NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2021-2022

EE6203 – COMPUTER CONTROL SYSTEMS

November / December 2021 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 7 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. Properties and Table of Z-Transform are included in Appendix A on pages 6 and 7.

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1. (a) Consider the following sampled-data signal:

$$x(kT) = \begin{cases} \cos\left(\frac{\pi}{2}kT\right), & k \geq 0 \\ 0, & k < 0 \end{cases}$$

where T is the sampling period, whose value is 0.5 second, and k is an integer. Also $y(kT) = \sum_{h=0}^{k} x(hT)$. Find the Z-transform of $y(kT)$. Discuss whether the Final Value Theorem can be used to determine $\lim_{k\to\infty} x(kT)$ with justifications.

(9 Marks)

(b) Solve the following difference equation

$$x(k+2) - x(k+1) + x(k) = 1(k), \text{ with } x(0) = 0 \text{ and } x(1) = 0,$$

where $1(k)$ is the unit-step function.

(11 Marks)

2. Consider the control system shown in Figure 1.

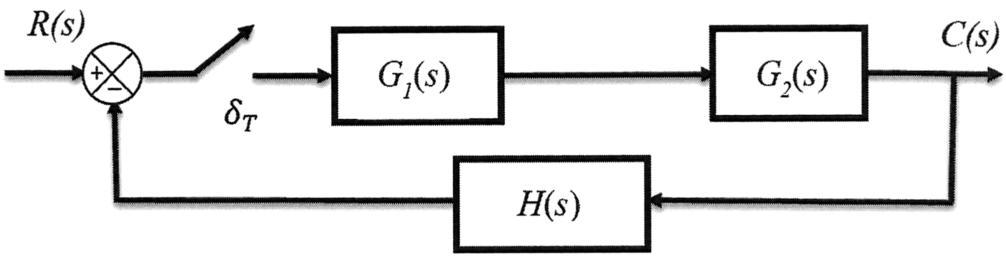

Figure 1

(a) Show that the system has a pulse transfer function. (6 Marks)

(b) Suppose that $G_1(s) = \frac{1-e^{-Ts}}{s}$, $G_2(s) = \frac{10(0.5s+1)}{s^2}$, $H(s)$=1 and the sampling period T is 0.2 second. Determine the pulse transfer function. (10 Marks)

(c) Determine the stability of the system using Jury Test. (4 Marks)

3. (a) An engineering process can be described by the following state-space representation:

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. The system is sampled with a zero-order hold at a sampling period of T seconds. Determine a discretized state-space model for the system. (7 Marks)

(b) A discrete-time system has a state-space representation shown below:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1.5 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Note: Question No. 3 continues on page 3.

where $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively.

(i) If $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, determine $u(0)$ and $u(1)$ such that $\begin{bmatrix} x_1(2) \\ x_2(2) \end{bmatrix} = \begin{bmatrix} 1 \\ \beta \end{bmatrix}$, where β is a real number. Express your answer in terms of β.

(ii) Determine the transfer function $\dfrac{Y(z)}{U(z)}$ of the system.

(7 Marks)

(c) Consider a discrete-time system described by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C}\mathbf{x}(k) + du(k)$$
$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}, d = 1.$$

A unity feedback closed-loop system is formed with the following

$$u(k) = r(k) - y(k),$$

where $r(k)$ is the reference input. Obtain a state-space model of the closed-loop system with $r(k)$ as the input and $y(k)$ as the output.

(6 Marks)

4. The discrete-time state-space representation obtained by sampling a continuous-time system with a zero-order hold at a sampling period $T = 0.1$ second is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where $\mathbf{A} = \begin{bmatrix} \alpha & -1 \\ -4 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & 2 \end{bmatrix}$, α is a real number and $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively.

(a) (i) Determine the condition on α for the system to be controllable.

Note: Question No. 4 continues on page 4.

(ii) With $\alpha = 2$, design a state-feedback controller of the following form

$$u(k) = -\mathbf{K}\mathbf{x}(k)$$

such that the closed-loop poles are at $z_{1,2} = \pm j0.25$.

(8 Marks)

(b) (i) With $\alpha = 2$, design an estimator of the following form

$$\overline{\mathbf{x}}(k+1) = \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_O\left(y(k) - \mathbf{C}\overline{\mathbf{x}}(k)\right)$$

such that the estimator poles are at $z_{e1,2} = 0.4 \pm j0.4$.

(ii) Determine the equivalent s-plane poles of $z_{e1,2}$ in part (i).
What is the damping ratio of the s-plane poles?

(7 Marks)

(c) Consider a system described by the following state equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\,u(k),$$

and associated with a performance index given by

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + ru^2(k)\right).$$

The control law such that J is minimised is of the following form

$$u^*(k) = -\mathbf{K}\mathbf{x}(k),$$

where the optimal control gain is given by

$$\mathbf{K} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B} + r\right)^{-1}\mathbf{B}^T\mathbf{S}\mathbf{A}$$

with $\mathbf{S} > 0$ being a solution of the following equation

$$\mathbf{S} = \mathbf{A}^T\mathbf{S}\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\mathbf{S}\mathbf{B}\left(r + \mathbf{B}^T\mathbf{S}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{S}\mathbf{A}.$$

Now, consider the following system and associated performance index

$$x(k+1) = x(k) + 0.8u(k)$$

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(8x^2(k) + 1.6u^2(k)\right)$$

Note: Question No. 4 continues on page 5.

Determine the optimal control law $u^*(k)$ such that J is minimised. Determine also the resulting closed-loop state equation.

(5 Marks)

5. (a) For a second-order analog filter with a damping ratio of 0.8, a natural frequency of 5 rad/s and a unity DC gain, find its bilinear digital filter approximation with the same DC gain and a sampling period of 1 second.

(5 Marks)

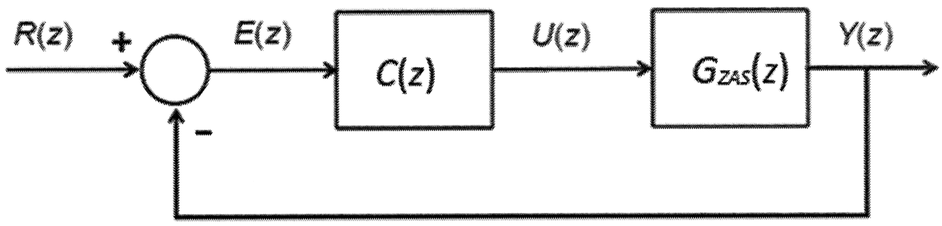

Figure 2

(b) Consider the closed-loop system described in Figure 2. With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = \frac{0.6z + 0.4}{(z - 0.5)(z - 0.4)}$$

It is required that the output $Y(z)$ tracks the unit-step input $R(z)$ with zero steady-state error. To have a ripple-free deadbeat controller to meet this requirement, firstly, show that $C(z)$ must contain an integrator, and secondly, design $C(z)$.

(11 Marks)

(c) Implement the controller $C(z)$ obtained in part (b) with the standard programming approach and depict the relevant block diagram.

(4 Marks)

Appendix A

Properties and Table of Z-Transform

Discrete function	z Transform
$x(k+4)$	$z^4X(z)-z^4x(0)-z^3x(1)-z^2x(2)-zx(3)$
$x(k+3)$	$z^3X(z)-z^3x(0)-z^2x(1)-zx(2)$
$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
$x(k+1)$	$zX(z)-zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ $1, \quad k=0$ $0, \quad k\neq 0$	1
2.	—	—	$\delta_0(n-k)$ $1, \quad n=k$ $0, \quad n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

Note: Transform Table continues on page 7.

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2 e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\sin\omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\cos\omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$	$e^{-akT}\sin\omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$e^{-akT}\cos\omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

$x(t)=0$, for $t<0$.

$x(kT)=x(k)=0$, for $k<0$.

Unless otherwise noted, $k=0,1,2,3,\cdots$

END OF PAPER

EE6203 COMPUTER CONTROL SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.