NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

EE6203 – COMPUTER CONTROL SYSTEMS

November / December 2022 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 8 pages.
2. Answer all 5 questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Unless specifically stated, all symbols have their usual meanings.
6. The Transform Table is included in Appendix A on pages 7 to 8.

---

1. (a) Consider the following signal:

$$x(t) = \begin{cases} e^{-2t}\sin\left(\frac{\pi}{2}t\right) + 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Justify whether $x(t)$ is a continuous-time signal. The signal is sampled with a sampling period of 0.5 second. Find the Z-transform of $x(t)$. Discuss whether the Final Value Theorem can be used to determine $\lim_{k\to\infty} x(kT)$ with justifications and use it to find $\lim_{k\to\infty} x(kT)$, if it can be applied.

(10 Marks)

(b) Solve the following difference equation:

$$x(k+2) - x(k) = 1(k)$$

where $1(k)$ is the unit-step function and $x(k)$=0 for k< 0. Discuss $\lim_{k\to\infty} x(kT)$.

(10 Marks)

2. Consider the control system shown in Figure 1. The sampling period T is 1 second.

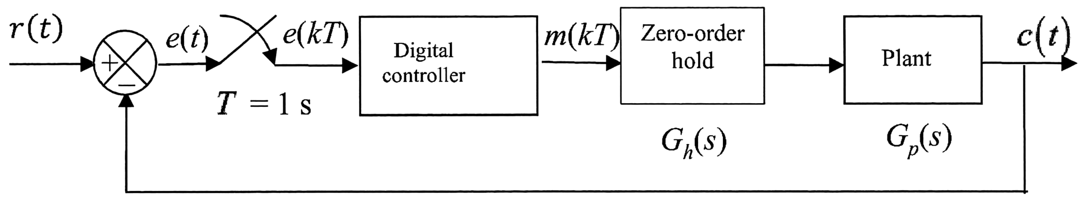

Figure 1

(a) Show that the system has a pulse transfer function.

(8 Marks)

(b) The input $e(kT)$ and output $m(kT)$ of the designed digital controller satisfy the following difference equation

$$m(kT) - m[(k-1)T] = 2.2e(kT) - 1.4e[(k-1)T] + 0.2e[(k-2)T]$$

Show that the controller is a PID controller.

(5 Marks)

(c) The transfer function of the plant is given by

$$G_p(s) = \frac{2}{s+2}$$

Find the pulse transfer function of the closed-loop control system, if it exists

(7 Marks)

3. (a) An engineering process can be described by the following set of differential equations:

$$m_1(t) - m_2(t) = \frac{dp(t)}{dt} + k_1\, p(t)$$

$$m_2(t) = k_2 \frac{d\phi(t)}{dt}$$

$$k_3\, p(t) = k_4 \frac{d\phi(t)}{dt} + k_5 \frac{d^2\phi(t)}{dt^2}$$

where k_1, k_2, k_3, k_4, k_5 are non-zero constants and $p(t)$, $m_1(t)$, $m_2(t)$, $\phi(t)$ are the system variables. If the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$, input variable $u(t)$ and output variable $y(t)$ are defined as

$$x_1(t) = \phi(t),\ x_2(t) = \frac{d\phi(t)}{dt},\ x_3(t) = p(t),\ u(t) = m_1(t),\ y(t) = \phi(t)$$

obtain the state-space model for the system.

(5 Marks)

Note: Question No. 3 continues on page 3.

(b) A continuous-time system has a state-space representation given by:

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the state variables, $u(t)$ is the input variable and $y(t)$ is the output variable.

(i) If the continuous-time system is sampled with a zero-order hold at a sampling period of $T = 0.1$ second, determine the discretised state-space model for the system.

(ii) Determine the values of the sampling period T that will ensure that the transfer function $\dfrac{Y(z)}{U(z)}$ of the discretised system obtained in part 3(b)(i) has stable poles, if any.

(10 Marks)

(c) Consider a discrete-time system described by the following model:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C}\mathbf{x}(k) + du(k)$$
$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, d = 0$$

where $\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the state variables, input and output variables, respectively. Suppose that a closed-loop system is formed with a controller as follows:

$$u(k+1) = -a\,u(k) + \left(r(k) - y(k)\right)$$

where $r(k)$ is the reference input and a is a non-zero constant. If the third state variable is chosen as $x_3(k) = u(k)$, obtain a state-space model of the closed-loop system with $\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$, $r(k)$ as the input and $y(k)$ as the output.

(5 Marks)

4. (a) A discrete-time system has a state-space representation given by:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}u(k)$$

$$y(k) = \mathbf{C}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

where $\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the state variables, input and output variables, respectively.

(i) A state-feedback controller of the following form

$$u(k) = -\mathbf{K}\mathbf{x}(k)$$

is to be implemented. Determine $\mathbf{K}$ for deadbeat control.

(ii) Suppose a new set of state variables are defined as follows:

$$\hat{x}_1(k) = x_1(k) + x_2(k)$$
$$\hat{x}_2(k) = x_2(k)$$

and a state-feedback controller of the following form

$$u(k) = -\hat{\mathbf{K}}\hat{\mathbf{x}}(k); \quad \hat{\mathbf{x}}(k) = \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix}$$

is to be implemented. Determine $\hat{\mathbf{K}}$ for deadbeat control.

(10 Marks)

(b) Consider a plant with the following state-space representation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C}\mathbf{x}(k)$$

An estimator of the following form is to be implemented:

$$\bar{\mathbf{x}}(k+1) = \mathbf{A}\bar{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_O\left(y(k) - \mathbf{C}\bar{\mathbf{x}}(k)\right)$$

Note: Question No. 4 continues on page 5.

Obtain an expression for the transfer function $\dfrac{\overline{\mathbf{X}}(z)}{U(z)}$ in its simplest form.

(5 Marks)

(c) Consider a system which is described by the following state equation

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}u(k)$$

with an associated performance index given by

$$J=\frac{1}{2}\sum_{k=0}^{N-1}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k)+ru^2(k)\right)$$

The optimal control law that minimises J is of the following form

$$u^*(k)=-\mathbf{K}(k)\mathbf{x}(k)$$

where the design equations are

$$\mathbf{K}(k)=\left(\mathbf{B}^T\mathbf{S}(k+1)\mathbf{B}+r\right)^{-1}\mathbf{B}^T\mathbf{S}(k+1)\mathbf{A}$$
$$\mathbf{S}(k)=\left[\mathbf{A}-\mathbf{B}\mathbf{K}(k)\right]^T\mathbf{S}(k+1)\left[\mathbf{A}-\mathbf{B}\mathbf{K}(k)\right]+\mathbf{K}^T(k)r\mathbf{K}(k)+\mathbf{Q}$$

Now, consider the following system and associated performance index

$$x(k+1)=5x(k)+u(k)$$
$$J=\frac{1}{2}\sum_{k=0}^{2}\left(8x^2(k)+u^2(k)\right)$$

Determine the optimal feedback gains that minimize J and hence obtain the values of $u^*(k)$ as a function of $x(0)$ for $k=0,1$ and 2.

(5 Marks)

5. Consider the closed loop system in Figure 2. With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = K\frac{0.8z + 0.2}{(z-1)(z-0.4)}$$

where K is a non-zero constant.

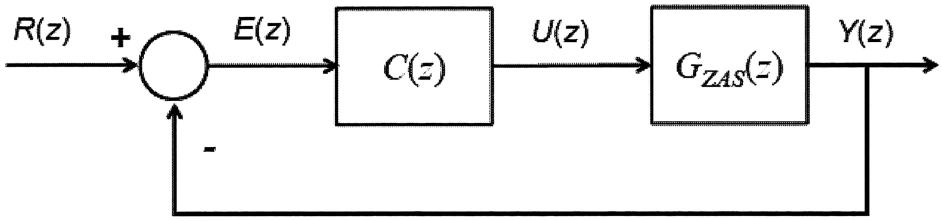

Figure 2

(a) If the controller is a proportional controller with a gain K_p, determine the range of K_pK so that the closed-loop system is stable.

(8 Marks)

(b) It is required that the output Y(z) tracks a unit-step input $R(z)$ without any steady state error. Design a ripple-free controller $C(z)$ to meet this requirement.

(8 Marks)

(c) Implement the controller $C(z)$ obtained in 5(b) with the standard programming approach and show the relevant block diagram.

(4 Marks)

Appendix A

Properties and Table of Z Transform

Discrete function	z Transform
$x(k+4)$	$z^4X(z)-z^4x(0)-z^3x(1)-z^2x(2)-zx(3)$
$x(k+3)$	$z^3X(z)-z^3x(0)-z^2x(1)-zx(2)$
$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
$x(k+1)$	$zX(z)-zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ $1, \quad k=0$ $0, \quad k\neq 0$	1
2.	—	—	$\delta_0(n-k)$ $1, \quad n=k$ $0, \quad n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

Note: Transform Table continues on page 8.

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2 e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\sin\omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\cos\omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$	$e^{-akT}\sin\omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$e^{-akT}\cos\omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

$x(t)=0,\;$ for $t<0.$

$x(kT)=x(k)=0,\;$ for $k<0.$

Unless otherwise noted, $k=0,1,2,3,\cdots$

END OF PAPER

EE6203 COMPUTER CONTROL SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.