NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2023-2024

EE6203 – COMPUTER CONTROL SYSTEMS

November / December 2023 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 8 pages.
2. Answer all 4 questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. Unless specifically stated, all symbols have their usual meanings.
6. The Transform Table is included in Appendix A on pages 7 to 8.

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1. (a) A signal $x(t)$ is given as

$$x(t) = \begin{cases} 0, & t < 0 \\ 1 + \sin(\pi t), & 0 \le t < 0.8 \\ 1, & t \ge 0.8 \end{cases}$$

Justify whether $x(t)$ is a continuous-time signal. Determine its Z-transform, where the sampling starts from $t=0$ with a sampling period $T=0.5$ second.

(5 Marks)

(b) Solve the following difference equation:

$$y(k+2)+(\beta-1)y(k+1)-\beta\, y(k)= \delta(k-1)$$

where $y(k)=0$ for $k<0$, $\delta(k)$ is the unit impulse function and β is a real number.

(10 Marks)

Note: Question No. 1 continues on page 2.

(c) Discuss the convergence of your solution $y(k)$ obtained in part (b) when $k \to \infty$ for different values of β and the applicability of the Final Value Theorem for these values of β. When it is applicable, verify whether the theorem holds.

(10 Marks)

2. Consider the closed loop system in Figure 1. With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = \frac{0.5z + 0.2}{(z - 0.5)(z - 0.4)}$$

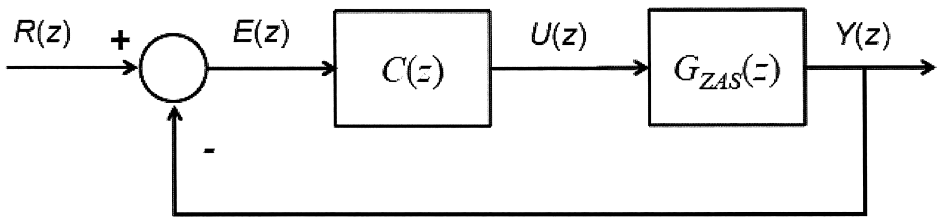

Figure 1

(a) Suppose that the controller is a PI controller with $C(z) = 1 + \frac{1}{1-z^{-1}}$. Determine whether the closed loop system is stable. With a constant reference input, find the steady state tracking error of the closed loop system

(12 Marks)

(b) It is required that the output $Y(z)$ should track a unit-step input $R(z)$ without any steady state tracking error. Design a ripple-free controller $C(z)$ to meet this requirement.

(11 Marks)

(c) If

$$G_{ZAS}(z) = \frac{0.5z+0.2}{(z-1.5)(z-0.4)}$$

can we still design a ripple-free controller $C(z)$ to achieve the objective that the output $Y(z)$ tracks a unit-step input $R(z)$ without any steady state tracking error? Justify your answer.

(2 Marks)

3. (a) The block diagram representation of a system is shown in Figure 2.

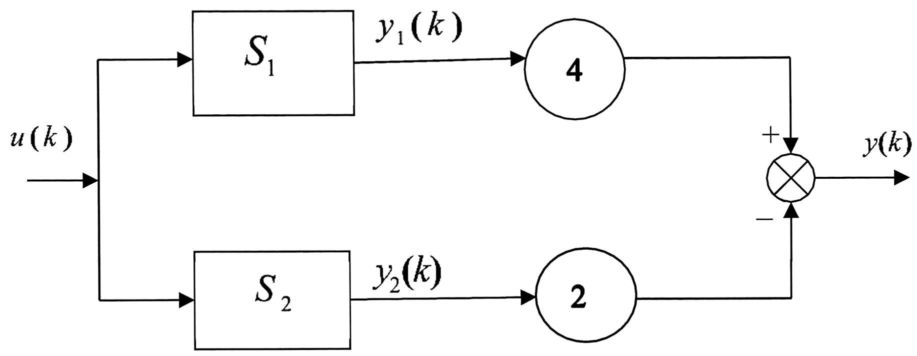

Figure 2

The state-space representations for the systems S_1 and S_2 are given, respectively, by

$$S_1 : \begin{cases} x_1(k+1) = 2x_1(k) + u(k) \\ y_1(k) = x_1(k) \end{cases}$$

$$S_2 : \begin{cases} x_2(k+1) = x_2(k) + u(k) \\ y_2(k) = 3x_2(k) \end{cases}$$

(i) Obtain a state-space representation of the overall system with the state vector $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, input $u(k)$ and output $y(k)$.

(ii) Determine the transfer function from the state-space representation obtained in part (i).

(7 Marks)

(b) A continuous-time system has a state-space representation which is described by

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Note: Question No. 3 continues on page 4.

where $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $u(t)$ and $y(t)$ are the states, input and output variables, respectively. The system is sampled with a zero-order hold. Determine

(i) A discretized state-space model for the system if the sampling period $T = \dfrac{\pi}{2}$ seconds.

(ii) The inputs $u(0)$ and $u(1)$, if they exist, that can drive the discretized system obtained in part (i) from $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to a desired final state $\begin{bmatrix} x_1(2) \\ x_2(2) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

(iii) Whether the discretized state-space model with a sampling period $T = \pi$ seconds is controllable or uncontrollable.

(12 Marks)

(c) Consider a discrete-time system described by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.2753 \\ 0 & 0.4503 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.06233\beta \\ 0.2803\beta \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where β is a process variable and $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively. If a closed-loop system is formed using,

$$u(k) = r(k) - y(k)$$

where $r(k)$ is the reference input, determine whether β has any effect on the observability of the closed-loop system.

(6 marks)

4. (a) Consider the following discrete-time system.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \qquad \text{... (4.1)}$$

$$y(k) = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \qquad \text{... (4.2)}$$

where $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively.

(i) A state-feedback law of the following form

$$u(k) = -\mathbf{K} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

is to be designed. Find $\mathbf{K}$ such that the desired closed-loop poles are located at $z_{1,2} = 0.9 \pm j0.1$.

(ii) For the system as given in (4.1) and (4.2), design a deadbeat observer of the following form,

$$\overline{\mathbf{x}}(k+1) = \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_{\mathrm{O}}\left(y(k) - \mathbf{C}\overline{\mathbf{x}}(k)\right).$$

(14 Marks)

(b) For the system as given in (4.1) and (4.2) in part (a), design a control law of the following form,

$$u(k) = -\mathbf{K} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + k_r r(k)$$

such that the final value of the output variable $y(k)$ is unity for a unit-step reference input $r(k)$ and the desired closed-loop poles are located at $z_{1,2} = 0.9 \pm j0.1$.

(5 Marks)

Note: Question No. 4 continues on page 6.

(c) Consider a system which is described by the following state equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k),$$

where an associated performance index is given by

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + ru^2(k)\right).$$

The control law that minimizes J is of the following form

$$u^*(k) = -\mathbf{K}\mathbf{x}(k),$$

where the optimal control gain is given by

$$\mathbf{K} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B} + r\right)^{-1}\mathbf{B}^T\mathbf{S}\mathbf{A}$$

and $\mathbf{S} > 0$ solves the following equation,

$$\mathbf{S} = \mathbf{A}^T\mathbf{S}\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\mathbf{S}\mathbf{B}\left(r + \mathbf{B}^T\mathbf{S}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{S}\mathbf{A}.$$

Now, consider the following system and associated performance index:

$$x(k+1) = 2x(k) + 3u(k)$$

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(x^2(k) + 5u^2(k)\right)$$

(i) Determine an optimal control law that minimizes J.

(ii) With the optimal control law obtained in part (i) applied, comment on the final value of $x(k)$ in the resulting closed-loop system for any non-zero value of $x(0)$. Explain your answer clearly.

(6 Marks)

Appendix A

Properties and Table of Z Transform

Discrete function	z Transform
$x(k+4)$	$z^4X(z)-z^4x(0)-z^3x(1)-z^2x(2)-zx(3)$
$x(k+3)$	$z^3X(z)-z^3x(0)-z^2x(1)-zx(2)$
$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
$x(k+1)$	$zX(z)-zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1, $k=0$ 0, $k\neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1, $n=k$ 0, $n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

Note: Transform Table continues on page 8.

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2\ e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\sin\omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\cos\omega kT$	$\frac{1\text{-}z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$	$e^{-akT}\sin\omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$e^{-akT}\cos\omega kT$	$\frac{1\text{-}e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

$x(t)=0,$ for $t<0$.

$x(kT)=x(k)=0,$ for $k<0$.

Unless otherwise noted, $k=0,1,2,3,\cdots$

END OF PAPER

EE6203 COMPUTER CONTROL SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.