Small-Signal MOSFET Amplifiers

E2002 Analog Electronics
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Module Goals

Understanding of concepts related to:

- Biasing of Transistors (BJT and MOSFET)
- dc and ac equivalent circuits for small-signal amplifier
- Small-signal models of BJT and MOSFET
- Amplifier characteristics such as voltage gain, input and output resistances
- Analysis of three broad classes of single-stage amplifiers

  - Inverting amplifiers – common-emitter and common-source configurations

  - Followers – common-collector and common-drain configurations

  - Noninverting amplifiers – common-base and common-gate configurations

References

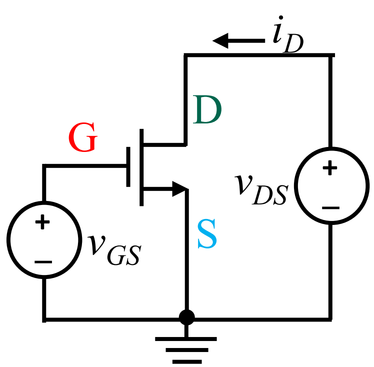
Text Book

1. Richard C. Jaeger and Travis N. Blalock, "Microelectronic Circuit Design", 4th Edition, McGraw Hill, 2011, Chapters 4, 5, 13 and 14.

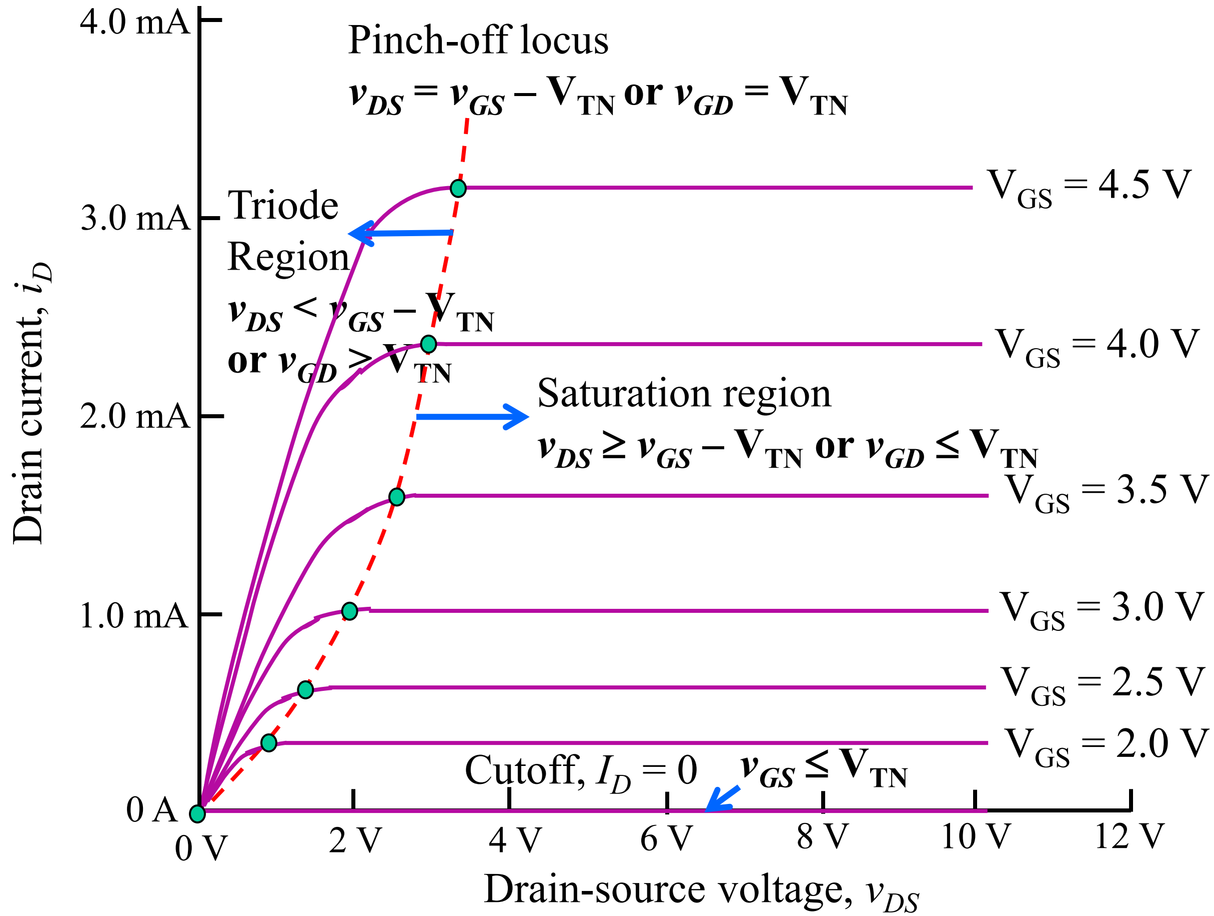
References

1. Allan R. Hambley, "Electronics", 2nd Edition, Prentice Hall, 2000
2. Donald A. Neamen, "Electronic Circuit Analysis and Design", 2nd Edition, McGraw-Hill, 2002

MOSFET Regions of Operation



V_{TN}: Threshold Voltage of NMOS



MOSFET Biasing for Different Regions of Operation

In EE2002, only enhancement mode MOSFET is considered.

Region	NMOS	PMOS
Cutoff	$V_{GS} < V_{TN}$ $I_D = 0$	$\lvert V_{GS}\rvert < \lvert V_{TP}\rvert$ $I_D = 0$
Triode	$V_{DS} < V_{GS} - V_{TN}$ $I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$	$\lvert V_{DS}\rvert < \lvert V_{GS}\rvert - \lvert V_{TP}\rvert$ $I_D = K_p\left(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert - \frac{\lvert V_{DS}\rvert}{2}\right)\lvert V_{DS}\rvert$
Saturation	$V_{DS} \geq V_{GS} - V_{TN}$ $I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2$	$\lvert V_{DS}\rvert \geq \lvert V_{GS}\rvert - \lvert V_{TP}\rvert$ $I_D = \frac{K_p}{2}\left(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert\right)^2$

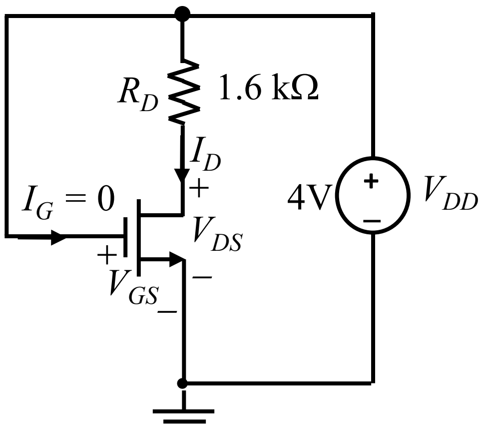
$K_n = \mu_n C_{ox}\left(\frac{W}{L}\right)$ and $K_p = \mu_p C_{ox}\left(\frac{W}{L}\right)$

MOSFET Bias Analysis Approach

- Assume saturation region (unless operation region is obvious)
- Use circuit analysis to find V_{GS}
- Use V_{GS} to calculate I_D, and I_D to find V_{DS}
- Check validity of operation region assumption
- Change assumption and analyze again if required.

NOTE: An enhancement-mode MOSFET with $V_{DS} = V_{GS}$ is always in saturation because $|V_{DS}|$ always greater than $(|V_{GS}| - |V_{TX}|)$, where X is N or P for threshold voltage of NMOS or PMOS, respectively.

MOSFET Bias Analysis: Triode Region



$K_n = 250\ \mu A/V^2$
$V_{TN} = 1$ V

Assume transistor is saturated,
$V_{GS} = V_{DD} = 4$ V.

$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 = \frac{250\mu}{2}(4-1)^2 = 1.13 \text{ mA}$$

$$V_{DD} = I_D R_D + V_{DS}$$
$$V_{DS} = 4 - 1.6 \times 1.13 = 2.19 \text{ V}$$

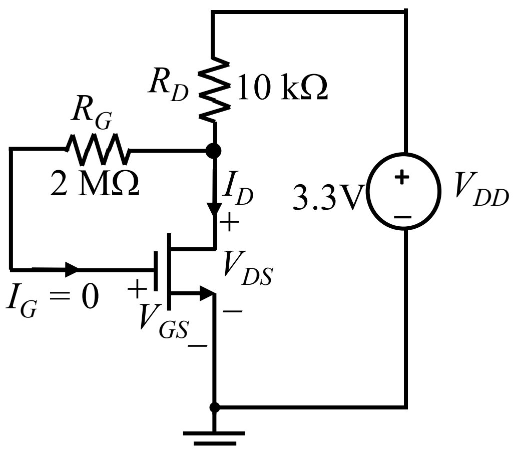
But $V_{DS} = 2.19 < V_{GS} - V_{TN} = 4 - 1 = 3$,
Saturation region assumption is incorrect.

Using triode region equation, $I_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$

$V_{DS} = 4 - 1600 \times 250\mu\ (4 - 1 - V_{DS}/2)V_{DS}$
$V_{DS} =$ **2.3 V** and $I_D =$ **1.06 mA**

$V_{DS} = 2.3 < V_{GS} - V_{TN} = 3$, transistor is in triode region.

MOSFET Bias Analysis: nMOS Two-Resistor Biasing



$K_n = 260\ \mu A/V^2$
$V_{TN} = 1$ V

Since $I_G = 0$, $V_{DS} = V_{GS}$
Transistor is saturated

$$V_{DS} = V_{DD} - I_D R_D$$

$$V_{GS} = 3.3 - \frac{260\mu}{2}(V_{GS} - 1)^2 \times 10000 = 3.3 - 1.3(V_{GS} - 1)^2$$

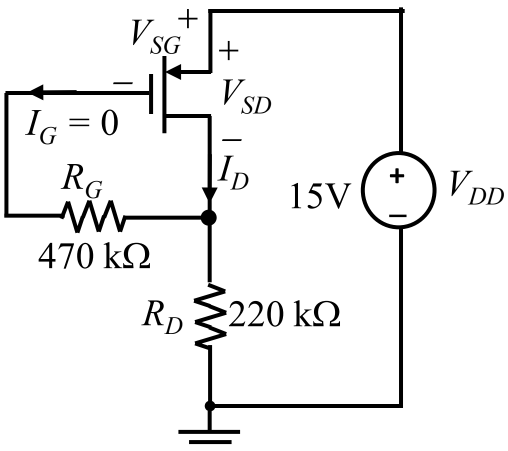
$$1.3V_{GS}^2 - 1.6V_{GS} - 2 = 0$$

$$V_{GS} = \frac{1.6 \pm \sqrt{1.6^2 - 4 \times 1.3 \times -2}}{2 \times 1.3}$$

$$V_{GS} = -0.77 \text{ V or } 2 \text{ V}$$

$V_{GS} = -0.77$ V implies MOSFET is cutoff and contradict the observation.
$V_{GS} = 2$ V and $V_{DS} = V_{GS} = 2$ V.
$I_D = 130\mu \times (2 - 1)^2 = 130\ \mu$A.

MOSFET Bias Analysis: pMOS Two-Resistor Biasing



$K_p = 50\ \mu A/V^2$
$V_{TP} = -2$ V

Since $I_G = 0$, $V_{SD} = V_{SG}$
Transistor is saturated

$$V_{SD} = V_{DD} - I_D R_D$$

$$|V_{GS}| = 15 - \frac{50\mu}{2}\left(|V_{GS}| - |V_{TP}|\right)^2 \times 220k = 15 - 5.5\left(|V_{GS}| - 2\right)^2$$

$$5.5|V_{GS}|^2 - 10|V_{GS}| - 11 = 0$$

$$|V_{GS}| = \frac{10 \pm \sqrt{10^2 - 4 \times 5.5 \times -11}}{2 \times 5.5}$$

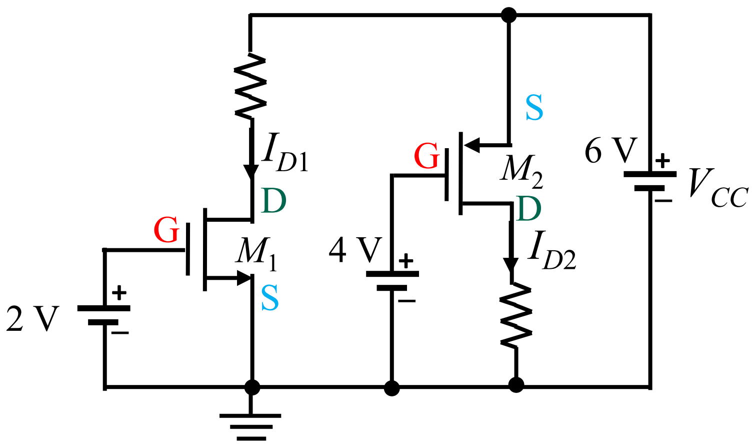
$$|V_{GS}| = 0.37 \text{ V or } 3.45 \text{ V}$$

Since $|V_{GS}| = 0.37$ V $< |V_{TP}| = 2$ V,
$|V_{GS}| = 3.45$ V or $V_{SG} = 3.45$ V
$I_D = 25\mu \times (3.45 - 2)^2 = 52.5\ \mu A$
$V_{SD} = V_{SG} = 3.45$ V

Introduction to Amplifiers

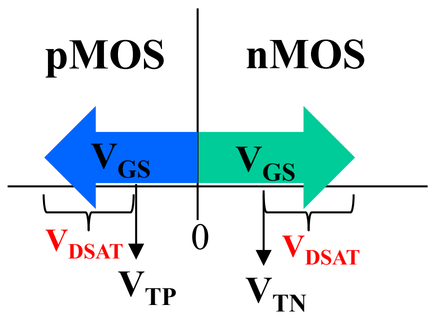
- BJT is an excellent amplifier when biased in forward-active region
- MOSFET can be used as amplifier when biased in saturation region.
- In these regions, transistors can provide high voltage, current and power gains.
- Bias refers to setting the ‘quiescent’ (idle) current when there is no signal presence. It sets the transistor in the desired operation region.
- Q-point (determined by DC analysis) also determines
  - Small-signal parameters of transistor
  - Voltage gain, input resistance, output resistance
  - Maximum input and output signal amplitudes
  - Power consumption
  - Efficiency (o/p signal power vs DC i/p power)

Biasing MOSFET for linear amplification



MOSFET is biased in **saturation region** (corresponding to the forward-active of BJT) for small-signal amplifier.

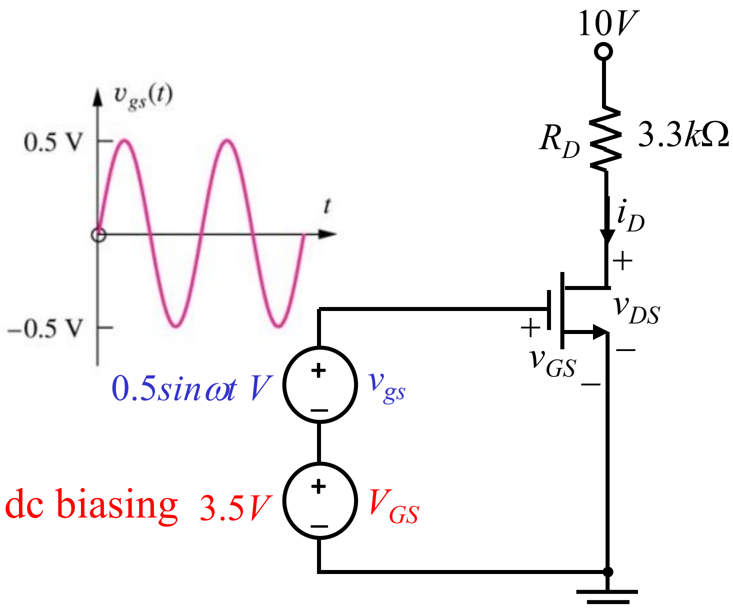
pMOS behaves like nMOS with all the polarities reversed.

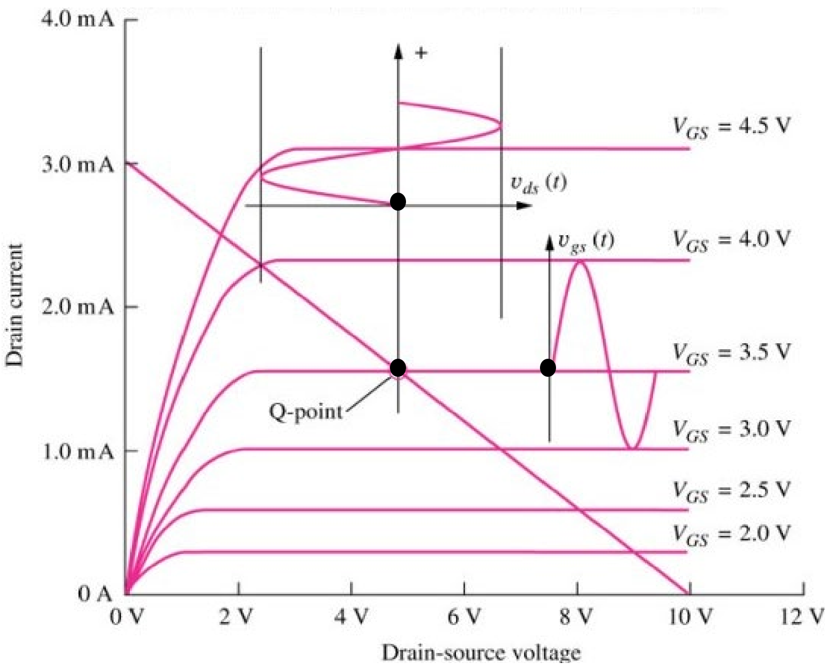


nMOS	pMOS
$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$	$I_D = \frac{K_p}{2}(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert)^2$
with channel length modulation:	with channel length modulation:
$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2 (1+\lambda V_{DS})$	$I_D = \frac{K_p}{2}(\lvert V_{GS}\rvert - \lvert V_{TP}\rvert)^2 (1+\lambda \lvert V_{DS}\rvert)$

$$V_{SAT} = \lvert V_{GS}\rvert - \lvert V_{TX}\rvert > 0$$
$$X = N \text{ or } P$$

MOSFET Amplifier





Q-point is set at (I_D, V_{DS}) = (1.56 mA, 4.8 V) with V_{GS} =3.5 V.

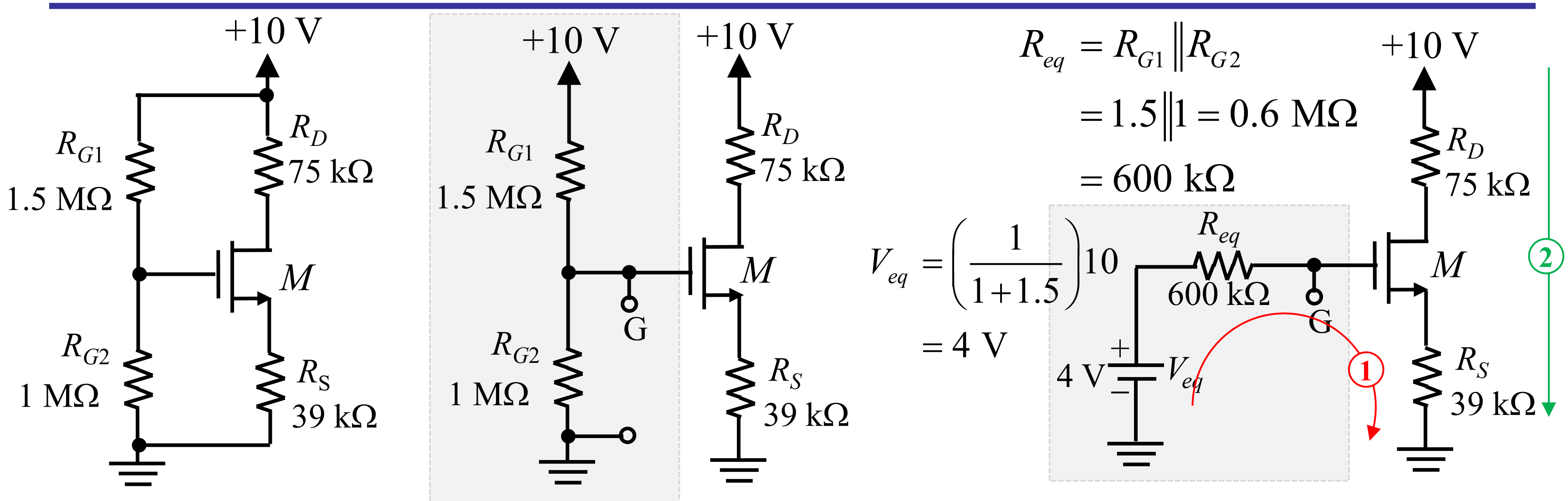
Total gate-source voltage is: $v_{GS} = V_{GS} + v_{gs}$

1 V *p-p* change in $v_{GS} \Rightarrow$ 1.25 mA *p-p* change in $i_D \Rightarrow$ 4 V *p-p* change in v_{DS}.

DC and AC Analysis

- DC analysis:
  - Obtain dc equivalent circuit by replacing all capacitors by open circuits and inductors by short circuits. ac voltage sources by ground connections and ac current sources by open circuits.
  - Find Q-point from dc equivalent circuit by using appropriate large-signal transistor model.
- AC analysis:
  - Obtain ac equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, dc voltage sources by ground connections and dc current sources by open circuits.
  - Replace transistor by small-signal model
  - Use small-signal ac equivalent to analyze ac characteristics of amplifier.
- Combine end results of dc and ac analysis to yield total voltages and currents in the network.

DC Analysis Example: Four-Resistor MOSFET Biasing Circuit



KVL 1: Since $I_G = 0$, $V_{eq} = V_{GS} + I_D R_S$

$4 = V_{GS} + 0.5 \times 25\mu\,(V_{GS} - 1)^2 \times 39k$

$V_{GS}^2 + 0.05\,V_{GS} - 7.21 = 0$

$V_{GS} = -2.71$ or 2.66 V

Since $V_{GS} = -2.71 < V_{TN} = 1$,

$V_{GS} = 2.66$ V.

$\boldsymbol{I_D} = (4 - 2.66)/39k = \mathbf{34.4\ \mu A}$

KVL 2: $V_{DS} = 10 - I_D R_D - I_S R_S$

$\boldsymbol{V_{DS}} = 10 - 0.0344 \times (75 + 39) = \mathbf{6.08\ V}$

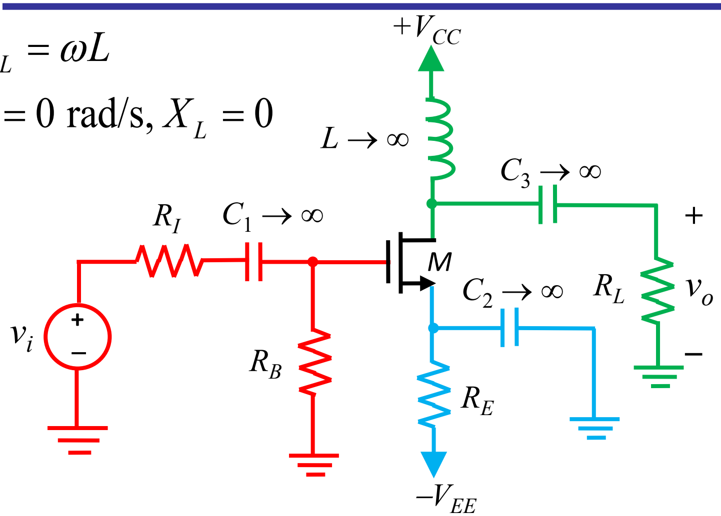
Since $V_{DS} > V_{GS} - V_{TN} = 1.66$,

M is in saturation region.

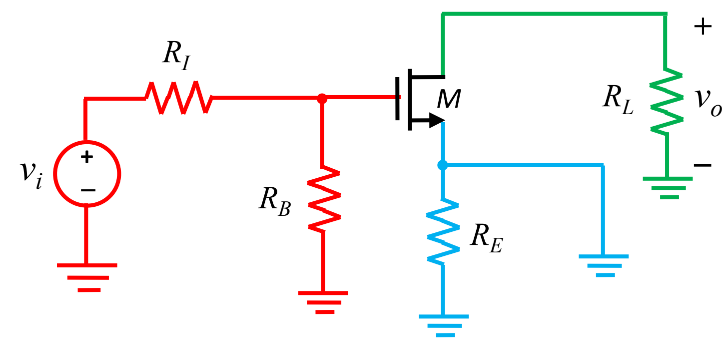
DC and AC Equivalents for MOSFET Amplifier

$X_L = \omega L$

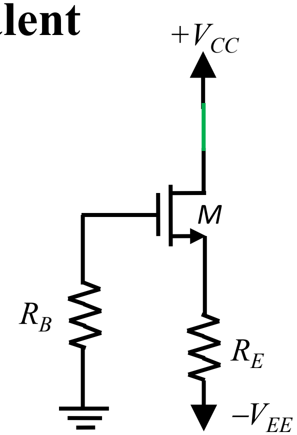
$\omega = 0$ rad/s, $X_L = 0$



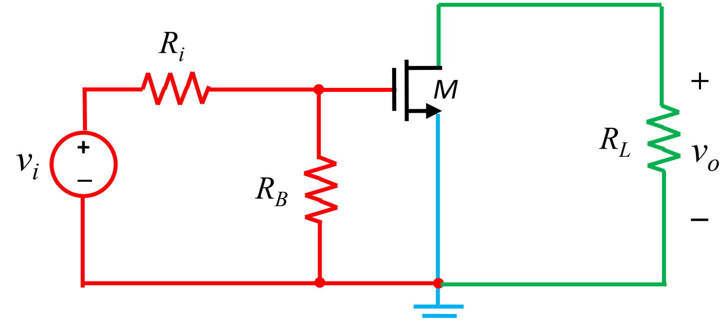
ac equivalent



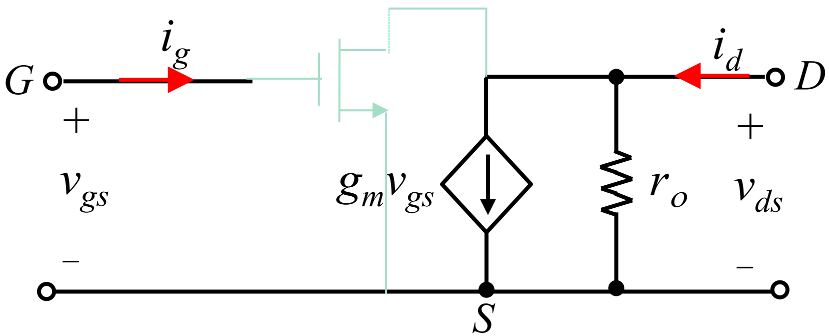
dc equivalent



Simplified ac equivalent



Small Signal Parameters of MOSFET



$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{Q-pt} = K_n\left(V_{GS} - V_{TN}\right)\left(1 + \lambda V_{DS}\right)$$

$$\text{where } K_n = \mu_n C_{OX}\left(\frac{W}{L}\right)$$

$$g_m = \frac{I_D}{\frac{V_{GS} - V_{TN}}{2}} = \sqrt{2K_n I_D}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \cong \frac{1}{\lambda I_D}$$

- Since gate is insulated from channel by gate-oxide input resistance = ∞.
- Small-signal parameters are controlled by the Q-point.
- For same operating point, MOSFET has lower transconductance than BJT. MOSFET transconductance is geometry dependent.

Small Signal Operation of MOSFET

$$i_D = \frac{K_n}{2}\left(v_{GS} - V_{TN}\right)^2 \text{ for } v_{DS} \geq v_{GS} - V_{TN}$$

$$i_D = \frac{K_n}{2}\left(V_{GS} + v_{gs} - V_{TN}\right)^2 = \frac{K_n}{2}\left[\left(V_{GS} - V_{TN}\right)^2 + 2v_{gs}\left(V_{GS} - V_{TN}\right) + v_{gs}^2\right]$$

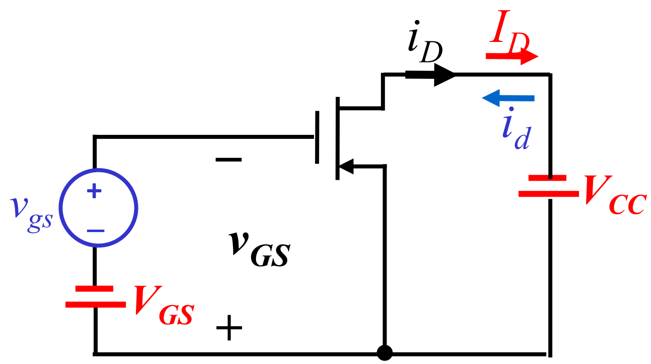
$$i_d = i_D - I_D = \frac{K_n}{2}\left[2v_{gs}\left(V_{GS} - V_{TN}\right) + v_{gs}^2\right]$$

For linearity, i_d should be proportional to v_{gs}

$$v_{gs} << 2\left(V_{GS} - V_{TN}\right) \Rightarrow v_{gs} \leq 0.2\left(V_{GS} - V_{TN}\right)$$

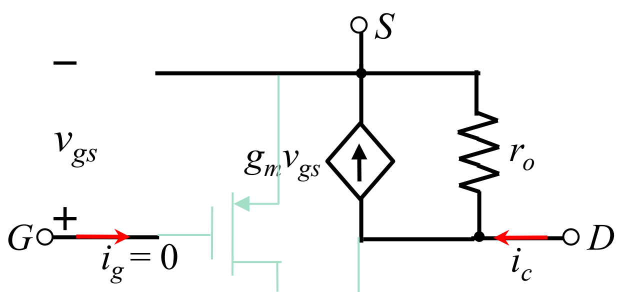
Since MOSFET can be biased with $(V_{GS} - V_{TN})$ equal to several volts, it can handle much larger values of v_{gs} than corresponding values of v_{be} for BJT.

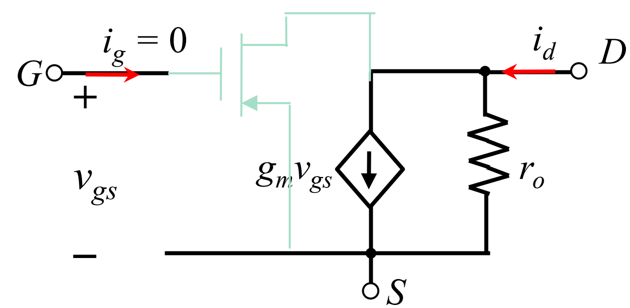
Small-Signal Model for PMOS



$$v_{GS} = V_{GS} - v_{gs}$$

$$i_D = I_D - i_d$$



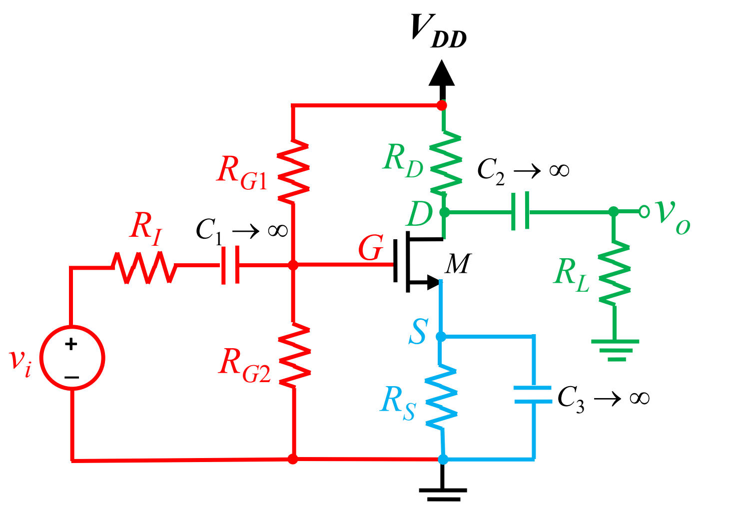


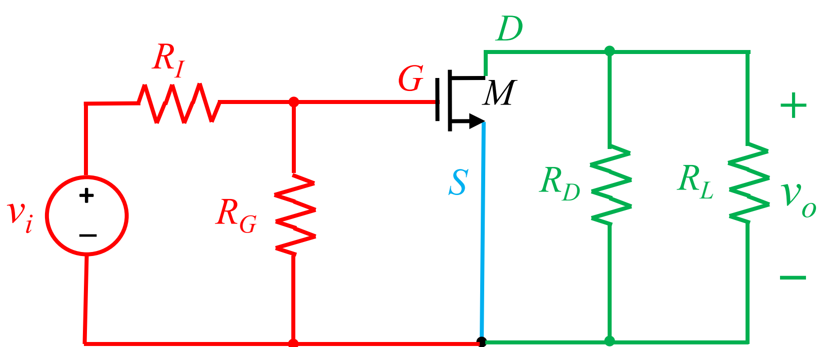
The small signal model for *p*MOS is exactly IDENTICAL to that of *n*MOS. This is not a mistake because the current direction is taken care of by the polarity of V_{GS}.

Summary of Small Signal Parameters

Parameter	BJT	n-MOSFET
g_m	$\dfrac{I_C}{V_T}$	$\dfrac{2I_D}{V_{GS}-V_{TN}}$ $K_n\left(V_{GS}-V_{TN}\right)\left(1+\lambda V_{DS}\right)\approx K_n\left(V_{GS}-V_{TN}\right)$ $\sqrt{2K_nI_D\left(1+\lambda V_{DS}\right)}\approx\sqrt{2K_nI_D}$
r_π	$\dfrac{\beta}{g_m}=\dfrac{\beta V_T}{I_C}$	∞
r_o	$\dfrac{V_A+V_{CE}}{I_C}\approx\dfrac{V_A}{I_C}$	$\dfrac{\frac{1}{\lambda}+V_{DS}}{I_D}\approx\dfrac{1}{\lambda I_D}$
Small-signal requirement	$v_{be}\le 0.005\ V$	$v_{gs}\le 0.2\left(V_{GS}-V_{TN}\right)$

Small Signal Analysis of Fully Bypass C-S Amplifier



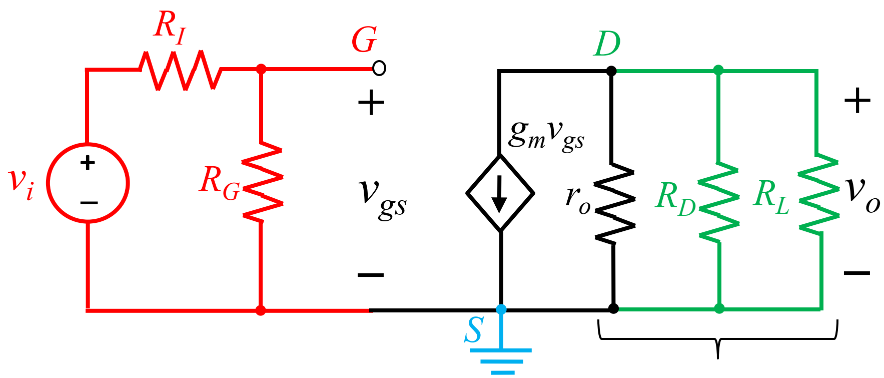


$R_G = R_{G1} \| R_{G2}$

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent ac grounds.

- The small signal parameters, g_m and r_o of the MOSFET is calculated at the Q-point, I_D and V_{DS}.

Fully Bypass C-S Amplifier: Voltage Gain



$$R_L' = r_0 \| R_D \| R_L$$

Overall voltage gain from source v_i to output voltage across R_L is:

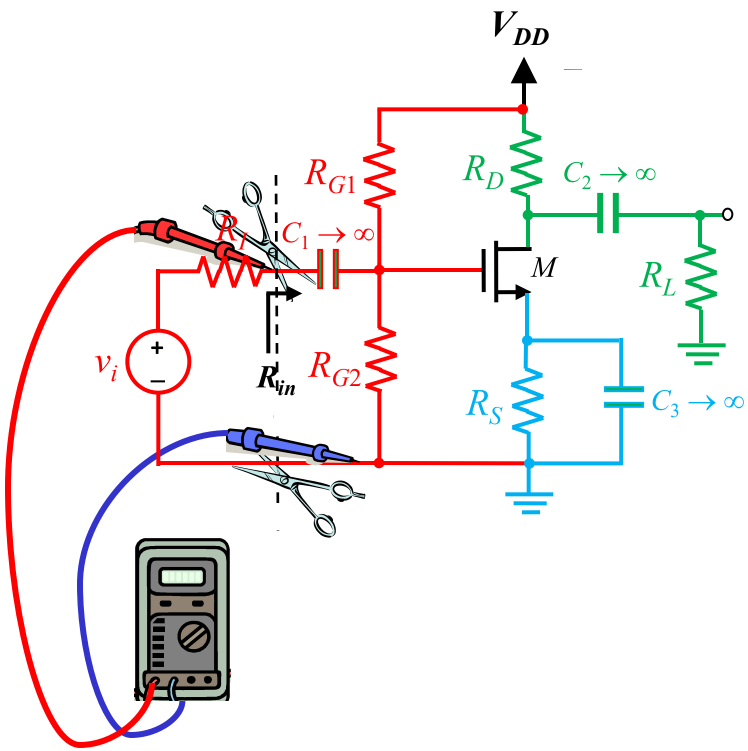
$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{v_g}{v_i}$$

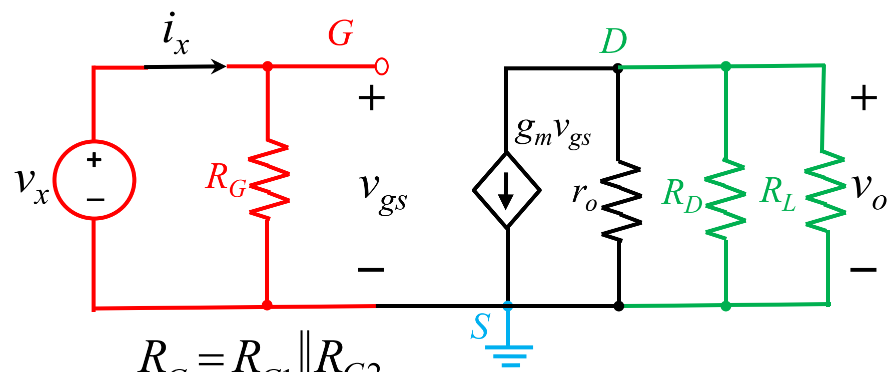
$$= -g_m R_L' \left(\frac{R_G}{R_I + R_G} \right)$$

Terminal voltage gain between gate and drain is:

$$A_{vt} = \frac{v_d}{v_g} = \frac{-g_m v_{gs} R_L'}{v_{gs}} = -g_m R_L'$$

Fully Bypass C-S Amplifier Input Resistance





$$R_G = R_{G1} \| R_{G2}$$

$$v_x = i_x R_G$$

$$R_{in} = \frac{v_x}{i_x} = R_G$$

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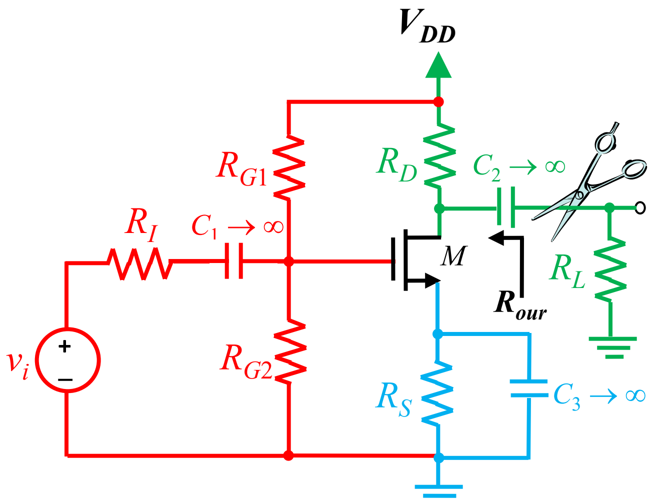
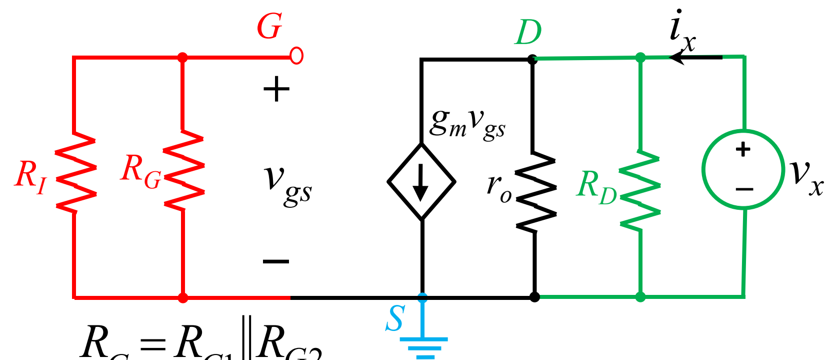
Fully Bypass C-S Amplifier Output Resistance





$R_G = R_{G1} \| R_{G2}$

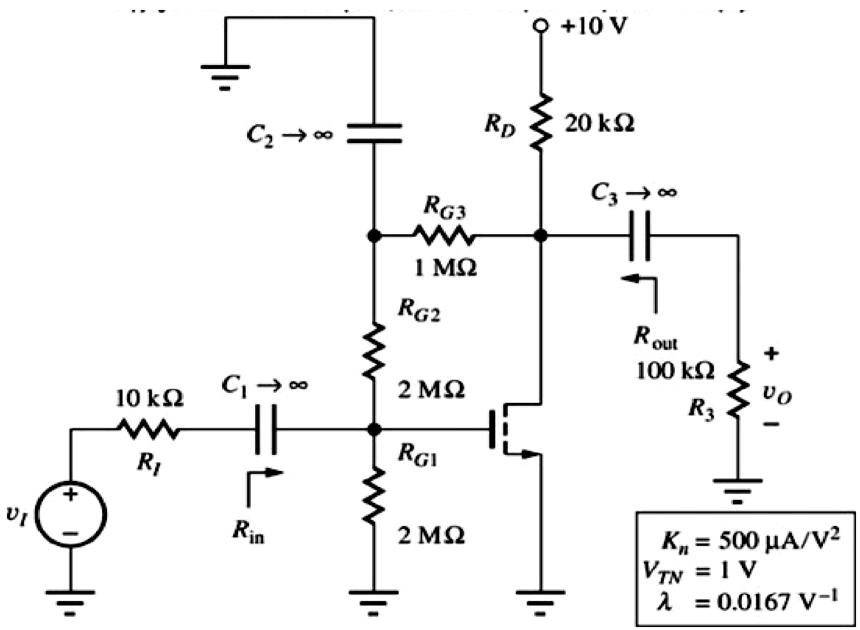
Since $v_{gs} = 0$, $g_m v_{gs} = 0$.

$$v_x = i_x \left(R_D \| r_o \right)$$

$$R_{out} = \frac{v_x}{i_x} = R_D \| r_o$$

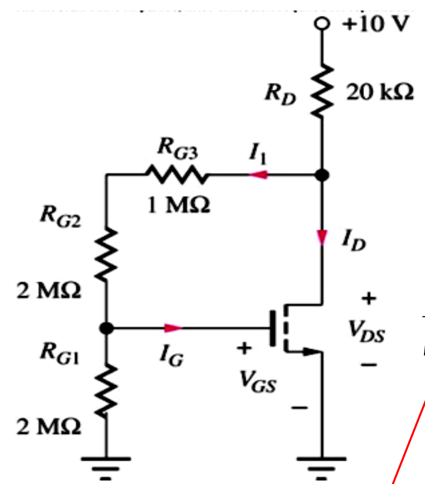
$$R_{out} \approx R_D \textbf{ if } r_o \gg R_D$$

Fully Bypass C-S Amplifier Example



Problem: Find voltage gain, input and output resistances.

Given K_n = 500 µA/V², V_{TN} = 1V, λ = 0.0167 V^{-1}



DC Analysis:

$$I_1 = \frac{V_{DS}}{5\times10^6}$$

$$V_{GS} = I_1 \times 2\times10^6 = 0.4V_{DS}$$

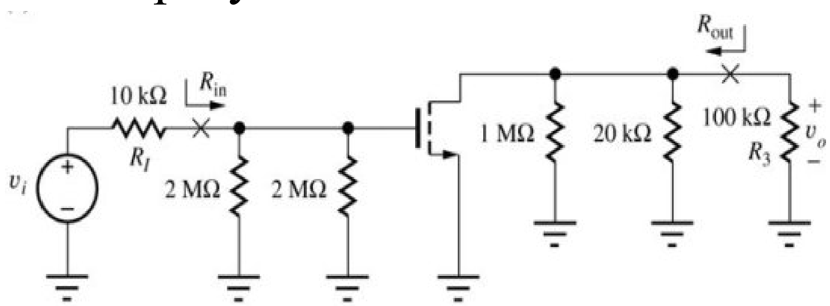
$$I_D = \frac{K_n}{2}\left(V_{GS} - V_{TN}\right)^2 \cdots\cdots (1)$$

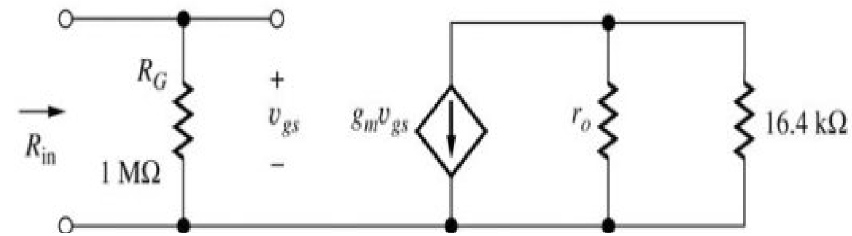
$$V_{DS} = 10 - 20\times10^3\left(I_D + I_1\right) \mathrm{L\ L}\ (2)$$

$$\therefore V_{DS} = 5V, V_{GS} = 2V, I_D = 250\mu A$$

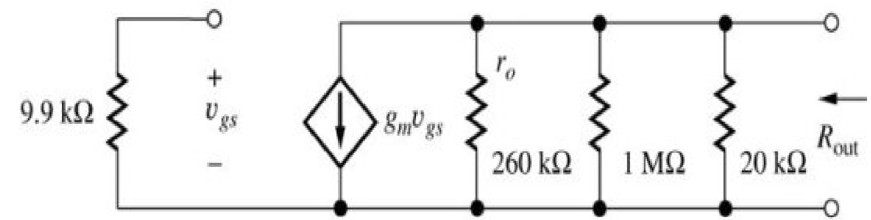
Fully Bypass C-S Amplifier Example (contd.)

Construct the ac equivalent and simplify it.





$$R_{in} = R_G = R_{G1} \| R_{G2} = 1M\Omega$$



$$g_m \approx \sqrt{2K_n I_D} = 5.20\times10^4\ S$$

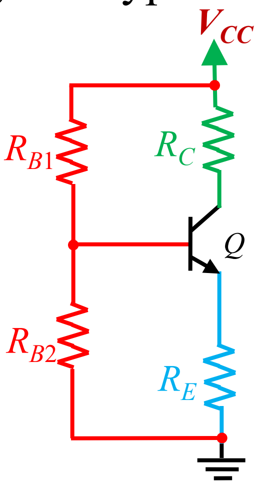
$$R_{\text{out}} = r_o \| R_D \| R_{G3} = 18.2\text{k}\Omega$$

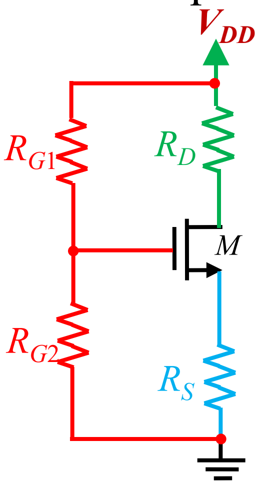
$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = 260\ \text{k}\Omega$$

$$A_v = \frac{v_o}{v_i} = -g_m(R_{\text{out}} \| R_3)\left[\frac{R_{\text{in}}}{R_I + R_{\text{in}}}\right] = -7.93$$

Amplifier Families

- Constraints for signal injection and extraction yield three families of amplifiers
  - Common-Emitter (C-E)/Common- Source (C-S)
  - Common-Base (C-B)/Common- Gate (C-G)
  - Common-Collector (C-C)/Common- Drain (C-D)
- All circuit examples here use the four-resistor bias circuits to establish Q-point of the various amplifiers
- Coupling and bypass capacitors are used to change the ac equivalent circuits.

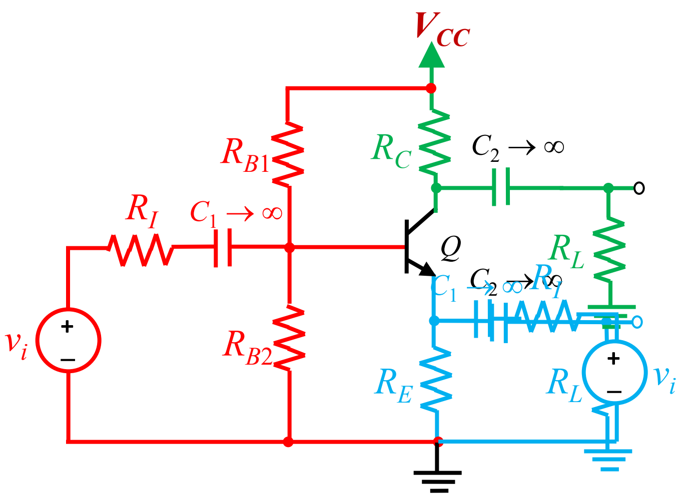


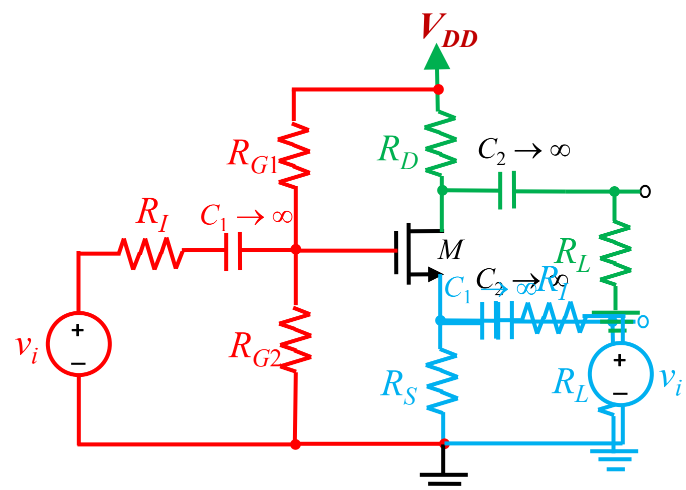


Amplifier Family

$$i_e \approx i_c \approx I_s \exp\left(\frac{v_b - v_e}{V_T}\right)$$

$$i_s = i_d \approx \frac{K_n}{2}\left(v_g - v_s - V_{TN}\right)^2$$





C-E: i/p at B, o/p at C

C-C: i/p at B, o/p at E

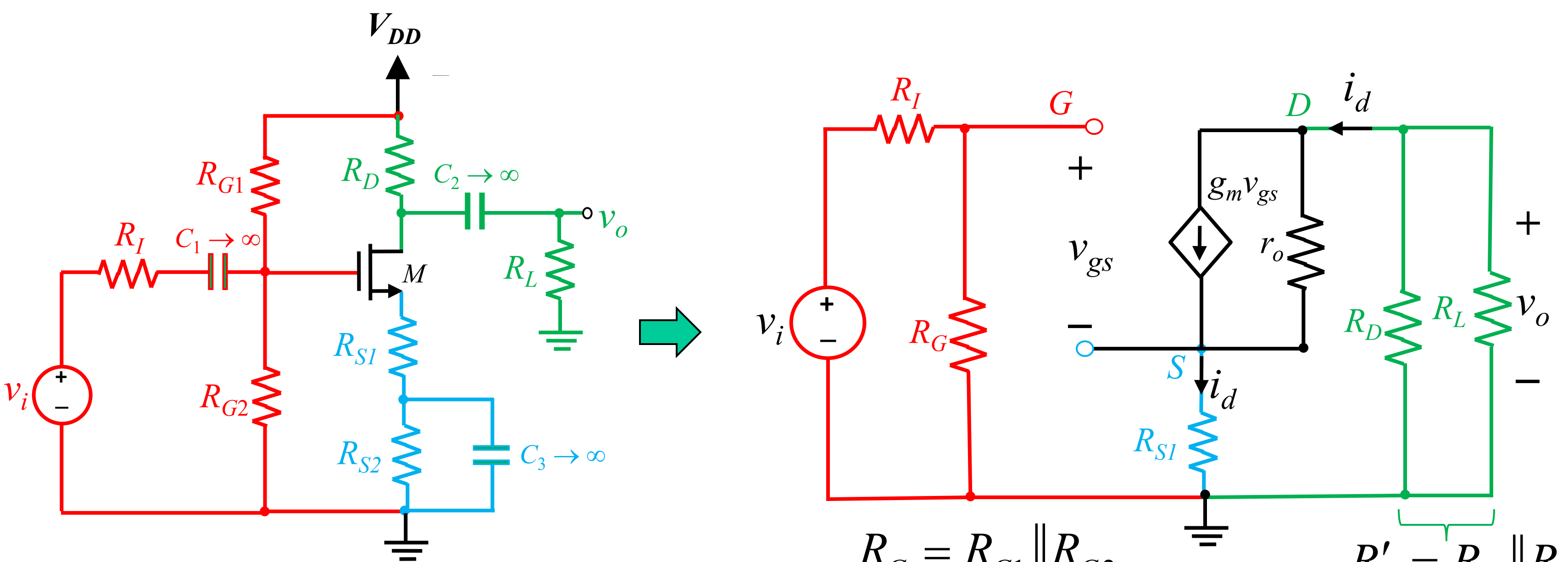
C-B: i/p at E, o/p at C

C-S: i/p at G, o/p at D

C-D: i/p at G, o/p at S

C-G: i/p at S, o/p at D

C-S Amplifier (Inverting Amplifier): Terminal Voltage Gain



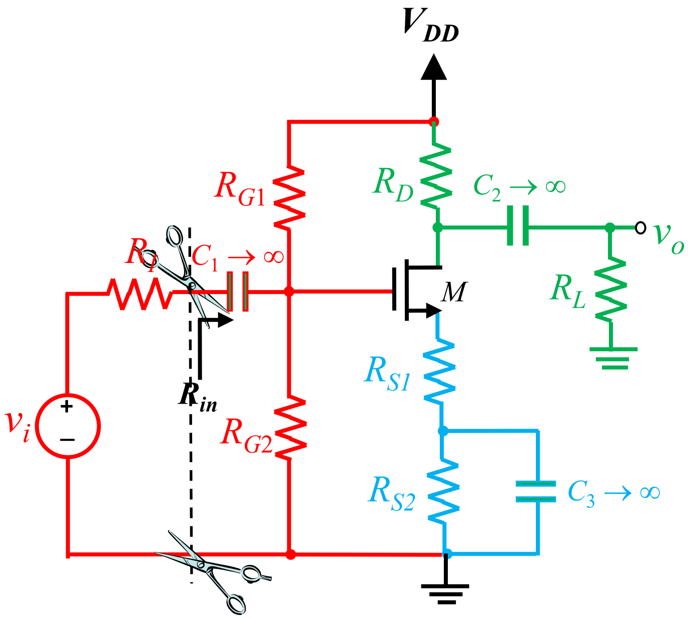
$R_G = R_{G1} \| R_{G2}$

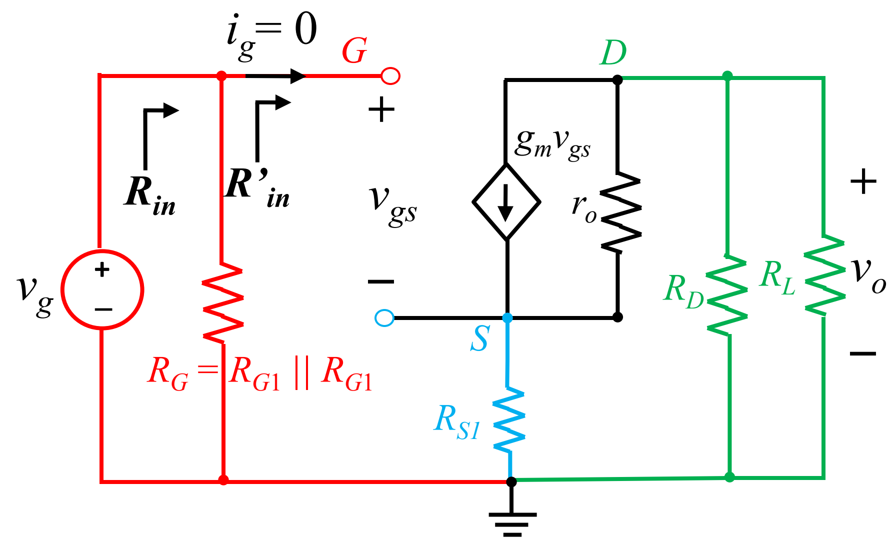
$R_L' = R_D \| R_L$

$\because i_{r_o} \ll g_m v_{gs}, i_d \approx g_m v_{gs}$

$$A_{vt} = \frac{v_d}{v_g} = \frac{-i_d R_L'}{v_{gs} + i_d R_{S1}} \approx \frac{-g_m v_{gs} R_L'}{v_{gs} + g_m v_{gs} R_{S1}} = \frac{-g_m R_L'}{1 + g_m R_{S1}}$$

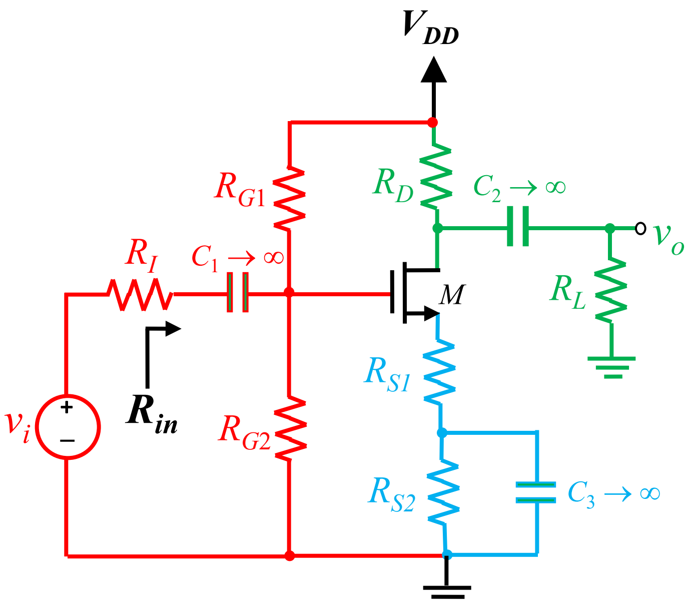
C-S Amplifier (Inverting Amplifier): Input Resistance

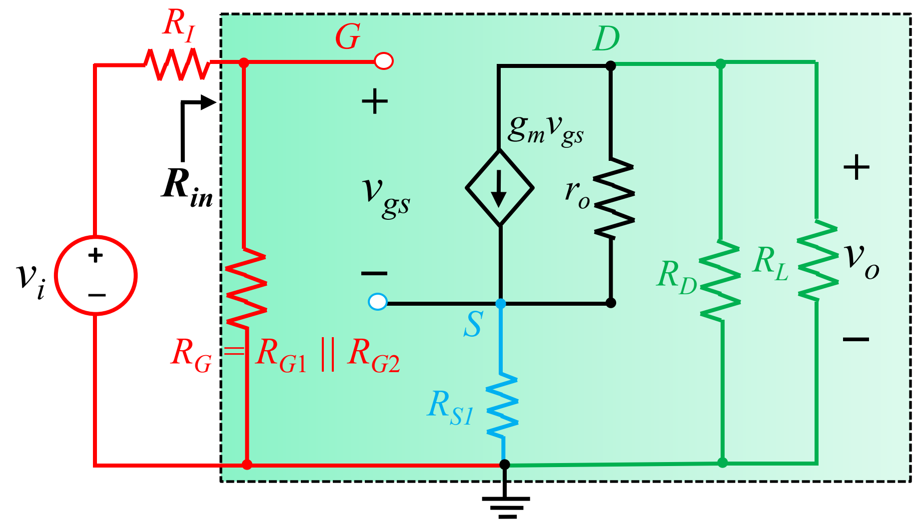


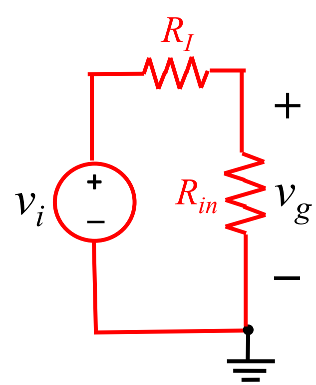


$$\because i_g = 0, R'_{in} = \frac{v_g}{i_g} = \infty \quad \Rightarrow \quad R_{in} = R'_{in} \| R_G = R_G$$

C-S Amplifier (Inverting Amplifier): Overall Voltage Gain

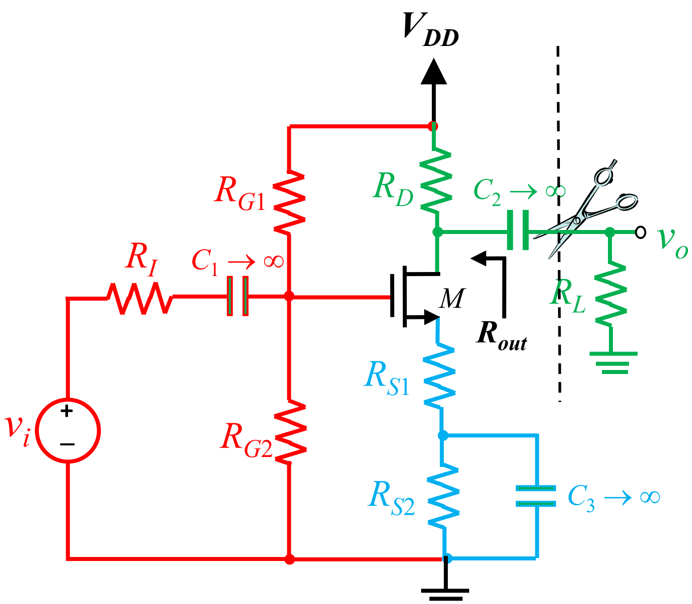


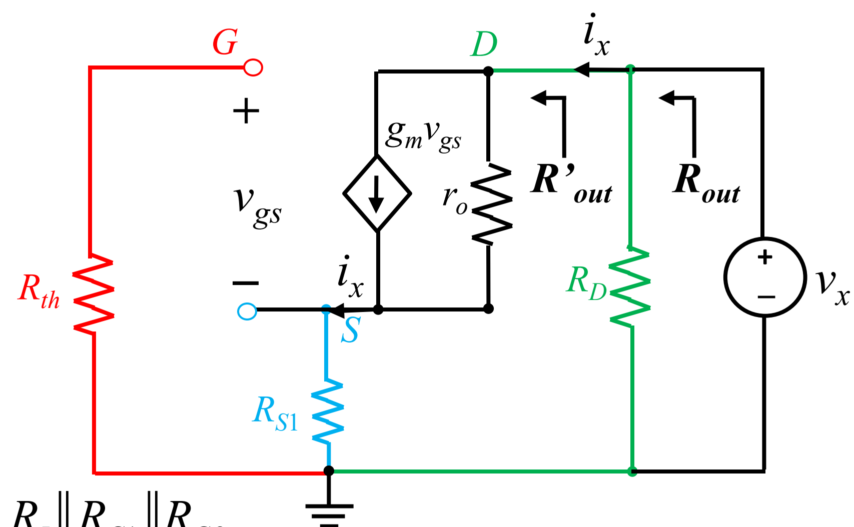




$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

C-S Amplifier (Inverting Amplifier): Output Resistance





$$R_{th} = R_I \| R_{G1} \| R_{G2}$$

$$v_x = (i_x - g_m v_{gs}) r_o + v_s$$

$$v_s = i_x R_{S1}$$

$$\because v_g = 0, v_{gs} = -v_s = -i_x R_{S1}$$

$$v_x = (i_x + g_m i_x R_{S1}) r_o + i_x R_{S1}$$

$$R'_{out} = \frac{v_x}{i_x} \approx (1 + g_m R_{S1}) r_o$$

$$R_{out} = R'_{out} \| R_D$$

C-S Amplifier (Inverting Amplifier): Input Signal Range

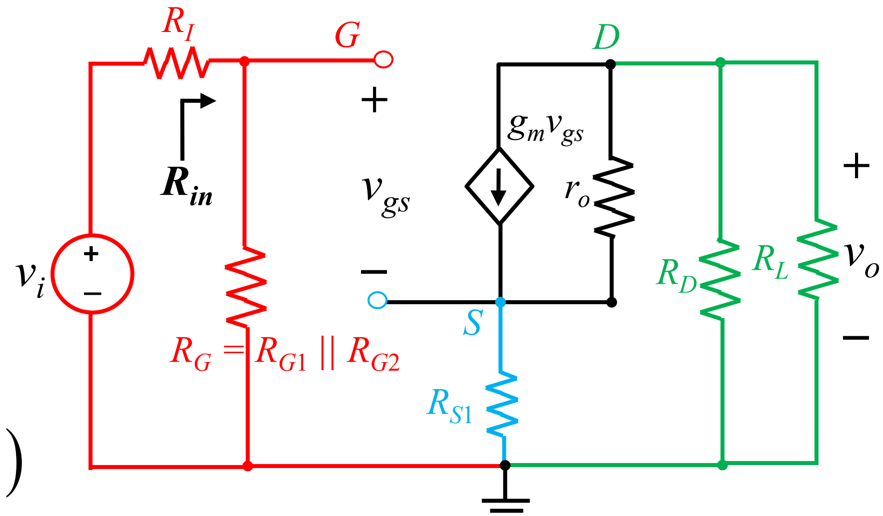
For MOSFET small-signal operation, $|v_{gs}| \leq 0.2(V_{GS} - V_{TN})$.

$$v_{gs} = v_g - v_s \approx v_g - g_m v_{gs} R_{S1}$$

$$v_{gs} = \frac{v_g}{1 + g_m R_{S1}}$$

$$\left|v_{gs}\right| \leq 0.2\left(V_{GS} - V_{TN}\right)$$

$$\Rightarrow \left|v_g\right| \leq 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_{S1}\right)$$



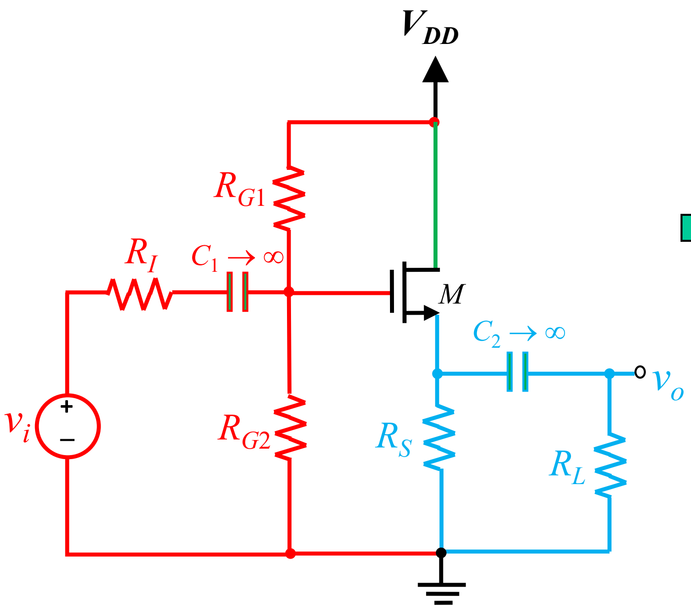
$$\because v_g = \left(\frac{R_{in}}{R_I + R_{in}}\right) v_i \Rightarrow \left|v_i\right| \leq 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_{S1}\right)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$

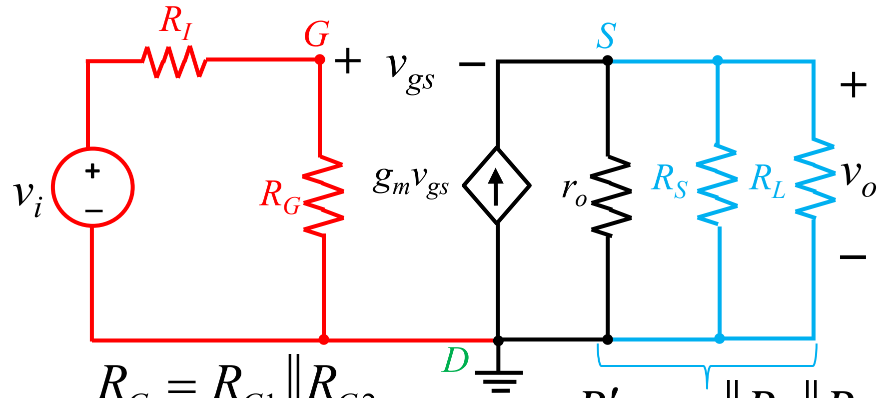
Presence of R_{S1} increase permissible value of v_i.

Summary: C-E and C-S

Amplifier		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\dfrac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\dfrac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\dfrac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\dfrac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\dfrac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\dfrac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \dfrac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\dfrac{1}{g_m}$	$\left[1+g_m (r_\pi \Vert R_{th})\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\dfrac{1}{g_m}$	$(1+g_m R_{th}) r_o$

C-D Amplifier (Voltage Follower): Terminal Voltage Gain





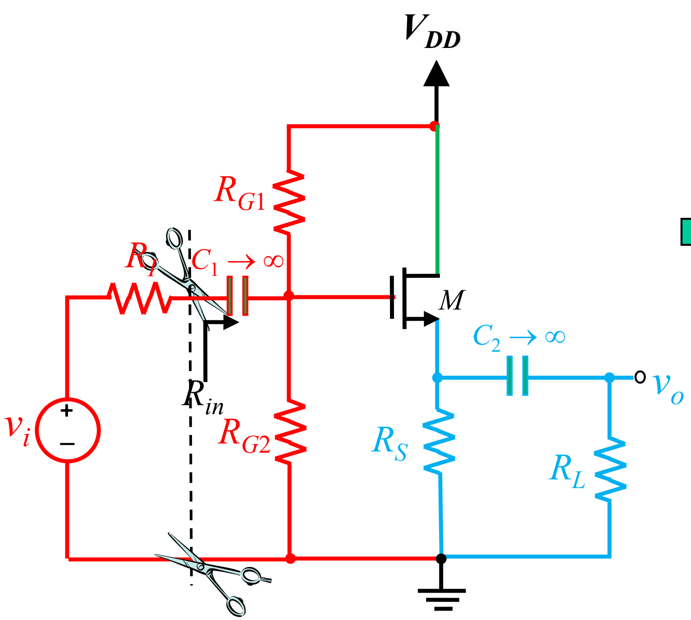
$$R_G = R_{G1} \| R_{G2}$$

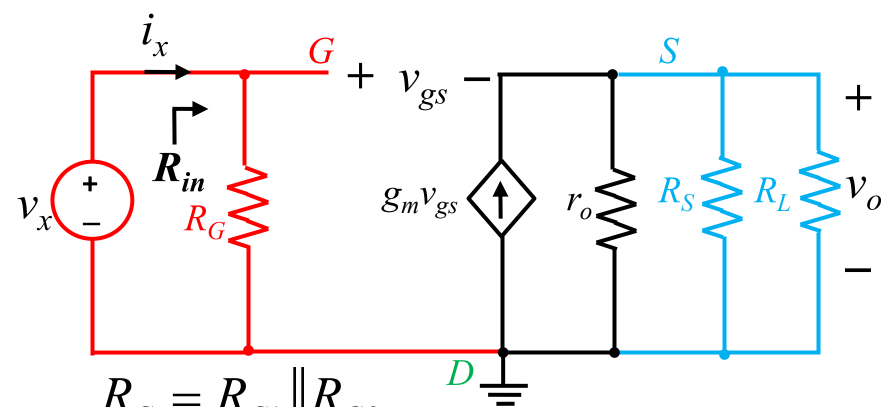
$$R_L' = r_o \| R_S \| R_L$$

$$A_{vt} = \frac{v_s}{v_g} = \frac{g_m v_{gs} R_L'}{v_{gs} + g_m v_{gs} R_L'} = \frac{g_m R_L'}{1 + g_m R_L'}$$

If $g_m R_L' >> 1, \; A_{vt} \approx 1 \Rightarrow v_o \approx v_g$

C-D Amplifier (Voltage Follower): Input Resistance

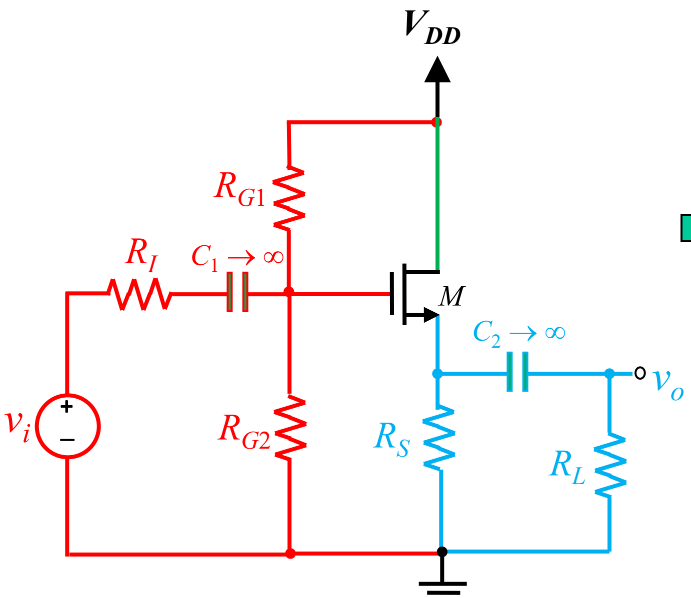


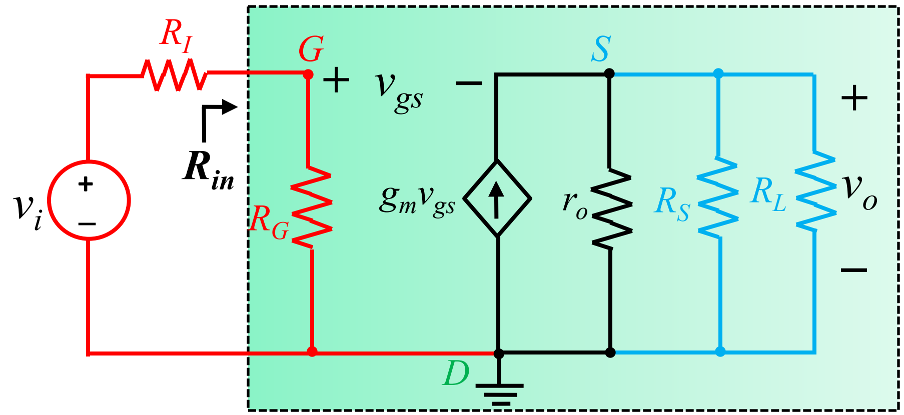


$$R_G = R_{G1} \| R_{G2}$$

$$v_x = i_x R_G \quad \Rightarrow \quad R_{in} = \frac{v_x}{i_x} = R_G$$

C-D Amplifier (Voltage Follower): Overall Voltage Gain

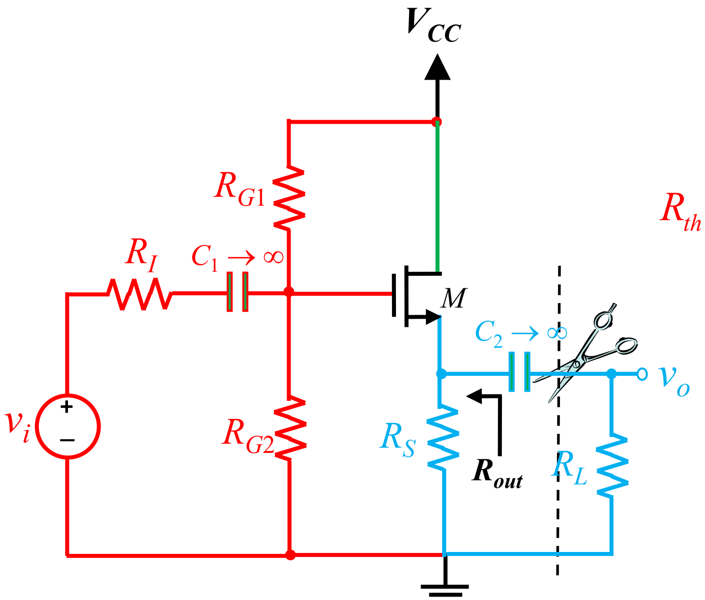


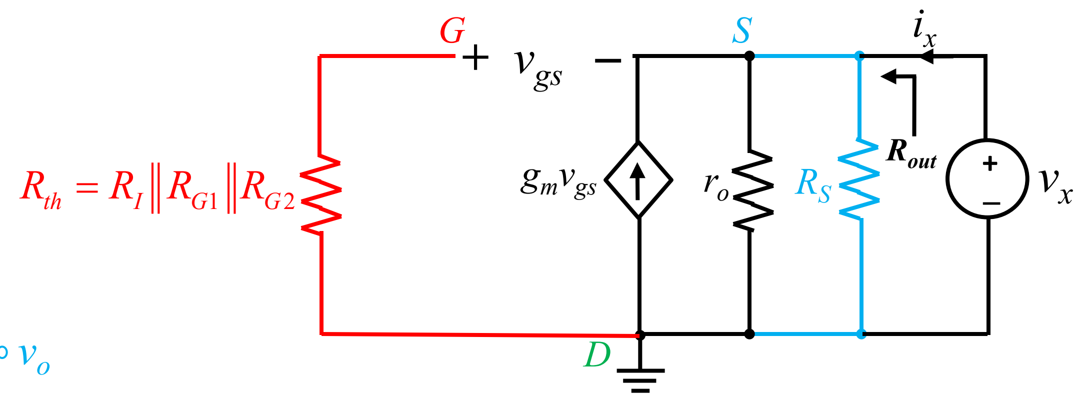


$$R_G = R_{G1} \| R_{G2}$$

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_G}{R_I + R_G}$$

C-D Amplifier (Voltage Follower): Output Resistance





$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} - g_m v_{gs}$$

$$v_{gs} = -v_x$$

$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} + g_m v_x = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m \right) v_x$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m \right)^{-1} = R_S \left\| r_o \right\| \frac{1}{g_m} \approx R_S \left\| \frac{1}{g_m} \right.$$

C-D Amplifier (Voltage Follower): Input Signal Range

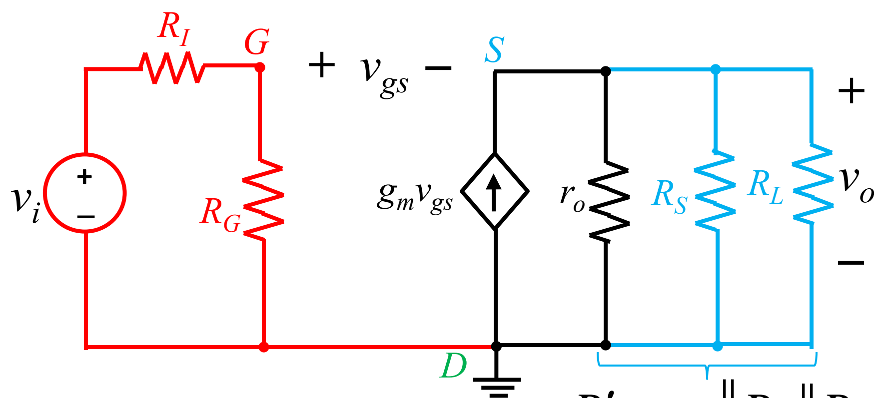
For MOSFET small-signal operation, $|v_{gs}| \leq 0.2(V_{GS} - V_{TN})$.

$$v_{gs} = v_g - v_s \approx v_g - g_m v_{gs} R_L'$$

$$v_{gs} = \frac{v_g}{1 + g_m R_L'}$$

$$\left|v_{gs}\right| \leq 0.2\left(V_{GS} - V_{TN}\right)$$

$$\Rightarrow \left|v_g\right| \leq 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_L'\right)$$



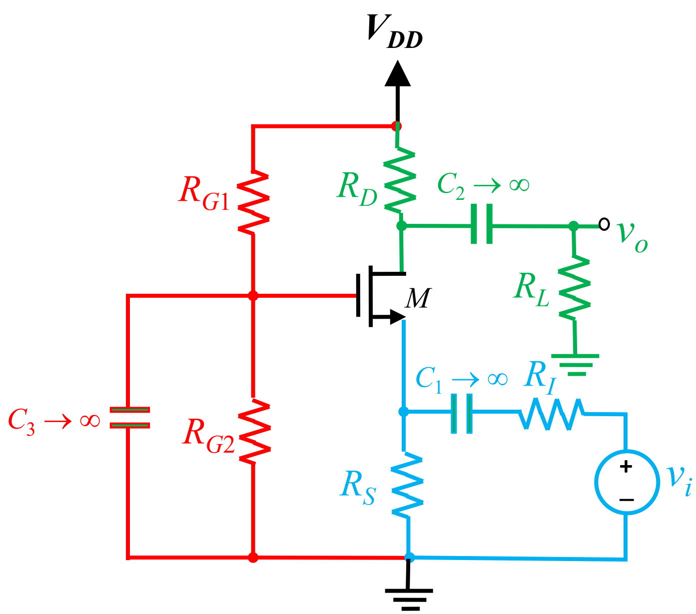
$$R_L' = r_o \| R_S \| R_L$$

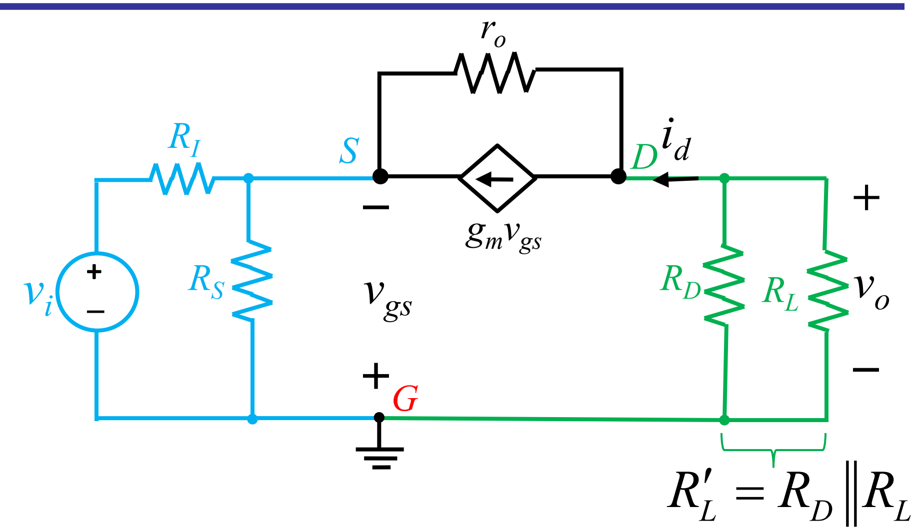
$$\because v_g = \left(\frac{R_G}{R_I + R_G}\right) v_i \Rightarrow \left|v_i\right| \leq 0.2\left(V_{GS} - V_{TN}\right)\left(1 + g_m R_L'\right)\left(\frac{R_I + R_G}{R_G}\right)$$

Summary: C-C and C-D

Parameter / Amplifier		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\dfrac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\dfrac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\dfrac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\dfrac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\dfrac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\dfrac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \dfrac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\dfrac{1}{g_m}$	$\left[1+g_m \left(r_\pi \Vert R_{th}\right)\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\dfrac{1}{g_m}$	$(1+g_m R_{th}) r_o$

C-G Amplifier (Noninverting Amplifier): Terminal Voltage Gain



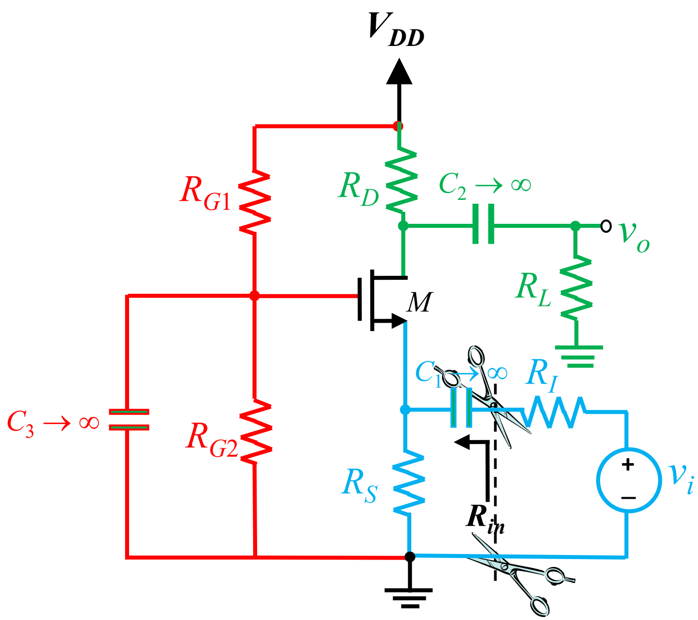


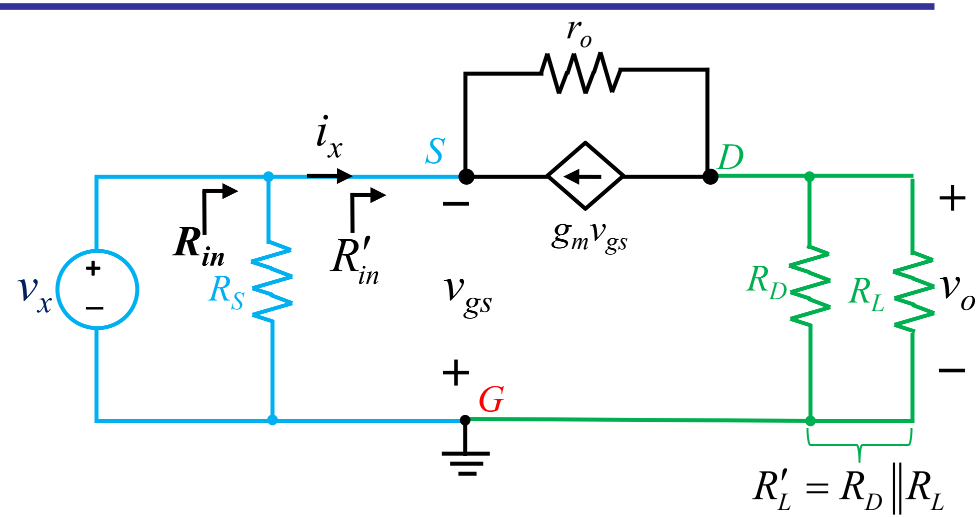
$$\because i_{r_o} << g_m v_{gs}, i_d \approx g_m v_{gs}$$

$$v_s = -v_{gs}$$

$$A_{vt} = \frac{v_d}{v_s} = \frac{-i_d R_L'}{-v_{gs}} \approx \frac{-g_m v_{gs} R_L'}{-v_{gs}} = g_m R_L'$$

C-G Amplifier (Noninverting Amplifier): Input Resistance





$$\because v_{gs} = -v_x,\ i_x \approx g_m v_x$$

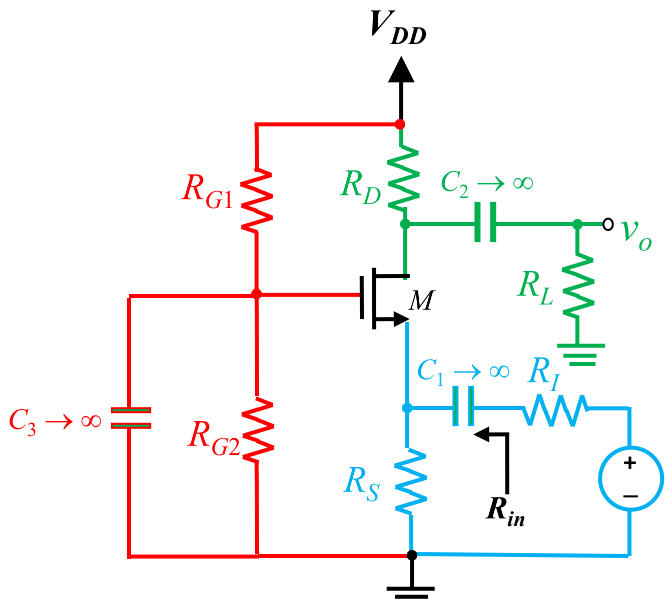
$$i_x = -g_m v_{gs} + \cancel{\frac{v_{sd}}{r_o}}$$

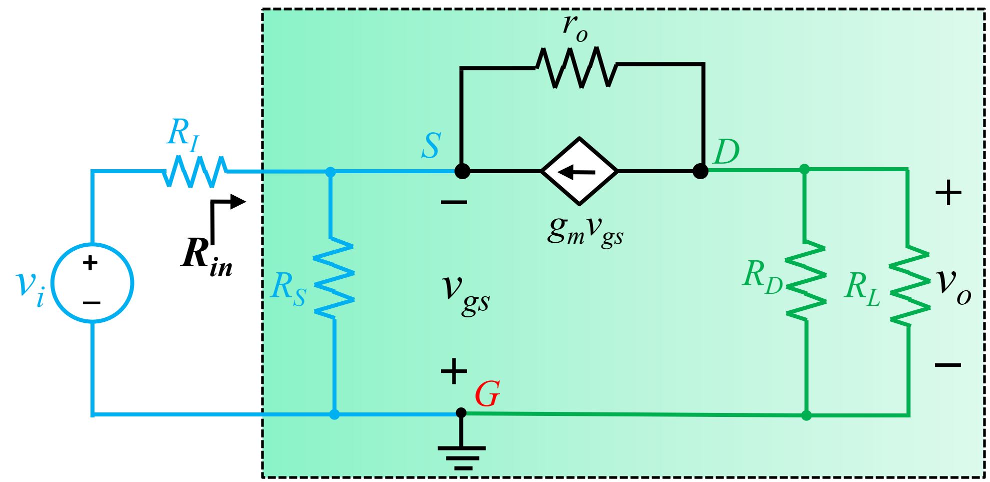
This current can be ignored due to large r_0

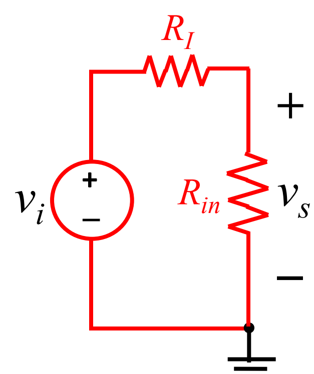
$$R'_{in} = \frac{v_x}{i_x} \approx \frac{1}{g_m}$$

$$R_{in} = R'_{in} \| R_S = \frac{R_S}{1 + g_m R_S}$$

C-G Amplifier (Noninverting Amplifier): Overall Voltage Gain



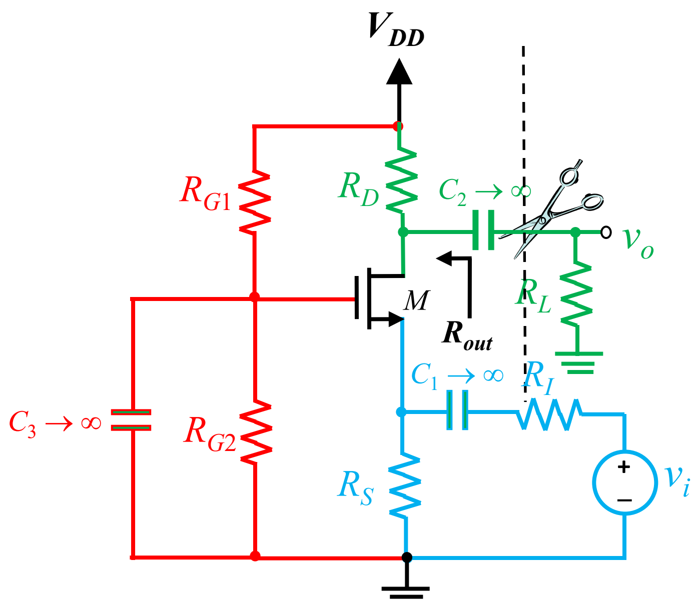


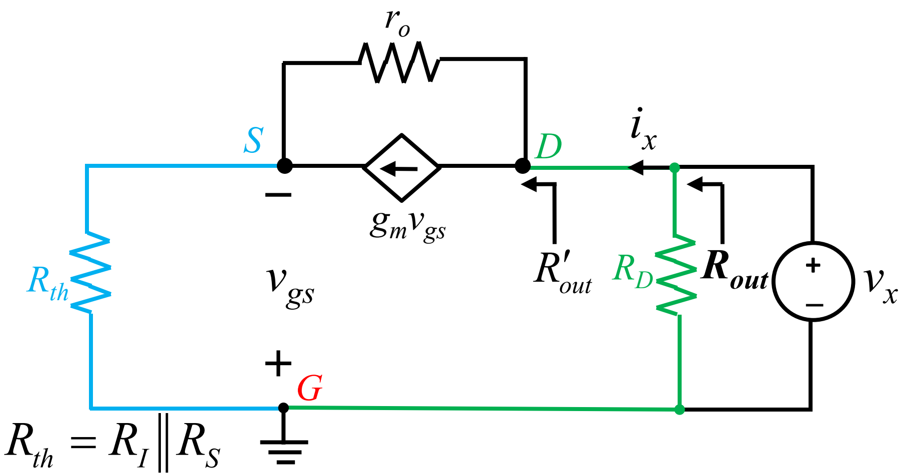


$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_s} \times \frac{v_s}{v_i} = A_{vt} \times \frac{R_{in}}{R_I + R_{in}}$$

$$R_{in} = R_S \,\Big\|\, \frac{1}{g_m} = \frac{R_S}{1 + g_m R_S}$$

C-G Amplifier (Noninverting Amplifier): Output Resistance





$R_{th} = R_I \| R_S$

$$v_x = \left(i_x - g_m v_{gs}\right) r_o + v_s$$

$$v_s = i_x R_{th}$$

$$v_{gs} = -v_s = -i_x R_{th}$$

$$v_x = \left(i_x + g_m i_x R_{th}\right) r_o + i_x R_{th}$$

$$\approx \left(1 + g_m R_{th}\right) i_x r_o$$

$$R'_{out} = \frac{v_x}{i_x} \approx \left(1 + g_m R_{th}\right) r_o$$

$$R_{out} = R'_{out} \| R_D$$

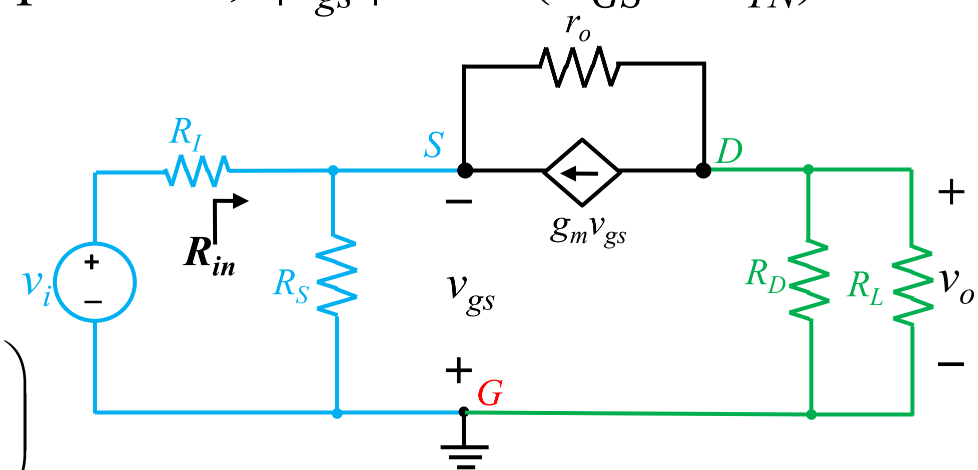
C-G Amplifier (Noninverting Amplifier): Input Signal Range

For MOSFET small-signal operation, $|v_{gs}| \le 0.2(V_{GS} - V_{TN})$.

$$v_{gs} = -\left(\frac{R_{in}}{R_I + R_{in}}\right) v_i$$

$$\left|v_{gs}\right| \le 0.2\left(V_{GS} - V_{TN}\right)$$

$$\Rightarrow \left|v_i\right| \le 0.2\left(V_{GS} - V_{TN}\right)\left(\frac{R_I + R_{in}}{R_{in}}\right)$$



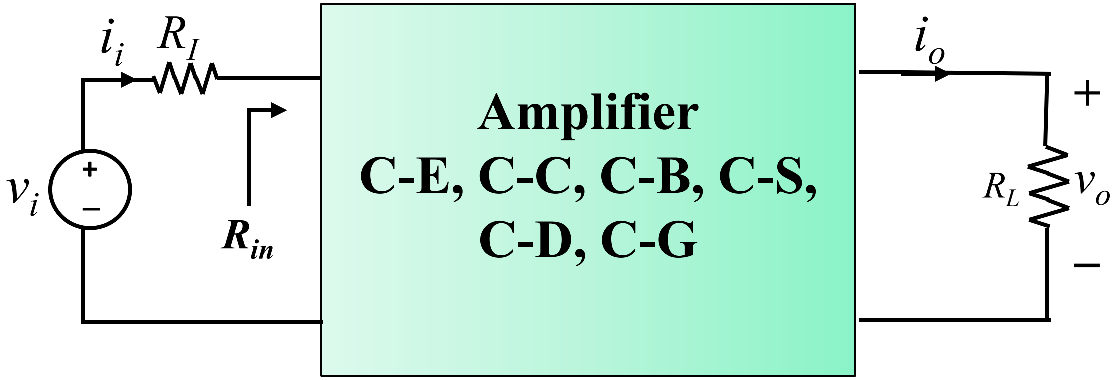
$$\because R_{in} = \frac{R_S}{1 + g_m R_S} \Rightarrow \frac{R_I + R_{in}}{R_{in}} = 1 + g_m R_I + \frac{R_I}{R_S}$$

If $R_S \gg R_I$, $|v_i| \le 0.2(V_{GS} - V_{TN})(1 + g_m R_I)$

Summary: C-B and C-G

Amplifier		Terminal Voltage Gain A_{vt}	Input Resistance R'_{in}	Output Resistance R'_{out}
BJT	C-E	$\dfrac{-g_m R'_L}{1+g_m R_{E1}}$	$r_\pi + (\beta+1) R_{E1}$	$\left(1+\dfrac{\beta R_{E1}}{r_\pi + R_{th} + R_{E1}}\right) r_o$
MOSFET	C-S	$\dfrac{-g_m R'_L}{1+g_m R_{S1}}$	∞	$(1+g_m R_{S1}) r_o$
BJT	C-C	$\dfrac{g_m R'_L}{1+g_m R'_L}$	$r_\pi + (\beta+1) R'_L$	$r_o \Big\Vert \left(\dfrac{r_\pi + R_{th}}{\beta+1}\right)$
MOSFET	C-D	$\dfrac{g_m R'_L}{1+g_m R'_L}$	∞	$r_o \Big\Vert \dfrac{1}{g_m}$
BJT	C-B	$g_m R'_L$	$\dfrac{1}{g_m}$	$\left[1+g_m (r_\pi \Vert R_{th})\right] r_o$
MOSFET	C-G	$g_m R'_L$	$\dfrac{1}{g_m}$	$(1+g_m R_{th}) r_o$

Current Gain



$$A_i = \frac{i_o}{i_i} = \frac{\dfrac{v_o}{R_L}}{\dfrac{v_i}{R_I + R_{in}}} = \frac{v_o}{v_i} \times \frac{R_I + R_{in}}{R_L} = A_v \times \frac{R_I + R_{in}}{R_L}$$