NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2022-2023

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April / May 2023 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A robotic manipulator with seven joints is shown in Figure 1.

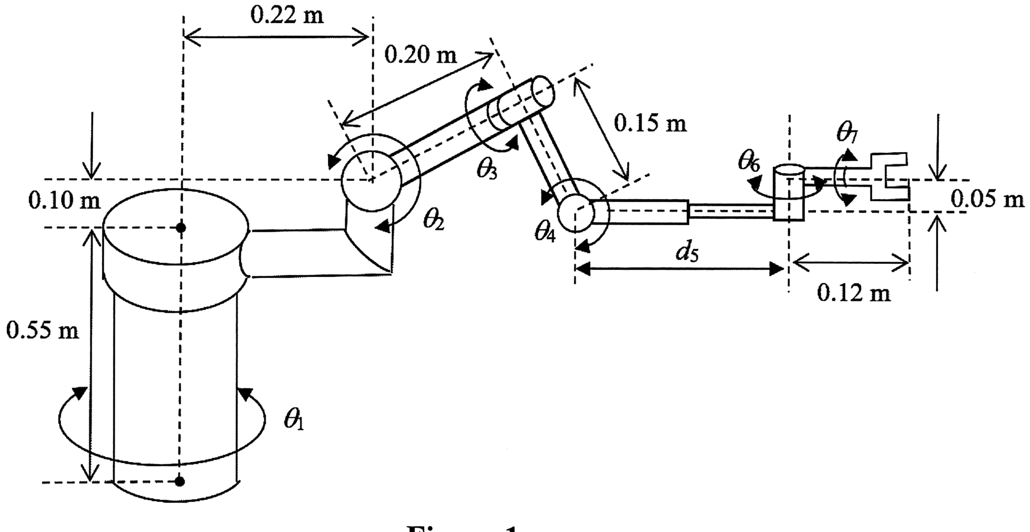

Figure 1

(a) Obtain the link coordinate diagram by using the Denavit-Hartenberg (D-H) algorithm.

(12 Marks)

Note: Question No. 1 continues on page 2.

(b) Derive the kinematic parameters of the robot based on the coordinate diagram obtained in part (a).

(8 Marks)

2. The dynamic equations of a robot, which is in contact with a frictionless surface, are given as follows:

$$(3 + 2\cos(q_2))\ddot{q}_1 + (1 + \cos(q_2))\ddot{q}_2 + (\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2)\sin(q_2) + 3\dot{q}_1 + 2g\cos(q_1) + g\cos(q_1 + q_2) = u_1$$

$$(1 + \cos(q_2))\ddot{q}_1 + \ddot{q}_2 + 2\sin(q_2)\dot{q}_1^2 + 5\dot{q}_2 + g\cos(q_1 + q_2) = u_2$$

$$2\ddot{q}_3 + 7\dot{q}_3 + 2g + f = u_3$$

where q_1, q_2, q_3 are the joint variables, u_1, u_2, u_3 are the control inputs and $g = 9.8\, m/s^2$ is the acceleration due to gravity. The first two joints possess unmodelled resonance at 12 rad/sec and the third joint possesses unmodelled resonance at 16 rad/sec. The contact force exerted on the environment is given by:

$$f = 10(q_3 - 0.1).$$

(a) Design a hybrid position and force controller for the robot so that the first two joints are critically damped, and the third joint is overdamped with a damping ratio of 1.2. The gains should be as large as possible, and the system should not excite all the unmodelled resonances.

(14 Marks)

(b) The controller designed in part (a) is now implemented on the robot in a space station, without any modification. Explain the possible effects and derive the error equations.

(6 Marks)

3. (a) A mobile robot with two castor wheels, one standard wheel and one steered standard wheel, is shown in Figure 2 on page 3. A local reference frame (x_r, y_r) and a steered angle β are assigned to the mobile robot as shown in Figure 2. The radius of each standard wheel is 7cm and the radius of each castor wheel is 3cm. If the rotational velocities of the steered standard wheel, standard wheel and the two castor wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_s$, $\dot{\varphi}_{c1}$ and $\dot{\varphi}_{c2}$, respectively, derive the rolling and sliding constraints of the mobile robot.

(10 Marks)

Note: Question No. 3 continues on page 3.

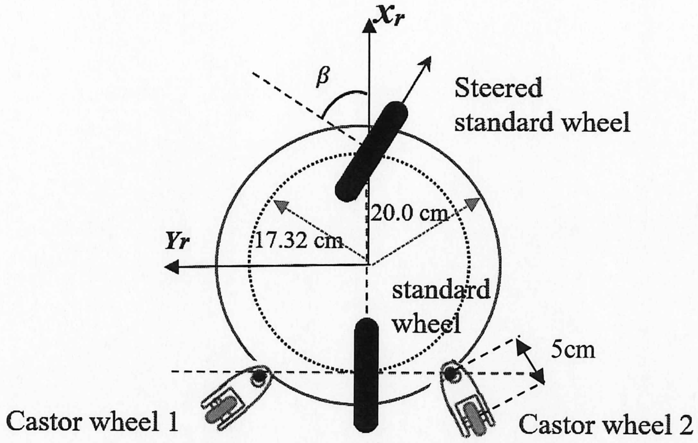

Figure 2

(b) A robot manipulator with three joint variables q_1, q_2, q_3 are mounted on a mobile robot. The link-coordinate homogeneous transformation matrix from the base coordinate to the tool coordinate of the robotic manipulator is given as:

$$T_{base}^{tool} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2(q_3+0.1) \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2(q_3+0.1) \\ -S_2 & 0 & C_2 & C_2(q_3+0.1) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $S_1 = \sin(q_1)$, $S_2 = \sin(q_2)$, $C_1 = \cos(q_1)$, $C_2 = \cos(q_2)$.

(i) Solve the inverse kinematic problem using an analytic method to express $(q_1, q_2, q_3)^T$ in terms of the position of the end effector $(x, y, z)^T$. (Note: orientation is not required).

(ii) If the robot is used to pick up an object with a centroid given as $x = 0.15$, $y = 0.25$, $z = 0.025$ units, calculate the joint configuration. Discuss the problems associated with this task.

(10 Marks)

4. As shown in Figure 3, a moving camera takes two images of the same object at two poses. Two coordinate frames represented by F_a and F_b are attached to the projection centre of the camera at the two poses, respectively. Seven feature points O_1, O_2, O_3, O_4, O_5, O_6 and O_7 are on three different lines on the same plane. Let H denote the Euclidean homography matrix from F_a to F_b.

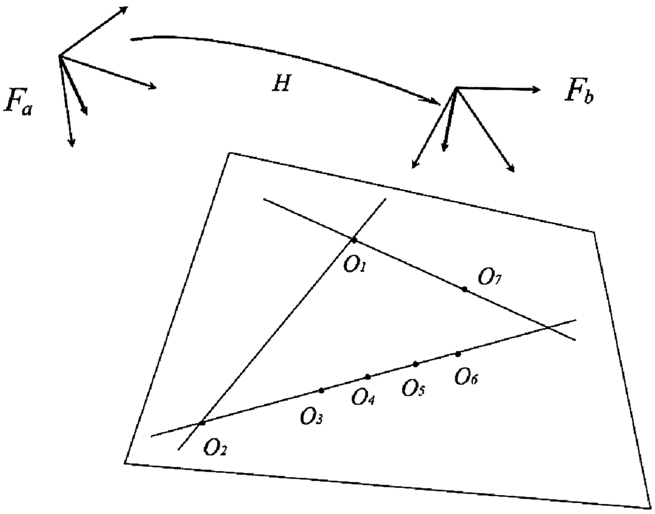

Figure 3

(a) Six feature points O_1, O_2, O_3, O_4, O_5 and O_6 can be detected in the image taken at the pose attached to F_a. Their corresponding normalized coordinates in F_a are given by:

$m_{1a} = \left[a_{1x}, a_{1y}, 1\right]^T$, $m_{2a} = \left[a_{2x}, a_{2y}, 1\right]^T$, $m_{3a} = \left[a_{3x}, a_{3y}, 1\right]^T$, $m_{4a} = \left[a_{4x}, a_{4y}, 1\right]^T$, $m_{5a} = \left[a_{5x}, a_{5y}, 1\right]^T$, $m_{6a} = \left[a_{6x}, a_{6y}, 1\right]^T$.

Six feature points O_1, O_3, O_4, O_5, O_6 and O_7 can be detected in the image taken at the pose attached to F_b. Their corresponding normalized coordinates in F_b are given by:

$m_{1b} = \left[b_{1x}, b_{1y}, 1\right]^T$, $m_{3b} = \left[b_{3x}, b_{3y}, 1\right]^T$, $m_{4b} = \left[b_{4x}, b_{4y}, 1\right]^T$, $m_{5b} = \left[b_{5x}, b_{5y}, 1\right]^T$, $m_{6b} = \left[b_{6x}, b_{6y}, 1\right]^T$, $m_{7b} = \left[b_{7x}, b_{7y}, 1\right]^T$.

Let h_{33} denote the third row third column element of the matrix H. Find the scaled homography matrix $\dfrac{H}{h_{33}}$.

(15 Marks)

(b) Scaled homography matrix can be used to extract rotation matrix information. When computing a rotation matrix from a scaled homography matrix, the solution may not be unique during the initial computation. How can you determine the correction solution?

(5 Marks)

5. A state variable x_k can be modelled by $x_{k+1} = x_k$. Five sensors are provided to measure the state variable x_k. The outputs of the sensors are y_{1k}, y_{2k}, y_{3k}, y_{4k} and y_{5k}, which are governed by the models:

$$y_{1k} = x_k + v_{1k}, \quad y_{2k} = x_k + v_{2k}, \quad y_{3k} = x_k + v_{3k}, \quad y_{4k} = x_k + v_{4k}, \quad y_{5k} = x_k + v_{5k}$$

where v_{1k}, v_{2k}, v_{3k}, v_{4k} and v_{5k} are zero mean Gaussian sensor noises with variances given by $2a^2$, $2a^2$, $4a^2$, $4a^2$ and $4a^2$, respectively.

Let $\hat{x}_k$ represent the estimate of x_k, and let the estimation error be $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error $\tilde{x}_k$ and the noise terms v_{1k}, v_{2k}, v_{3k}, v_{4k} and v_{5k} are uncorrelated, and that $E\left[\tilde{x}_{k+1}\right] = 0$ for any k. Let the estimation error variance be $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

Two estimation predictors are designed, using sensors 1 and 2, and sensors 3, 4 and 5, respectively, as follows:

Predictor 1:

$$\hat{x}_{k+1} = \hat{x}_k + L_{1k}\left(y_{1k} - \hat{x}_k\right) + L_{1k}\left(y_{2k} - \hat{x}_k\right)$$

Predictor 2:

$$\hat{x}_{k+1} = \hat{x}_k + L_{2k}\left(y_{3k} - \hat{x}_k\right) + L_{2k}\left(y_{4k} - \hat{x}_k\right) + L_{2k}\left(y_{5k} - \hat{x}_k\right)$$

where L_{1k} and L_{2k} represent Kalman gains.

(a) Design the update laws for the Kaman gains L_{1k} and L_{2k} to minimize the corresponding estimation error variance.

(12 Marks)

(b) Between Predictor 1 and Predictor 2, which one gives a smaller estimation error variance. Justify your answer.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.