EE 6225 – Multivariable Control System

Part I – Advanced Process Control

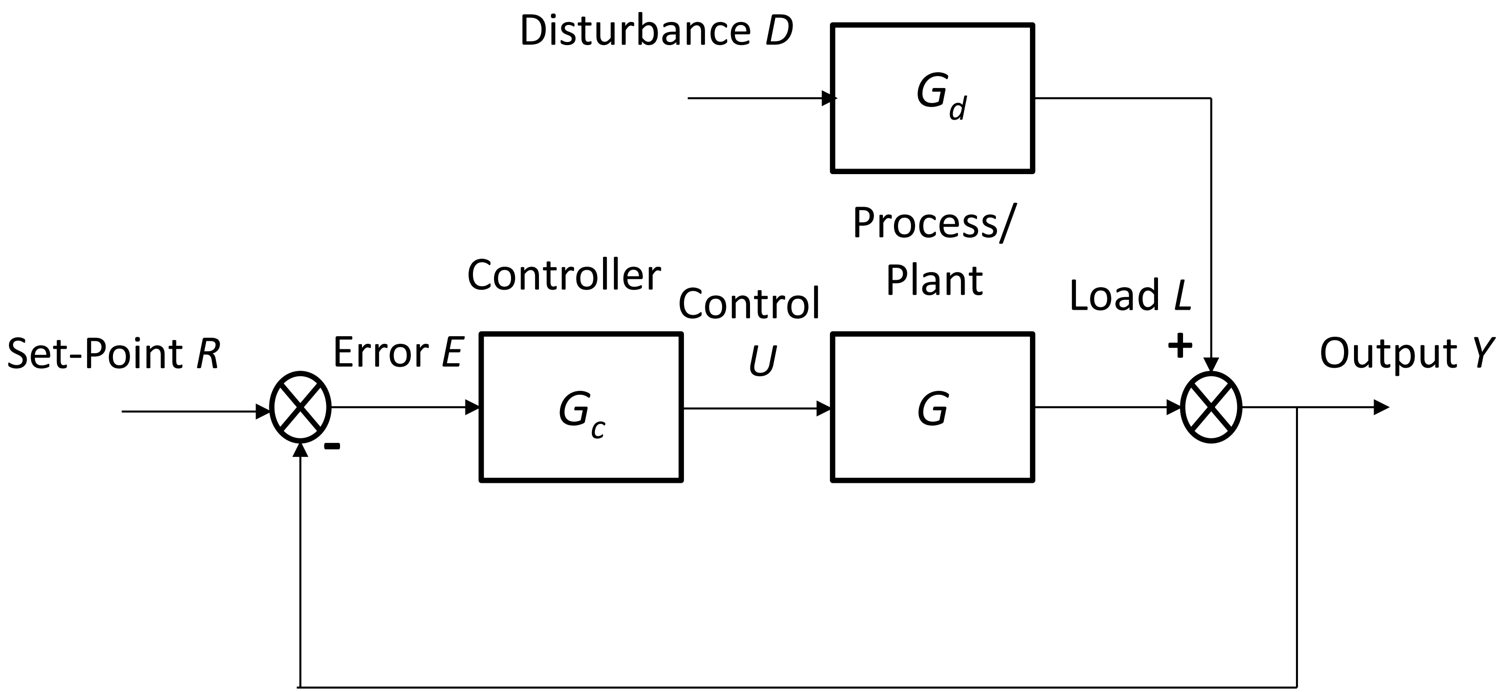
CA1 Assignment

(i) Due Date: 20 Apr 2025

(ii)Submit softcopy solution to NTULearn EE6225 Course Assignments Folder and include Your Name and Matriculation Number

1(a). Direct Synthesis Method (DSM)

- Consider the following block diagram of a feedback control system



1(a). Direct Synthesis Method (DSM)

The Second Order Plus Time Delay (SOPTD) process is given by:

$$G(s) = \frac{2e^{-s}}{(5s+1)(2s+1)}$$

The disturbance D to output Y is

$$G_d(s) = \frac{e^{-0.5s}}{(10s+1)(5s+1)}$$

Apply the Direct Synthesis Method (DSM) to design a controller using the following desired closed loop transfer function

$$G_c = \frac{1}{\tilde{G}}\left(\frac{\left(\frac{Y}{R}\right)_d}{1-\left(\frac{Y}{R}\right)_d}\right) \quad \text{where} \quad \left(\frac{Y}{R}\right)_d = \frac{e^{-s}}{\tau_c s+1}$$

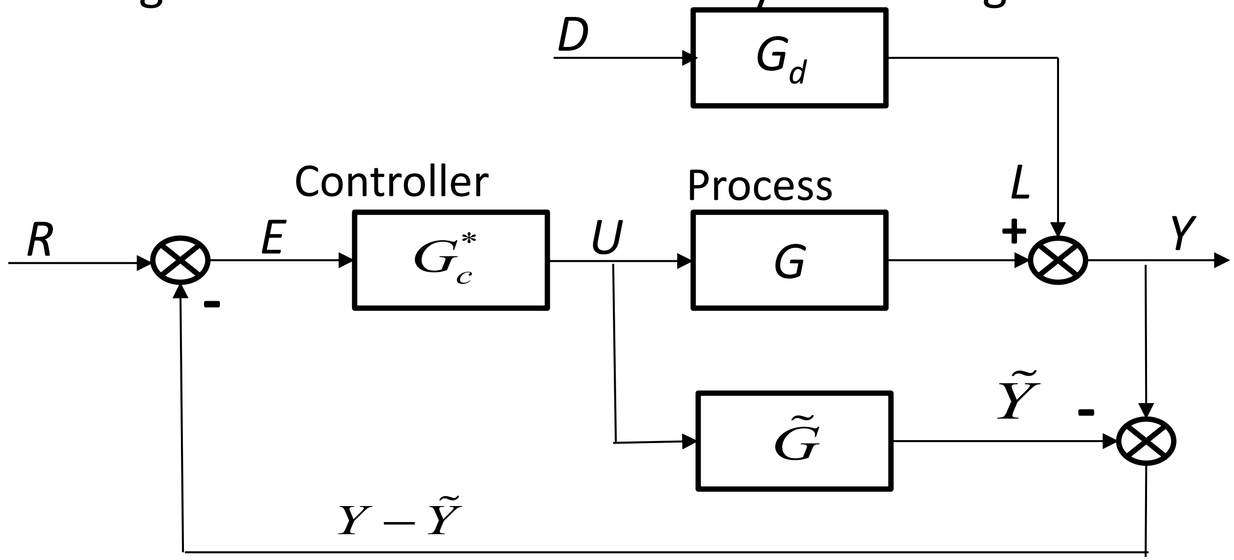
(i) Assume $\tilde{G} = G$, design a PID controller G_c for $\tau_c = 1$ sec and 10 sec.

(Hint: Using first order Taylor series to approximate $e^{-\theta s} \approx 1-\theta s$ in the denominator of G_c; the $e^{-\theta s}$ in the numerator will be cancelled by identical term in $\tilde{G}$)

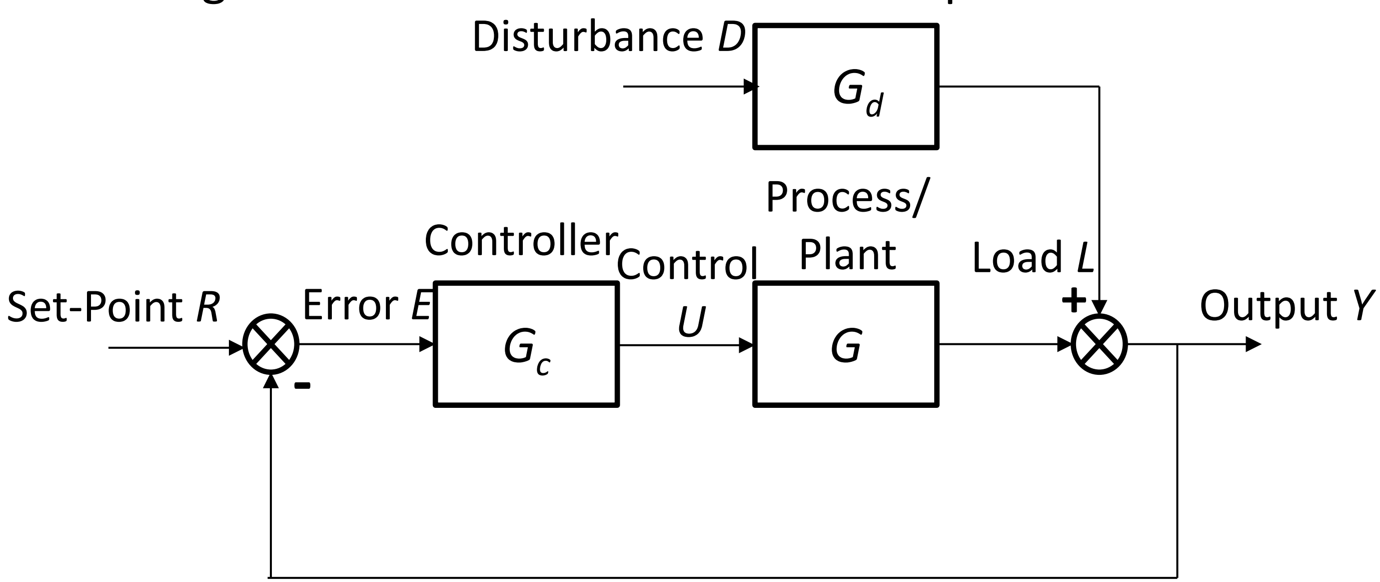
(ii) Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t=0$ sec and a step disturbance input D at $t=60$ sec.

1(b). 1 Degree of Freedom Internal Model Control (IMC)

- (a) Block diagram of a feedback control system using IMC



- (b) Block diagram of a feedback control with equivalent standard controller



1(b). 1 Degree of Freedom Internal Model Control (IMC)

The Second Order process with right Half Plane (RHP) zero is given by:

$$G(s) = \frac{2(-s+1)}{(5s+1)(2s+1)}$$

The disturbance D to output Y is

$$G_d(s) = \frac{e^{-s}}{(10s+1)(5s+1)}$$

An IMC controller G_c^* is given by

$$G_c^* = \frac{1}{\tilde{G}_-} f \quad \text{where } \tilde{G} = \tilde{G}_+ \tilde{G}_-,\ \tilde{G}_+ \text{ contains time delays and RHP zeros with } \tilde{G}_+(s=0)=1$$

and $f = \dfrac{1}{\tau_c s + 1}$

(i) Assume $\tilde{G} = G$, design an IMC Controller G_c^* for $\tau_c = 5$ sec.

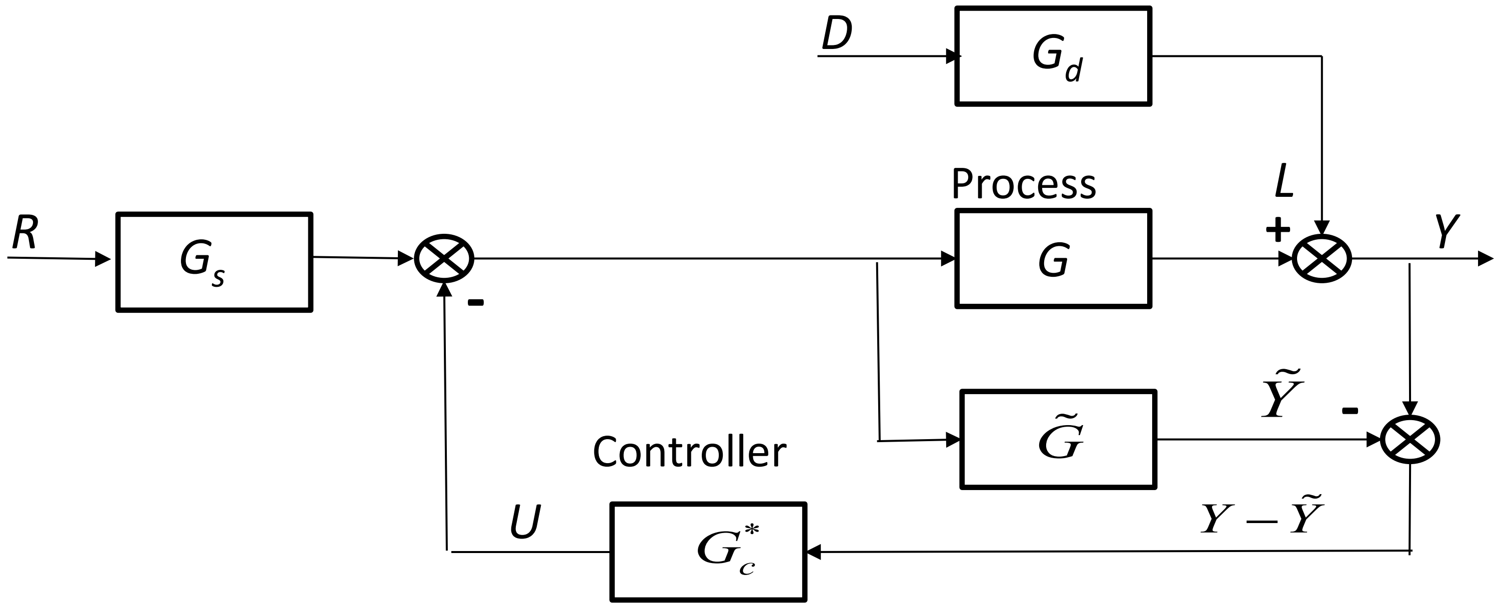
(ii) Derive the equivalent standard controller $G_c = \dfrac{G_c^*}{1 - G_c^* \tilde{G}}$ and determine the PID parameters

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right).$$ Check if you get the same G_c if you use DSM with $\left(\dfrac{Y}{R}\right)_d = \tilde{G}_+ f$.

(iii) Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 60$ sec.

1(c). 2 Degree of Freedom Internal Model Control (IMC)

- (a) Block diagram of a feedback control system using 2 degree of freedom controller design (the controller is place in feedback path so that setpoint filter G_s can be designed independently of G_c* from $R(s)$ to $Y(s)$)



$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G_c^*(s)\left(G(s)-\tilde{G}(s)\right)}G_s(s),\quad \frac{Y(s)}{D(s)}=\frac{1-G_c^*(s)\tilde{G}(s)}{1+G_c^*(s)\left(G(s)-\tilde{G}(s)\right)}G_d(s)$$

Assume $\tilde{G}(s)=G(s)$,

$$Y(s)=G(s)G_s(s)R(s) \text{ and } Y(s)=\left(1-G_c^*(s)\tilde{G}(s)\right)G_d(s)D(s)$$

1(c). 2 Degree of Freedom Internal Model Control (IMC)

The First Order Plus Time Delay (FOPTD) is given by:

$$G(s) = G_d(s) = \frac{e^{-2s}}{(10s+1)} \quad (\text{that is, we assume that the disturbance is at the plant input})$$

An IMC controller G_c^* is given by

$$G_c^* = \frac{1}{\tilde{G}_-} f \quad \text{where } \tilde{G} = \tilde{G}_+ \tilde{G}_- \text{ and } f = \frac{\lambda s + 1}{(\tau_c s + 1)^r}$$

(i) Assume $\tilde{G} = G$, design an IMC Controller G_c^* for $\tau_c = 2$ sec, $r = 1$ and $\lambda = 0$.

Set $G_s = 1$. Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 20$ sec.

(ii) Next design another IMC Controller with $\tau_c = 2$ sec, $r = 2$ and λ is determined by setting

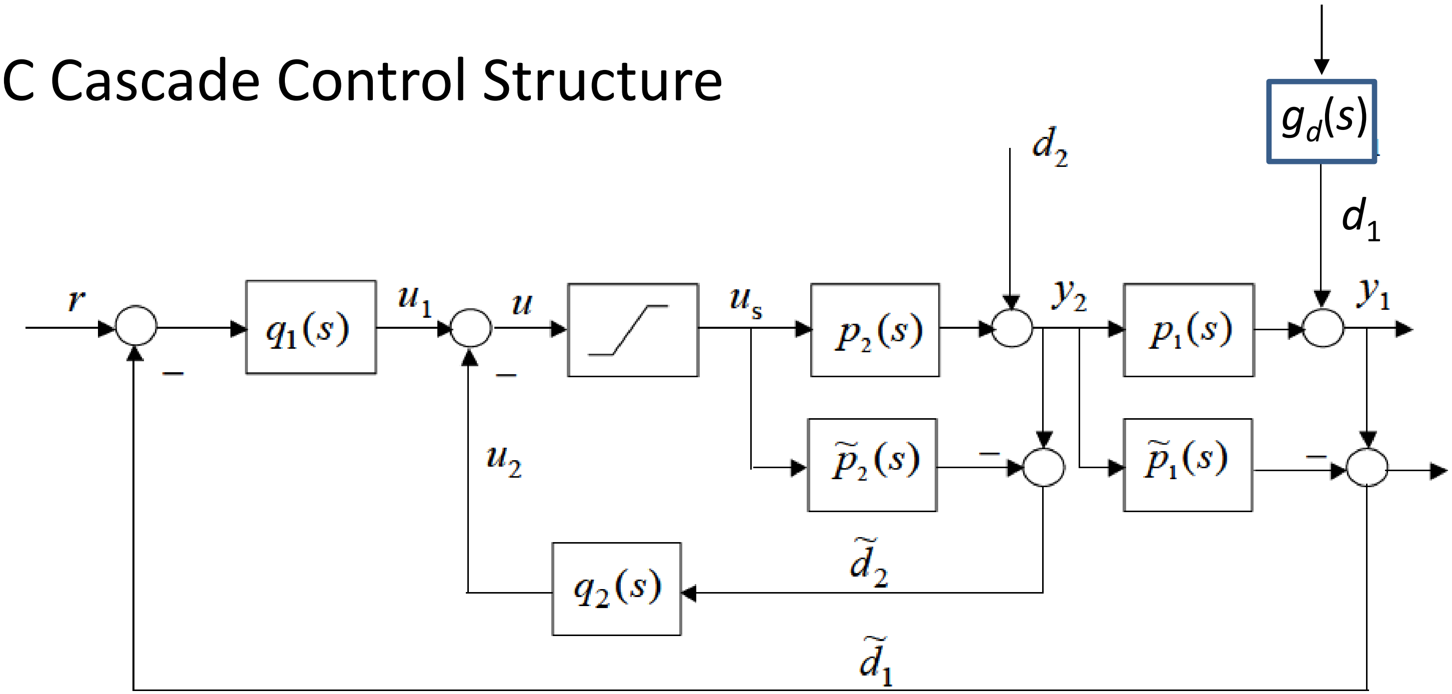
$$\left(1 - G(s) G_c^*\right) = 0 \text{ at the pole of } G_d(s) \Rightarrow \left.\left(1 - G(s) \frac{1}{\tilde{G}_-(s)} \frac{\lambda s + 1}{(\tau_c s + 1)^r}\right)\right|_{s=-1/10, \tau_c=2, r=2} = 0$$

Choose $G_s = \frac{1}{\tau_s s + 1}$ and judiciously select τ_s. Using MatLab/Simulink or any simulation software, plot the output response Y to a step input R at time $t = 0$ sec and a step disturbance input D at $t = 20$ sec.

(iii) Compare the responses obtained in (i) and (ii).

1(d). Cascade Structure and Feedforward Control

- IMC Cascade Control Structure



$$y_2(s) = \frac{p_2(s)u_1(s) + (1 - \tilde{p}_2(s)q_2(s))d_2(s)}{(1 + (p_2(s) - \tilde{p}_2(s))q_2(s)}.$$

$$y_1(s) = \frac{p_1 p_2 q_1 r(s) + (1 - \tilde{p}_2 q_2) p_1 d_2(s) + (1 - \tilde{p}_1 p_2 q_1 + (p_2 - \tilde{p}_2) q_2) d_1(s)}{(1 + (p_1 - \tilde{p}_1) p_2 q_1 + (p_2 - \tilde{p}_2) q_2)}.$$

1(d). Cascade Structure and Feedforward Control

The Second Order Plus Time Delay (SOPTD) is given by:

$$p(s) = p_1(s)\,p_2(s)$$

where by

$$p_1(s) = \frac{e^{-20s}}{15s+1} \text{ and } p_2(s) = \frac{2e^{-4s}}{s+1}$$

and

$$g_d(s) = \frac{e^{-30s}}{(10s+1)(0.3s+1)}$$

(i) Design the inner controller q_2 for $p_2(s)$ using the 1 Degree of Freedom IMC design method as in Part 1(b) with $f = \dfrac{1}{4s+1}$. Next design the outer controller q_1 for $p(s)$ using the 1 Degree of Freedom IMC design method as in Part 1(b) with $f = \dfrac{1}{(18s+1)^2}$. Using MatLab/Simulink or any simulation software, if d_2=0, plot the output response y to a step input r at time $t = 0$ sec and a step disturbance input to $g_d(s)$ at $t = 40$ sec.

1(d). Cascade Structure and Feedforward Control

(ii) Design the inner controller q_2 for $p_2(s)$ with $f = \dfrac{\lambda s+1}{(4s+1)^2}$ and λ is determined by setting

$$(1-p_2(s)q_2(s)) = 0 \text{ at the pole of } p_1(s) \Rightarrow \left.\left(1-p_2(s)\frac{1}{\tilde{p}_{2\_}(s)}\frac{\lambda s+1}{(4s+1)^2}\right)\right|_{s=-1/15} = 0$$

Use the same outer controller q_1 for $p(s)$ designed in Part 1(d)(i). Using MatLab/Simulink or any simulation software, if d_2=0, plot the output response y to a step input r at time $t=0$ sec and a step disturbance input to $g_d(s)$ at $t=40$ sec.

(iii) Design a feedforward controller against the distance d_1. Together with the designed IMC controllers designed in Part 1 (d)(ii), if d_2=0, plot the output response y to a step input r at time $t=0$ sec and a step disturbance input to $g_d(s)$ at $t=40$ sec.

(iv) Compare the responses obtained in (i), (ii) and (iii).

1(e) Automobile Cruise Control

1. In this problem, we develop a simple model of an automobile cruise control system. The control objective is to maintain a speed preset by the driver. If we Figure 2.18. Mass and damper system neglect the inertia of the wheels, and assume that friction (which is proportional to the car.s speed) is what is opposing the motion of the car, then the plant description is reduced to the simple mass and damper system shown in Figure 1.1.

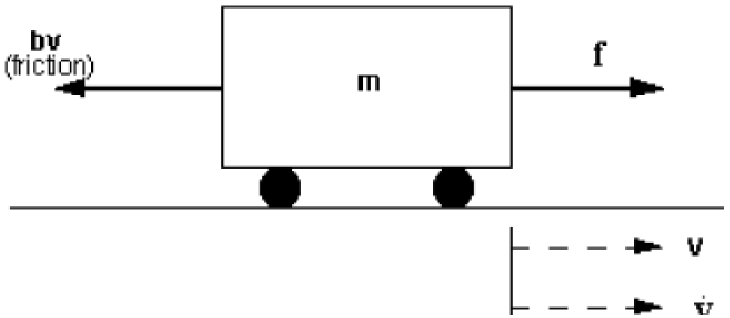


Figure 1.1

Using Newton's law of motion, the model equations for this system are

$$m\frac{dv}{dt}+bv=f$$

$$y=v$$

where f is the force produced by the engine and v is the measured velocity of the automobile. For this example, Let us assume that mass m = 1000 kg, viscous coefficient b = 50 Newton seconds/meter.

1(e) Automobile Cruise Control

The plant transfer function can be easily derived to be as follows:

$$\frac{Y(s)}{F(s)} = \frac{1}{ms+b} = \frac{0.001}{s+0.05} \qquad (1.1)$$

(a) Design a Proportional-Integral-Derivative (PID) controller for the closed loop system in Figure 1.2 with plant description given in Equation (1.1). The design specifications are to achieve a rise time of 5 sec, with an allowable overshoot of 10% on the velocity and a 2% steady-state error to a step input command of 10 m/s.

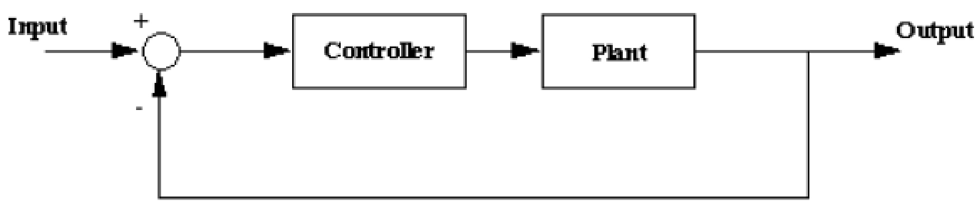


Figure 1.2

The standard PID controller is given is given as:

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \ ,$$

where K_P, K_I and K_D are the proportional, integral and derivative gains respectively which are to be determined. Plot the velocity error, the PID controller output and the actual velocity.