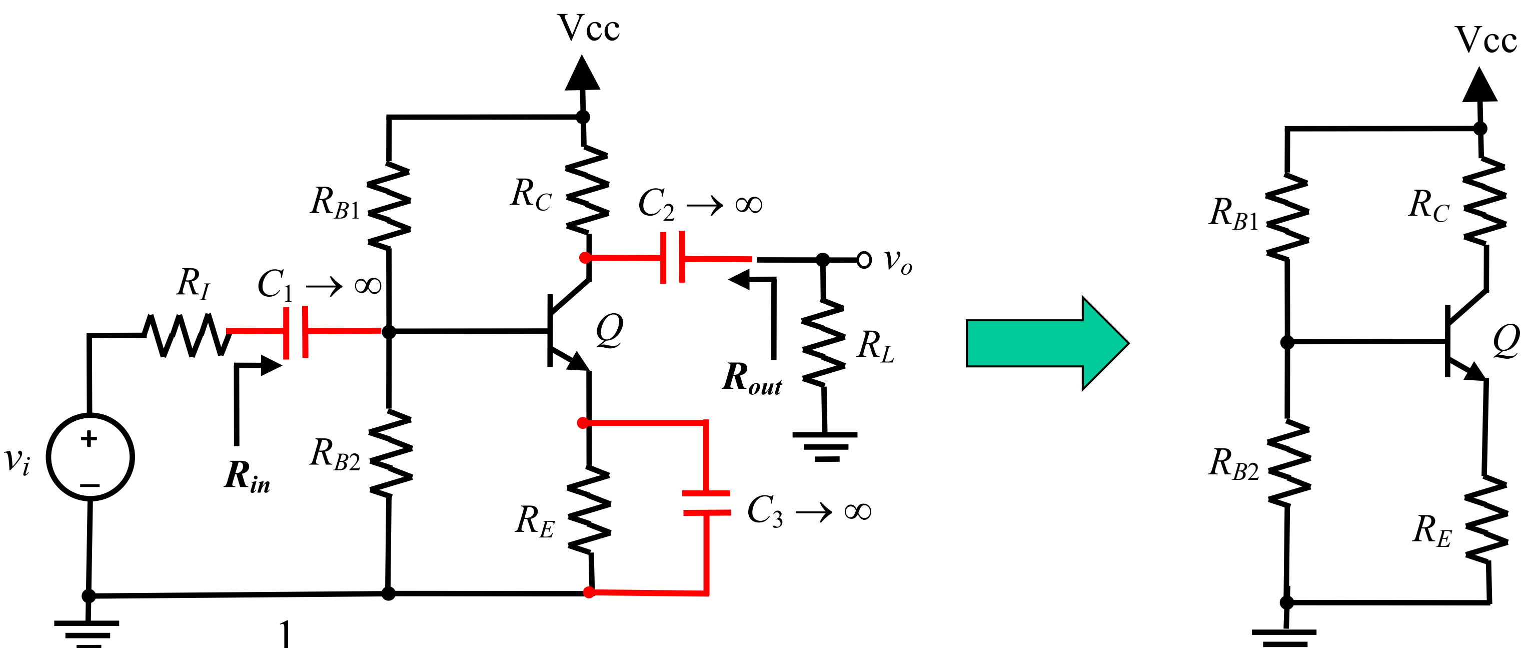
bypass capacitor, provides low impedance path for ac current from emitter to ground, effectively eliminating R_E from circuit when ac signals are considered (R_E is required for good Q-point stability).

- AC coupling through capacitors is used to inject ac input signal and extract output signal without disturbing the Q-point
- Capacitors provide negligible impedance at frequencies of interest and provide open circuits at dc.

DC and AC Analysis

- DC analysis:
  - Obtain dc equivalent circuit by replacing all capacitors by open circuits and inductors by short circuits. ac voltage sources by ground connections and ac current sources by open circuits.
  - Find Q-point from dc equivalent circuit by using appropriate large-signal transistor model.
- AC analysis:
  - Obtain ac equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, dc voltage sources by ground connections and dc current sources by open circuits.
  - Replace transistor by small-signal model
  - Use small-signal ac equivalent to analyze ac characteristics of amplifier.
- Combine end results of dc and ac analysis to yield total voltages and currents in the network.

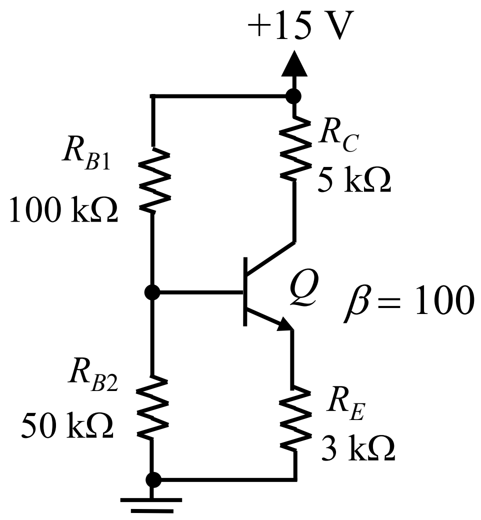
DC Equivalent for BJT Amplifier

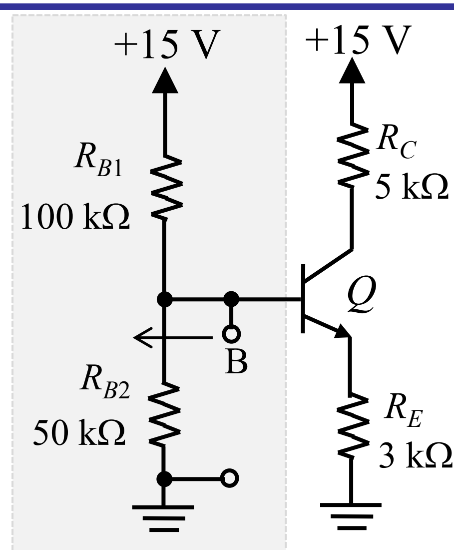


$$X_C = \frac{1}{\omega C} \qquad \omega = 0 \text{ rad/s (DC)}, X_c = \infty \text{ (open circuit)}$$

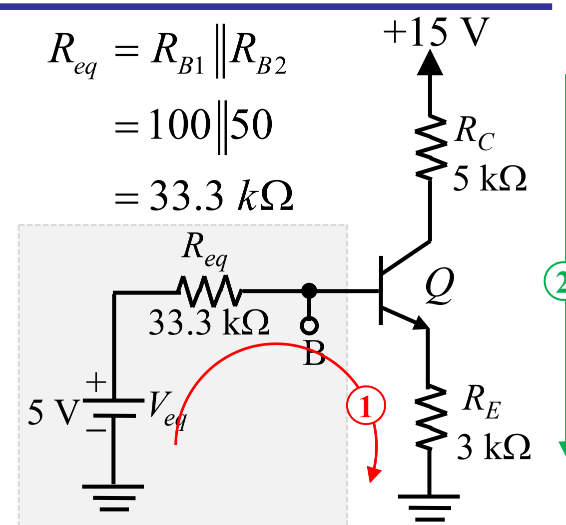
All capacitors in original amplifier circuits are replaced by open circuits, disconnecting v_I, R_I, and R_3 from circuit.

DC Analysis Example: Four-Resistor BJT Biasing Circuit





$$R_{eq} = R_{B1} \| R_{B2}$$
$$= 100 \| 50$$
$$= 33.3\ k\Omega$$



$$V_{eq} = \left(\frac{R_{B2}}{R_{B1} + R_{B2}}\right) V_{CC}$$
$$= \frac{50}{100 + 50} \times 15 = 5\ \text{V}$$

KVL 1: $V_{eq} = I_B R_{eq} + V_{BE} + I_E R_E$

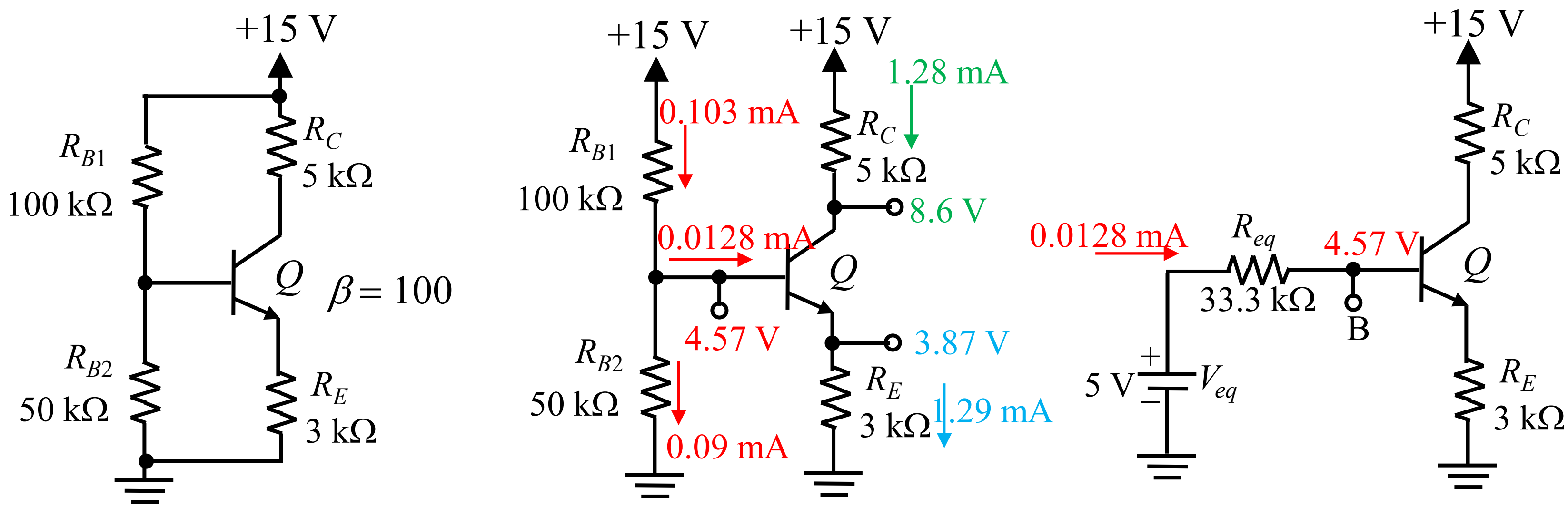
$5 = 33.3 I_B + 0.7 + 101 \times I_B \times 3$

$I_B = 0.0128$ mA

$I_C = \beta I_B =$ **1.28 mA**, $I_E = (\beta + 1) I_B = 1.29$ mA

KVL 2: $V_{CE} = 15 - I_C R_C - I_E R_E = 15 - 1.28 \times 5 - 1.29 \times 3 =$ **4.73 V**

DC Analysis Example: Four-Resistor BJT Biasing Circuit (cont.)



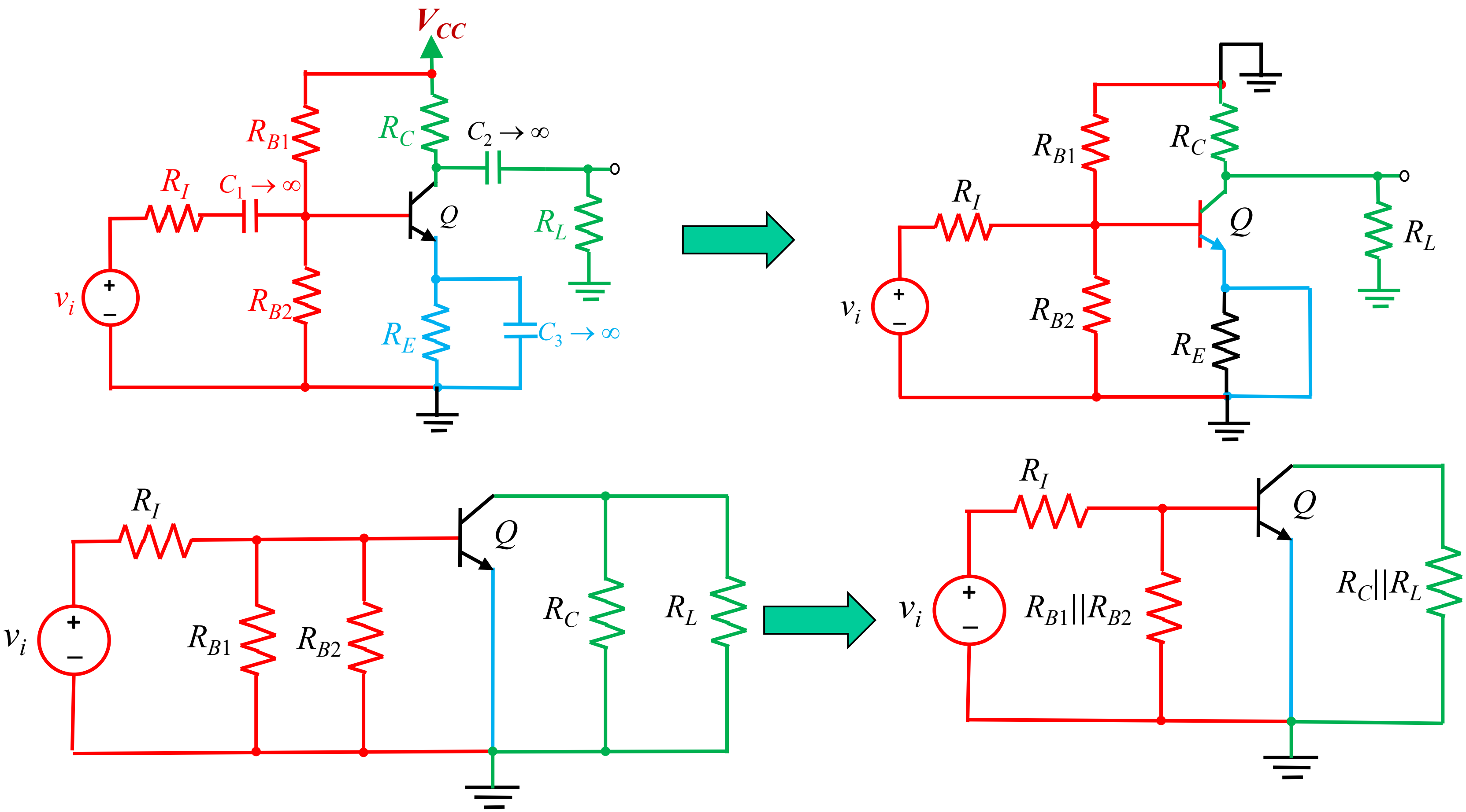
$V_B = V_{BE} + I_E R_E$ or $V_{eq} - I_B R_{eq} = 0.7 + 3.87 = 4.57$ V

$V_C = V_{CC} - I_C R_C = 15 - 1.28 \times 5 = 8.6$ V

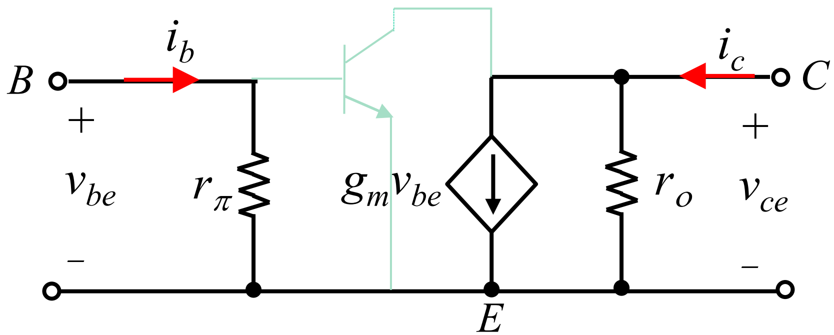
$V_{BC} = V_B - V_C = 4.57 - 8.6 = -4.03$ V.

BCJ is reverse baised, Q is indeed in active mode as had been assumed.

AC Equivalent for BJT Amplifier



Hybrid-Pi Model of BJT



- The hybrid-pi small-signal model is the intrinsic low-frequency representation of the BJT.
- Small-signal parameters are controlled by the Q-point and are independent of geometry of BJT

Transconductance:

$$g_m = \frac{I_C}{V_T} \approx 40 I_C$$

$$\text{where } V_T = \frac{kT}{q} \approx 25\ mV$$

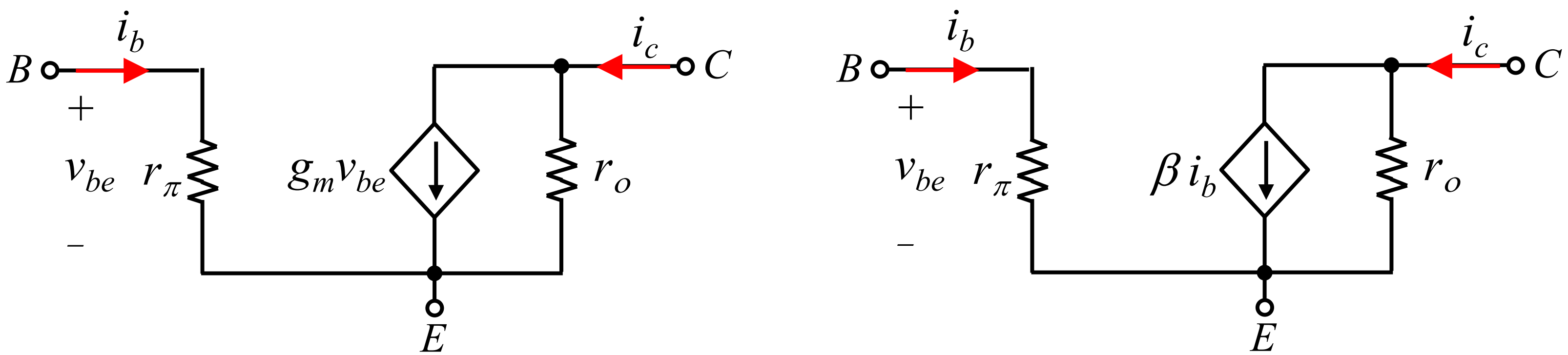
Input resistance:

$$r_\pi = \frac{\beta}{g_m}$$

Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} \textbf{ if } V_A \gg V_{CE}$$

Equivalent Forms of Small-Signal Model for BJT



- Voltage -controlled current source $g_m v_{be}$ can be transformed into current-controlled current source,

$$v_{be} = i_b r_\pi$$

$$g_m v_{be} = g_m i_b r_\pi = \beta i_b$$

$$i_c = \beta i_b + \frac{v_{ce}}{r_o} \approx \beta i_b$$

- Basic relationship $i_c = \beta\, i_b$ is useful in both dc and ac analysis when BJT is in forward-active region.

Small Signal Operation of BJT

$$i_C \approx I_S \exp\left(\frac{v_{BE}}{V_T}\right) = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_T}\right)$$

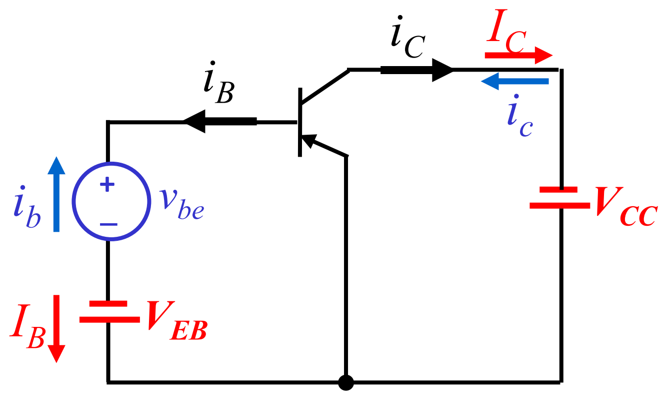
$$\therefore i_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\exp\left(\frac{v_{be}}{V_T}\right) = I_C\left[1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_{be}}{V_T}\right)^3 + \cdots\right]$$

$$i_c = i_C - I_C = I_C\left[\frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \cdots\right]$$

For linearity, i_c should be proportional to v_{be}

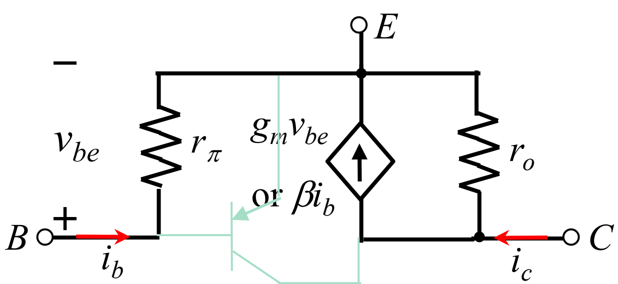
$$\frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 << \frac{v_{be}}{V_T} \Rightarrow \left|v_{be}\right| << 2V_T = 0.05\,V \Rightarrow \left|v_{be}\right| \le 0.005\,V$$

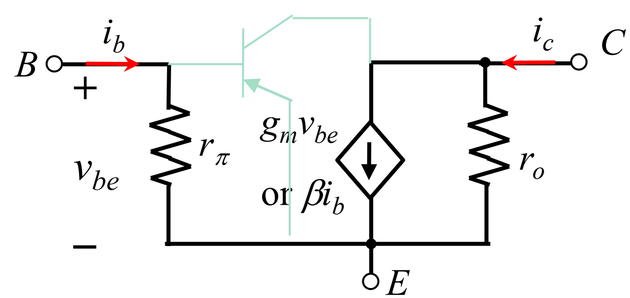
Small-Signal Model for *pnp* BJT



$$i_B = I_B - i_b$$

$$i_C = I_C - i_c$$



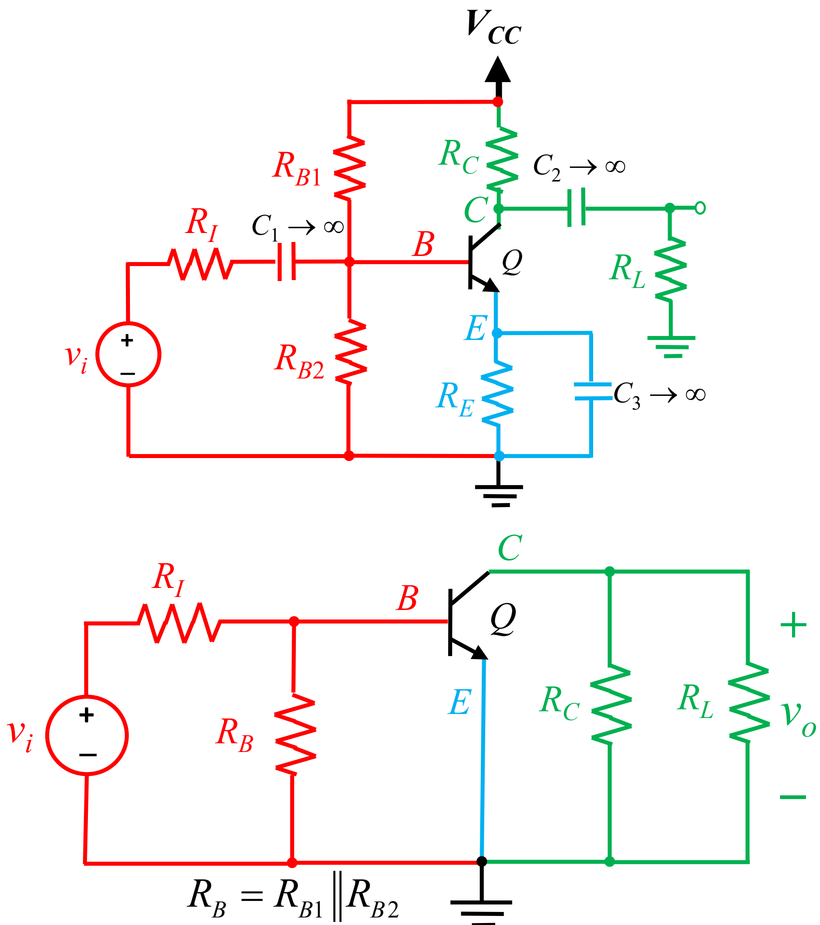


The small signal model for *pnp* transistor is exactly IDENTICAL to that of *npn*. This is not a mistake because the current direction is taken care of by the polarity of V_{BE}.

Summary of Small Signal Parameters

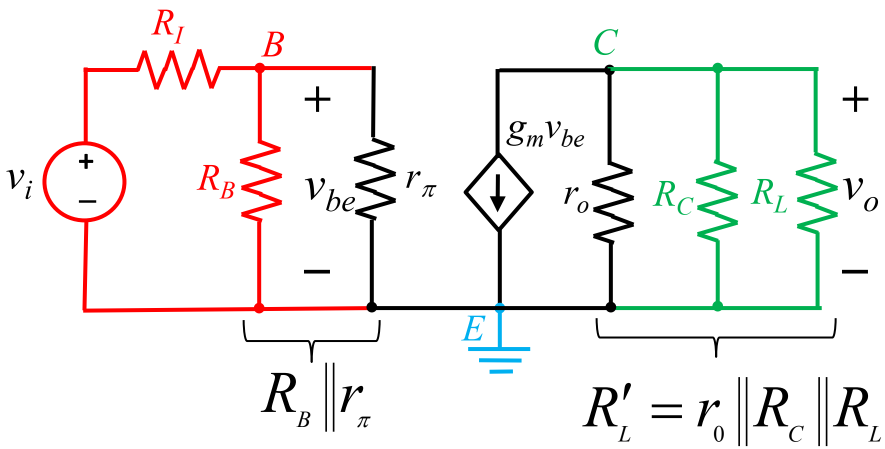
Parameter	BJT	n-MOSFET
g_m	$\dfrac{I_C}{V_T}$	$\dfrac{2I_D}{V_{GS}-V_{TN}}$ $K_n\left(V_{GS}-V_{TN}\right)\left(1+\lambda V_{DS}\right)\approx K_n\left(V_{GS}-V_{TN}\right)$ $\sqrt{2K_nI_D\left(1+\lambda V_{DS}\right)}\approx\sqrt{2K_nI_D}$
r_π	$\dfrac{\beta}{g_m}=\dfrac{\beta V_T}{I_C}$	∞
r_o	$\dfrac{V_A+V_{CE}}{I_C}\approx\dfrac{V_A}{I_C}$	$\dfrac{\frac{1}{\lambda}+V_{DS}}{I_D}\approx\dfrac{1}{\lambda I_D}$
Small-signal requirement	$v_{be}\le 0.005\ V$	$v_{gs}\le 0.2\left(V_{GS}-V_{TN}\right)$

Small Signal Analysis of Fully Bypass C-E Amplifier



- The ac equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage source is ac ground.
- Assume that Q-point is already known.

Fully Bypass C-E Amplifier: Voltage Gain



Overall voltage gain from source v_i to output voltage across R_I is:

$$A_v = \frac{v_o}{v_i} = \left(\frac{v_o}{v_{be}}\right)\left(\frac{v_{be}}{v_i}\right)$$

$$= A_{vt}\left(\frac{v_{be}}{v_i}\right)$$

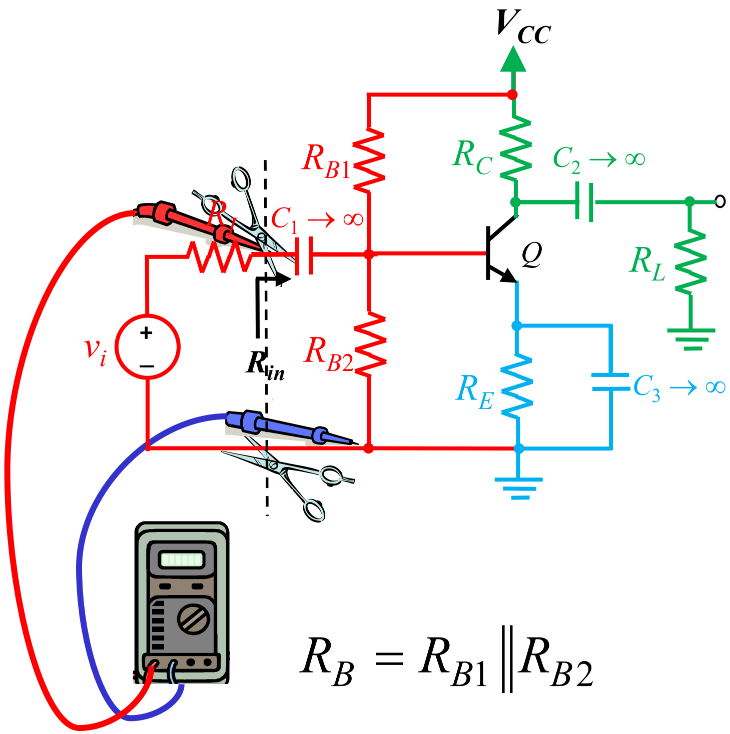
Terminal voltage gain between base and collector is:

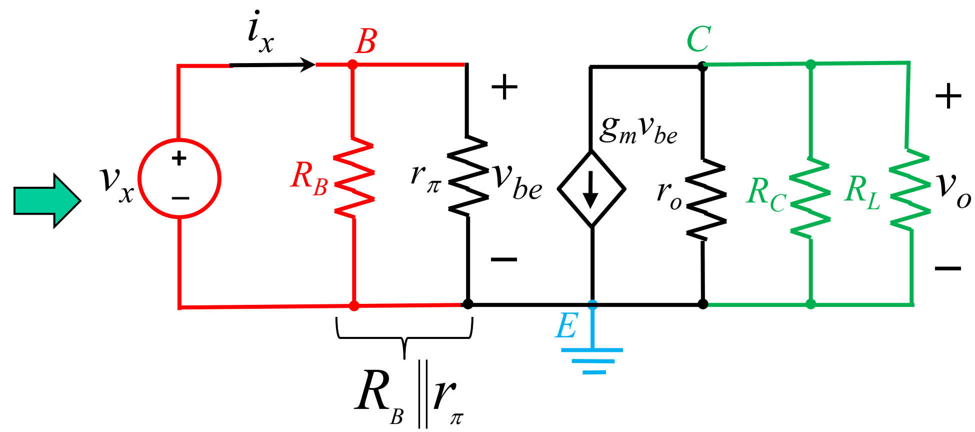
$$A_{vt} = \frac{v_c}{v_b} = \frac{v_o}{v_{be}}$$

$$= \frac{-g_m v_{be} R_L'}{v_{be}} = -g_m R_L'$$

$$\therefore A_v = -g_m R_L' \left(\frac{R_B \| r_\pi}{R_I + R_B \| r_\pi}\right)$$

Fully Bypass C-E Amplifier Input Resistance



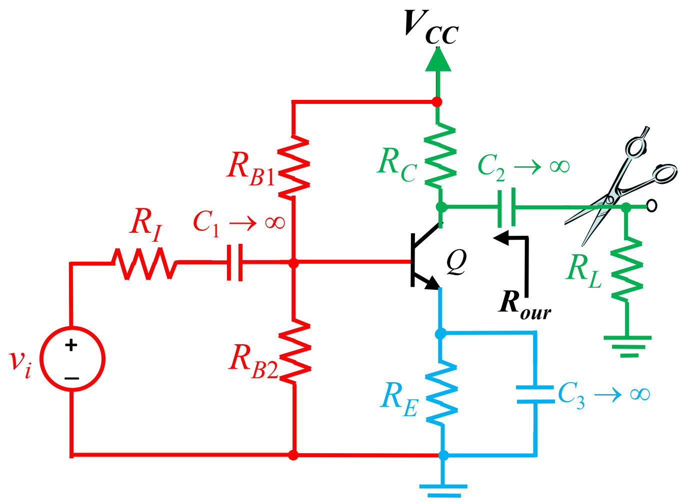


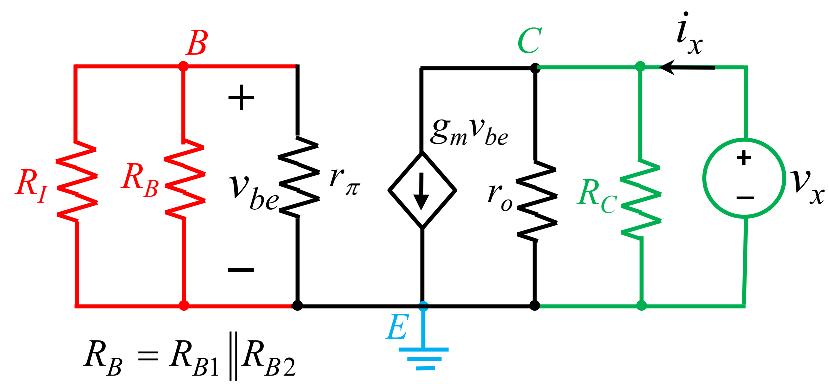
$$R_B = R_{B1} \| R_{B2}$$

$$v_x = i_x (R_B \| r_\pi)$$

$$R_{in} = \frac{v_x}{i_x} = R_B \| r_\pi$$

Fully Bypass C-E Amplifier Output Resistance





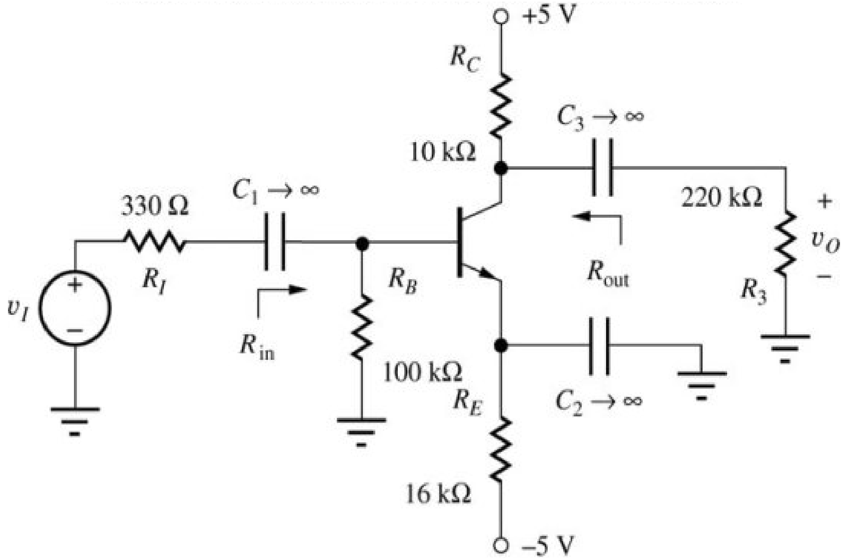
$$i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o} + g_m v_{be}$$

$$v_{be} = 0 \Rightarrow i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o}$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_C} + \frac{1}{r_o}\right)^{-1} = R_C \| r_o$$

$$R_{out} \approx R_C \text{ if } r_o \gg R_C$$

Fully Bypass C-E Amplifier Example



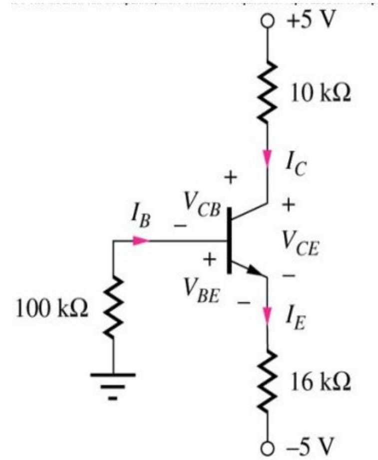
Problem: Find voltage gain, input and output resistances.

Given $\beta = 65$, $V_A = 50$ V

Assume $V_{BE} = 0.7$ V, and BJT biased for small signal operating conditions.

Find the Q-point from dc equivalent circuit

$$10^5 I_B + V_{BE} + (\beta + 1) I_B (1.6 \times 10^4) = 5$$



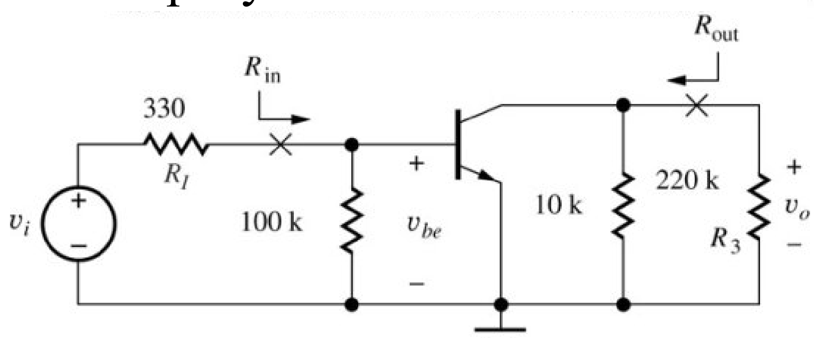
$$I_B = 3.71 \mu A$$
$$I_C = 65 I_B = 241 \mu A$$
$$I_E = 66 I_B = 245 \mu A$$

$$5 - 10^4 I_C - V_{CE} - (1.6 \times 10^4) I_E - (-5) = 0$$

$$V_{CE} = 3.67\ V$$

Analysis of Fully Bypass C-E Amplifier (contd.)

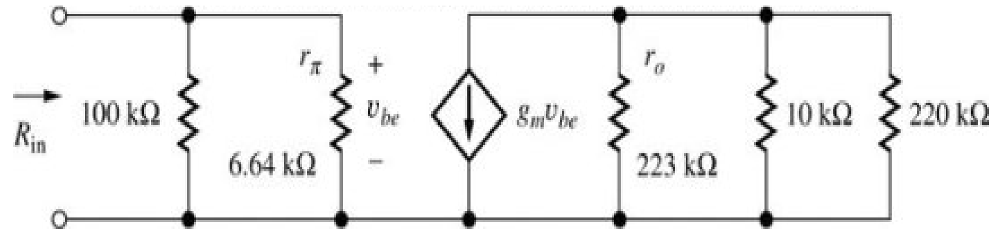
Construct the ac equivalent and simplify it.



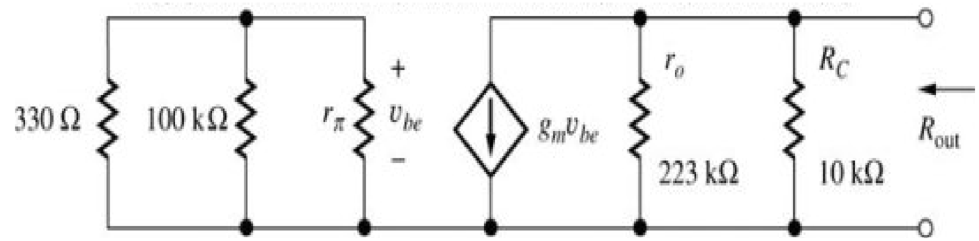
$$g_m = 40 I_C = 9.64 \times 10^{-3}\ S$$

$$r_\pi = \frac{\beta}{g_m} = 6.64\ k\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = 223\ k\Omega$$



$$R_{in} = R_B \| r_\pi = 6.23\ k\Omega$$

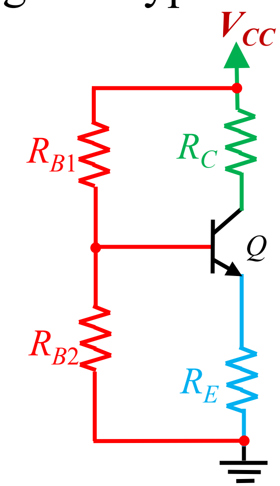


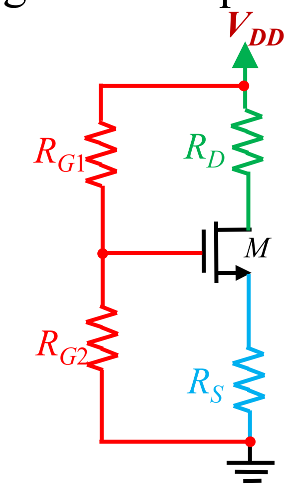
$$R_{out} = R_C \| r_o = 9.57\ k\Omega$$

$$A_v = \frac{v_o}{v_i} = -g_m \left(R_{out} \| R_3\right)\left(\frac{R_{in}}{R_I + R_{in}}\right) = -84.0$$

Amplifier Families

- Constraints for signal injection and extraction yield three families of amplifiers
  - Common-Emitter (C-E)/Common- Source (C-S)
  - Common-Base (C-B)/Common- Gate (C-G)
  - Common-Collector (C-C)/Common- Drain (C-D)
- All circuit examples here use the four-resistor bias circuits to establish Q-point of the various amplifiers
- Coupling and bypass capacitors are used to change the ac equivalent circuits.

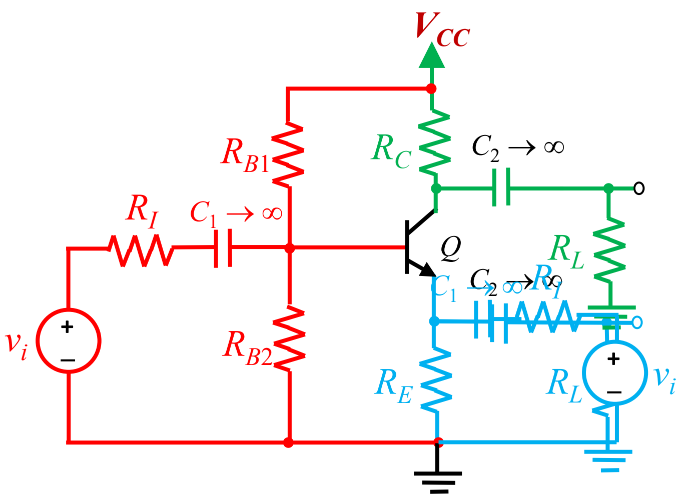


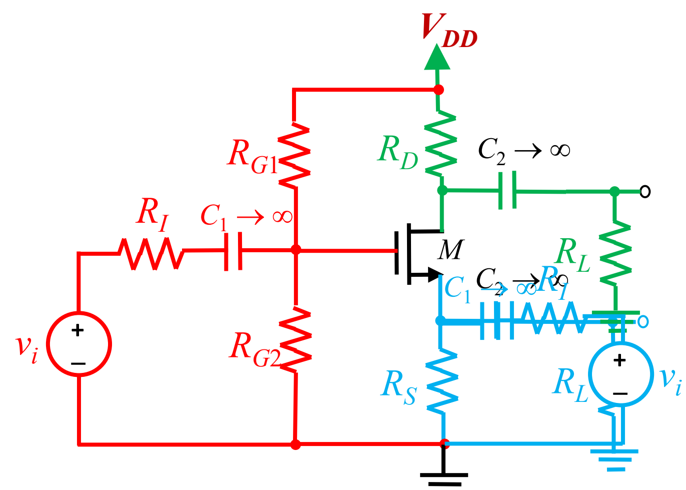


Amplifier Family

$$i_e \approx i_c \approx I_s \exp\left(\frac{v_b - v_e}{V_T}\right)$$

$$i_s = i_d \approx \frac{K_n}{2}\left(v_g - v_s - V_{TN}\right)^2$$





C-E: i/p at B, o/p at C

C-C: i/p at B, o/p at E

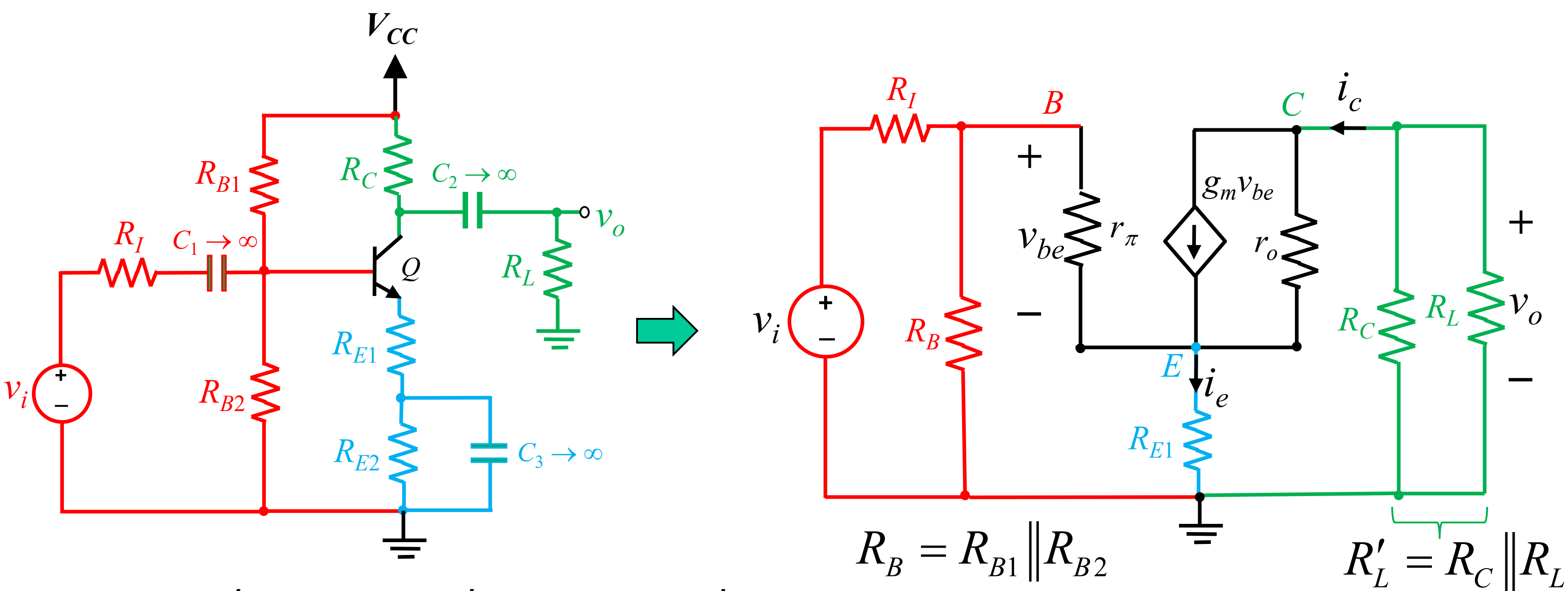
C-B: i/p at E, o/p at C

C-S: i/p at G, o/p at D

C-D: i/p at G, o/p at S

C-G: i/p at S, o/p at D

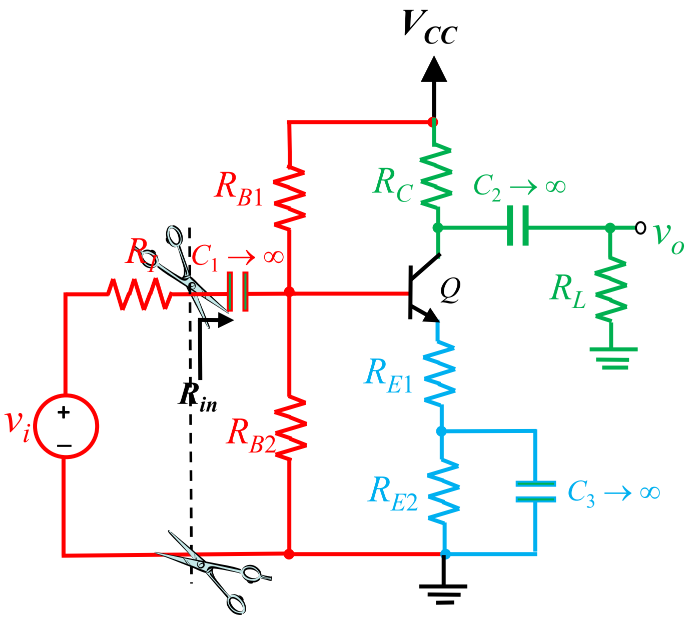
C-E Amplifier (Inverting Amplifier): Terminal Voltage Gain

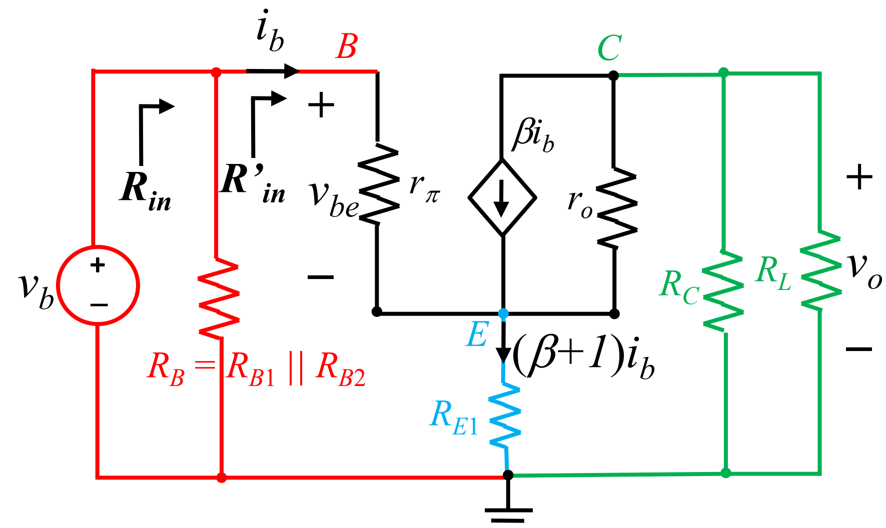


$\because i_{r_o} \ll g_m v_{be}, i_c \approx g_m v_{be} \approx i_e$

$$A_{vt} = \frac{v_c}{v_b} = \frac{-i_c R_L'}{v_{be} + i_e R_{E1}} \approx \frac{-g_m v_{be} R_L'}{v_{be} + g_m v_{be} R_{E1}} = \frac{-g_m R_L'}{1 + g_m R_{E1}}$$

C-E Amplifier (Inverting Amplifier): Input Resistance

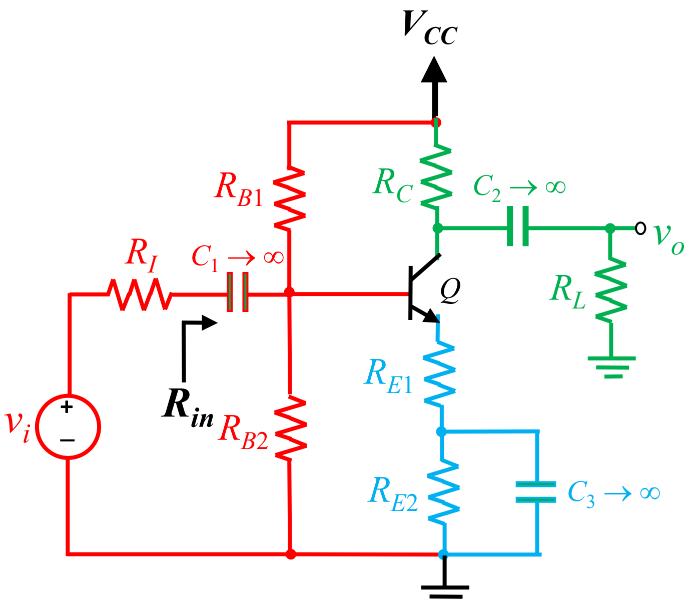


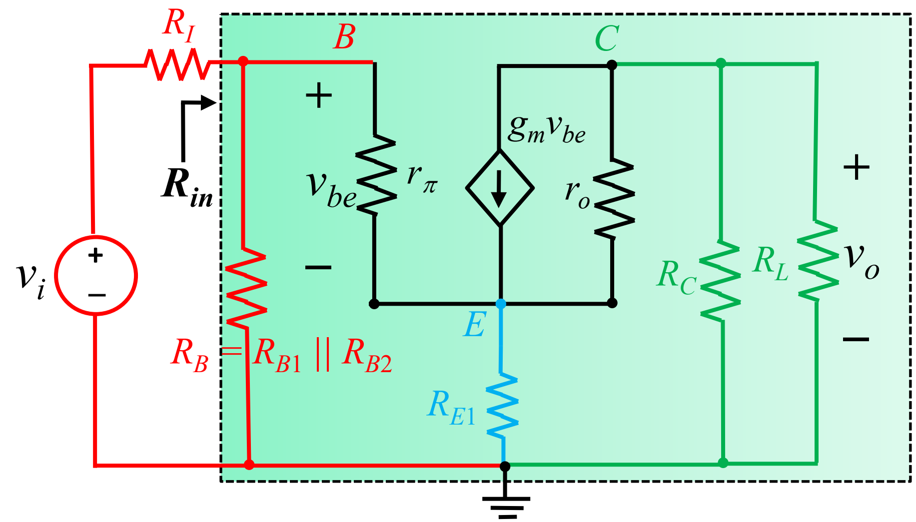


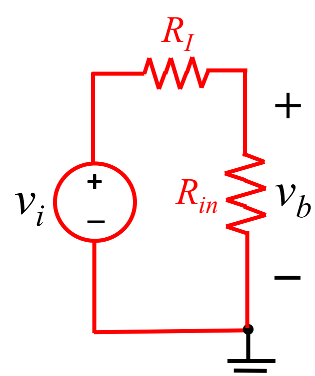
$$v_b = i_b r_\pi + (\beta+1) i_b R_{E1}$$

$$R'_{in} = \frac{v_b}{i_b} = r_\pi + (\beta+1) R_{E1} \quad \Rightarrow \quad R_{in} = R'_{in} \| R_B$$

C-E Amplifier (Inverting Amplifier): Overall Voltage Gain

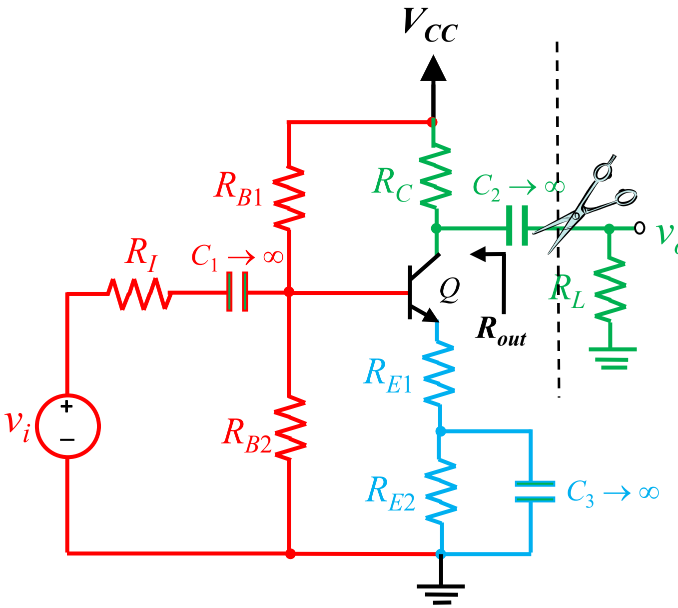


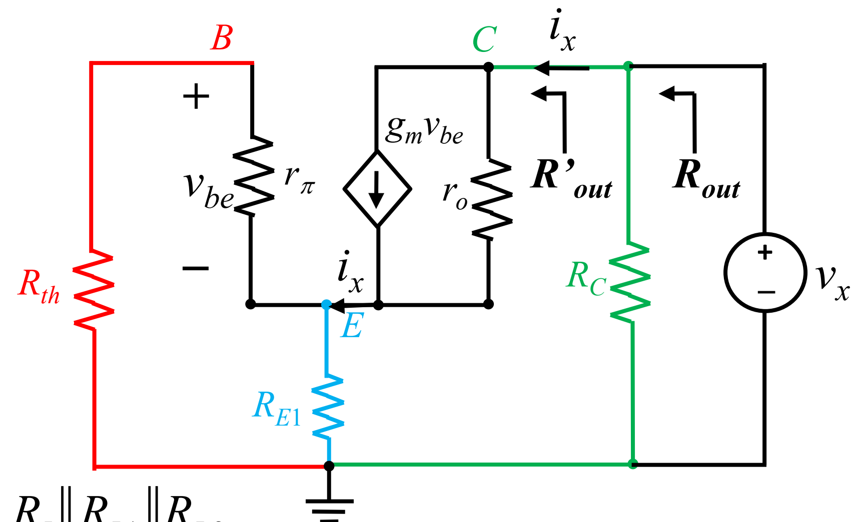




$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-E Amplifier (Inverting Amplifier): Output Resistance





$$R_{th} = R_I \| R_{B1} \| R_{B2}$$

$$v_x = \left(i_x - g_m v_{be}\right) r_o + v_e$$

$$v_e = i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\} \qquad v_{be} = -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) v_e = -\left(\frac{r_\pi}{r_\pi + R_{th}}\right) i_x \left\{\left(r_\pi + R_{th}\right) \| R_{E1}\right\}$$

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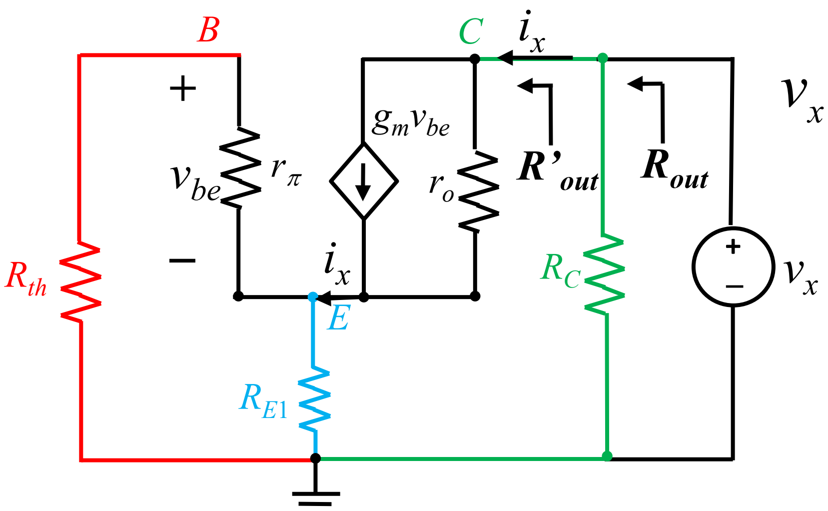
C-E Amplifier (Inverting Amplifier): Output Resistance (Continue)



$$v_x = \left(i_x - g_m v_{be}\right) r_o + v_e$$

$$= \left(i_x + g_m \left(\frac{r_\pi}{r_\pi + R_{th}} \right) i_x \left\{ \left(r_\pi + R_{th} \right) \| R_{E1} \right\} \right) r_o$$

$$+ i_x \left\{ \left(r_\pi + R_{th} \right) \| R_{E1} \right\}$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + \frac{g_m r_\pi R_{E1}}{r_\pi + R_{th} + R_{E1}} \right) r_o = \left(1 + \frac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}} \right) r_o$$

$$R_{out} = R'_{out} \| R_C$$

C-E Amplifier (Inverting Amplifier): Input Signal Range

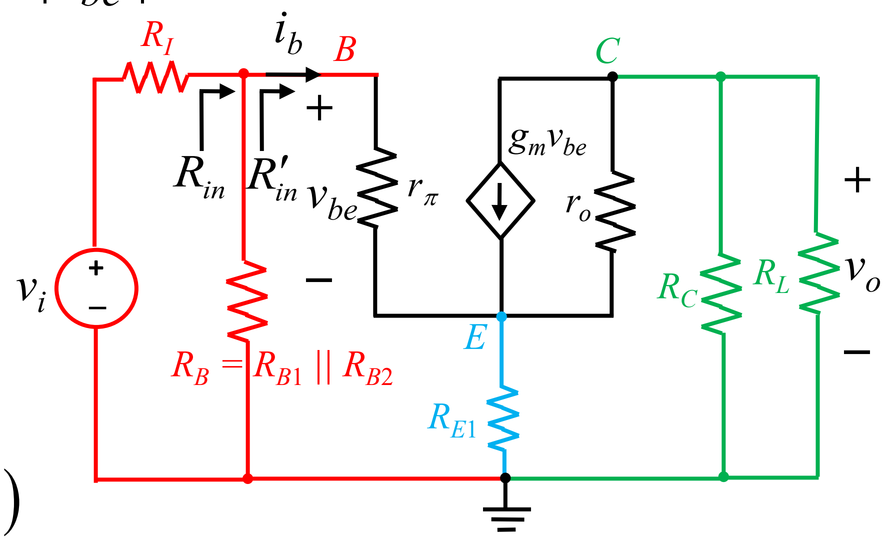
For BJT small-signal operation, $|v_{be}| \le 5\text{mV}$.

$$v_{be} = i_b r_\pi = \frac{v_b r_\pi}{R'_{in}} = \frac{v_b r_\pi}{r_\pi + (\beta+1) R_{E1}}$$

$$|v_{be}| \le 0.005$$

$$\Rightarrow |v_b| \le 0.005 \left(\frac{r_\pi + (\beta+1) R_{E1}}{r_\pi} \right)$$

$$\because \beta + 1 \approx g_m r_\pi, |v_b| \le 0.005 (1 + g_m R_{E1})$$



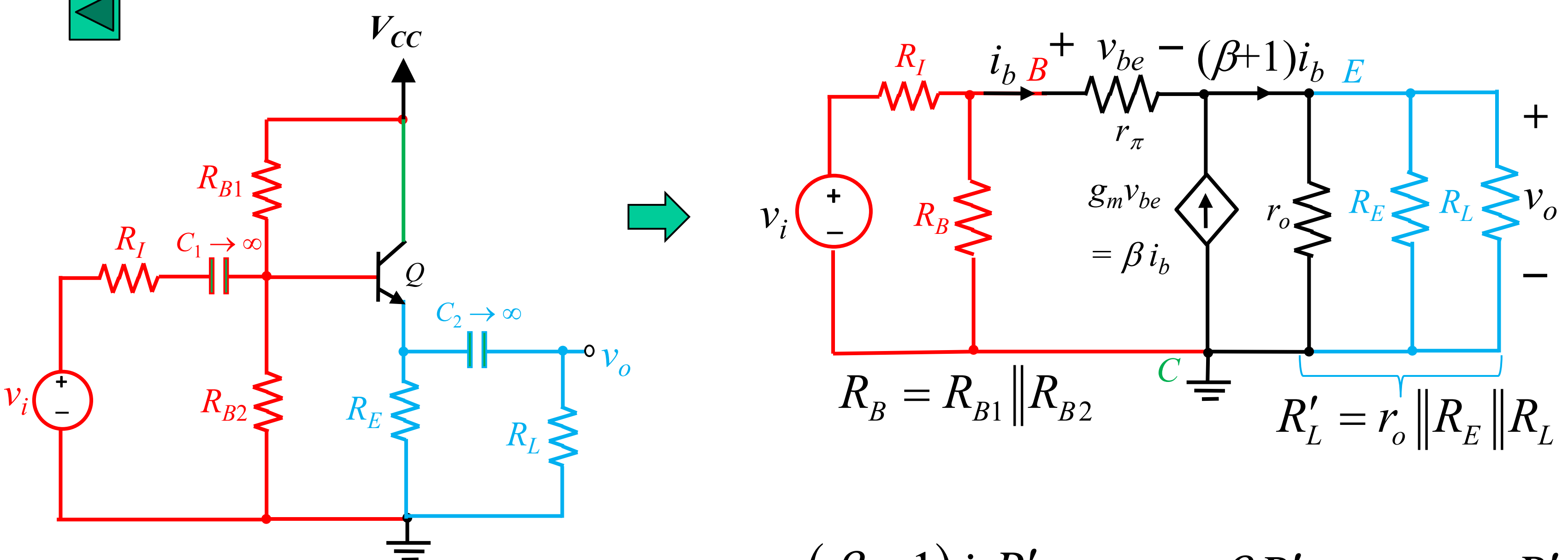
$$\because v_b = \left(\frac{R_{in}}{R_I + R_{in}} \right) v_i \Rightarrow |v_i| \le 0.005 (1 + g_m R_{E1}) \left(\frac{R_I + R_{in}}{R_{in}} \right)$$

If $g_m R_{E1} >> 1$, $|v_i|$ can be increased beyond 5 mV limit.

Summary: C-E and C-S

Amplifier \ *Parameter*		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\dfrac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\dfrac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\dfrac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\dfrac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\dfrac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\dfrac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \dfrac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\dfrac{1}{g_m}$	$\left[1+g_m (r_\pi \Vert R_{th})\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\dfrac{1}{g_m}$	$(1+g_m R_{th}) r_o$

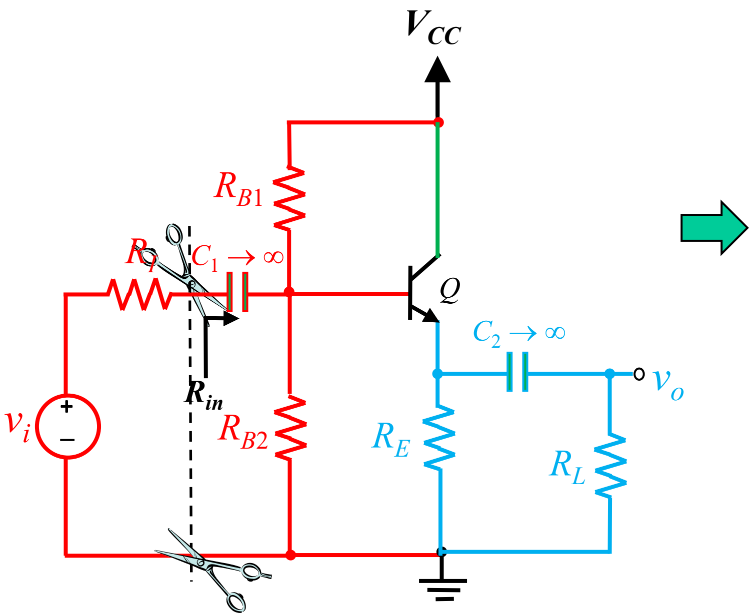
C-C Amplifier (Voltage Follower): Terminal Voltage Gain

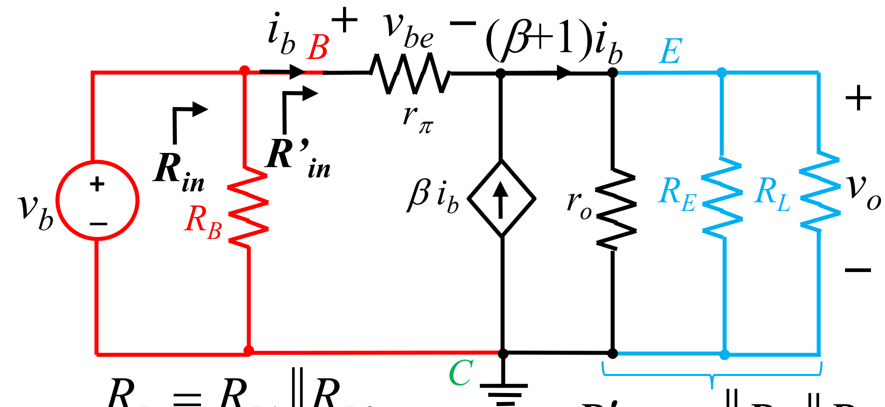


$$A_{vt} = \frac{v_e}{v_b} = \frac{(\beta+1)i_b R_L'}{i_b r_\pi + (\beta+1) i_b R_L'} \approx \frac{\beta R_L'}{r_\pi + \beta R_L'} = \frac{g_m R_L'}{1 + g_m R_L'}$$

$$\text{If } g_m R_L' >> 1,\ A_{vt} \approx 1 \Rightarrow v_o \approx v_b$$

C-C Amplifier (Voltage Follower): Input Resistance





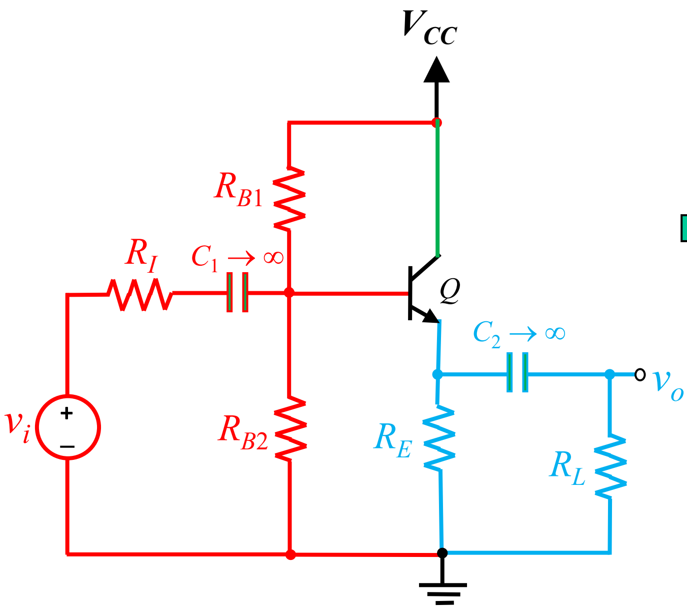
$$R_B = R_{B1} \| R_{B2}$$

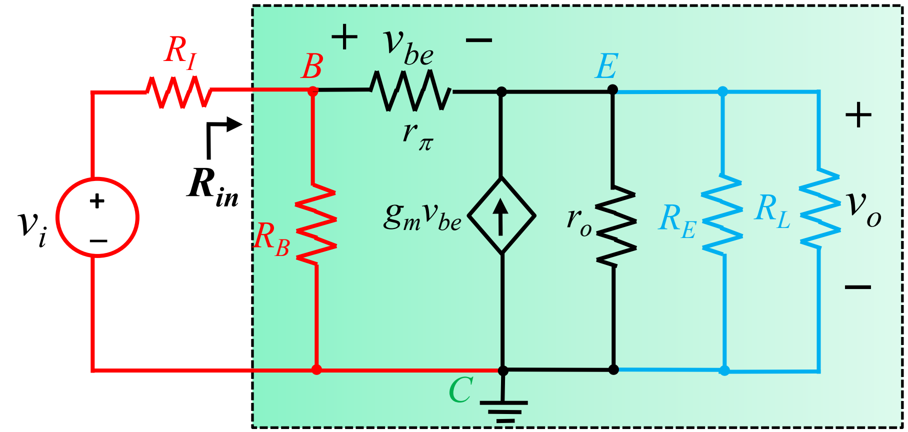
$$R'_L = r_o \| R_E \| R_L$$

$$v_b = i_b r_\pi + (\beta+1) i_b R'_L$$

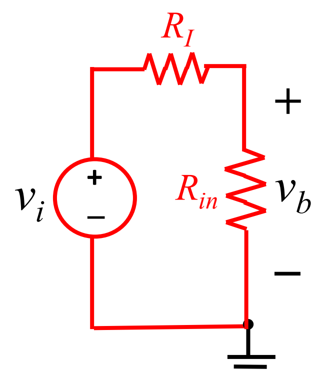
$$R'_{in} = \frac{v_b}{i_b} = r_\pi + (\beta+1) R'_L \Rightarrow R_{in} = R'_{in} \| R_B$$

C-C Amplifier (Voltage Follower): Overall Voltage Gain



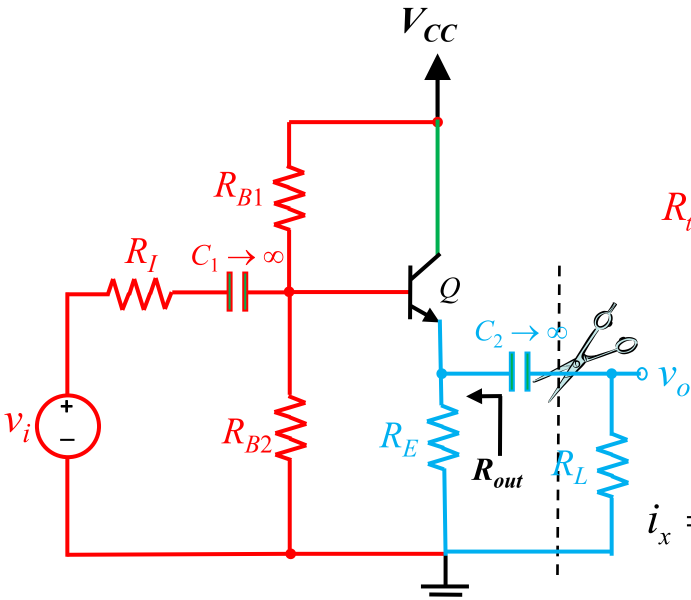


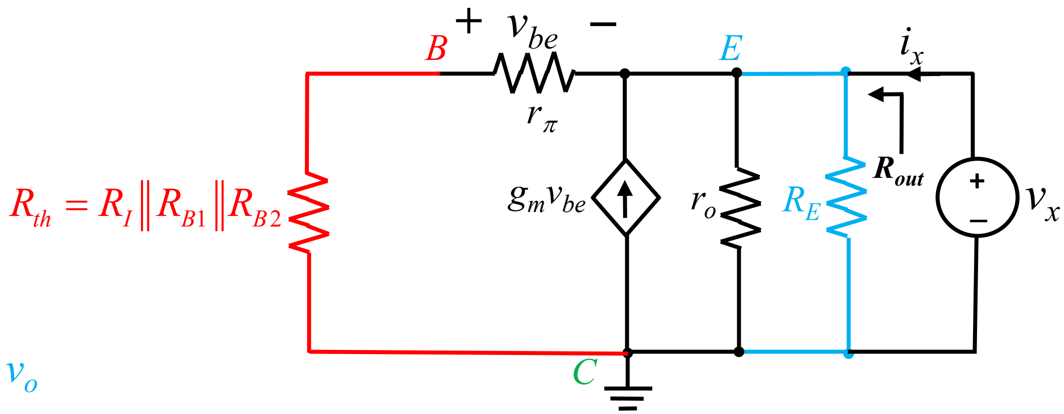
$$R_B = R_{B1} \| R_{B2}$$



$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_b} \times \frac{v_b}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-C Amplifier (Voltage Follower): Output Resistance





$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_o} + \frac{g_m r_\pi v_x}{r_\pi + R_{th}} + \frac{v_x}{r_\pi + R_{th}} = \left(\frac{1}{R_E} + \frac{1}{r_o} + \frac{\beta}{r_\pi + R_{th}} + \frac{1}{r_\pi + R_{th}} \right) v_x$$

$$i_x = \frac{v_x}{R_E} + \frac{v_x}{r_o} - g_m v_{be} + \frac{v_x}{r_\pi + R_{th}}$$

$$v_{be} = -\left(\frac{r_\pi}{r_\pi + R_{th}} \right) v_x$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_E} + \frac{1}{r_o} + \frac{\beta + 1}{r_\pi + R_{th}} \right)^{-1}$$

$$= R_E \left\| r_o \right\| \left(\frac{r_\pi + R_{th}}{\beta + 1} \right) \approx R_E \left\| \left(\frac{r_\pi + R_{th}}{\beta + 1} \right) \right.$$

C-C Amplifier (Voltage Follower): Input Signal Range

For BJT small-signal operation, $|v_{be}| \le 5\text{mV}$.

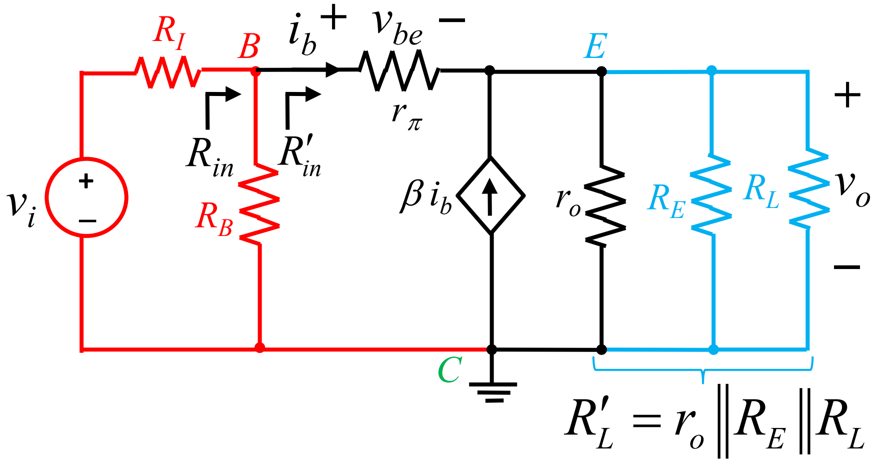
$$v_{be} = i_b r_\pi = \frac{v_b r_\pi}{R'_{in}} = \frac{v_b r_\pi}{r_\pi + (\beta+1)R'_L}$$

$$|v_{be}| \le 0.005$$

$$\Rightarrow |v_b| \le 0.005\left(\frac{r_\pi + (\beta+1)R'_L}{r_\pi}\right)$$

$$\because \beta + 1 \approx g_m r_\pi, |v_b| \le 0.005(1 + g_m R'_L)$$

$$\because v_b = \left(\frac{R_{in}}{R_I + R_{in}}\right) v_i \Rightarrow |v_i| \le 0.005(1 + g_m R'_L)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$

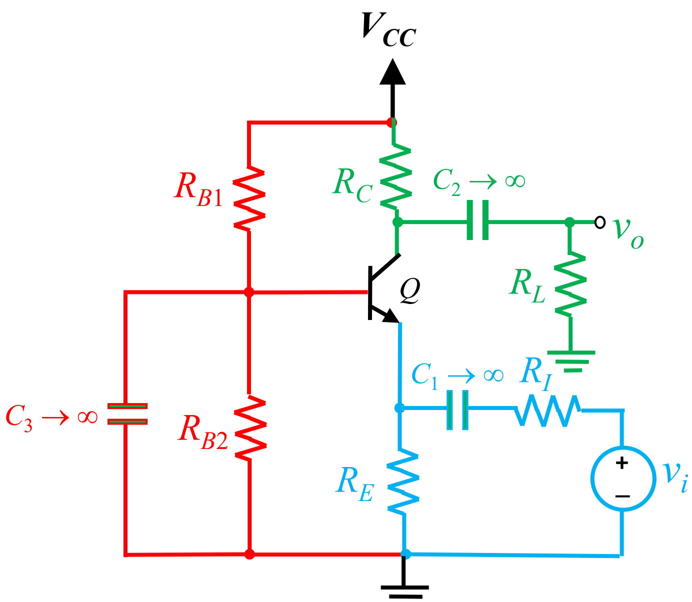


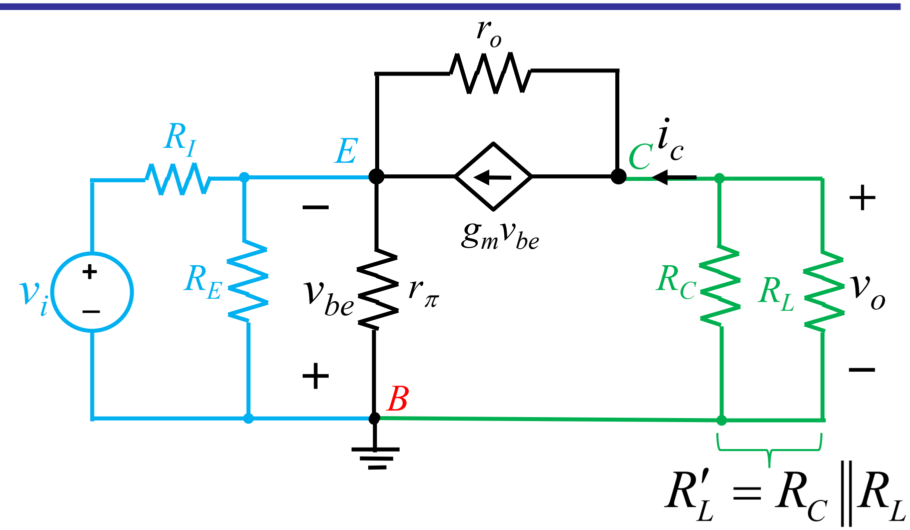
If $g_m R'_L >> 1$, $|v_i|$ can be increased beyond 5 mV limit.

Summary: C-C and C-D

Amplifier \ Parameter		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\dfrac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\dfrac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\dfrac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\dfrac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\dfrac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\dfrac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \dfrac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\dfrac{1}{g_m}$	$\left[1+g_m (r_\pi \Vert R_{th})\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\dfrac{1}{g_m}$	$(1+g_m R_{th}) r_o$

C-B Amplifier (Noninverting Amplifier): Terminal Voltage Gain





$$\because i_{r_o} << g_m v_{be}, i_c \approx g_m v_{be}$$

$$v_e = v_{eb} = -v_{be}$$

$$A_{vt} = \frac{v_c}{v_e} = \frac{-i_c R'_L}{-v_{be}} \approx \frac{-g_m v_{be} R'_L}{-v_{be}} = g_m R'_L$$

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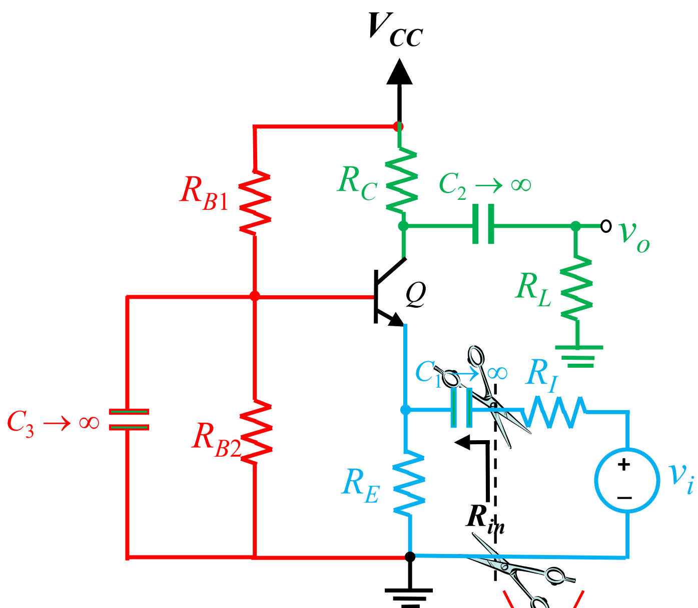
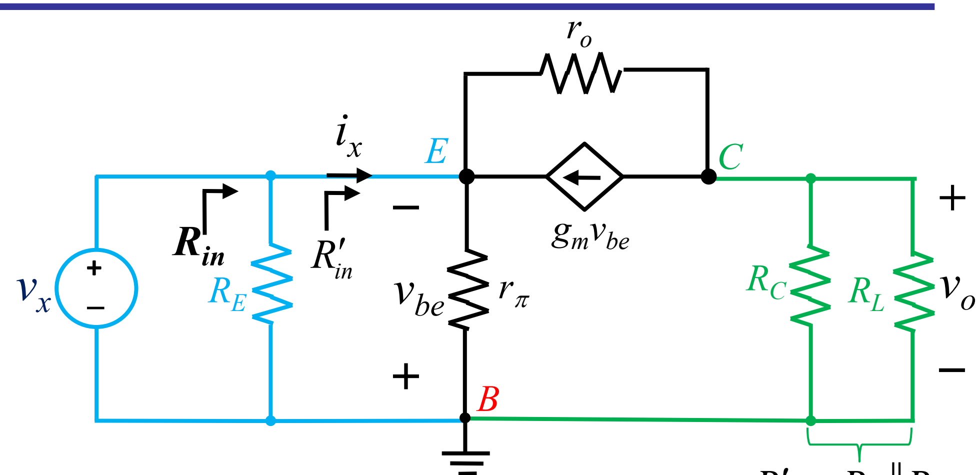
C-B Amplifier (Noninverting Amplifier): Input Resistance





$$R'_L = R_C \| R_L$$

$$i_x = -\frac{v_{be}}{r_\pi} - g_m v_{be} + \cancel{\frac{v_{ec}}{r_o}}$$

This current is small because of large r_0

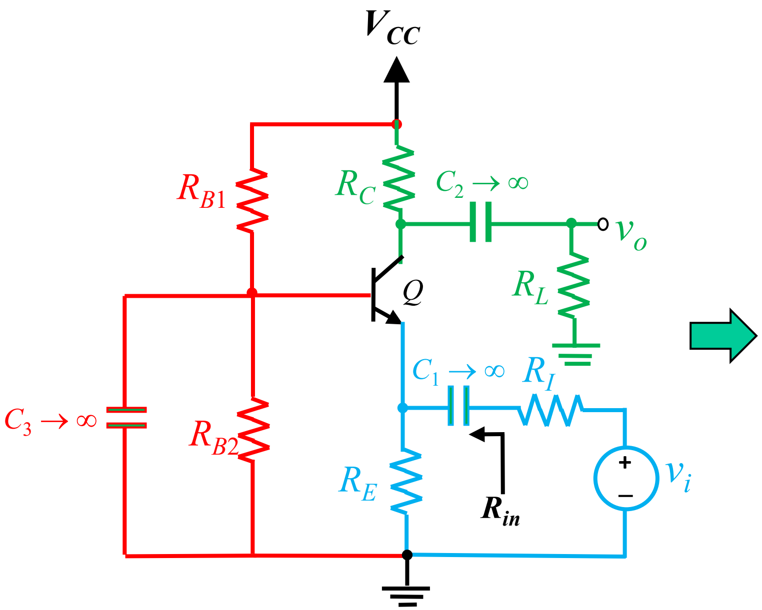
$$\because v_{be} = -v_x,\ i_x = \frac{v_x}{r_\pi} + g_m v_x$$

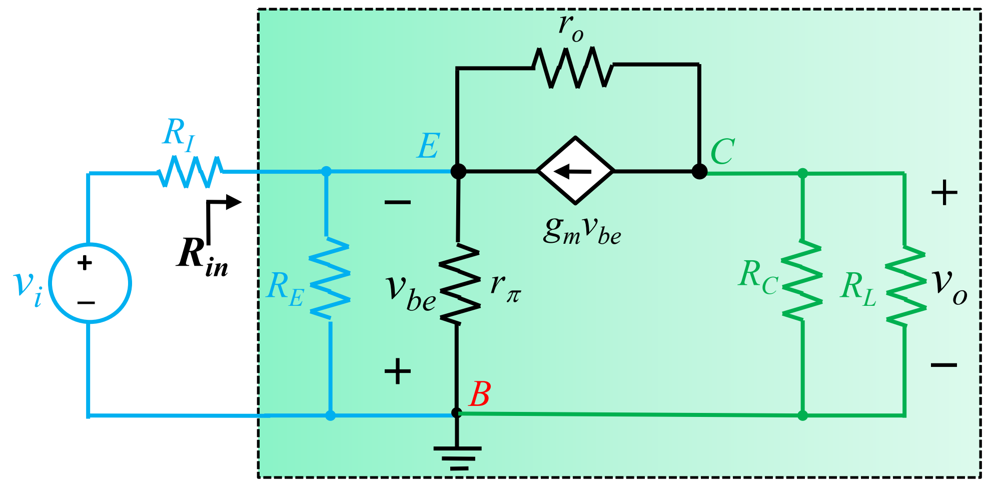
$$i_x = \left(\frac{1+g_m r_\pi}{r_\pi}\right) v_x = \left(\frac{1+\beta}{r_\pi}\right) v_x$$

$$R'_{in} = \frac{v_x}{i_x} = \left(\frac{r_\pi}{\beta+1}\right) \approx \frac{r_\pi}{\beta} = \frac{1}{g_m}$$

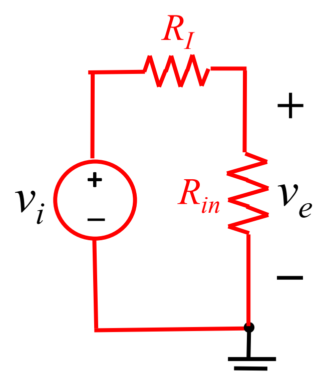
$$R_{in} = R'_{in} \| R_E = \frac{R_E}{1+g_m R_E}$$

C-B Amplifier (Noninverting Amplifier): Overall Voltage Gain



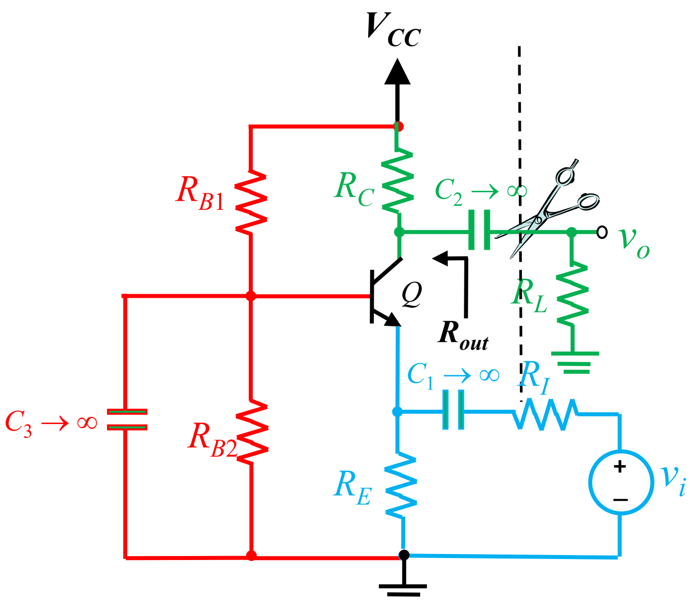


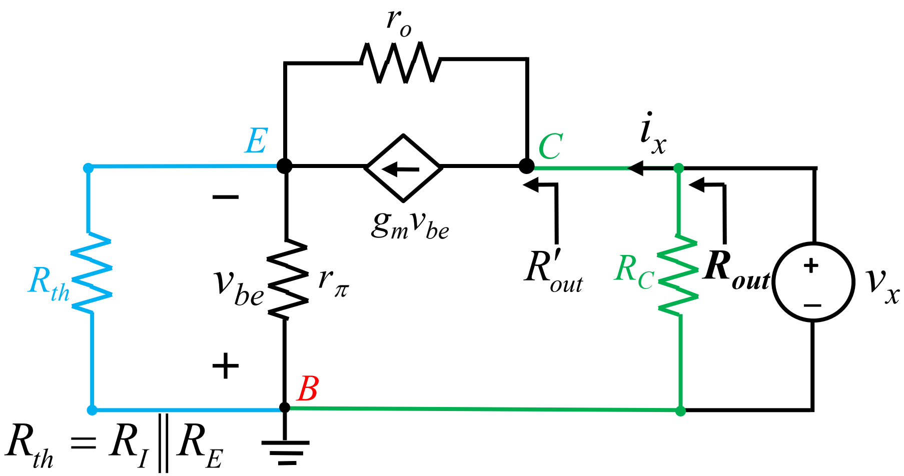
$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_e} \times \frac{v_e}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$



$$R_{in} = R_E \| \frac{1}{g_m} = \frac{R_E}{1 + g_m R_E}$$

C-B Amplifier (Noninverting Amplifier): Output Resistance





$$R_{th} = R_I \| R_E$$

$$v_x = \left[i_x + g_m i_x \left(r_\pi \| R_{th} \right) \right] r_o + i_x \left(r_\pi \| R_{th} \right)$$

$$v_x = \left(i_x - g_m v_{be} \right) r_o + v_e$$

$$v_e = i_x \left(r_\pi \| R_{th} \right)$$

$$v_{be} = -v_e = -i_x \left(r_\pi \| R_{th} \right)$$

$$R'_{out} = \frac{v_x}{i_x} = \left[1 + g_m \left(r_\pi \| R_{th} \right) \right] r_o + r_\pi \| R_{th}$$

$$\approx \left[1 + g_m \left(r_\pi \| R_{th} \right) \right] r_o$$

$$R_{out} = R'_{out} \| R_C$$

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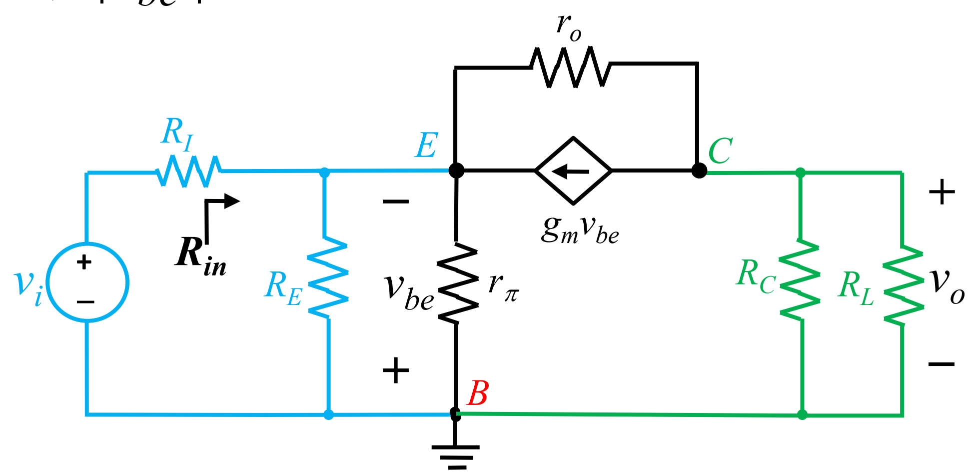
C-B Amplifier (Noninverting Amplifier): Input Signal Range

For BJT small-signal operation, $|v_{be}| \le 5\text{mV}$.

$$v_{be} = -v_e = -\left(\frac{R_{in}}{R_I + R_{in}}\right) v_i$$

$$|v_{be}| \le 0.005$$

$$\Rightarrow |v_i| \le 0.005\left(\frac{R_I + R_{in}}{R_{in}}\right)$$



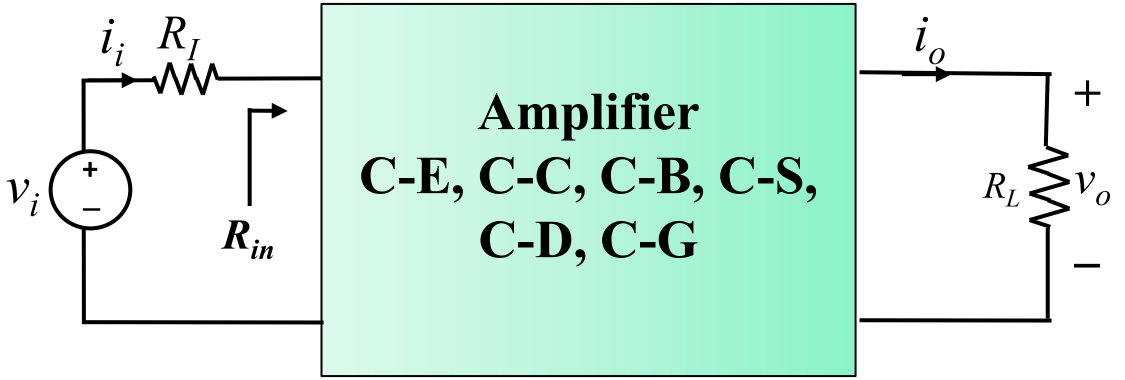
$$\because R_{in} = \frac{R_E}{1 + g_m R_E} \Rightarrow \frac{R_I + R_{in}}{R_{in}} = 1 + g_m R_I + \frac{R_I}{R_E}$$

If $R_E >> R_I$, $|v_i| \le 0.005\left(1 + g_m R_I\right)$

Summary: C-B and C-G

Amplifier \ Parameter		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\frac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\frac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\frac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\frac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\frac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\frac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \frac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\frac{1}{g_m}$	$\left[1+g_m (r_\pi \Vert R_{th})\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\frac{1}{g_m}$	$(1+g_m R_{th}) r_o$

Current Gain



$$A_i = \frac{i_o}{i_i} = \frac{\dfrac{v_o}{R_L}}{\dfrac{v_i}{R_I + R_{in}}} = \frac{v_o}{v_i} \times \frac{R_I + R_{in}}{R_L} = A_v \times \frac{R_I + R_{in}}{R_L}$$