EE4341/EE6341 Advanced Analog Circuits - Power Amplifiers

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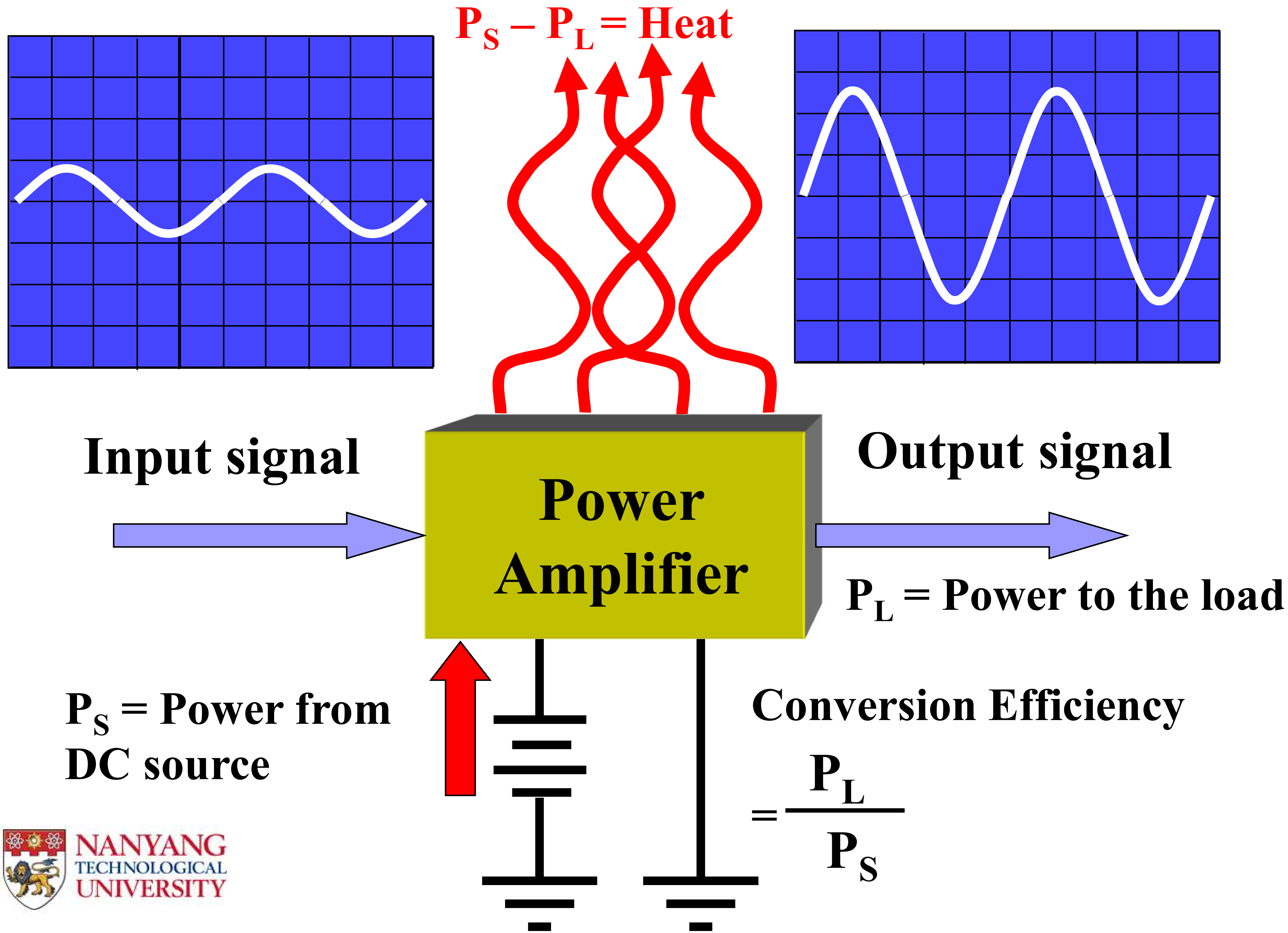
Power Amplifier



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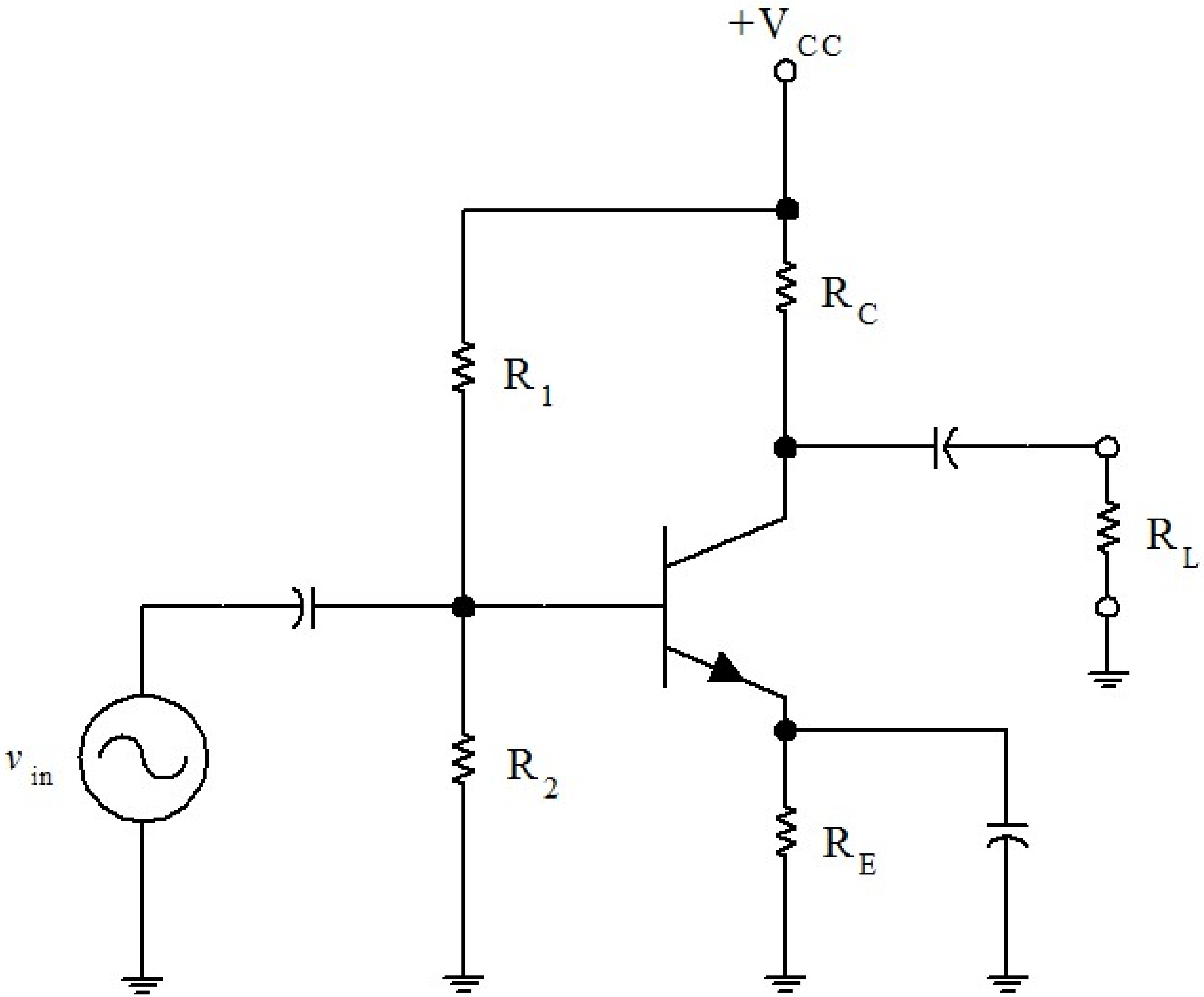
Power Amplifier

- To deliver a large amount of power to a load (for examples, audio amplifiers and RF transmitters).

- To deliver the required power to the load efficiently with lowest possible power dissipation in the power amplifier itself.

- To operate in the linear region to minimize the distortion of the output signal waveform, usually measured in terms of total harmonic distortion (THD).

Typical Amplifier Circuit

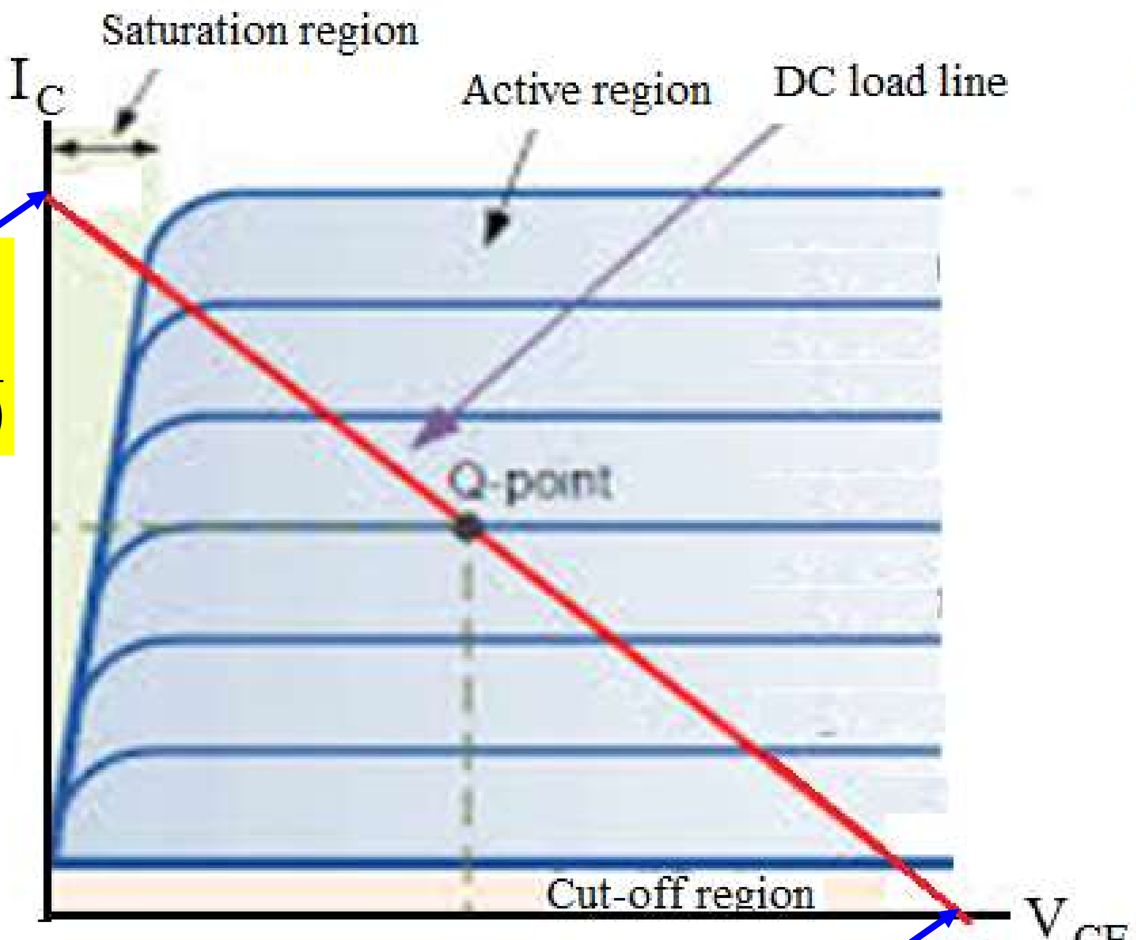


Under DC condition:

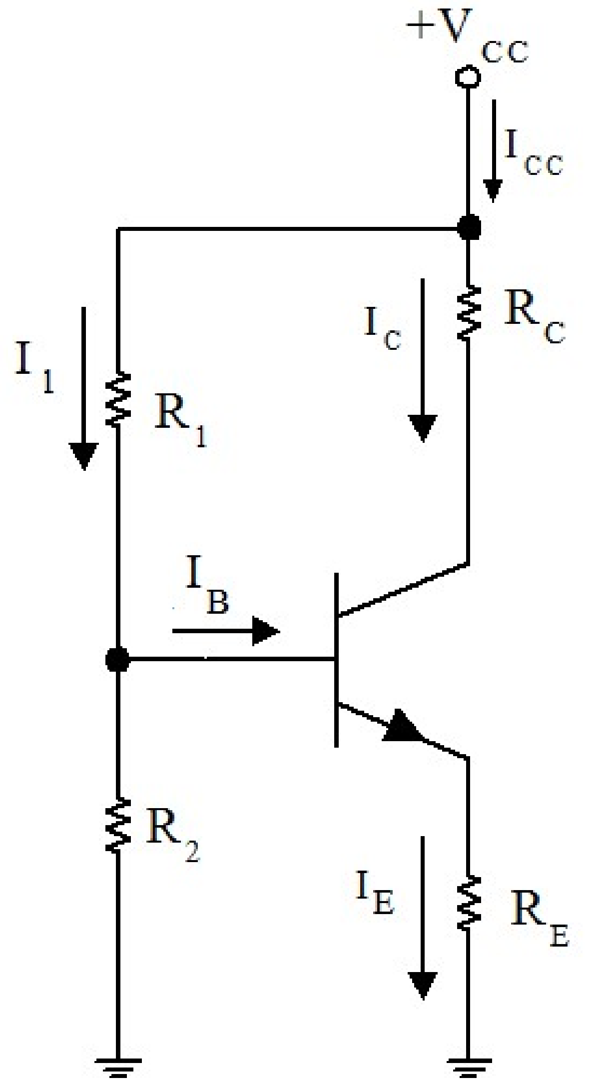
If β is large, $I_C \approx I_E$ $\qquad V_{CE} = V_{CC} - I_C\left(R_C + R_E\right)$

When $V_{CE} = 0$

$$I_C = \frac{V_{CC}}{\left(R_C + R_E\right)}$$



When $I_C = 0, V_{CE} = V_{CC}$



For maximum output voltage swing, the biasing is chosen at the mid-point of the DC load line.

The Q-point is chosen so that:

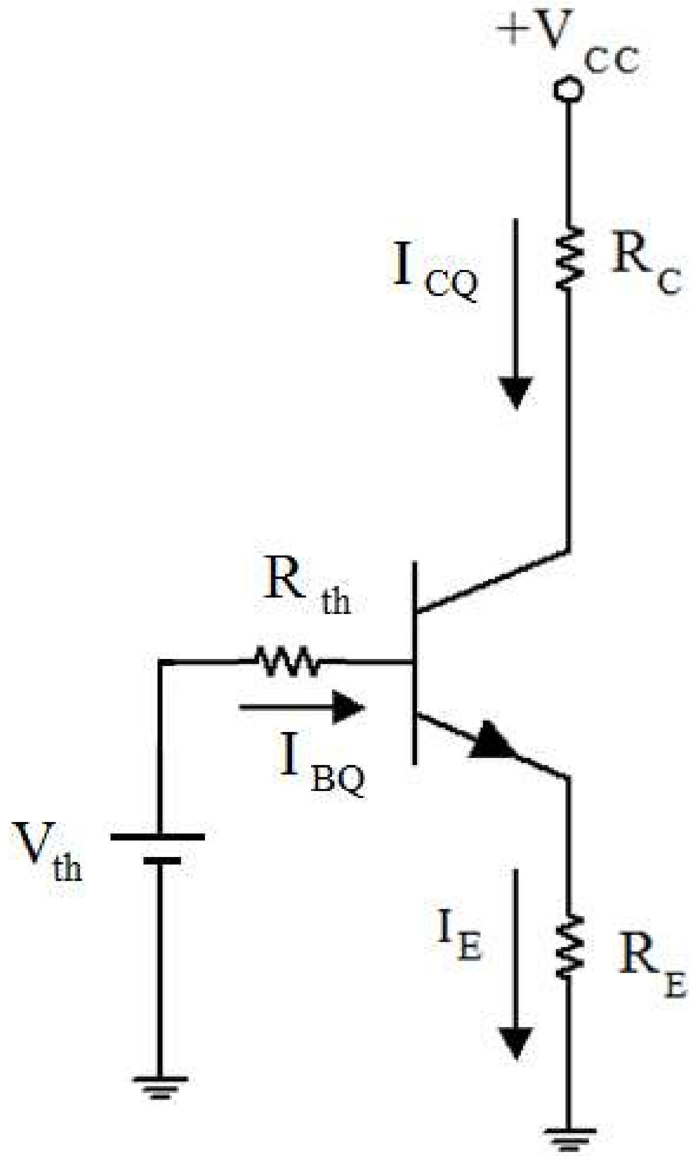
$$V_{CEQ} = \frac{V_{CC}}{2} \qquad I_{CQ} = \frac{1}{2} \times \frac{V_{CC}}{(R_C + R_E)}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{V_{CC}}{2\beta(R_C + R_E)}$$

$$V_{th} = V_{CC}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$R_{th} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{th} = I_{BQ}R_{th} + V_{BE} + (\beta + 1)I_{BQ}R_E$$



From the above equations, R_1 and R_2 can be determined to provide the required Q-point.

Amplifier Efficiency

The **conversion efficiency** is defined as: $\eta = \dfrac{\overline{P_L}}{\overline{P_S}}$

where $\overline{P_L}$ = Average ac power delivered to the load

$\overline{P_S}$ = Average power supplied from dc power source

$$\overline{P_L}(max) = \frac{1}{2}V_p I_p = \frac{1}{2}\left(\frac{V_{CC}}{2}\right)(I_{CQ}) = \frac{V_{CC}I_{CQ}}{4}$$

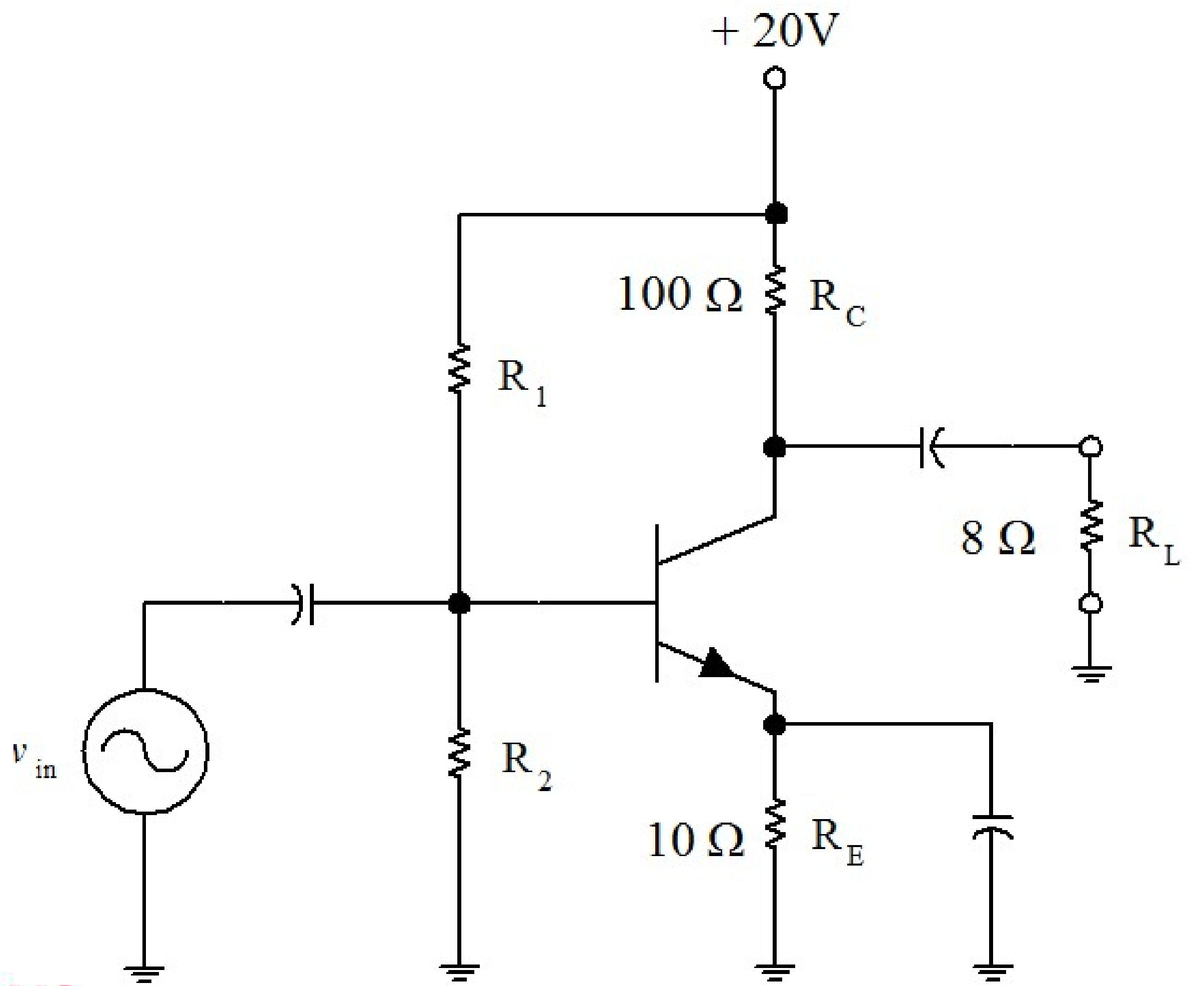
Assuming that both maximum voltage and current swings are possible

$$\overline{P_S} = V_{CC}I_{CC} = V_{CC}\left(I_{CQ} + I_1\right) \approx V_{CC}I_{CQ} \quad \text{assume } I_1 << I_{CQ}$$

$$\eta(max) = \frac{\overline{P_L}(max)}{\overline{P_S}} = \left(\frac{V_{CC}I_{CQ}}{4}\right)\left(\frac{1}{V_{CC}I_{CQ}}\right) = 25\%$$

This is the highest efficiency that can be achieved

Exercise #1: For the given amplifier circuit, determine the value of R_1 and R_2 to provide maximum output voltage swing. Calculate its conversion efficiency if $v_i = 0.2\ V_{peak}$. Assume $\beta = 25$, $V_T = 26$ mV, $V_{BE} = 0.7$V and $V_{CE,sat} = 0.2$V.



DC biasing circuit:

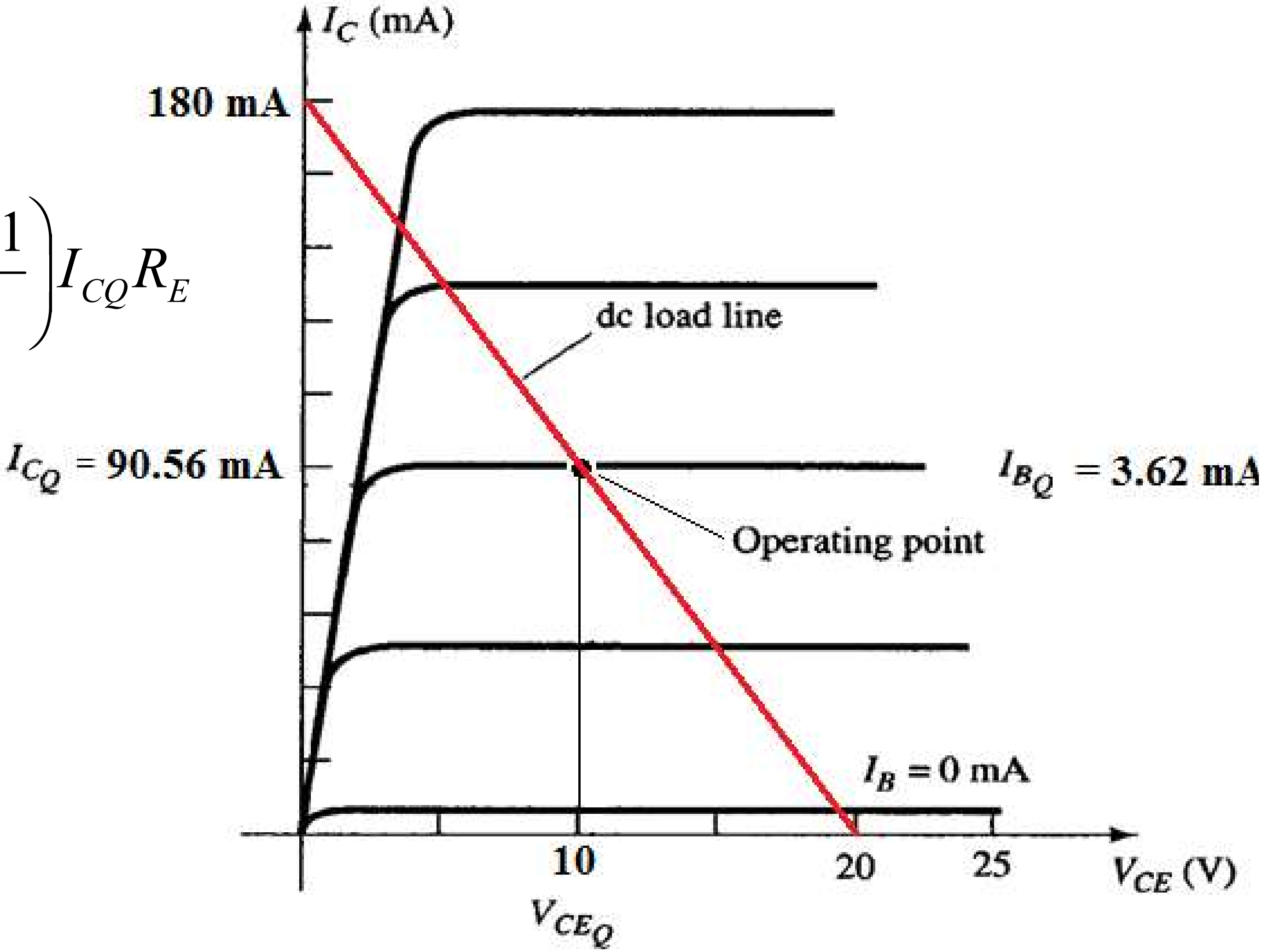
$$V_{CEQ} = \frac{V_{CC}}{2} = 10\text{ V}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + \left(\frac{\beta+1}{\beta}\right)I_{CQ}R_E$$

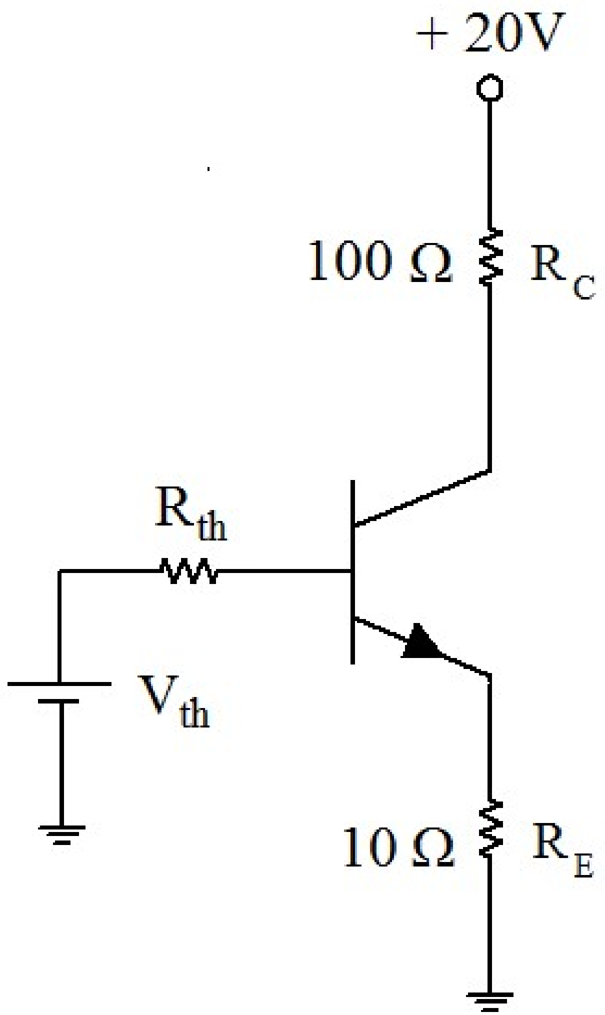
$$I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C + \left(\frac{\beta+1}{\beta}\right)R_E}$$

$$= \frac{20-10}{100+1.04\times 10}$$

$$= 90.56\text{ mA}$$



$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{90.56\text{ mA}}{25} = 3.62\text{ mA}$$



$$V_{th} = I_{BQ}R_{th} + V_{BE} + (\beta + 1)I_{BQ}R_E$$

$$I_{BQ} = \frac{V_{th} - V_{BE}}{R_{th} + (\beta + 1)R_E}$$

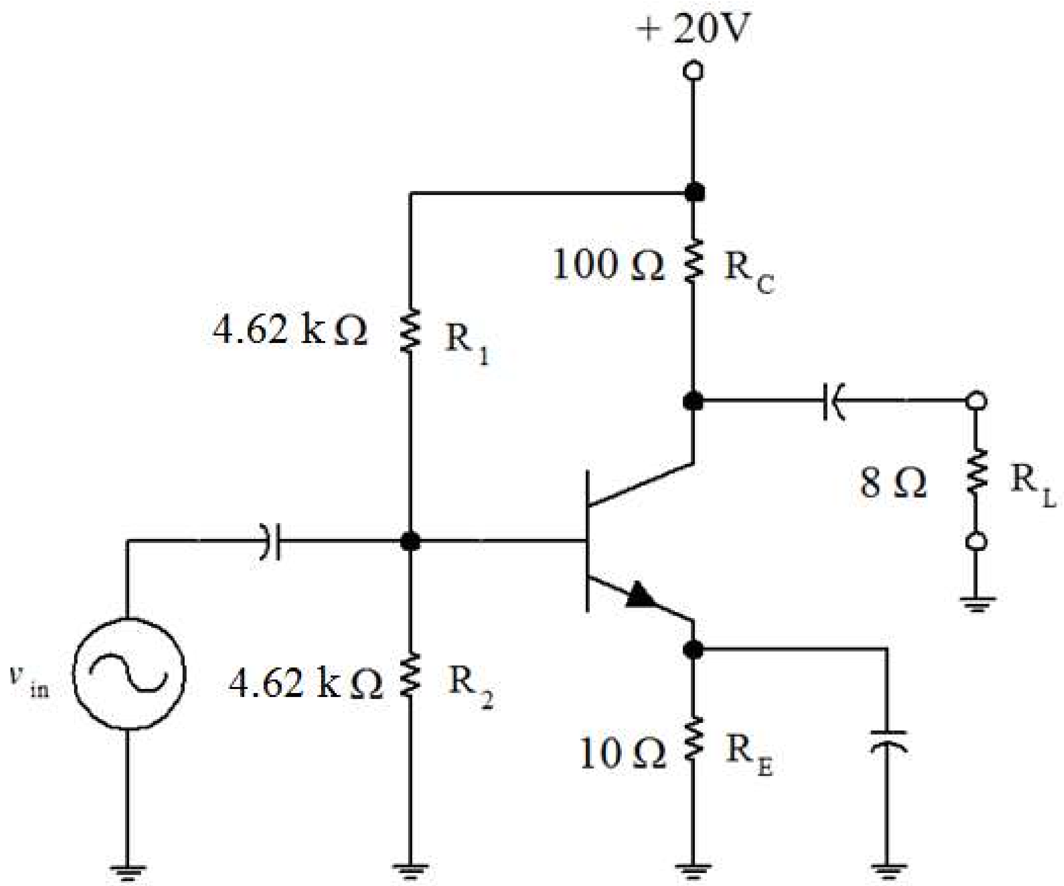
$$R_{th} = \frac{V_{th} - V_{BE}}{I_{BQ}} - (\beta + 1)R_E$$

Let make $R_1 = R_2$

$$\therefore V_{th} = 0.5V_{CC} = 10\ \text{V}$$

$$R_{th} = \frac{10 - 0.7}{3.62 \times 10^{-3}} - 26 \times 10 = 2.31\ \text{k}\Omega$$

$$\therefore R_1 = R_2 = 2 \times 2.31\ \text{k}\Omega = 4.62\ \text{k}\Omega$$

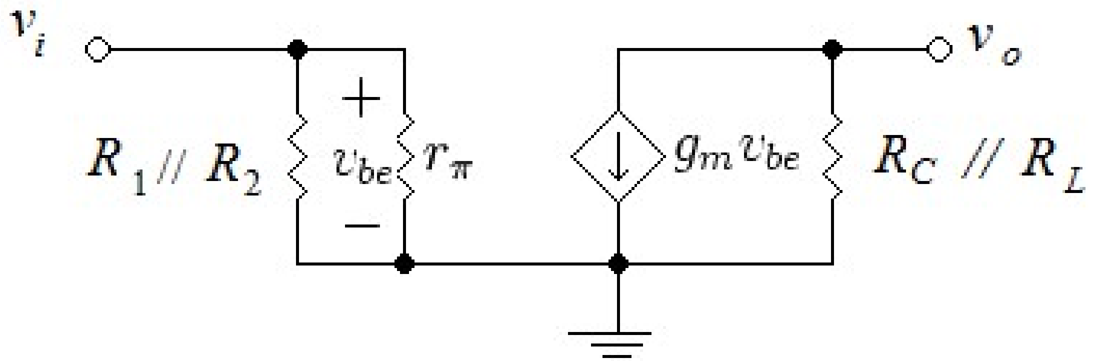


$$g_m = \frac{I_{CQ}}{V_T} = \frac{90.56\text{ mA}}{26\text{ mV}} = 3.48\text{ S}$$

$$v_{o,peak} = g_m v_{i,peak}\left(R_L // R_C\right)$$
$$= 3.48 \times 0.2 \times (7.41)$$
$$= 5.15\text{ V}$$

$$i_{o,peak} = \frac{v_{o,peak}}{R_L} = \frac{5.15}{8} = 643.75\text{ mA}$$

AC small signal model:



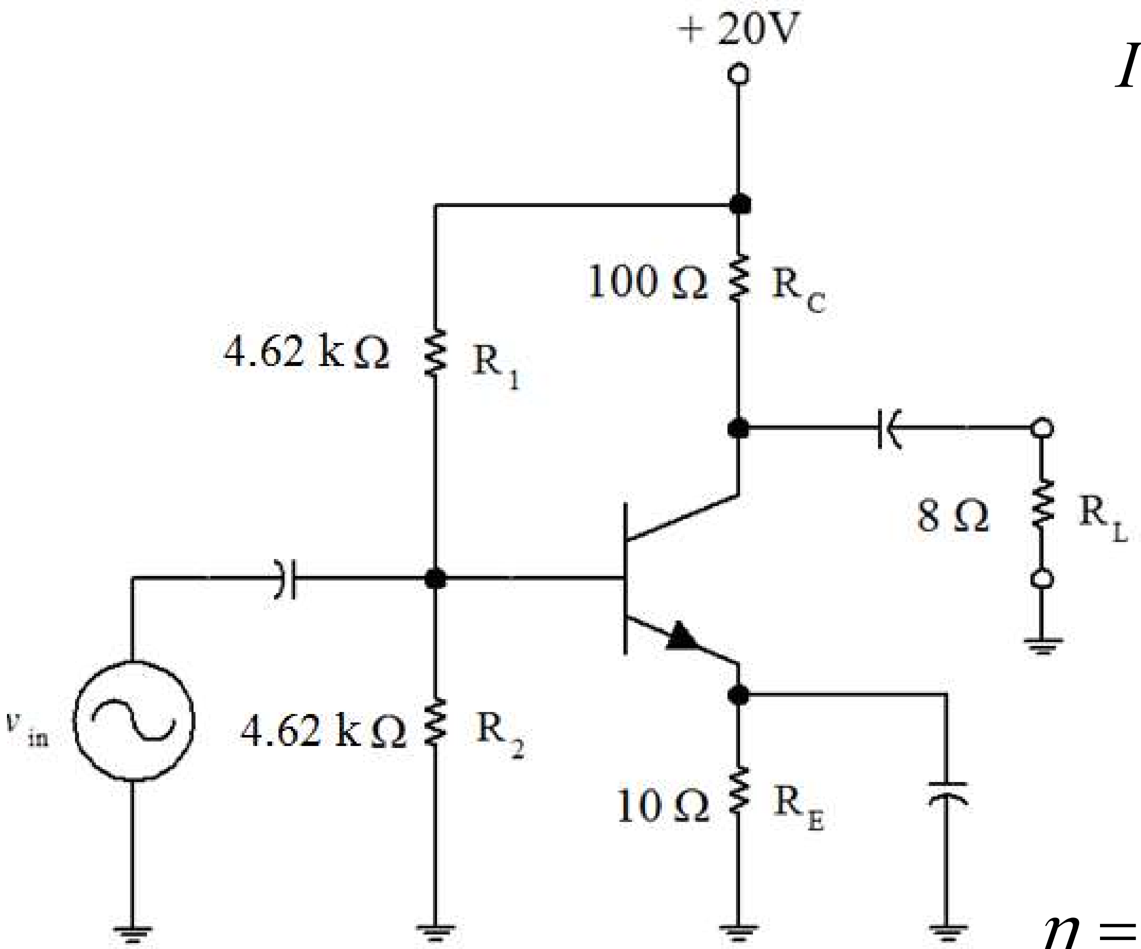
Maximum peak current = $I_{CQ} \approx 90$ mA , so it is impossible for the peak voltage to be 5.15 V!

Maximum peak current = $I_{CQ} \approx 90$ mA

$$v_{o,peak} = i_{o,peak}\left(R_L // R_C\right) = 90 \text{ mA} \times \left(7.41\right) = 0.67 \text{ V}$$

Maximum peak voltage is limited to 0.67V not 10 V!

$$\overline{P_L} = \frac{1}{2} i_{o,peak} v_{o,peak} = 0.5 \times 90\text{mA} \times 0.67\text{V} = 0.03 \text{ W}$$



$$I_{R1} = \frac{V_{CC} - V_{BE} - (\beta+1) I_{BQ} R_E}{R_1}$$

$$= \frac{20 - 0.7 - 26 \times 3.62m \times 10}{4.62k}$$

$$= 3.97 \text{ mA}$$

$$\overline{P_S} = V_{CC}\left(I_{CQ} + I_{R1}\right)$$

$$= 20 \times (90.56 + 3.97)m$$

$$= 7.19 \text{ W}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} \times 100\% = \frac{0.03}{7.91} \times 100 = 0.38\%$$

- The output peak voltage and current are load-dependent.
- It is good for voltage amplification but unable to deliver the load current.

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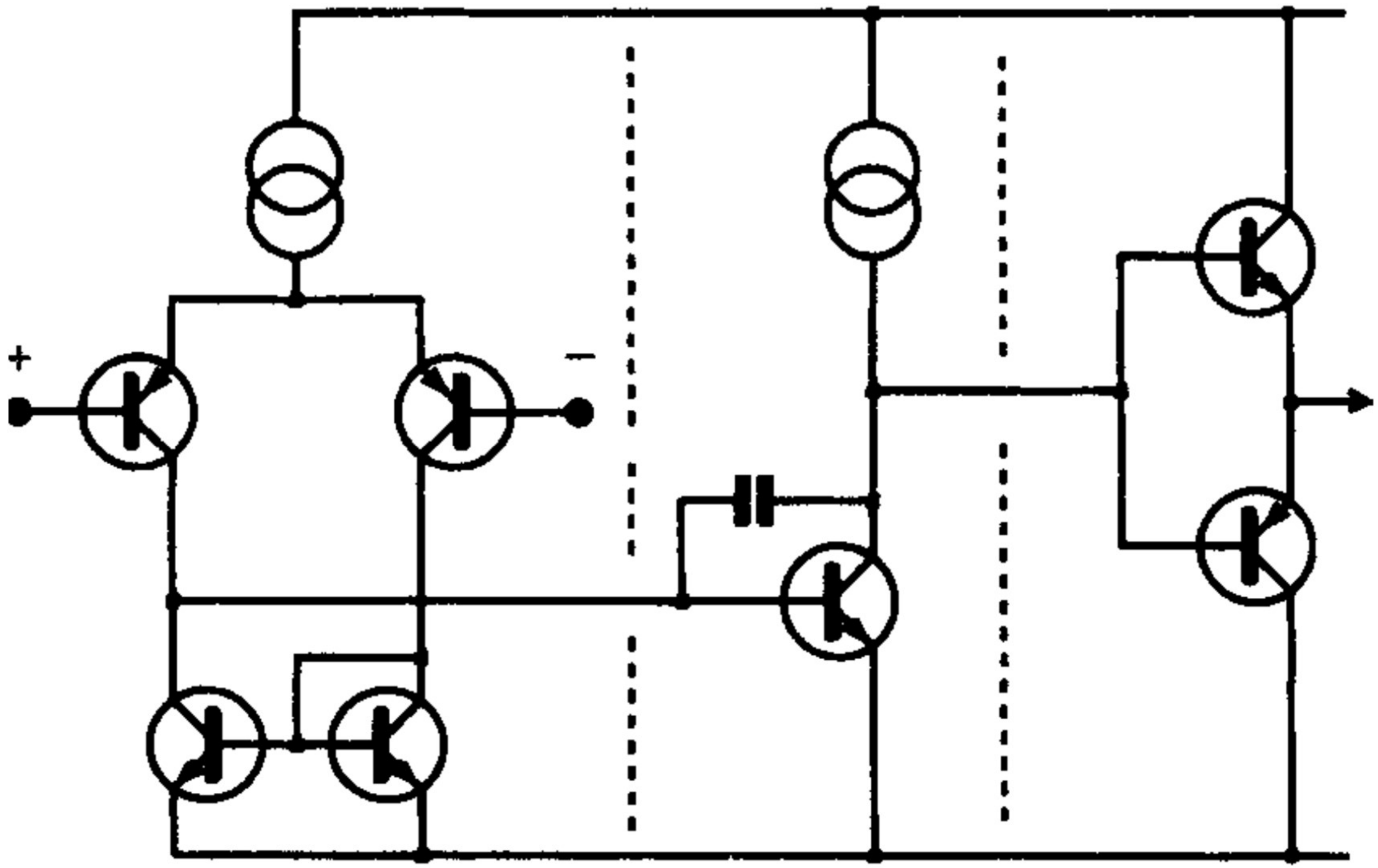
Power Amplifier Architecture

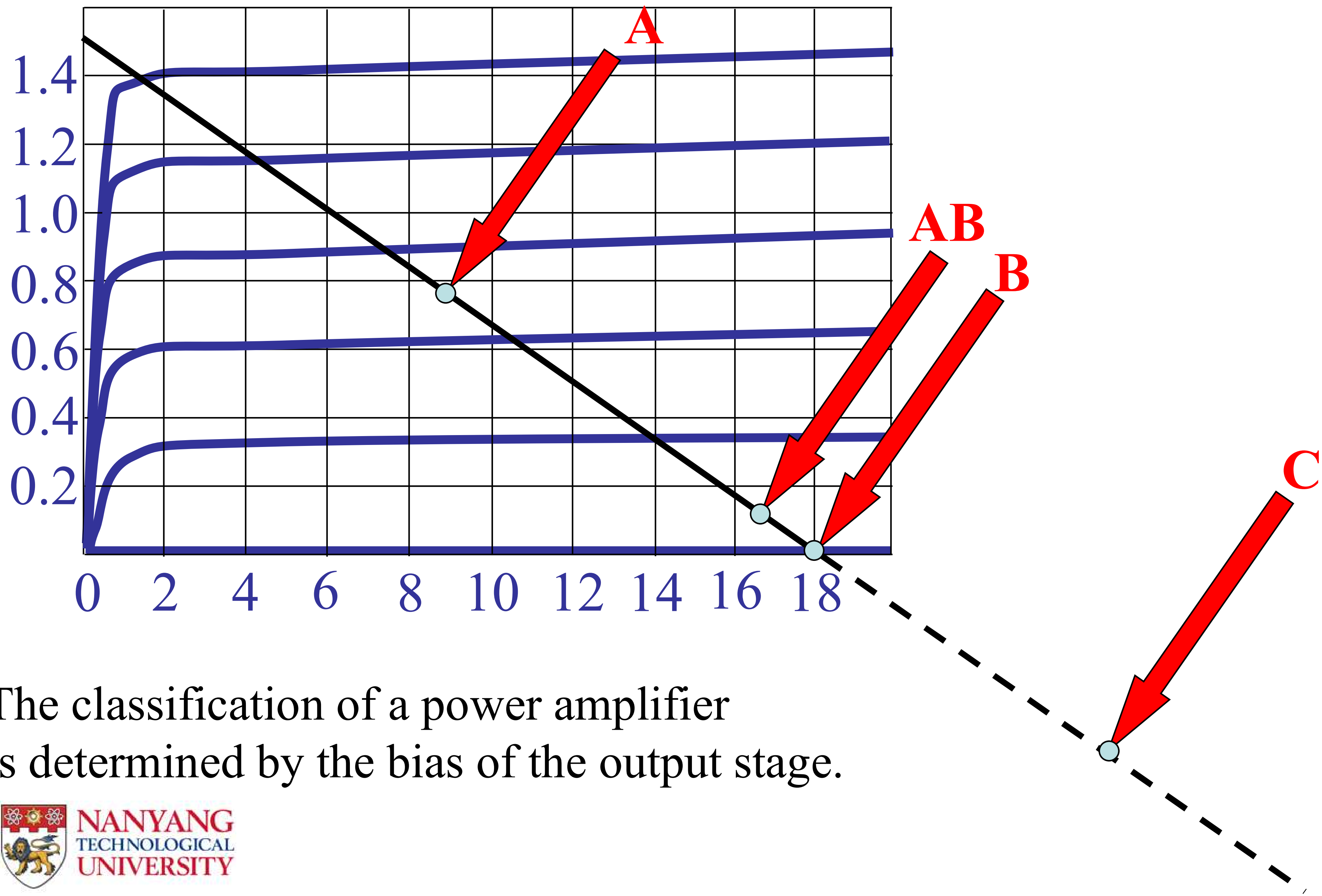


Different input to reject common-mode noise

Voltage amplifier to achieve maximum voltage swing

Output stage to deliver maximum current swing

Classification of Power Amplifiers



The classification of a power amplifier
is determined by the bias of the output stage.

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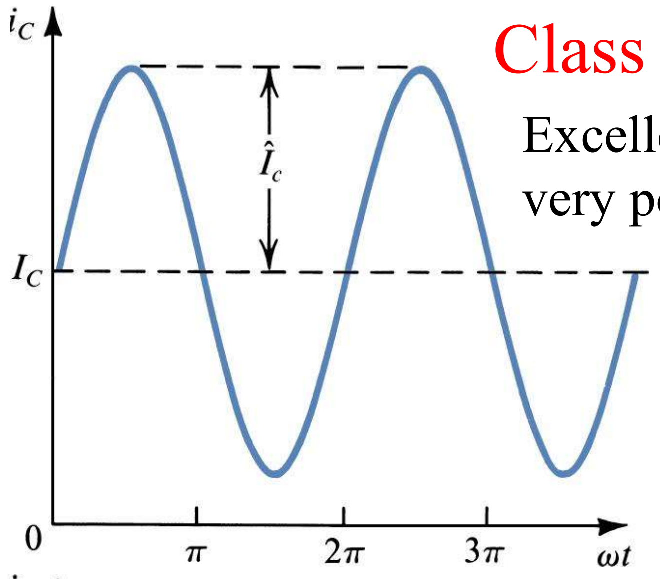
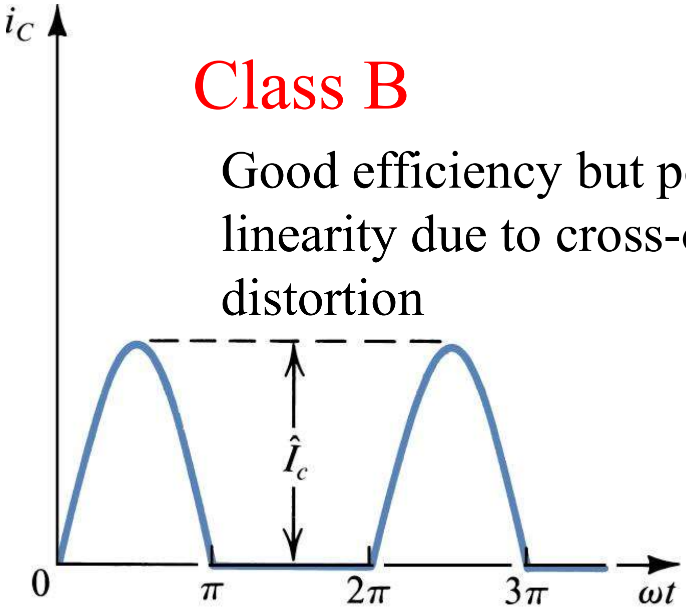
Classification of Power Amplifiers



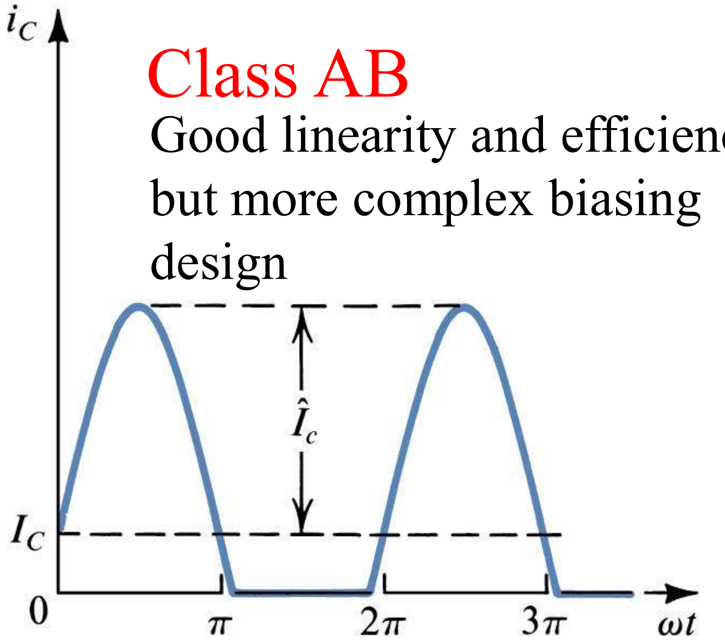
Class A

Excellent linearity but very poor efficiency



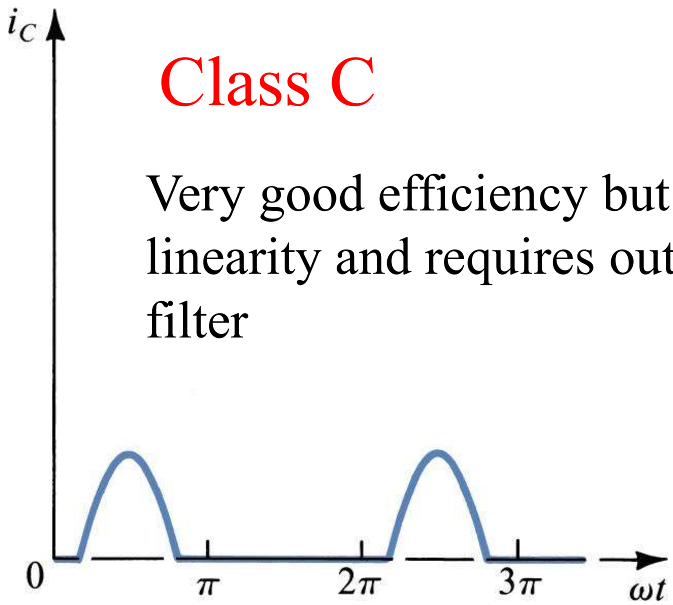
Class B

Good efficiency but poor linearity due to cross-over distortion



Class AB

Good linearity and efficiency but more complex biasing design

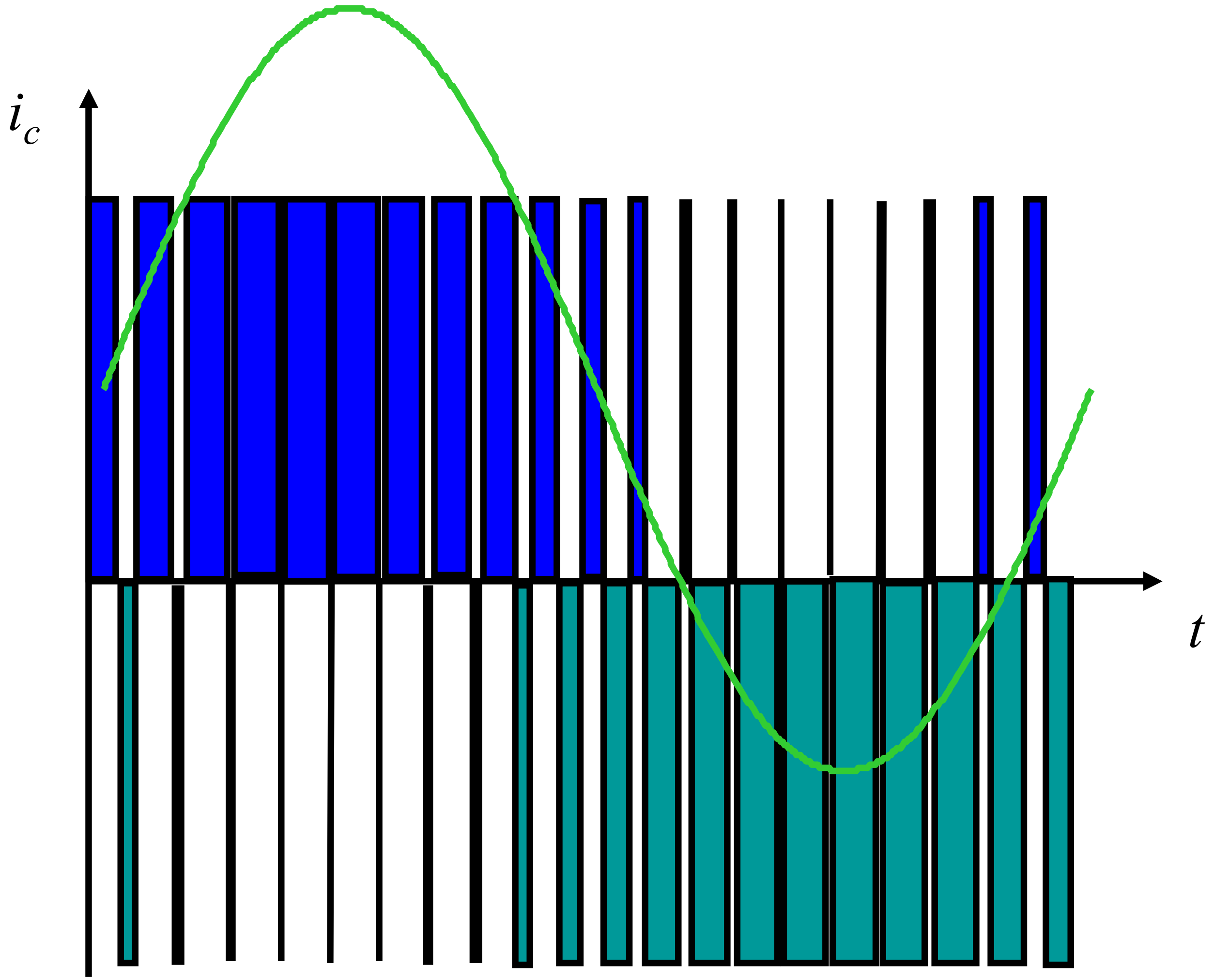


Class C

Very good efficiency but poor linearity and requires output filter

Power Amplifier using Digital Technique

There is another class of amplifier (Class D) which is based pulse-width modulation (PWM) technique.



- Superb efficiency
- More circuit blocks such as PWM, lower pass filter, etc.
- Generates significant electromagnetic interference (EMI) due to fast switching

Class A Emitter Follower

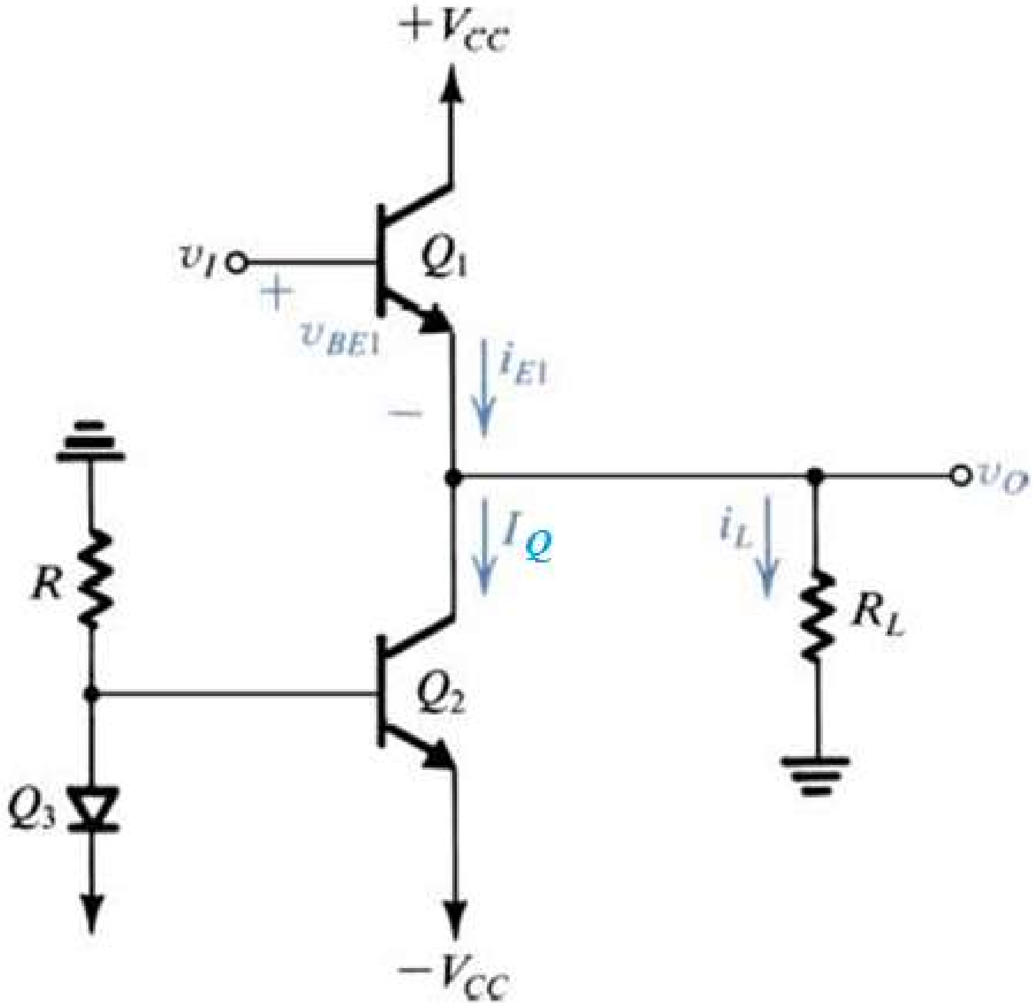
$$v_o = v_i - v_{be1}$$

$$v_{BE1} = \frac{kT}{q} ln\left(\frac{i_{c1}}{I_S}\right)$$

Q_2 is in active region and provide the biasing current I_Q

$$i_{C1} \approx i_{E1} = I_Q + i_L = I_Q + \frac{v_o}{R_L}$$

$$v_o = v_i - \frac{kT}{q} ln\left(\frac{I_Q + \frac{v_o}{R_L}}{I_S}\right)$$



$$v_o = v_i - \frac{kT}{q} ln\left(\frac{I_Q + \frac{v_o}{R_L}}{I_S}\right)$$

If $v_o > 0$:

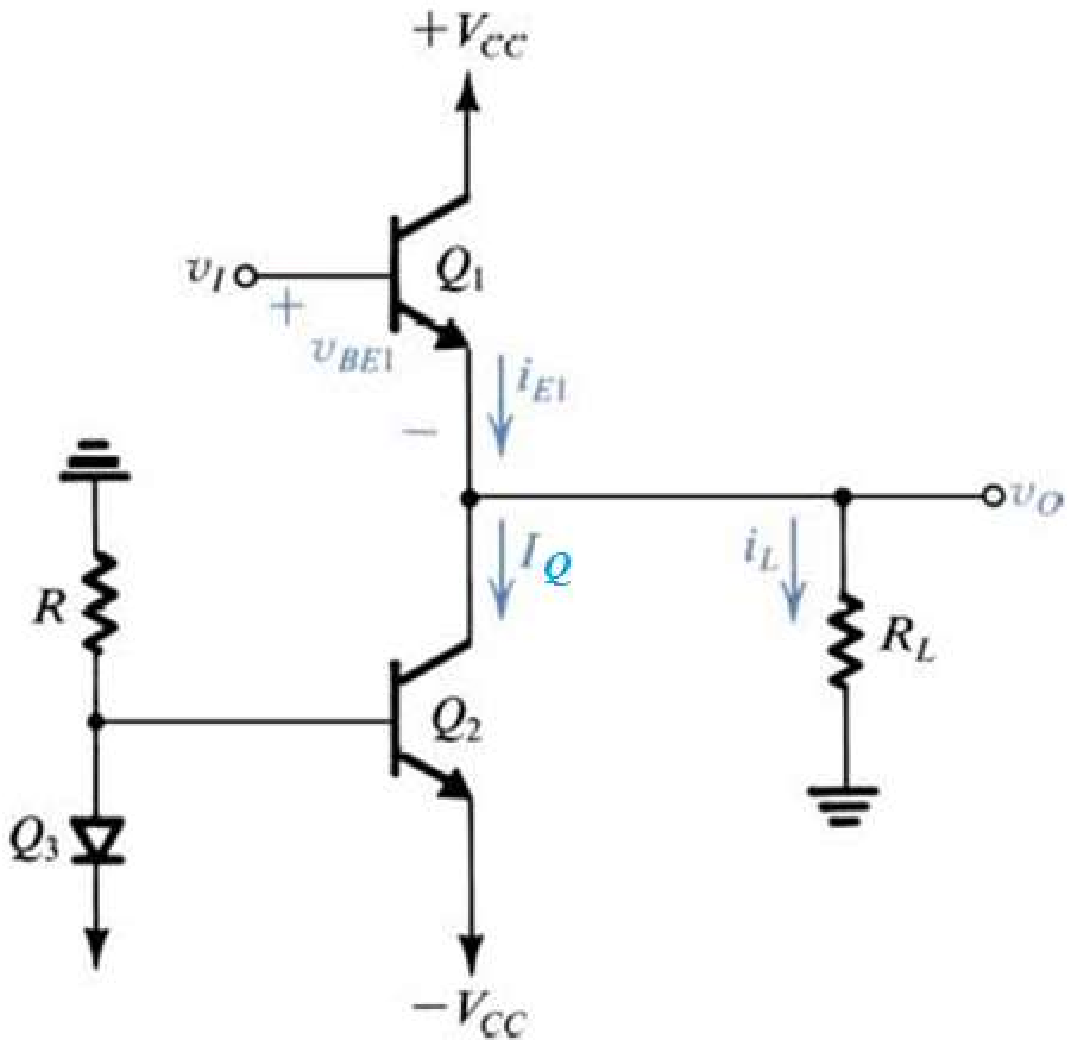
$$i_{C1} = I_Q + \frac{v_o}{R_L} > 0$$

Then v_o and v_i are related by:

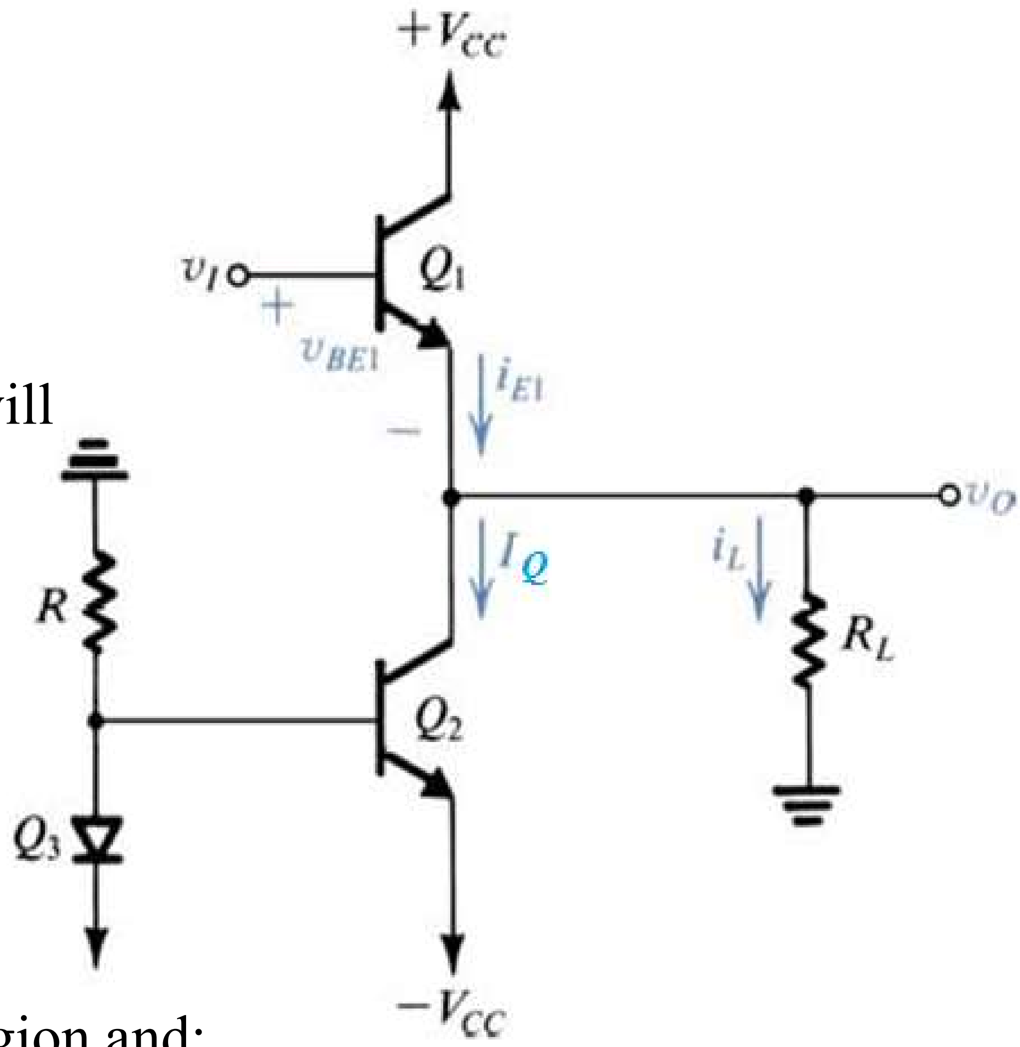
$$v_o = v_i - v_{BE1}$$

The maximum output voltage is limited by:

$$v_o = V_{CC} - V_{CE1,sat}$$



$$v_o = v_i - \frac{kT}{q} ln\left(\frac{I_Q + \frac{v_o}{R_L}}{I_S}\right)$$



When $v_o < 0$, i_L will be –ve, the output will depend of the value of R_L.

If R_L is small, Q_1 will be in cut-off if:

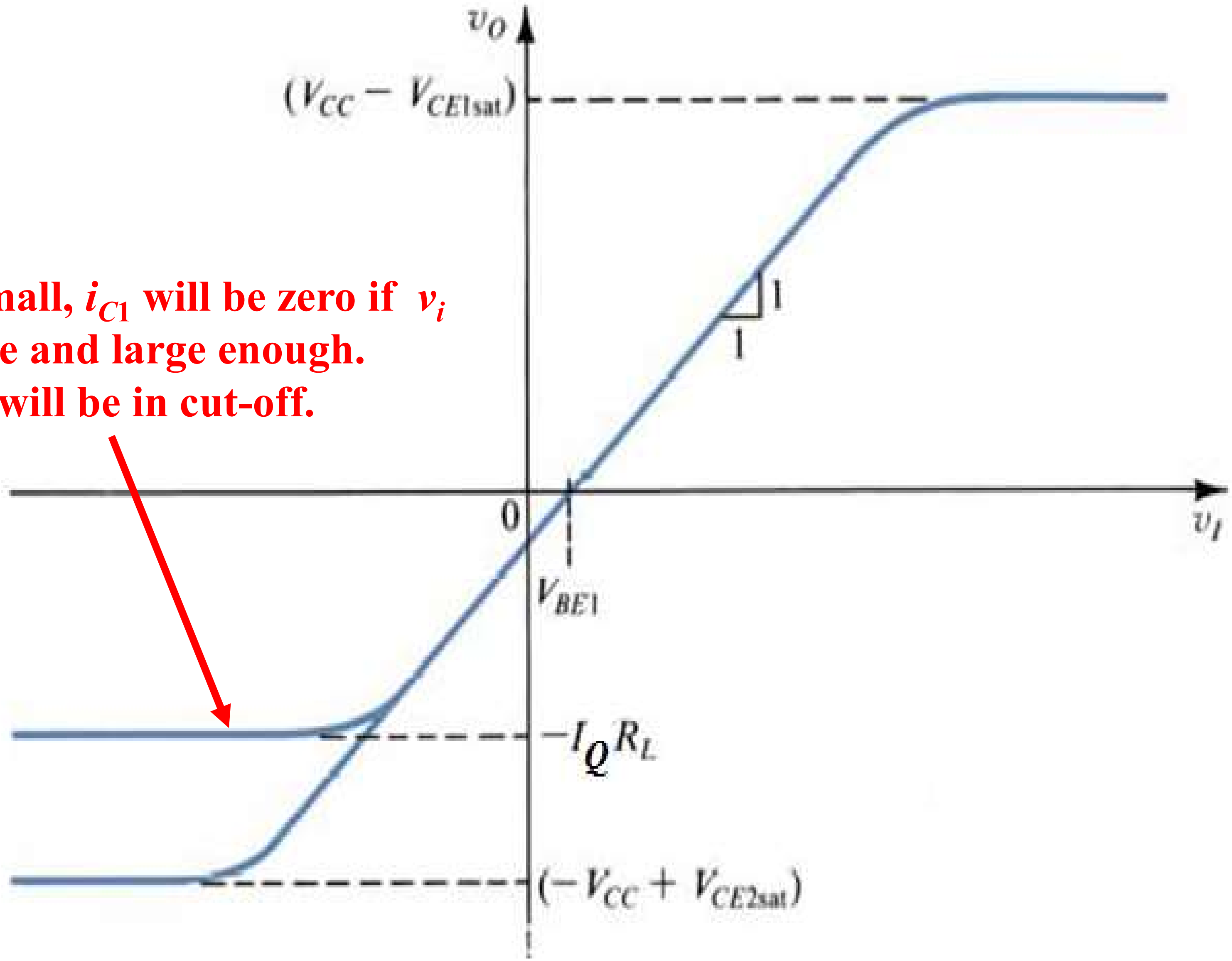
$$i_{C1} = I_Q + \frac{v_o}{R_L} = 0, \therefore I_Q = -\frac{v_o}{R_L}$$

$$v_o = -I_Q R_L$$

If R_L is large, Q_1 will remain in active region and:

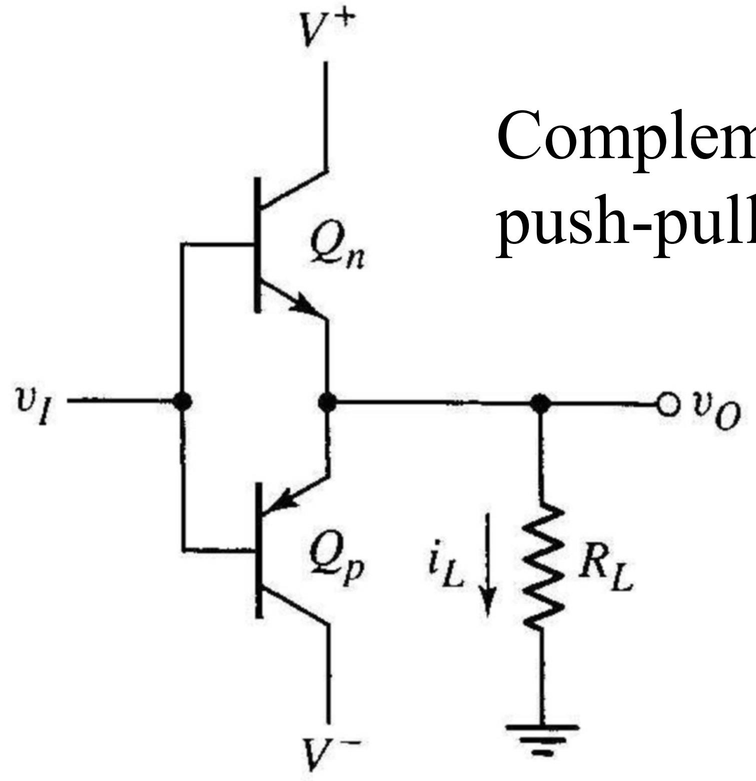
$$v_o = -V_{CC} + V_{CE2,sat}$$

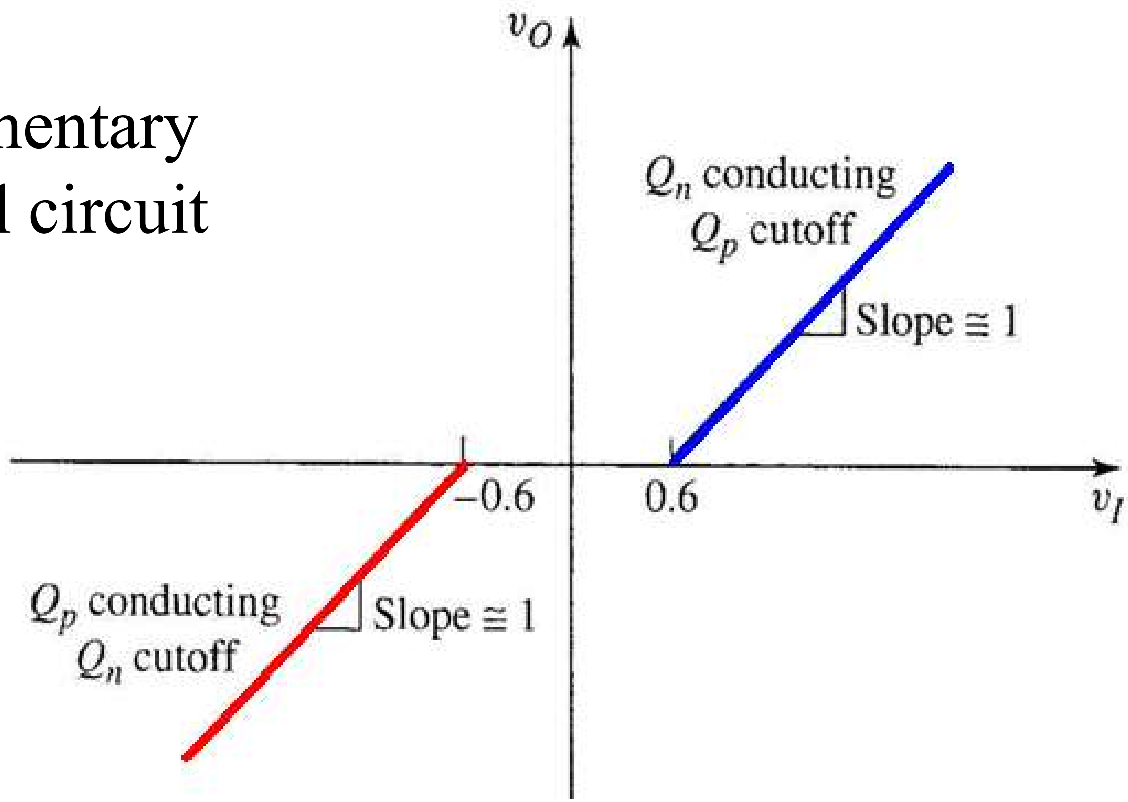
Transfer Characteristic



If R_L is small, i_{C1} will be zero if v_i is negative and large enough. Then Q1 will be in cut-off.

Class B Amplifiers

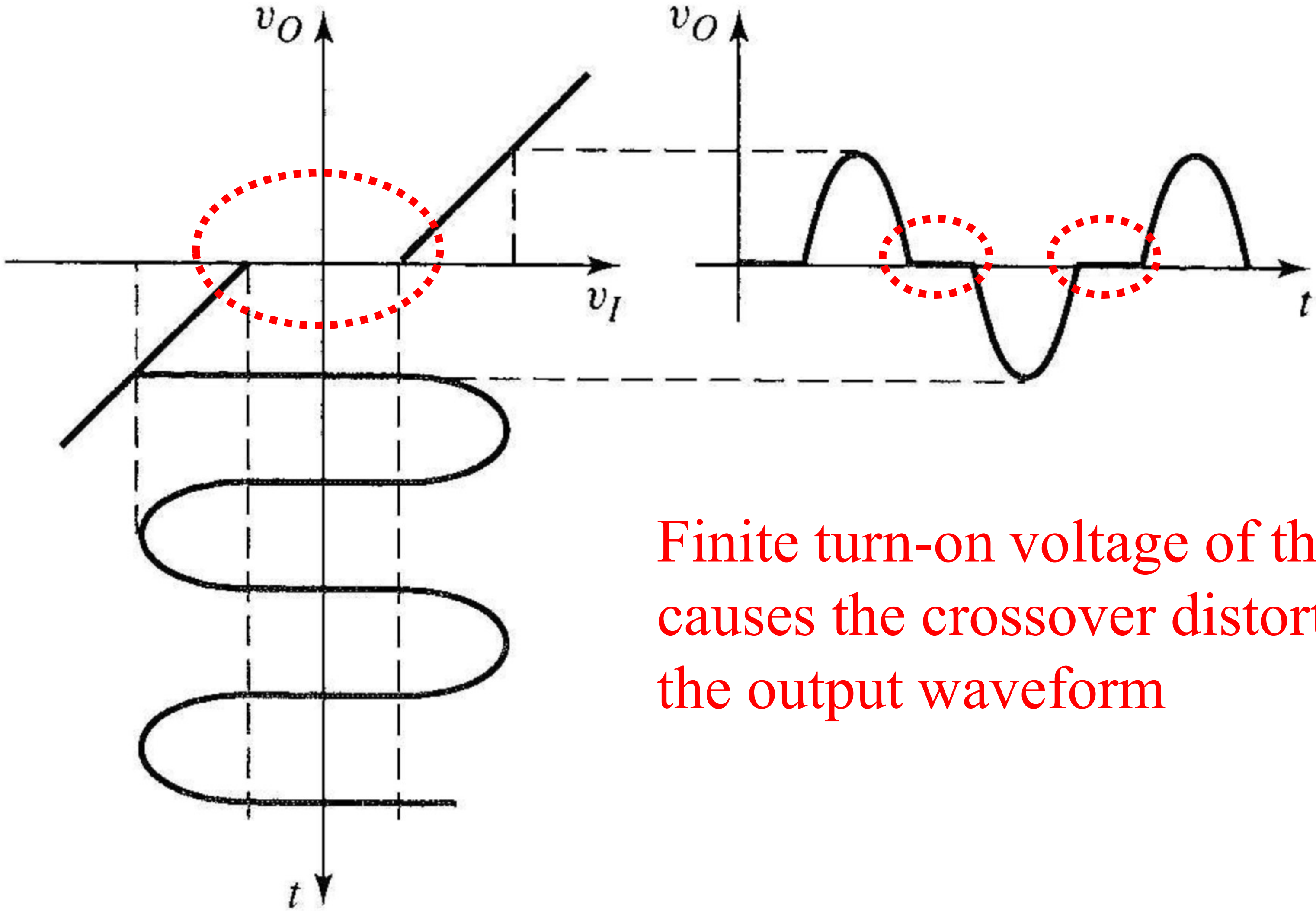




When V_i >0 and greater than 0.6 V, Q_n turns on and operates as an emitter follower.

When V_i <0 and less than −0.6 V, Q_p turns on and operates as an emitter follower.

Crossover Distortion



Finite turn-on voltage of the BJT causes the crossover distortion of the output waveform

Exercise #2: A 1 kHz sinusoidal signal with an amplitude of 2V was applied to the Class B amplifier. The supply voltage = ±10V. Due to crossover distortion, the output waveform is not a pure sinusoidal waveform. The harmonics measured as the output in frequency domain are recorded as follows. Please compute the total harmonic distortion (THD).

Frequency (Hz)	Fourier component	Normalized component
1.000E+03	1.151E+00	1.000E+00
2.000E+03	6.313E-03	5.485E-03
3.000E+03	2.103E-01	1.827E-01
4.000E+03	4.984E-03	4.331E-03
5.000E+03	8.064E-02	7.006E-02
6.000E+03	3.456E-03	3.003E-03
7.000E+03	2.835E-02	2.464E-02
8.000E+03	2.019E-03	1.754E-03
9.000E+03	6.679E-03	5.803E-03

Frequency (Hz)	Fourier component	Normalized component
1.000E+03	1.151E+00	1.000E+00
2.000E+03	6.313E-03	5.485E-03
3.000E+03	2.103E-01	1.827E-01
4.000E+03	4.984E-03	4.331E-03
5.000E+03	8.064E-02	7.006E-02
6.000E+03	3.456E-03	3.003E-03
7.000E+03	2.835E-02	2.464E-02
8.000E+03	2.019E-03	1.754E-03
9.000E+03	6.679E-03	5.803E-03

$$THD = \frac{\sqrt{V_2^{\ 2} + V_3^{\ 2} + V_4^{\ 2} + \dots + V_n^{\ 2}}}{V_1} \times 100$$

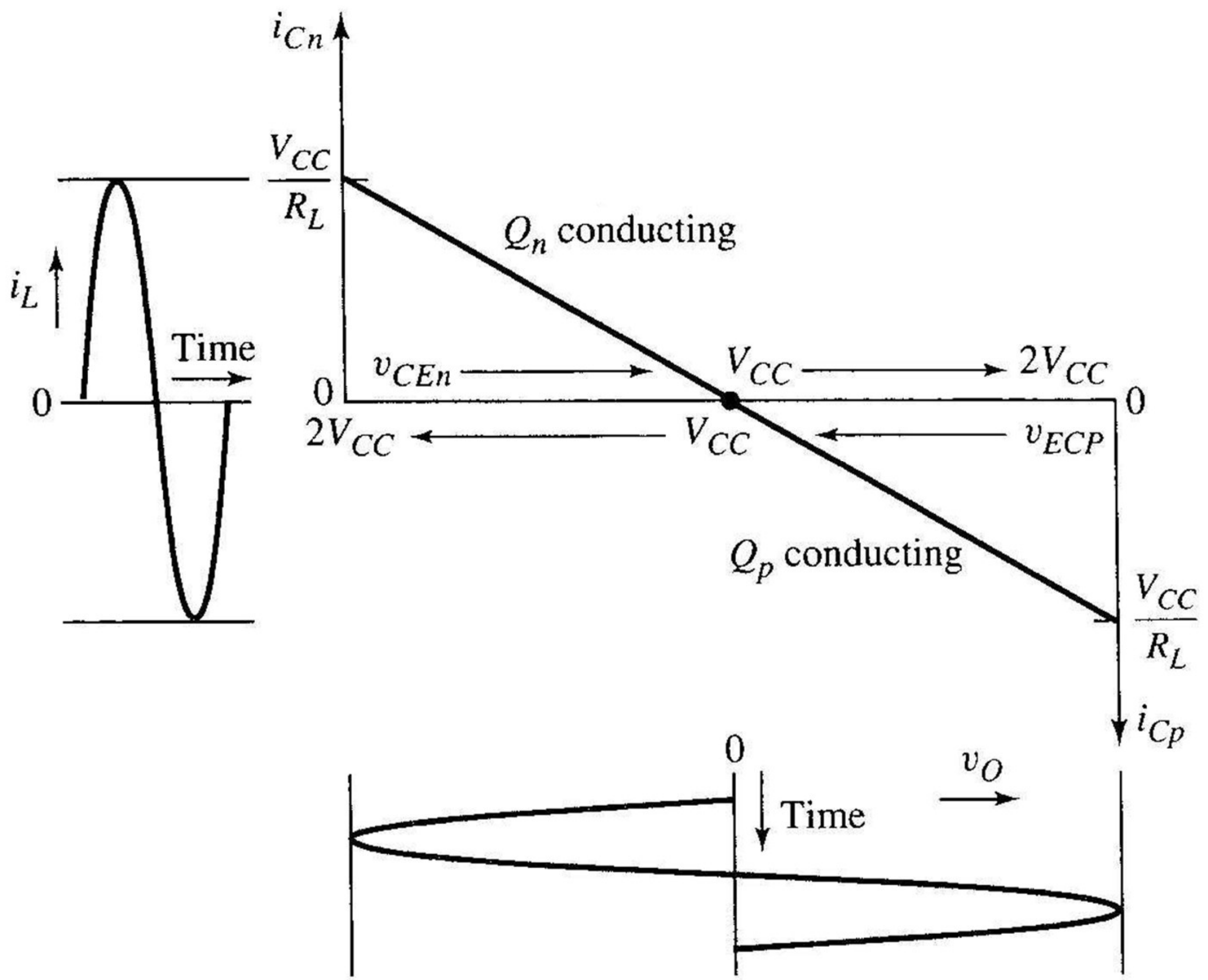
$$= \frac{\sqrt{0.006313^2 + 0.2103^2 + 0.004984^2 + 0.08064^2 + \cdots}}{1.151} \times 100$$

$$= \frac{0.2273}{1.151} \times 100$$

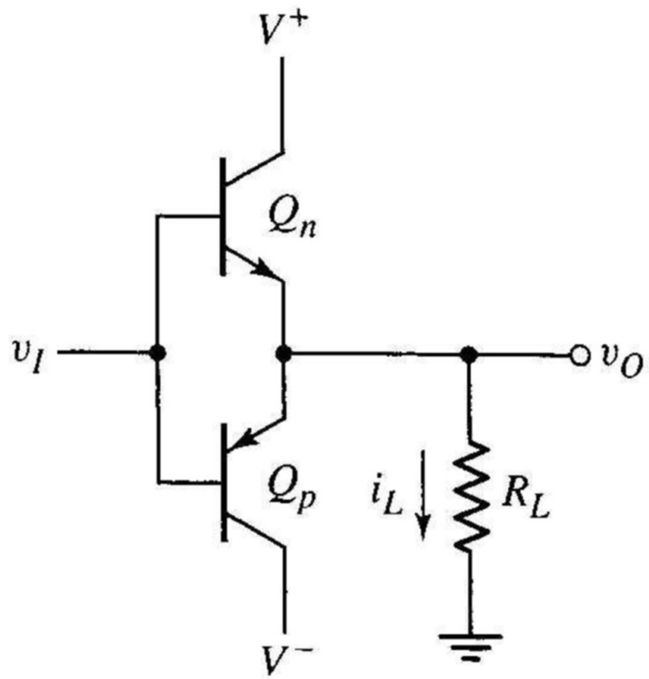
$$= 19.7\%$$

THD is a gauge of how good a power amplifier amplifies a signal without distortion. THD > 5% is usually not acceptable for audio applications.

Class B Amplifier Efficiency



For ease of analysis, assume the transistors are ideal with zero turn-on voltage.



$$v_o = V_p \sin \omega t$$

Average power to the load: $\overline{P_L} = \frac{1}{2} V_p I_p = \frac{1}{2} V_p \left(\frac{V_p}{R_L} \right) = \frac{V_p^{\ 2}}{2R_L}$

Average power supplied by each voltage source:

$$\overline{P}_{S+} = \overline{P}_{S-} = V_{CC} \overline{I}_S = V_{CC} \left(\frac{V_p}{\pi R_L} \right)$$

Total average power supplied by two voltage sources:

$$\overline{P}_S = 2V_{CC} \left(\frac{V_p}{\pi R_L} \right)$$

Conversion efficiency: $\eta = \frac{\overline{P}_L}{\overline{P}_S} = \frac{\frac{1}{2}\left(\frac{V_p^{\ 2}}{R_L} \right)}{2V_{CC}\left(\frac{V_p}{\pi R_L} \right)} = \frac{\pi}{4} \left(\frac{V_p}{V_{CC}} \right)$

Maximum conversion efficiency occurs when $V_p = V_{CC}$:

$$\eta(max) = \frac{\pi}{4} = 78.5\%$$ It is much higher than Class A amplifier.

In reality, to avoid distortion of output signal, the actual voltage swing is smaller and therefore the conversion efficiency is less than 78.5%.

$$v_{CEn} = V_{CC} - v_o = V_{CC} - V_p \sin \omega t$$

$$i_{Cn} = \frac{V_p}{R_L} \sin \omega t \ \text{ for } 0 \le \omega t \le \pi, \ i_{Cn} = 0 \ \text{ for } \pi \le \omega t \le 2\pi$$

$$P_n = v_{CEn} i_{Cn} = \left(V_{CC} - V_p \sin \omega t\right)\left(\frac{V_p}{R_L} \sin \omega t\right) \ \text{ for } 0 \le \omega t \le \pi$$

$$P_n = 0 \ \text{ for } \pi \le \omega t \le 2\pi$$

$$P_n = \frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^{\,2}}{R_L}\sin^2\omega t = \frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^{\,2}}{R_L}\left(\frac{1-\cos 2\omega t}{2}\right)$$

$$= \frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^{\,2}}{2R_L} + \frac{V_p^{\,2}\cos 2\omega t}{2R_L}$$

$$\overline{P_n} = \frac{1}{2\pi}\int_0^{\pi}\left(\frac{V_{CC}V_p}{R_L}\sin\omega t - \frac{V_p^{\,2}}{2R_L} + \frac{V_p^{\,2}\cos 2\omega t}{2R_L}\right)d\omega t$$

$$= \frac{1}{2\pi}\left\{\frac{V_{CC}V_p}{R_L}\left[-\cos\omega t\right]_0^{\pi} - \frac{V_p^{\,2}}{2R_L}\left[\omega t\right]_0^{\pi} + \frac{V_p^{\,2}}{2R_L}\left[\frac{1}{2}\sin 2\omega t\right]_0^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{2V_{CC}V_p}{R_L} - \frac{\pi V_p^{\,2}}{2R_L} + 0\right\} = \frac{V_{CC}V_p}{\pi R_L} - \frac{V_p^{\,2}}{4R_L}$$

Note: The average power dissipated in transistor Q_p is the same.

To determine the maximum average power dissipated in transistor Q_n:

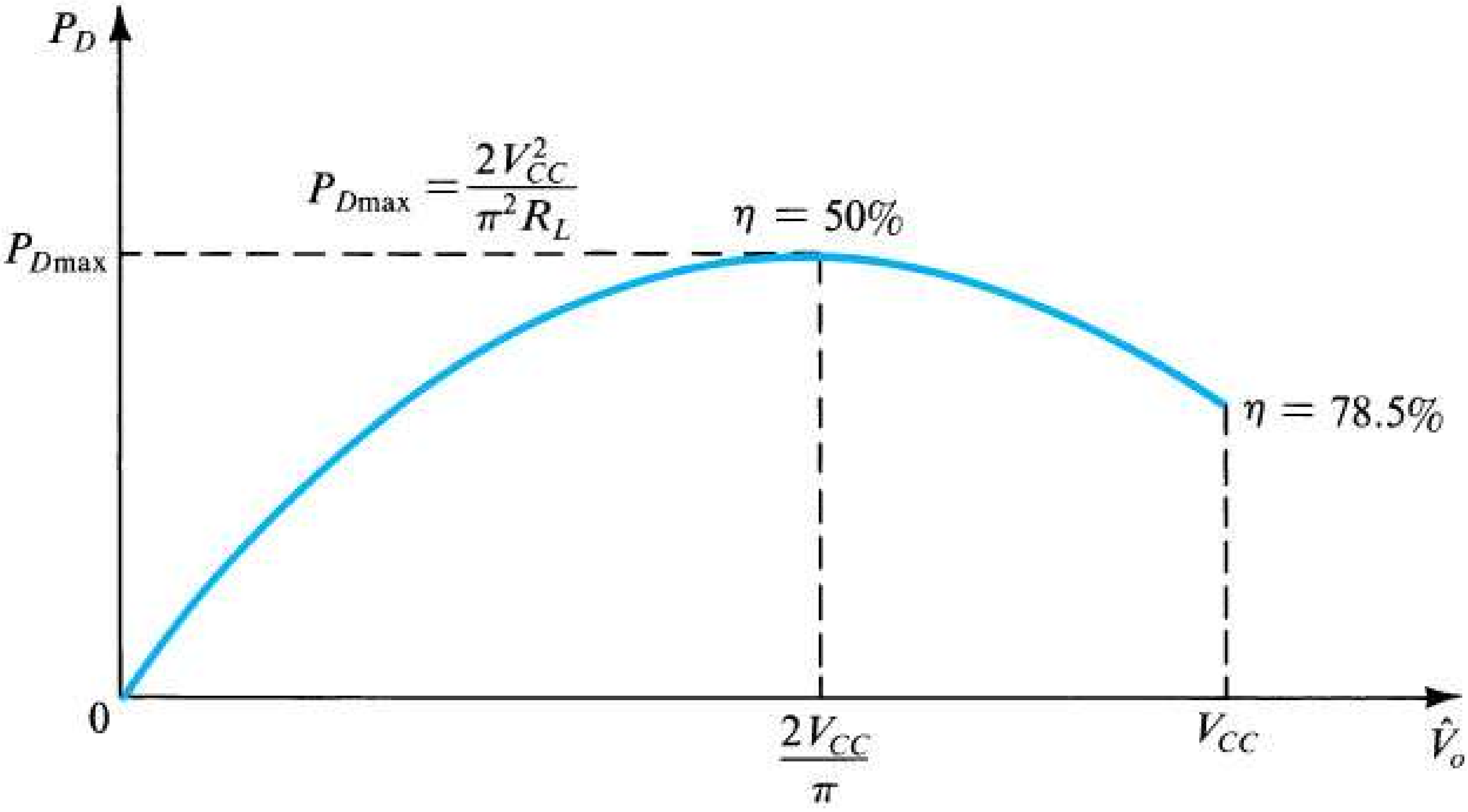
$$\frac{d\overline{P_n}}{dV_p} = \frac{V_{CC}}{\pi R_L} - \frac{V_p}{2R_L} = 0 \Rightarrow V_p = \frac{2V_{CC}}{\pi}$$

$$\overline{P_n}(max) = \frac{V_{CC}}{\pi R_L}\left(\frac{2V_{CC}}{\pi}\right) - \frac{1}{4R_L}\left(\frac{2V_{CC}}{\pi}\right)^2 = \frac{{V_{CC}}^2}{\pi^2 R_L}$$

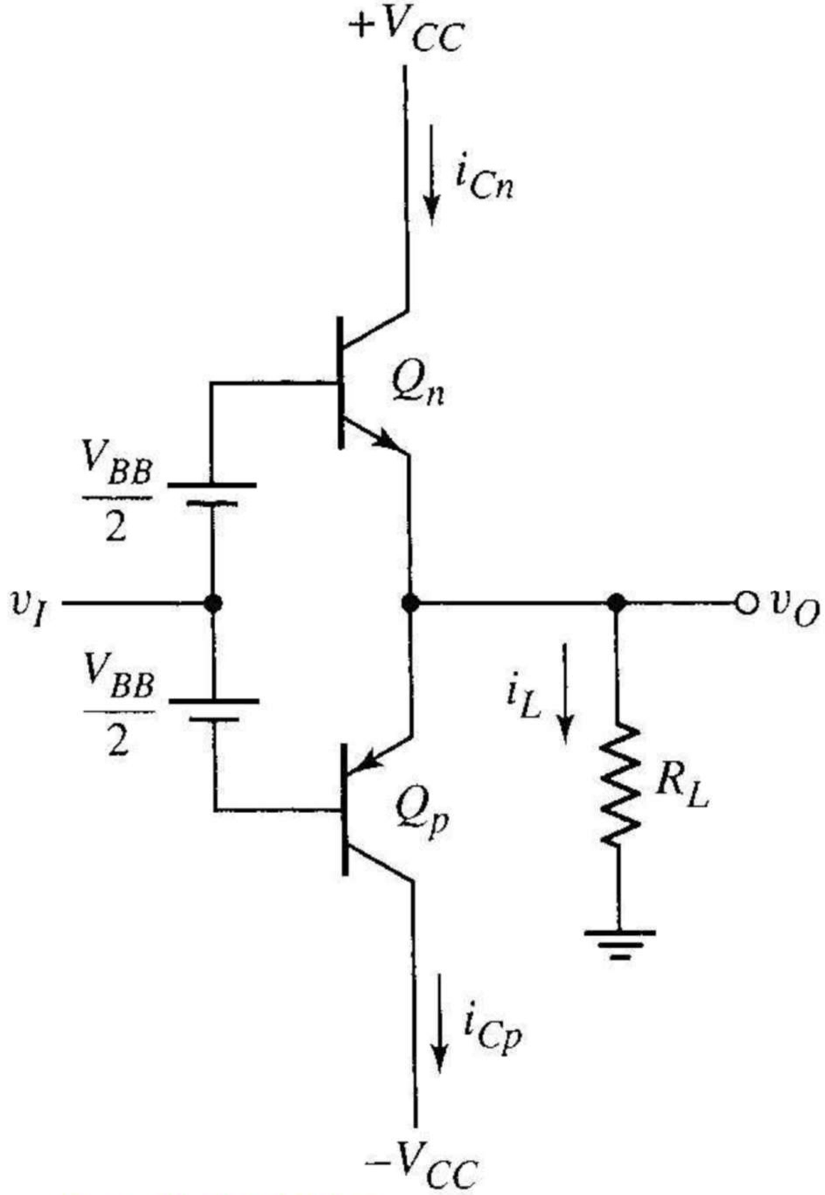
Conversion effciecny when $V_p = \dfrac{2V_{CC}}{\pi}$:

$$\eta = \frac{\pi}{4}\left(\frac{2V_{CC}}{\pi V_{CC}}\right) = \frac{1}{2} = 50\%$$

Class B Amplifier Conversion Efficiency



Class AB Amplifiers



Crossover distortion can be eliminated by applying a small bias on each transistor so that the transistor is already turn-on without an input signal.

When $v_i = 0$, $v_o = 0$, both Q_n and Q_p have the same biasing currents:

$$v_{BEn} = v_{EBp} = \frac{V_{BB}}{2}$$

$$i_{Cn} = i_{Cp} = I_{CQ}$$

$$I_{CQ} = I_S e^{\frac{V_{BB}}{2V_T}}$$

Class AB Amplifiers

When v_i is +ve, the output voltage increases and the collector current of Q_n also increases to supply the load current:

$$v_o = v_i + \frac{V_{BB}}{2} - v_{BEn} \qquad i_{Cn} = i_L + i_{Cp}$$

When v_i is –ve, Q_p turns on and sinking the current from the load and the output voltage goes –ve.

The collector currents of both transistors are governed by:

$$v_{BEn} + v_{EBp} = V_{BB}$$

$$V_T \ln\left(\frac{i_{Cn}}{I_S}\right) + V_T \ln\left(\frac{i_{Cp}}{I_S}\right) = 2V_T \ln\left(\frac{I_{CQ}}{I_S}\right)$$

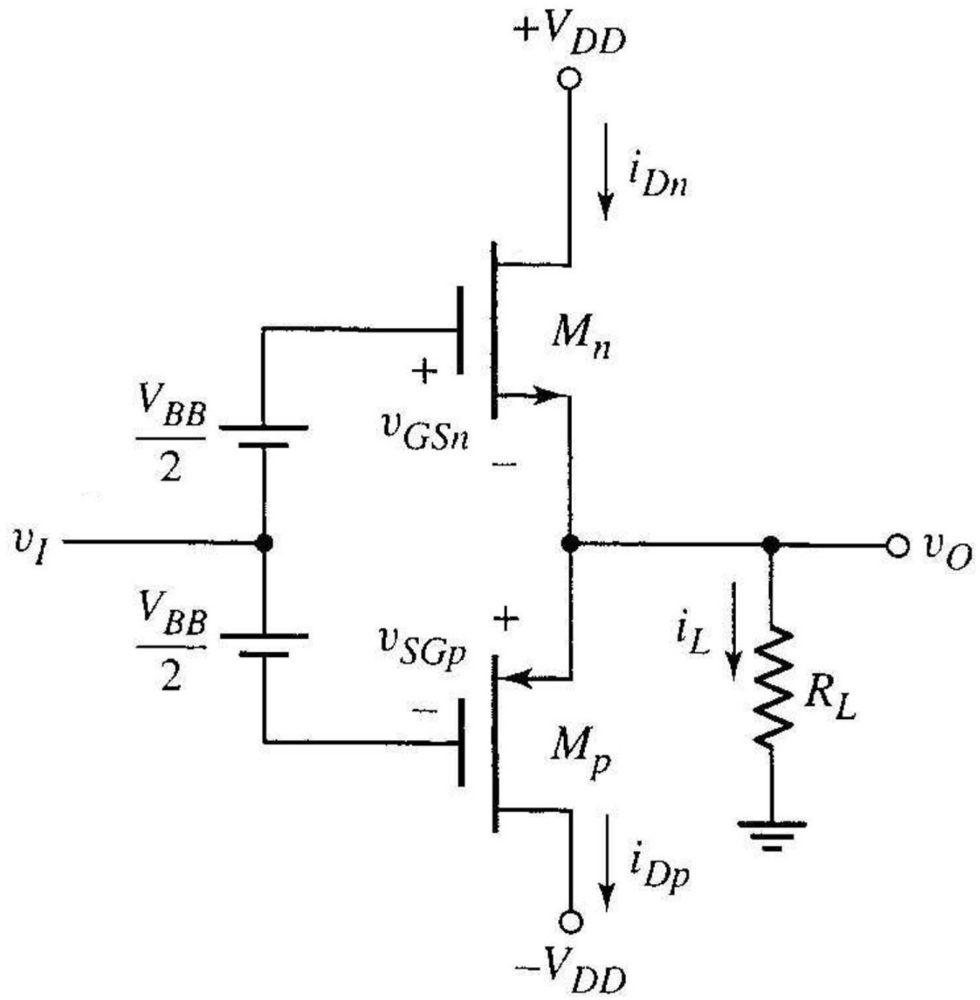
$$i_{Cn} i_{Cp} = I_{CQ}{}^2$$

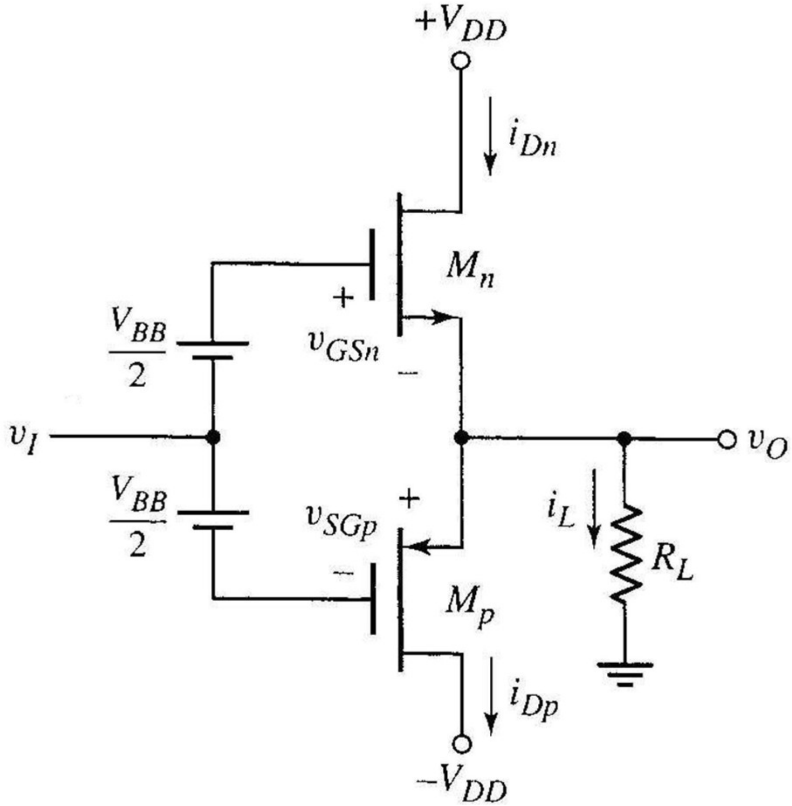
$$i_{Cn} i_{Cp} = I_{CQ}{}^2$$

$$i_{Cn}\left(i_{Cn} - i_L\right) = I_{CQ}{}^2$$

$$i_{Cn}{}^2 - i_L i_{Cn} = I_{CQ}{}^2$$

Exercise #3: Determine the required biasing for the MOSFET Class AB amplifier so that I_{DQ} = 20% of the load current when v_o = 5V. Power supply voltage = ±10V and R_L = 20 Ω. Assume that both transistors are matched with $K_n = K_p$ = 0.2 A/V^2 and $|V_{TN}| = |V_{TP}|$ = 1 V.





$$\text{When } v_o = 5\text{ V}, i_L = \frac{v_o}{R_L} = \frac{5}{20} = 250\ \text{mA}$$

$$I_{DQ} = 0.2 \times 250\,\text{mA} = 50\,\text{mA}$$

$$\text{When } v_o = 0, i_L = 0, i_{Dn} = i_{Dp} = I_{DQ} = 50\,\text{mA}$$

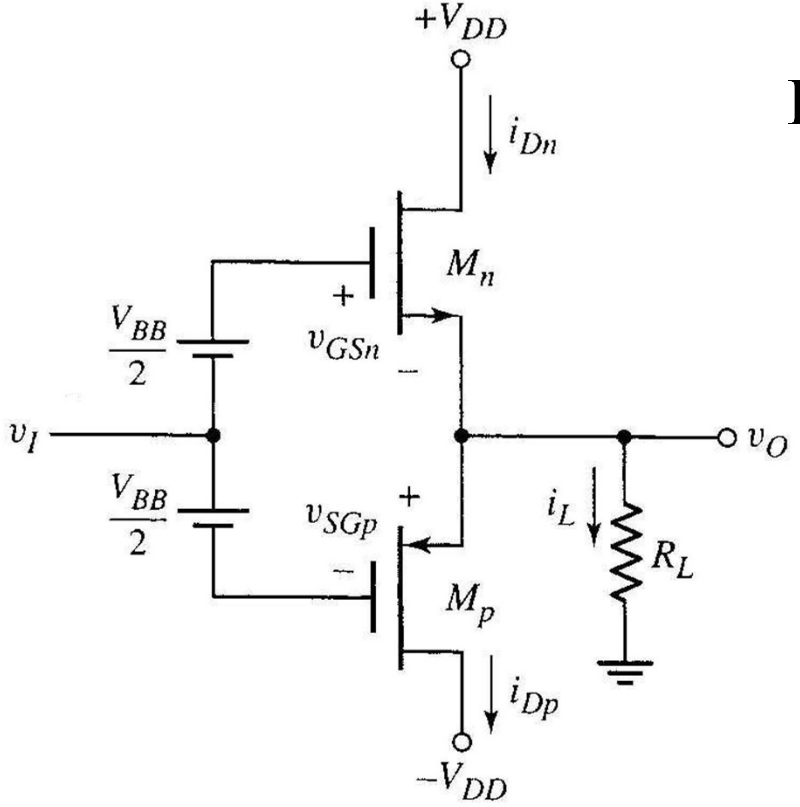
$$\therefore v_{GSn} = v_{SGp} = \frac{V_{BB}}{2}$$

$$i_{Dn} = i_{Dp} = K\left(\frac{V_{BB}}{2} - |V_T|\right)^2 = 50\,\text{mA}$$

$$(0.2)\left(\frac{V_{BB}}{2} - 1\right)^2 = 0.05\text{A} \Rightarrow \frac{V_{BB}}{2} = 1.5\text{ V}$$

$$v_{GSn} = v_{SGp} = \frac{V_{BB}}{2} = 1.5\ \text{V}$$

$$v_i = v_o + v_{GSn} - \frac{V_{BB}}{2} = 0 + 1.5 - 1.5 = 0\ \text{V}$$



For $v_o = 5\text{V}, \; i_{Dp} \approx 0, \; i_{Dn} \approx i_L = 250 \text{ mA}$

$$i_{Dn} = K\left(v_{GSn} - |V_T|\right)^2 = 0.25 \text{ A}$$

$$v_{GSn} = \sqrt{\frac{i_{Dn}}{K}} + |V_T| = \sqrt{\frac{0.25}{0.2}} + 1 = 2.12 \text{ V}$$

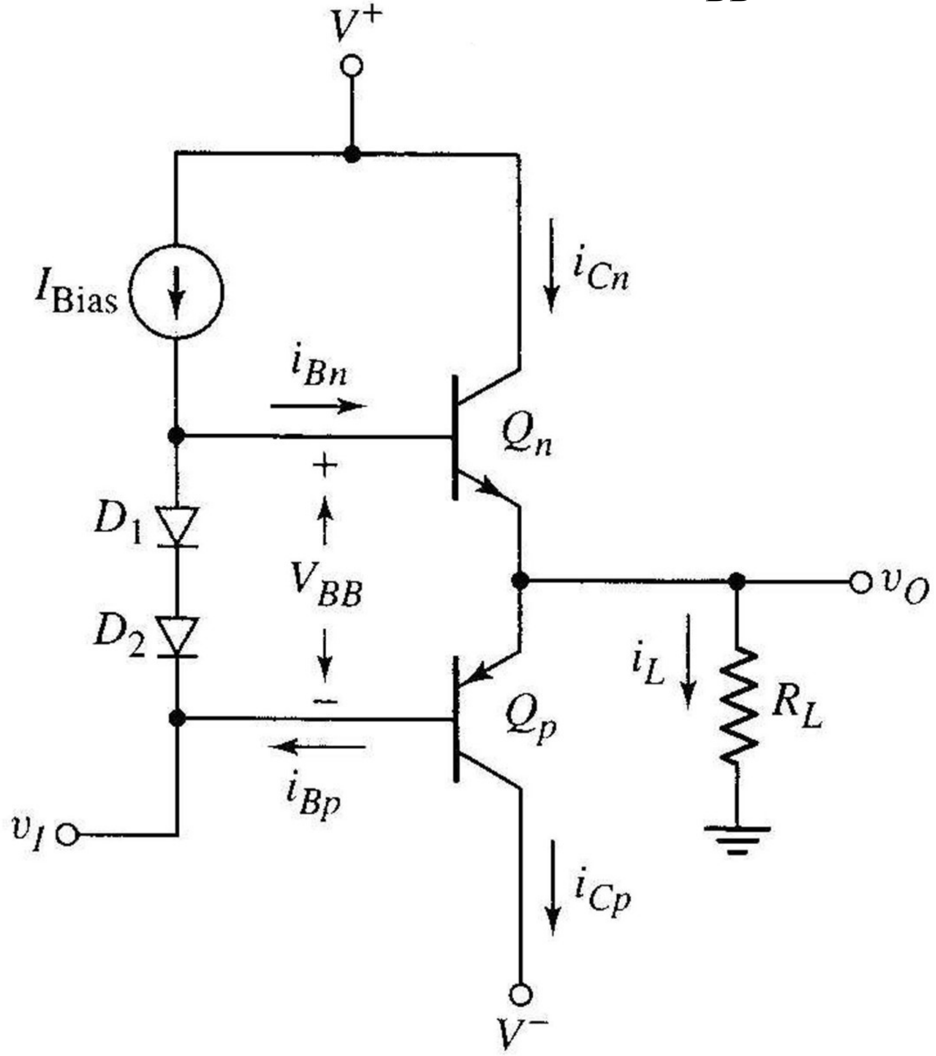
$$v_{SGp} = V_{BB} - v_{GSn} = 3 - 2.12 = 0.88 \text{ V}$$

$\because v_{SGp} = 0.88 \text{ V} < V_T = 1\text{V}$

It confirms that M_p is in cut off, $i_{Dp} = 0$ and $i_{Dn} = i_L$.

$$v_i = v_o + v_{GSn} - \frac{V_{BB}}{2} = 5 + 2.12 - 1.5 = 5.62 \text{ V}$$

Exercise #4: Design the following BJT Class AB amplifier so that the average power to the load is 5 W and R_L = 8 Ω. The peak output voltage is not more than 80% of V_{CC} to ensure good linearity. The diode current cannot be less than 5 mA to maintain a nearly constant V_{BB}.



$I_{SD} = 3\times10^{-14}$ A for D_1 and D_2

$I_{SQ} = 10^{-13}$ A for Q_n and Q_p

$\beta_n = \beta_p = 75$

$$\overline{P}_L = \frac{1}{2}\frac{V_p{}^2}{R_L} \Rightarrow V_p = \sqrt{2R_L\overline{P}_L} = \sqrt{2(8)(5)} = 8.94\text{V}$$

$$V_{CC} = \frac{V_p}{0.8} = \frac{8.94}{0.8} = 11.2\text{ V}$$

At the peak of output voltage,

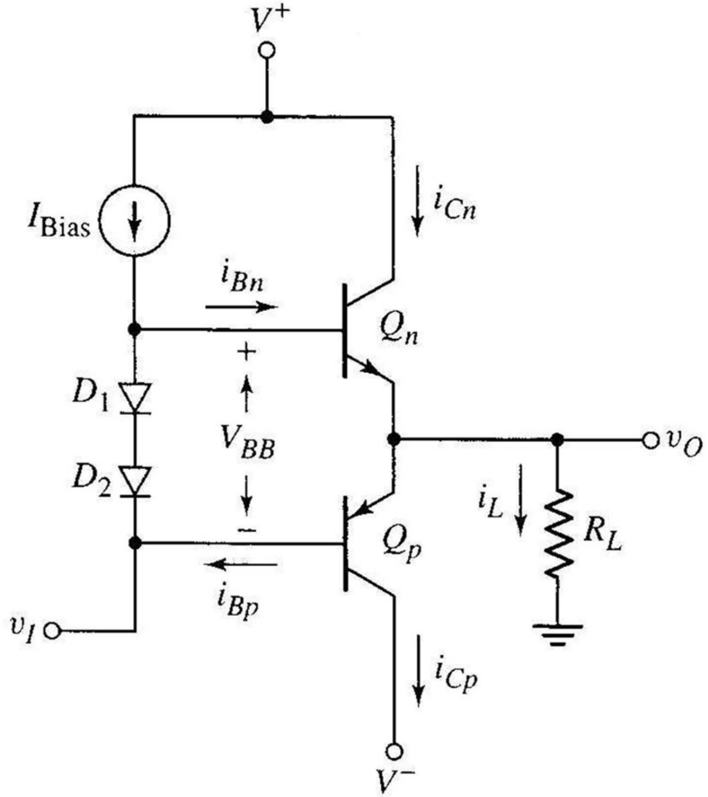
$$i_L = \frac{V_p}{R_L} = \frac{8.94}{8} = 1.12\text{ A}$$

$$i_{En} = i_L + i_{Ep} = 1.12 + i_{Ep}$$

$$\frac{i_{Cn}}{\alpha} = i_L + \frac{i_{Cp}}{\alpha} \Rightarrow i_{Cn} = \alpha i_L + i_{Cp} \Rightarrow i_{Cn} \approx i_L + i_{Cp}$$

$$V_{BB} = v_{BEn} + v_{EBp} \Rightarrow 2V_D = v_{BEn} + v_{EBp}$$

$$2V_T \ln\left(\frac{I_D}{I_{SD}}\right) = V_T \ln\left(\frac{i_{Cn}}{I_{SQ}}\right) + V_T \ln\left(\frac{i_{Cp}}{I_{SQ}}\right)$$



$$\therefore \frac{I_D{}^2}{I_{SD}{}^2} = \frac{i_{Cn}i_{Cp}}{I_{SQ}{}^2} \Rightarrow i_{Cn}i_{Cp} = \left(\frac{I_{SQ}}{I_{SD}}\right)^2 I_D{}^2$$

$$i_{Cn}i_{Cp} = \left(\frac{10^{-13}}{3\times10^{-14}}\right)^2 I_D{}^2$$

$$i_{Cn}i_{Cp} = 11.11 I_D{}^2$$

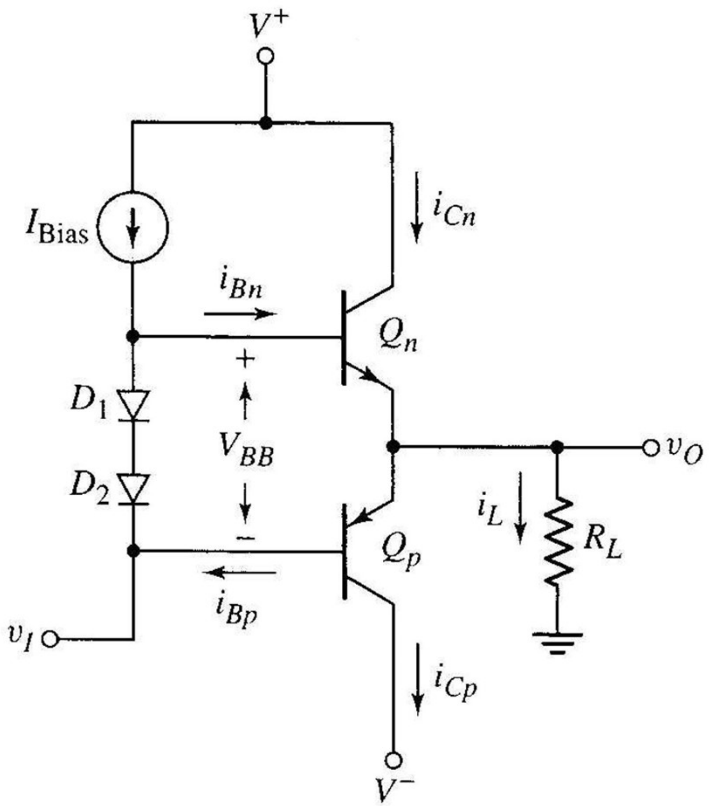
$$i_{Cn} i_{Cp} = 11.11 I_D^{\ 2}$$

$$i_{Cn}\left(i_{Cn} - i_L\right) = 11.11 I_D^{\ 2}$$

$$i_{Cn}\left(i_{Cn} - 1.12\right) = 11.11\left(5\times10^{-3}\right)^2 = 2.78\times10^{-4} \approx 0$$

$$i_{Cn} = 0 \ \text{ or } i_{Cn} = 1.12\text{A (take } i_{Cn} = 1.12\text{A)}$$

$$i_{Cp} = i_{Cn} - i_L = 1.12 - 1.12 = 0$$



$$i_{Bn} = \frac{i_{Cn}}{\beta} = \frac{1.12}{75} = 14.93 \ \text{mA}$$

$$I_{Bias} = I_D + i_{Bn} = 5 \text{ mA} + 14.93 \text{ mA} \approx 20 \text{ mA}$$

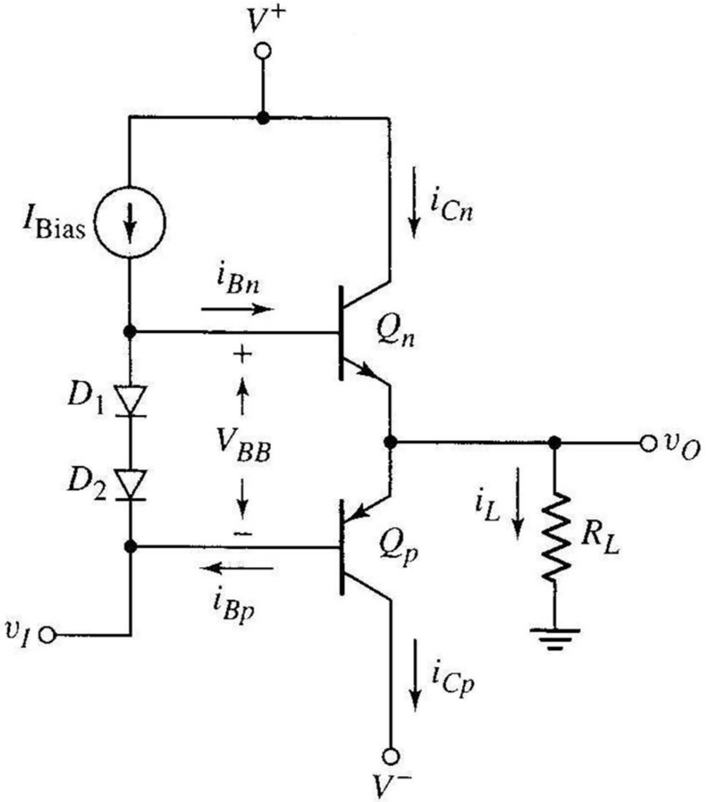
$$V_{BB} = 2V_T \ln\left(\frac{I_D}{I_{SD}}\right) = 2(0.026)\ln\left(\frac{5\times10^{-3}}{3\times10^{-14}}\right) = 1.344\text{V}$$

$$v_{BEn} = V_T \ln\left(\frac{i_{Cn}}{I_{SQ}}\right) = (0.026)\ln\left(\frac{1.12}{10^{-13}}\right) = 0.781 \text{ V}$$

$$v_{EBp} = V_{BB} - v_{BEn} = 1.344 - 0.781 = 0.563 \ \text{V}$$

$$i_{Cp} = I_{SQ} e^{\frac{V_{EBp}}{V_T}} = 10^{-13} e^{\frac{0.563}{0.026}} = 0.25 \text{mA}$$

When $v_o = 0$:



$$i_{Cn}\left(i_{Cn} - i_L\right) = 11.11 {I_D}^2$$

$$i_{Cn}\left(i_{Cn} - 0\right) = 11.11\left(20\times 10^{-3} - \frac{i_{Cn}}{75}\right)^2$$

$${i_{Cn}}^2 = 0.0044 - 0.006 i_{Cn} - 0.002 {i_{Cn}}^2$$

$$0.998 {i_{Cn}}^2 + 0.006 i_{Cn} - 0.0044 = 0$$

$$i_{Cn} = \frac{-0.006 \pm \sqrt{0.006^2 - 4\times 0.998 \times 0.0044}}{2\times 0.998}$$

$$i_{Cn} = 63.46\,\text{mA (take + ve value)}$$

$$i_{Cp} = i_{Cn} = 63.46\,\text{mA}$$

$$i_{Bn} = \frac{i_{Cn}}{\beta} = \frac{63.46\text{mA}}{75} = 0.846\ \text{mA}$$

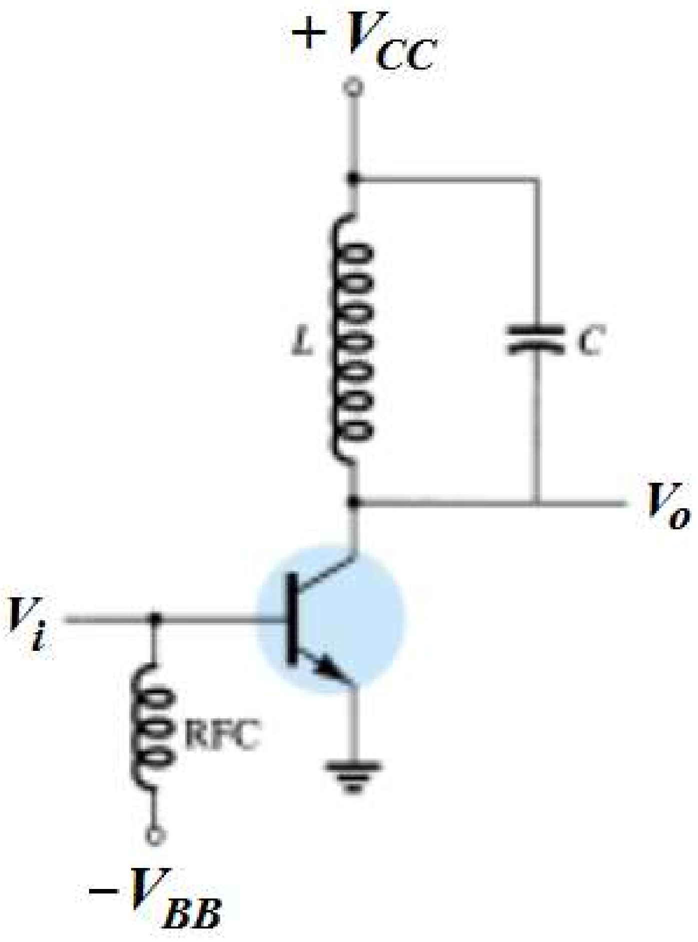
$$I_D = I_{Bias} - i_{Bn} = 20\,\text{mA} - 0.846\,\text{mA} \approx 19.15\,\text{mA}$$

$$V_{BB} = 2(0.026)\ln\left(\frac{19.15\times 10^{-3}}{3\times 10^{-14}}\right) = 1.413\text{V}$$

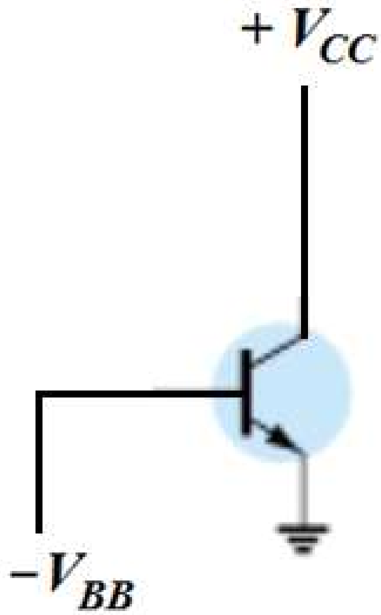
$$v_{BEn} = (0.026)\ln\left(\frac{63.46\times 10^{-3}}{10^{-13}}\right) = 0.707\ \text{V}$$

$$v_{EBp} = 1.413 - 0.707 = 0.707\ \text{V}$$

Class C Amplifiers



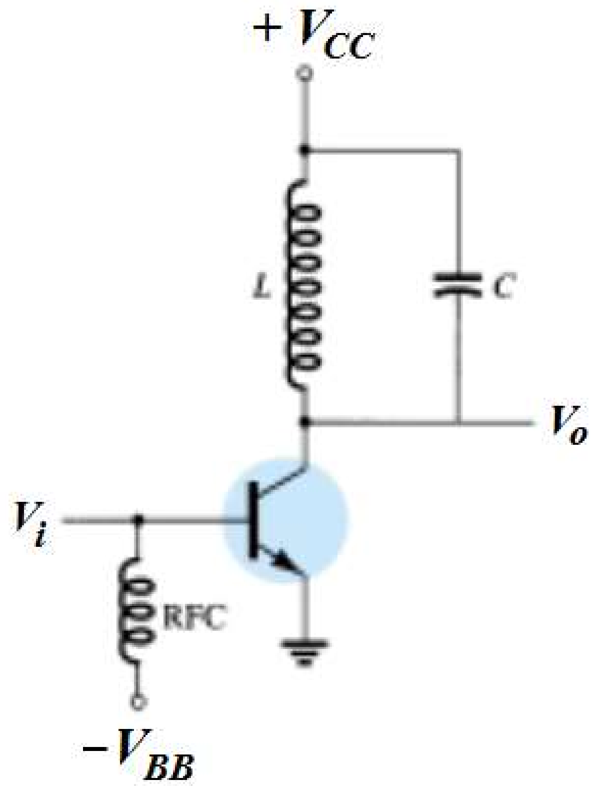
Under DC condition, both RFC (radio frequency choke) and L are shorted and C is opened.

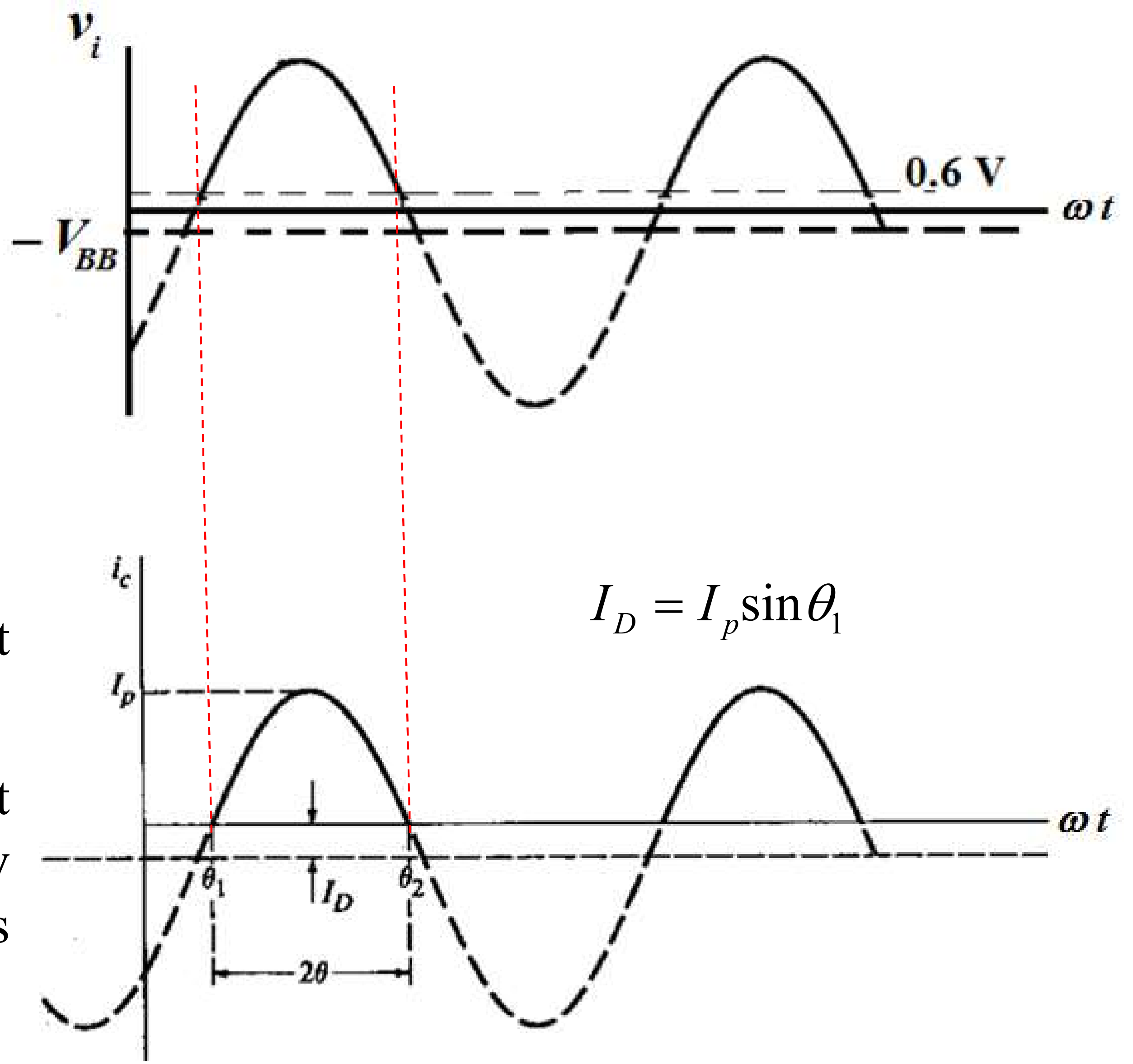


$$V_{BEQ} = -V_{BB}$$

$\therefore$ The BJT is reversed biased.

Class C Amplifiers





$$I_D = I_p \sin\theta_1$$

- The transistor is reversed bias at the Q point
- It conducts only when the input signal becomes sufficiently positive, i.e. it conducts less than half a cycle.

Fourier Series

For any periodic function, it can be expressed in terms of a DC component and a series of harmonics:

$$f(x) = a_o + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

Its average component is given by:

$$a_o = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

Its harmonics, n = 1, 2, 3 …. can be determined by:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

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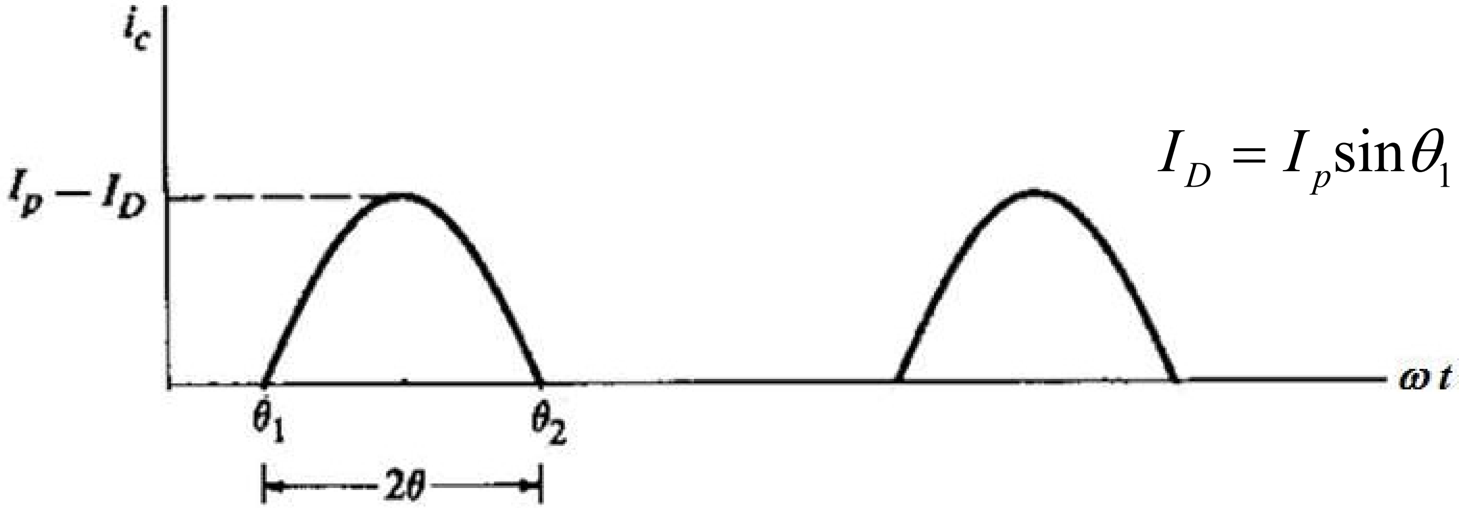


$$I_D = I_p \sin\theta_1$$

The collector current is given by: $i_C = \begin{cases} I_p \sin\omega t - I_D & \theta_1 \le \omega t \le \theta_2 \\ 0 & \text{otherwise} \end{cases}$

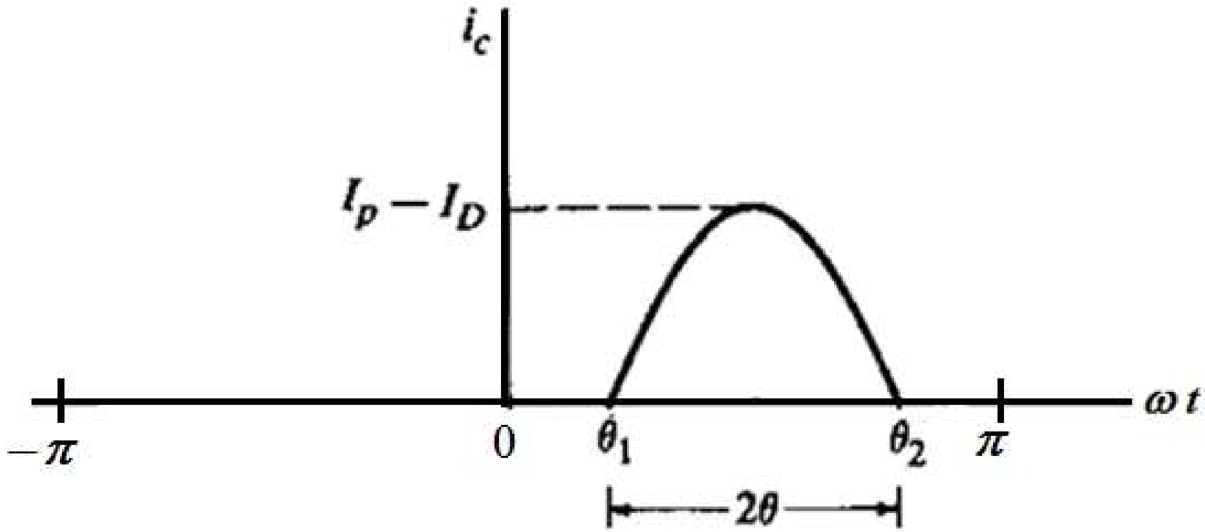
For any periodic function, it can be expressed mathematically by:

$$i_c(\omega t) = a_o + \sum_{n=1}^{\infty}\left(a_n\, cos\, n\omega t + b_n\, sin\, n\omega t\right)$$

Its average component is given by: $a_o = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} i_c(\omega t)\, d\omega t$

Its harmonic components (n = 1, 2, 3 ….) are given by:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_c(\omega t)\, cos\, n\omega t\, d\omega t \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_c(\omega t)\, sin\, n\omega t\, d\omega t$$

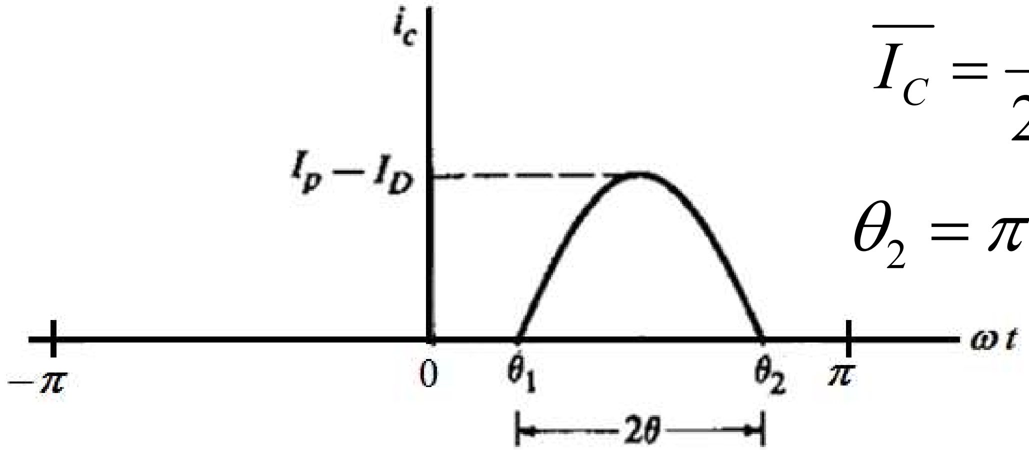


- The average collector current:

$$\overline{I_C} = a_o = \frac{1}{2\pi}\int_{-\pi}^{\pi}(I_p \sin \omega t - I_D)d\omega t$$

$$= \frac{1}{2\pi}\int_{\theta_1}^{\theta_2}(I_p \sin \omega t - I_D)d\omega t = \frac{1}{2\pi}\left[-I_p \cos \omega t - I_D \omega t\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2\pi}\left[I_p\left(\cos\theta_1 - \cos\theta_2\right) - I_D\left(\theta_2 - \theta_1\right)\right]$$



$$\overline{I_C} = \frac{1}{2\pi}\left[I_p\left(\cos\theta_1 - \cos\theta_2\right) - I_D\left(\theta_2 - \theta_1\right)\right]$$

$$\theta_2 = \pi - \theta_1 \,,\ \cos\theta_2 = \cos\left(\pi - \theta_1\right) = -\cos\theta_1$$

$$I_D = I_p \sin\theta_1$$

$$\therefore \overline{I_C} = \frac{1}{2\pi}\left[2I_p\cos\theta_1 - I_p\sin\theta_1\left(\theta_2 - \theta_1\right)\right]$$

The conduction angle can be defined as:

$$2\theta = \theta_2 - \theta_1 \quad \text{or} \quad \theta_1 = \frac{\pi}{2} - \theta$$

$$\overline{I_C} = \frac{1}{2\pi}\left[2I_p\cos\left(\frac{\pi}{2} - \theta\right) - 2\theta I_p \sin\left(\frac{\pi}{2} - \theta\right)\right] = \frac{I_p}{\pi}\left(\sin\theta - \theta\cos\theta\right)$$

If I_1 is the amplitude of the fundamental current components which is determined by the trigonometric Fourier series:

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(I_p \sin\omega t - I_D\right)\cos\omega t\, d\omega t$$

$$= \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(I_p \sin\omega t - I_D\right)\cos\omega t\, d\omega t = \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(I_p \sin\omega t \cos\omega t - I_D \cos\omega t\right)d\omega t$$

$$= \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(\frac{I_p \sin 2\omega t}{2} - I_D \cos\omega t\right)d\omega t = \frac{1}{\pi}\left[\frac{-I_p \cos 2\omega t}{4} - I_D \sin\omega t\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{4\pi}\left[I_p\left(\cos 2\theta_1 - \cos 2\theta_2\right) - I_D\left(\sin\theta_2 - \sin\theta_1\right)\right]$$

$$\theta_2 = \pi - \theta_1 :$$

$$\cos 2\theta_2 = \cos\left(2\pi - 2\theta_1\right) = \cos 2\theta_1\,,\ \ \sin\theta_2 = \sin\left(\pi - \theta_1\right) = \sin\theta_1$$

$$\therefore a_1 = 0$$

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(I_p \sin\omega t - I_D\right)\sin\omega t d\omega t$$

$$= \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(I_p \sin\omega t - I_D\right)\sin\omega t d\omega t = \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(I_p \sin^2\omega t - I_D \sin\omega t\right)d\omega t$$

$$= \frac{1}{\pi}\int_{\theta_1}^{\theta_2}\left(\frac{I_p\left(1 - \cos 2\omega t\right)}{2} - I_D \sin\omega t\right)d\omega t = \frac{1}{\pi}\left[\frac{I_p\omega t}{2} - \frac{I_p \sin 2\omega t}{4} + I_D \cos\omega t\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{\pi}\left[\frac{I_p}{2}\left(\theta_2 - \theta_1\right) - \frac{I_p}{4}\left(\sin 2\theta_2 - \sin 2\theta_1\right) + I_D\left(\cos\theta_2 - \cos\theta_1\right)\right]$$

$$\theta_2 = \pi - \theta_1 \text{ and } \theta_2 - \theta_1 = 2\theta$$

$$\sin 2\theta_2 = \sin\left(2\pi - 2\theta_1\right) = -\sin 2\theta_1 ,\ \cos\theta_2 = \cos\left(\pi - \theta_1\right) = -\cos\theta_1$$

$$\therefore b_1 = \frac{1}{\pi}\left[I_p\theta + \frac{I_p}{2}\sin 2\theta_1 - 2I_D \cos\theta_1\right]$$

$$b_1 = \frac{1}{\pi}\left[I_p\theta + \frac{I_p}{2}\sin 2\theta_1 - 2I_D\cos\theta_1 \right]$$

$$= \frac{1}{\pi}\left[I_p\theta + \frac{I_p}{2}\sin 2\theta_1 - 2I_p\sin\theta_1\cos\theta_1 \right]$$

$$= \frac{I_p}{\pi}\left[\theta + \sin\theta_1\cos\theta_1 - 2\sin\theta_1\cos\theta_1\right]$$

$$= \frac{I_p}{\pi}\left[\theta - \sin\theta_1\cos\theta_1\right] = \frac{I_p}{\pi}\left(\theta - \sin\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)\right)$$

$$= \frac{I_p}{\pi}\left(\theta - \cos\theta\sin\theta\right) = \frac{I_p}{\pi}\left(\theta - \frac{\sin 2\theta}{2}\right) = \frac{I_p}{2\pi}\left(2\theta - \sin 2\theta\right)$$

The fundamental current component is: $I_1 = \frac{I_p}{2\pi}\left(2\theta - \sin 2\theta\right)$

$$i_c(\omega t) = \underbrace{\frac{I_p}{\pi}\left(\sin\theta - \theta\cos\theta\right)}_{\text{DC current}} + \underbrace{\frac{I_p}{2\pi}\left(2\theta - \sin 2\theta\right)\sin\omega t}_{\text{Fundamental component of the current}}$$

$$I_1 = \frac{I_p}{2\pi}(2\theta - \sin 2\theta) \qquad \overline{I_C} = \frac{I_p}{\pi}(\sin\theta - \theta\cos\theta)$$

- The output power to load:

$$\overline{P_L} = \tfrac{1}{2} V_{CC} I_1 = \frac{V_{CC} I_p}{4\pi}(2\theta - \sin 2\theta)$$

- The power drawn from supply:

$$\overline{P_S} = V_{CC}\overline{I_C} = \frac{V_{CC} I_p}{\pi}(\sin\theta - \theta\cos\theta)$$

- Conversion efficiency:

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{V_{CC} I_p (2\theta - \sin 2\theta)}{4\pi} \times \frac{\pi}{V_{CC} I_p (\sin\theta - \theta\cos\theta)}$$

$$= \frac{2\theta - \sin 2\theta}{4(\sin\theta - \theta\cos\theta)}$$

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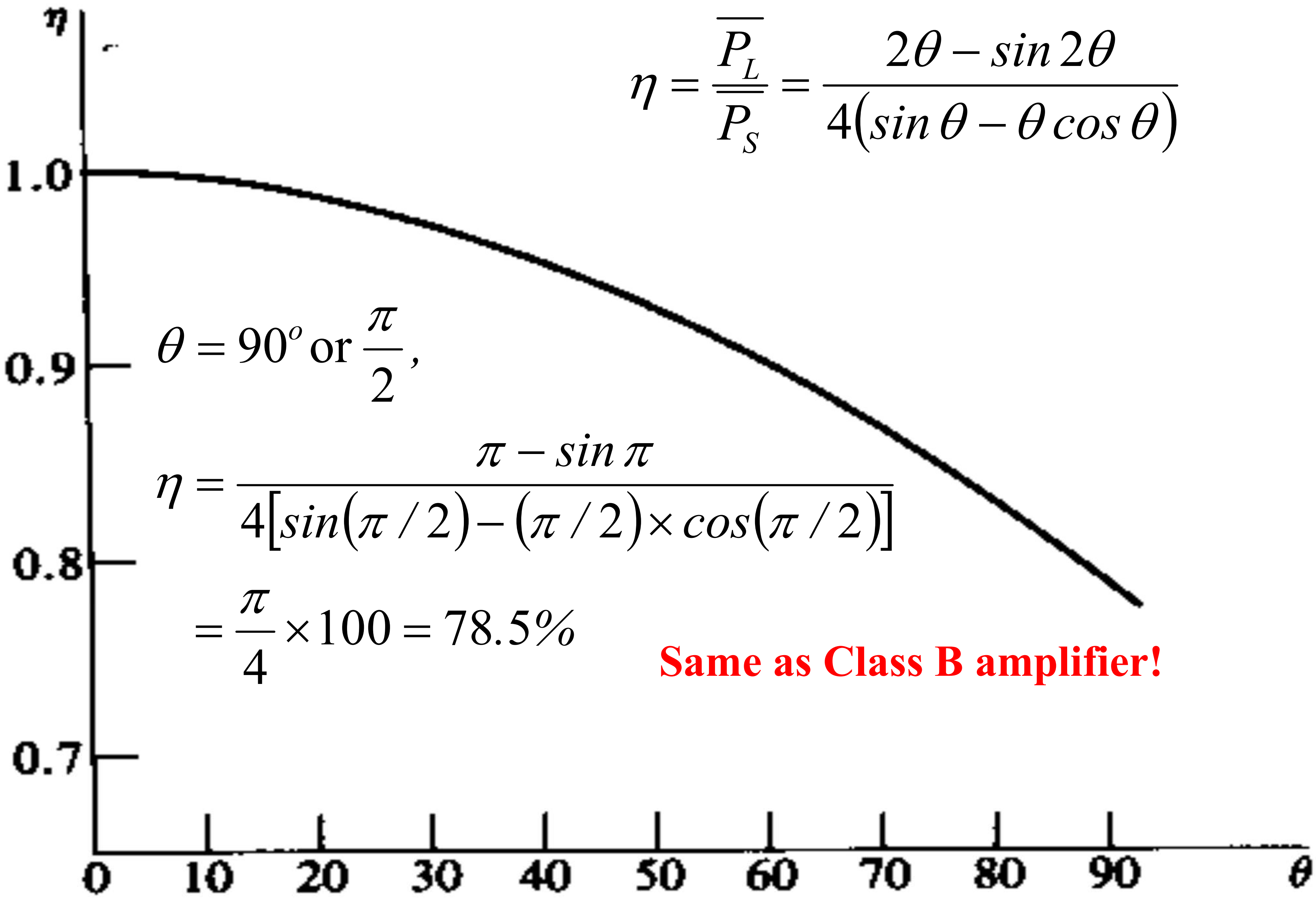
Class C Amplifier Conversion Efficiency



$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{2\theta - \sin 2\theta}{4(\sin\theta - \theta\cos\theta)}$$

$$\theta = 90^o \text{ or } \frac{\pi}{2},$$

$$\eta = \frac{\pi - \sin\pi}{4[\sin(\pi/2) - (\pi/2)\times\cos(\pi/2)]}$$

$$= \frac{\pi}{4}\times 100 = 78.5\%$$

Same as Class B amplifier!

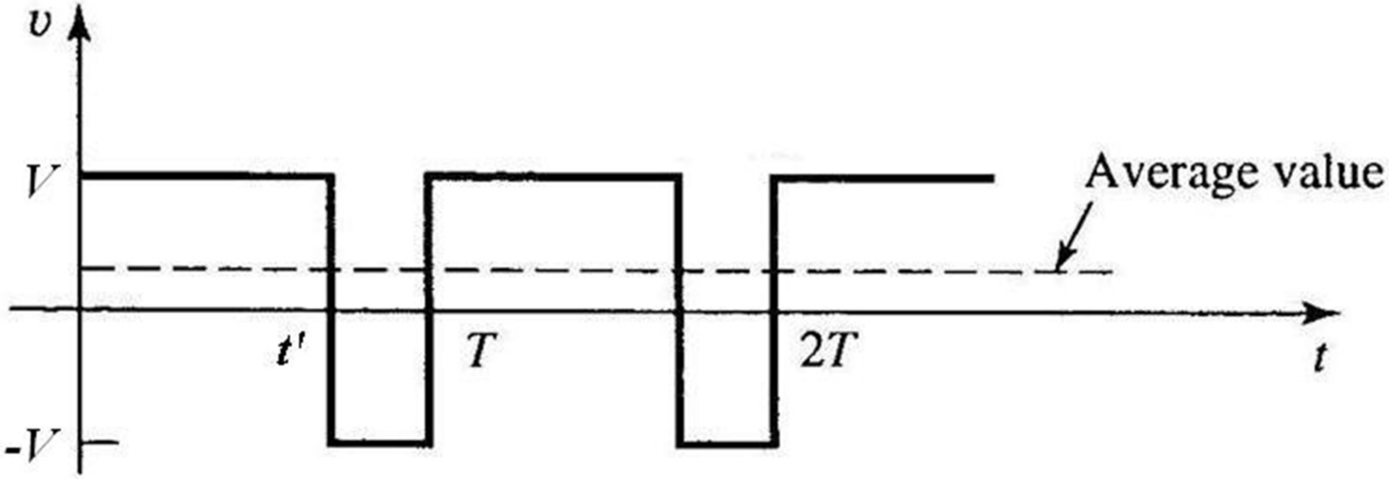
Class D Amplifiers

- Class D amplifier is a pulse-width modulated (PWM) circuit.

- Conversion efficiency as high as 90% is achievable and therefore commonly used for very high power (400-600 W) amplifier stage.

- As PWM requires switching frequencies of 200-500 kHz, electromagnetic interference (EMI) issue has to be considered in the design.

Concept of PWM



$$d = \frac{t'}{T}$$

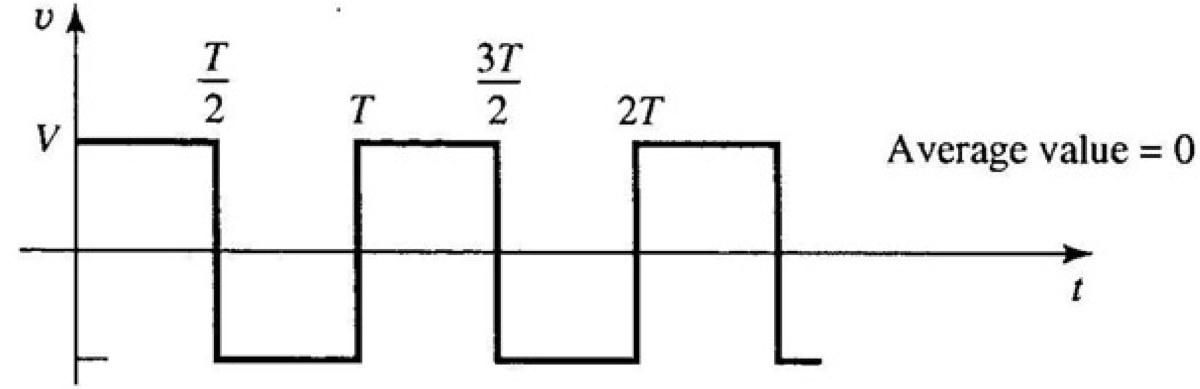
$$\therefore t' = dT$$

$$V_{av} = \frac{1}{T}\left[\int_0^{dT} V\,dt + \int_{dT}^{T} (-V)\,dt\right] = \frac{1}{T}\left[VdT + (-VT + VdT)\right]$$

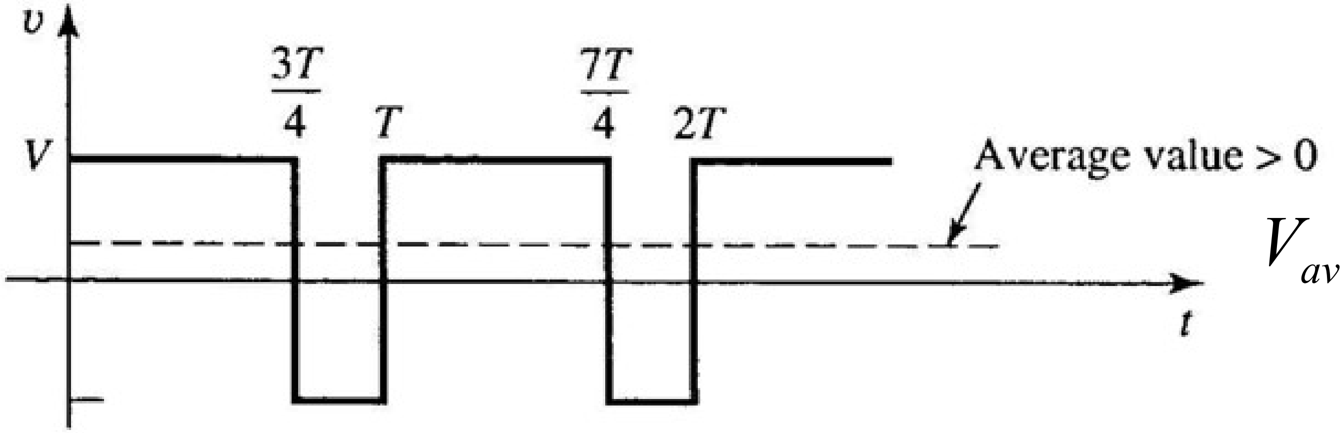
$$= \frac{1}{T}\left[2VdT - VT\right] = \frac{VT}{T}\left[2d - 1\right] = V\left[2d - 1\right]$$

Concept of PWM

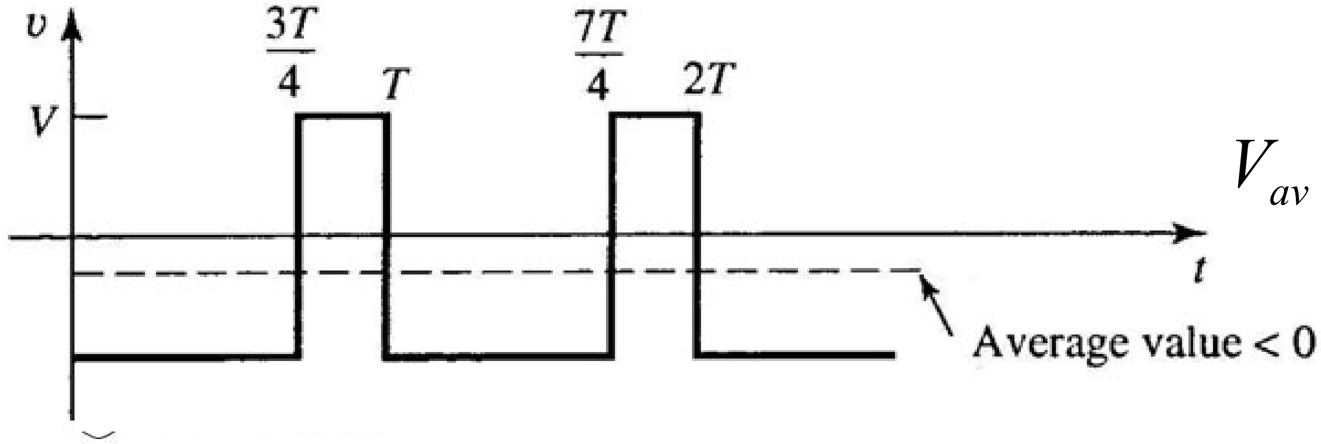
$$V_{av} = V[2d - 1]$$



$$V_{av} = V[2(0.5) - 1] = 0$$

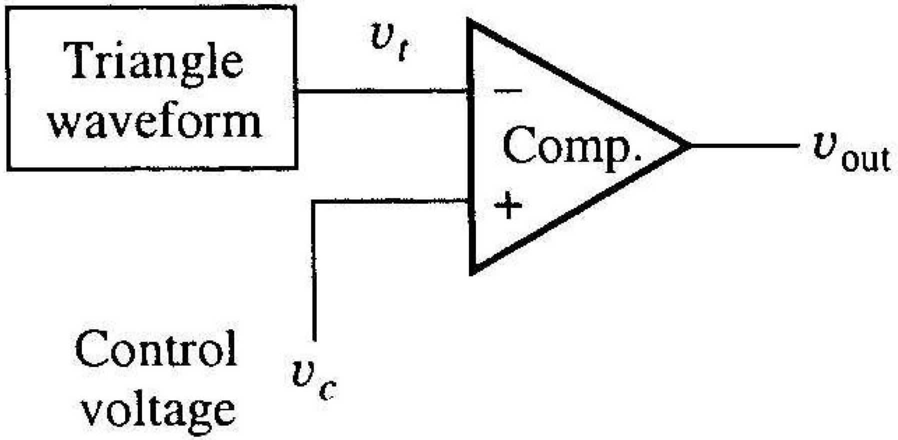


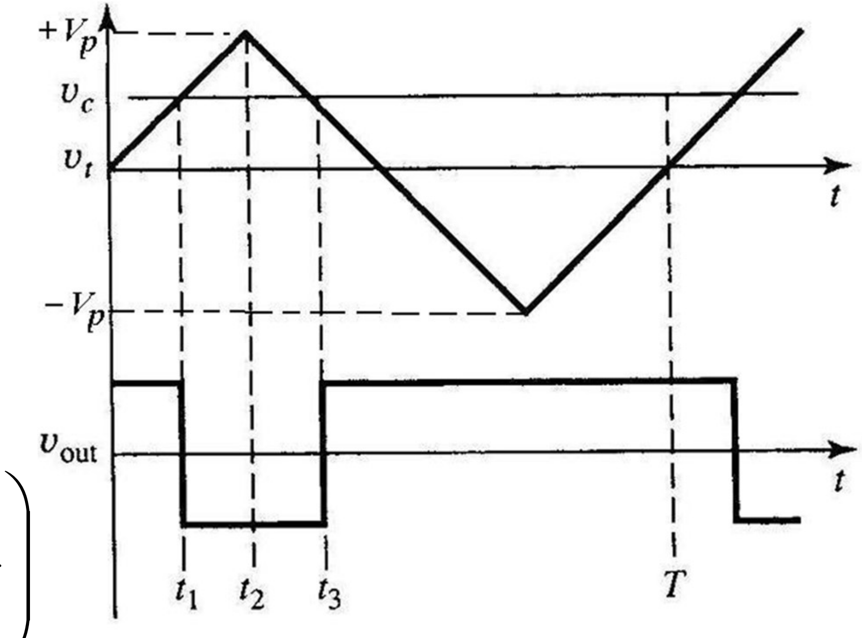
$$V_{av} = V[2(0.75) - 1] = 0.5V$$



$$V_{av} = V[2(0.25) - 1] = -0.5V$$

Generation of PWM Signal





$$t_1 = \frac{v_c}{V_p}\left(\frac{T}{4}\right),\ t_2 = \frac{T}{4}, t_2 - t_1 = \frac{T}{4}\left(1 - \frac{v_c}{V_p}\right)$$

$$t^- = 2(t_2 - t_1) = \frac{T}{2}\left(1 - \frac{v_c}{V_p}\right) \quad t^+ = T - t^- = \frac{T}{2}\left(1 + \frac{v_c}{V_p}\right)$$

$$d = \frac{t^+}{T} = \frac{(T/2)(1 + v_c/V_p)}{T} = 0.5 + \frac{v_c}{2V_p} = 0.5 + kv_c \qquad k = \frac{1}{2V_p}$$

$$V_{av} = V(2d - 1)$$

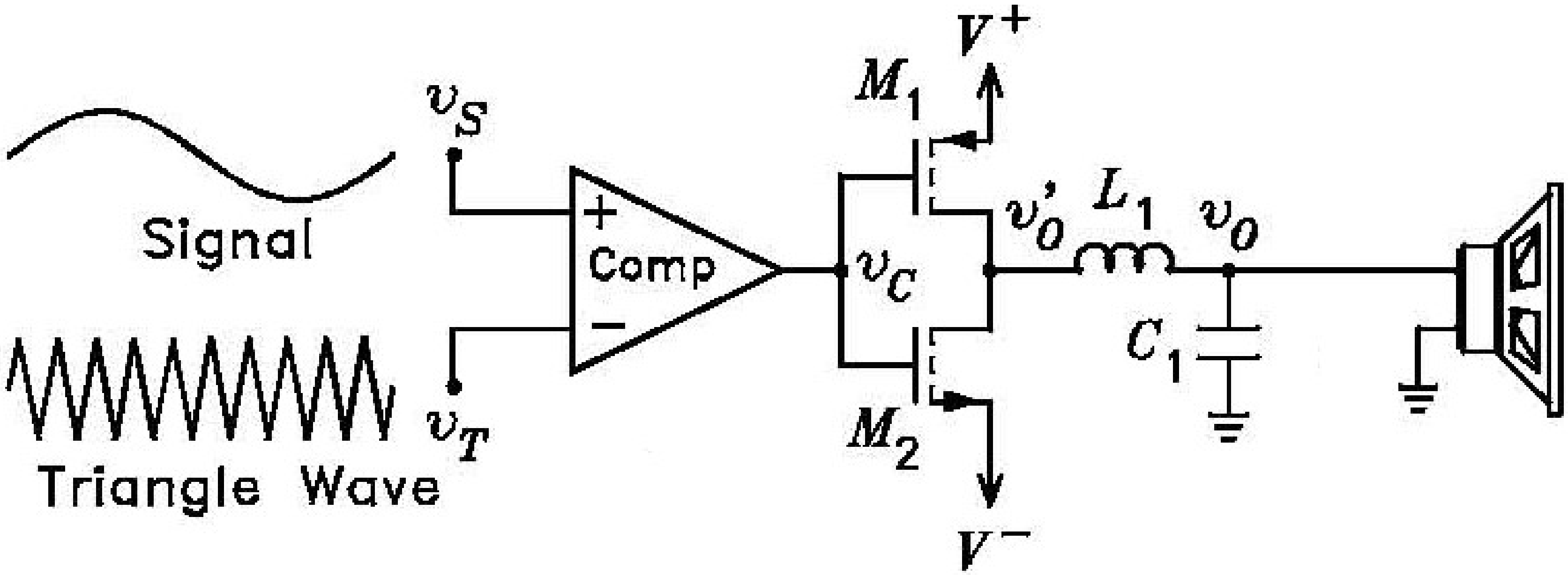
$$d = 0.5 + kv_c$$

$$\text{If } v_c = \sin\omega t, \text{ then} : d = 0.5 + k\sin\omega t$$

$$V_{av} = V(2d - 1) = V[2(0.5 + k\sin\omega t) - 1] = 2Vk\sin\omega t$$

- *d* can be varied sinusoidally by making v_c a sinusoidal signal instead of a constant DC voltage
- then the average signal of the PWM signal will be a sinusoidal signal too.

Class D Amplifier

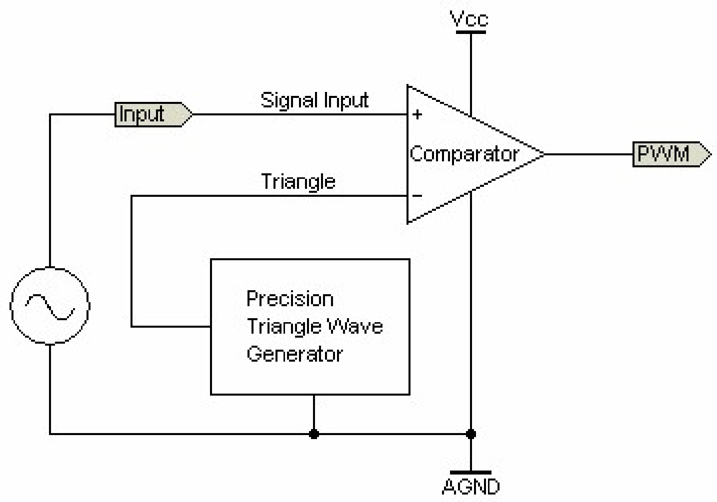


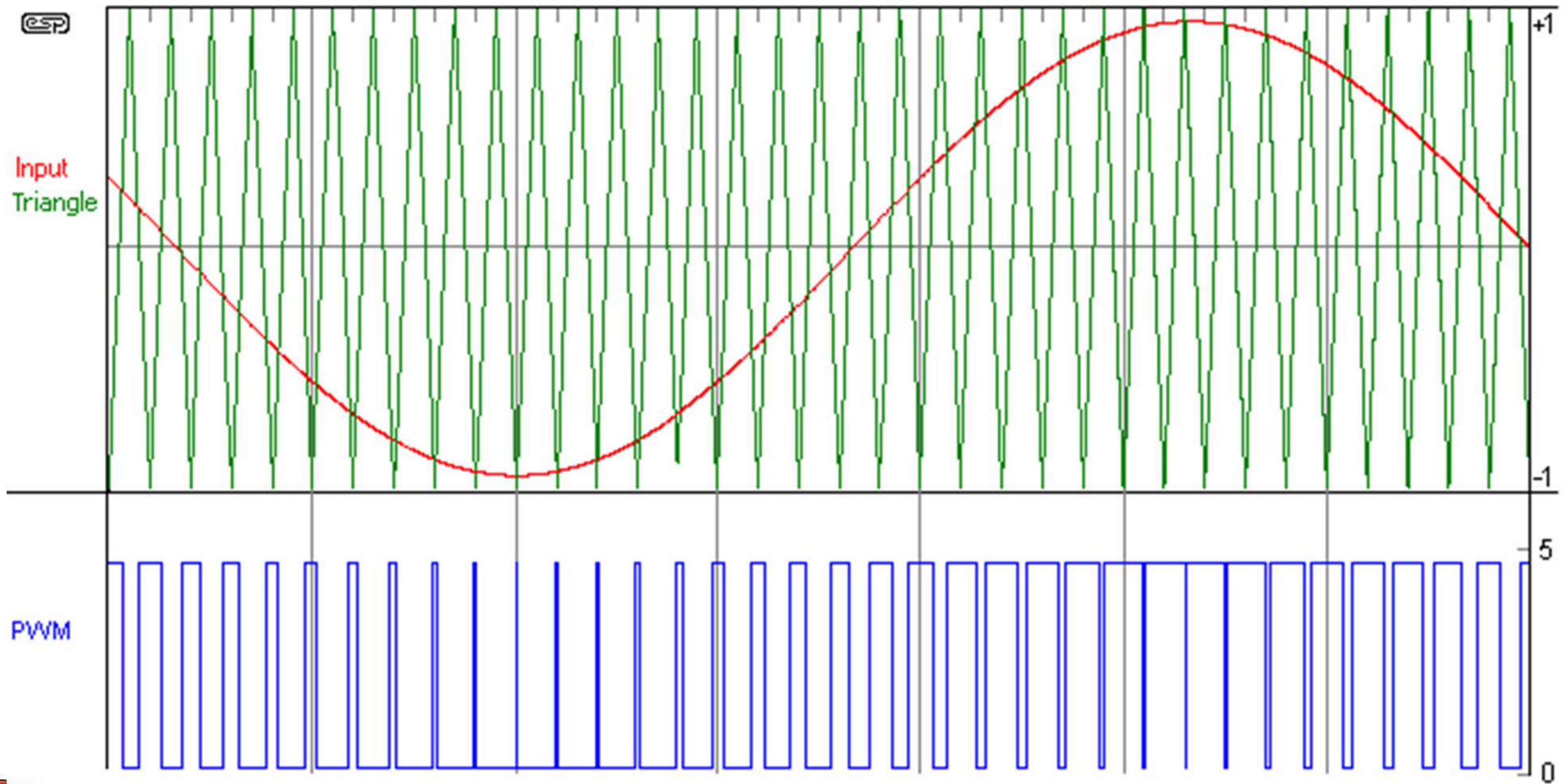
$$\text{If } d = 0.5 + k\sin\omega t$$

$$V_{av} = V(2d-1) = V(1+2k\sin\omega t - 1) = 2kV\sin\omega t$$

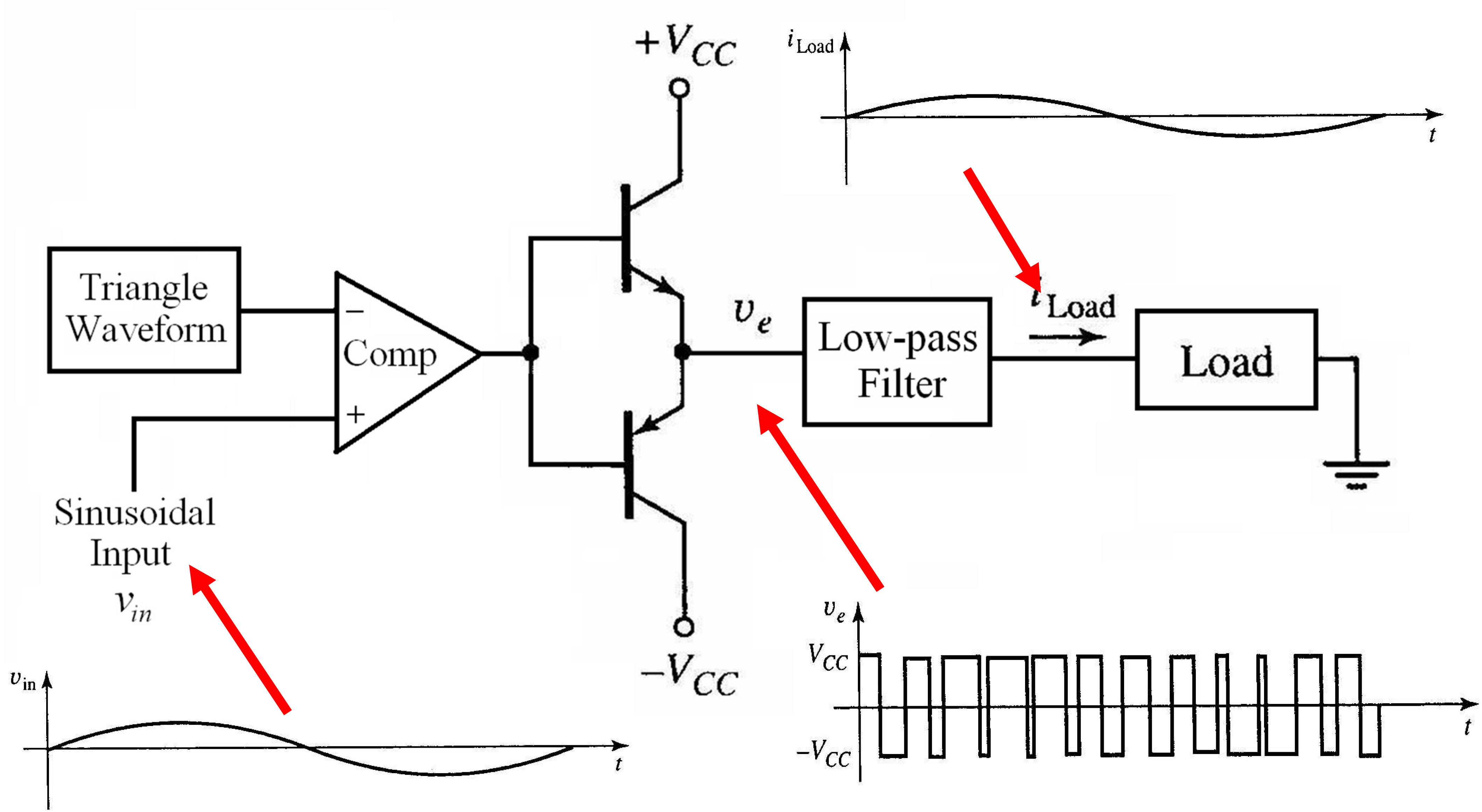
By having the low-pass filter, only the average component of the PWM waveform appeared at the load and other higher harmonics of ω are eliminated. Note: when there is no input sinusoidal signal to the PWM circuit, $d = 0.5$ and $V_{av} = 0$.

- The sine wave swings between –1 and +1V.
- It will produce 0% to 100% duty cycles
- 0% and 100% duty cycles correspond to -1 V and +1V, respectively. 50% corresponds to 0V input.





Class D Amplifier



Class D Amplifier Efficiency

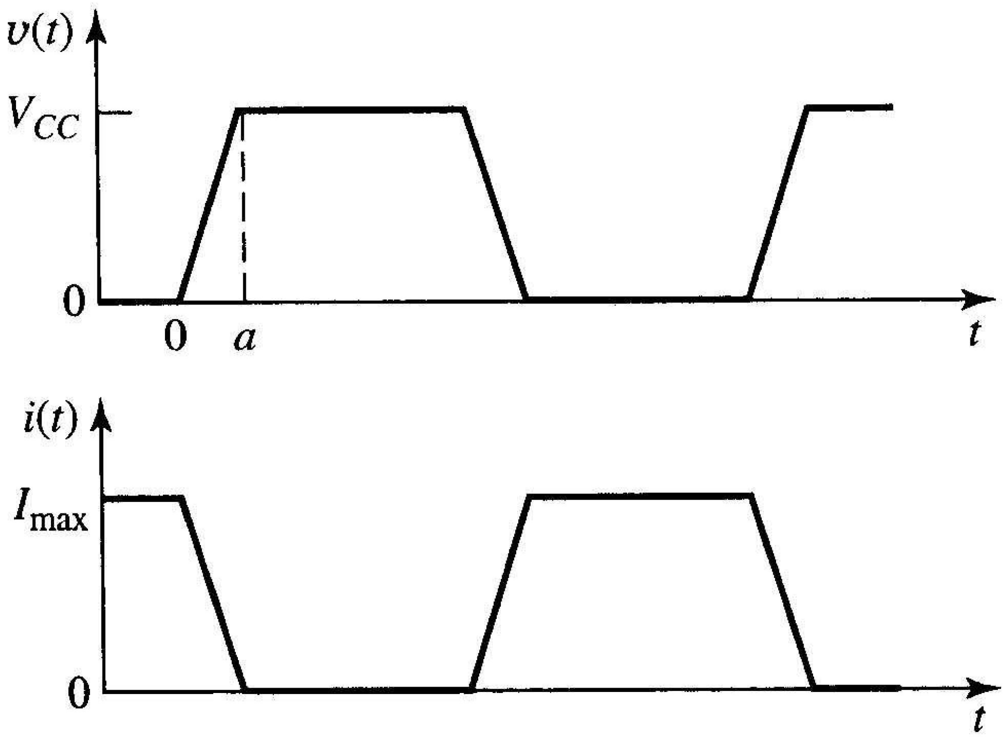
For 50% duty cycle (d = 0.5), the time-domain repetitive pulses with amplitude of $\pm V_{CC}$ has the following Fourier components in frequency-domain:

$$v_e = \frac{4V_{CC}}{\pi}\left(sin\,\omega_s t + \frac{1}{3} sin\,3\omega_s t + \frac{1}{5} sin\,5\omega_s t + \dots \right)$$

where ω_s is the switching frequency.

When a sinusoidal inputs signal is applied, d is modulated sinusoidally about the Q-point of 0.5. The maximum possible output load voltage and current are:

$$V_{max} = V_{CC} \quad I_{\max} = \frac{V_{CC}}{R_L} \quad P_L = \frac{{V_{CC}}^2}{2R_L}$$



The voltage across the transistor and the current through the transistor from $t = 0$ to $t = a$:

$$v(t) = \frac{t}{a} V_{CC}$$

$$i(t) = I_{max} - \frac{t}{a} I_{max} = \frac{V_{CC}}{R_L}\left(1 - \frac{t}{a}\right)$$

The power dissipation for each transistor over the transition from $t = 0$ to $t = a$:

$$p(t) = v(t)i(t) = \frac{V_{CC}^{\ 2}}{R_L}\left(\frac{t}{a} - \frac{t^2}{a^2}\right)$$

Each transistor makes two transitions in one period of the switching frequency. The average power dissipation for each transistor is:

$$P_T = 2 \times \frac{1}{T}\int_0^a \frac{V_{CC}^{\ 2}}{R_L}\left(\frac{t}{a} - \frac{t^2}{a^2}\right)dt = \frac{2}{T}\left(\frac{V_{CC}^{\ 2}}{R_L}\right)\left[\frac{t^2}{2a} - \frac{t^3}{3a^2}\right]_0^a = \frac{a}{T}\left(\frac{V_{CC}^{\ 2}}{3R_L}\right)$$

The total power dissipation for two transistors:

$$2P_T = \frac{a}{T}\left(\frac{2V_{CC}^{\ 2}}{3R_L}\right)$$

The conversion efficiency:

$$\eta = \frac{P_L}{P_L + 2P_T}$$

Note: the actual conversion efficiency is lower because of additional power dissipation in the low-pass filter and non-zero saturation voltage of the transistor.

Exercise #5: Determine the conversion efficiency for a Class D amplifier. Each switching transition is 5% of the period of the switching frequency. The power supplies are ± 24 V and the load resistance is 50 Ω. Neglect the power dissipation in the low-pass filter.

$$2P_T = \frac{a}{T}\left(\frac{2V_{CC}^{\ 2}}{3R_L}\right) = (0.05)\left(\frac{2\times 24^2}{3\times 50}\right) = 0.384 \text{ W}$$

$$P_L = \frac{V_{CC}^{\ 2}}{2R_L} = \frac{24^2}{100} = 5.76 \text{ W}$$

$$\eta = \frac{P_L}{P_L + 2P_T} = \frac{5.76}{5.76 + 0.384} = 93.8\%$$

If considering finite saturation voltage of the transistor $V_{CE(sat)}$ = 0.4V,

$$V_{max} = V_{CC} - V_{CE(sat)} = 24 - 0.4 = 23.6 \text{ V}$$

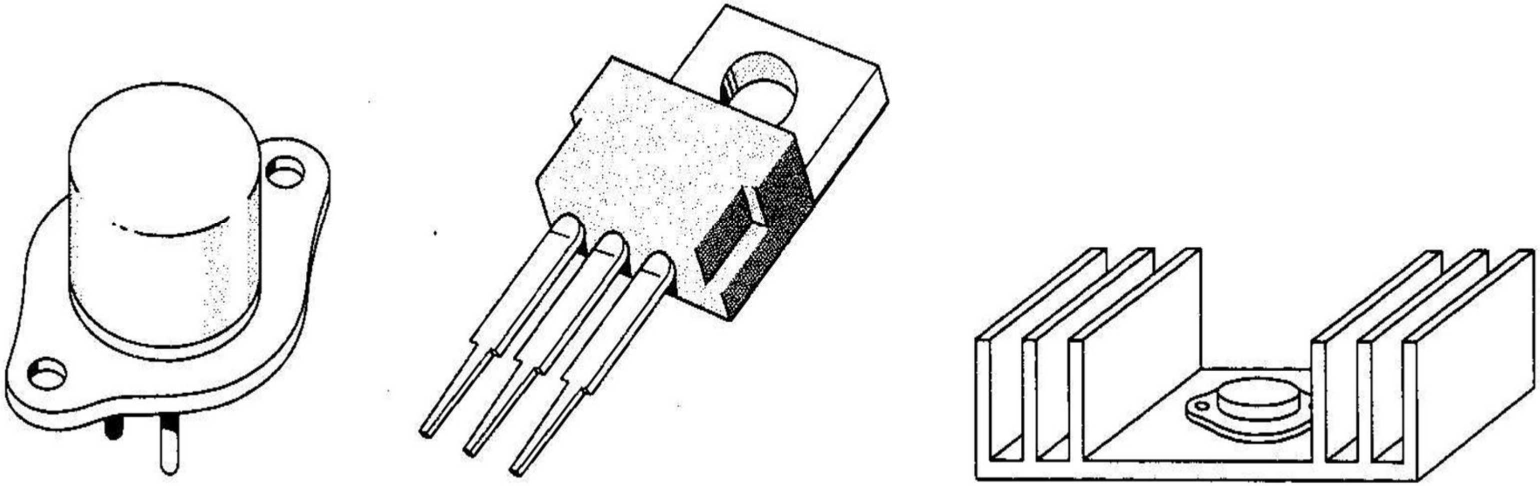
$$P_L = \frac{V_{max}^2}{2R_L} = \frac{(23.6)^2}{100} = 5.57 \text{ W}$$

$$I_{max} = \frac{V_{max}}{R_L} = \frac{23.6}{50} = 0.472 \text{ A}$$

$$2P_T = \frac{a}{T}\left(\frac{2V_{max}^2}{3R_L}\right) = (0.05)\left(\frac{2 \times 23.6^2}{3 \times 50}\right) = 0.371$$

$$\eta = \frac{P_L}{P_L + 2P_T + 0.9 I_{max} V_{CE(sat)}} = \frac{5.57}{5.57 + 0.371 + 0.9 \times 0.472 \times 0.4} = 91.15\%$$

Heat Sinks



Heat sink removes the heat from the device's junction to prevent permanent damage.

$$T_2 - T_1 = P\theta$$

where $(T_2 - T_1)$ is the temperature difference across an element, θ is the **thermal resistance** of the element in unit of $^oC/W$ and P is the thermal power through the element.

Exercise #6: Determine the maximum power dissipation in a transistor without and with heat sink. With the heat sink, determine the temperature of the transistor case and heat sink. The ambient temperature is 30^{o}C and the maximum device temperature is 150^{o}C. The transistor and the heat sink thermal resistance parameters are:

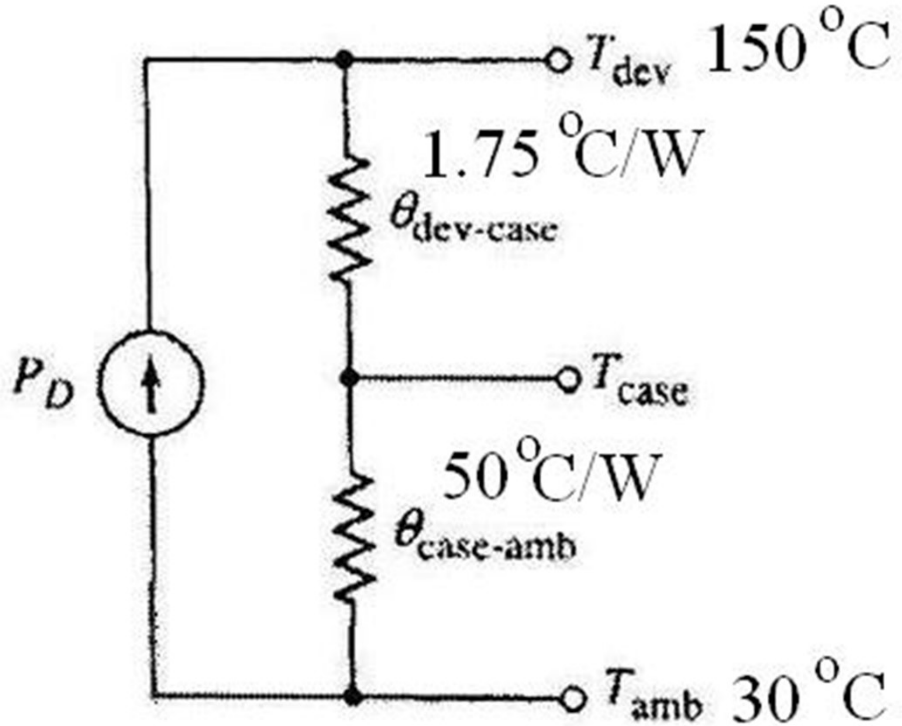
$$\theta_{dev-case} = 1.75^o \text{ C/W}$$

$$\theta_{case-\sin k} = 1^o \text{ C/W}$$

$$\theta_{\sin k-amb} = 5^o \text{ C/W}$$

$$\theta_{case-amb} = 50^o \text{ C/W}$$

When no heat sink is used, the maximum allowable power dissipation is:



$$P_{D,max} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-amb}}$$

$$= \frac{150 - 30}{1.75 + 50}$$

$$= 2.32 \text{ W}$$

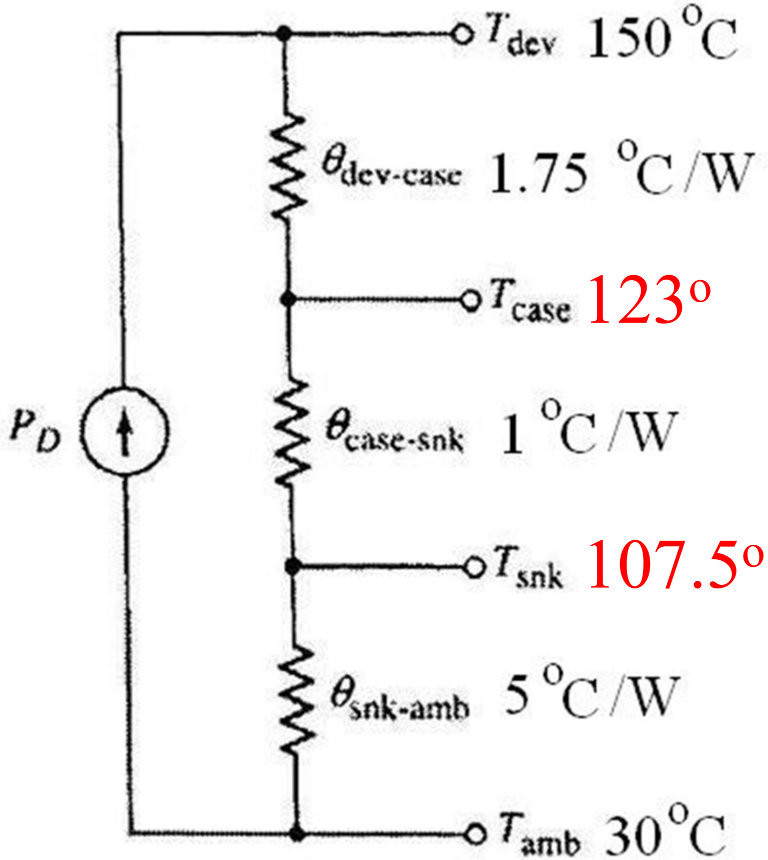
When a heat sink is used, the maximum allowable power dissipation is:

$$P_{D,max} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-sink} + \theta_{sink-amb}}$$

$$= \frac{150-30}{1.75+1+5}$$

$$= 15.5 \text{ W}$$

Note: The use of heat sink allows more power to be dissipated (increased from 2.32 W to .15.5W)



$$T_{sink} - T_{amb} = P_D \theta_{sink-amb}$$

$$T_{sink} = P_D \theta_{sink-amb} + T_{amb}$$

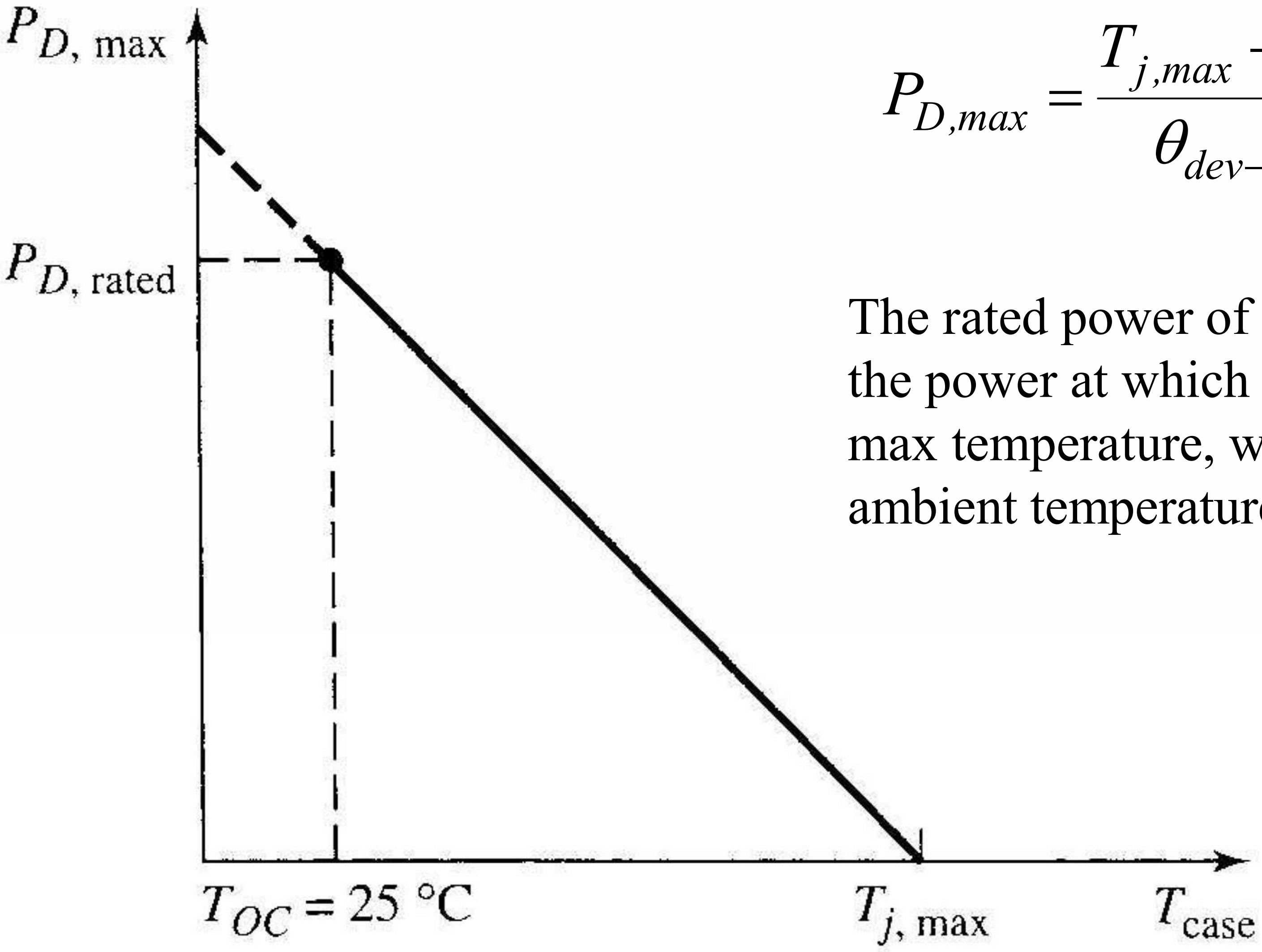
$$= (15.5)(5) + 30 = 107.5^{\circ}\text{C}$$

$$T_{case} - T_{amb} = P_D(\theta_{case-sink} + \theta_{sink-amb})$$

$$T_{case} = (15.5)(1+5) + 30 = 123^{\circ}\text{C}$$

Power Derating

The max safe power dissipation is determined by:



$$P_{D,max} = \frac{T_{j,max} - T_{case}}{\theta_{dev-case}}$$

The rated power of a device is defined as the power at which the device reaches its max temperature, while the case remains at ambient temperature.