DDS Algorithm

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Portfolio Theory

Introduction

Capital Asset Pricing Model (CAPM)

Sharpe Ratio as a Measure of Performance

The Expected return of a portfolio with k assets is simply the weighted sum of expected returns for each asset and is given by:

$$E[R_p] = \sum_{i=1}^{k} w_i r_i = \mathbf{w}^T \mathbf{r}$$

where $\mathbf{w}$ is a vector of stock weights and $\mathbf{r}$ is a vector of the expected returns for each stock. Defining these functions in matrix algebra notation emphasizes the use of vectorization within the DDS Algorithm. The variance of the expected return is then:

$$\begin{aligned} Var[E[R_p]] &= Var[\mathbf{w}^T \mathbf{r}] \\ &= \mathbf{w}^T Var[\mathbf{r}] \mathbf{w} \\ &= \mathbf{w}^T \Sigma \mathbf{w} \end{aligned}$$

Generally use simple returns when we have many assets. The portfolios Sharpe Ratio is then given as:

$$Sharpe = \frac{E[R_p] - r_f}{SD[R_p]}$$

The Sharpe ratio specifies the Additional expected return for a given level of risk. A higher Sharpe Ratio suggest a more optimal portfolio.

Methodology

Optimization Problem

Optimization is a common theme that underpins the fitting of Statistical models. From the maximization of the log-likelihood function under Linear Models, to the minimization of errors under Neural Network classifiers, optimization is fundamentally important to obtaining objective solutions to many problems in the world of Data Science and Statistical research.

The general Optimization problem can be defined as follows:

$$x^* = argminf(x; y)$$
$$x \in \Omega$$

In this case, one would like to maximize the Sharpe ratio such that the sum of weights in the portfolio equals one. This optimization problem can be expressed as follows:

$$Max \left(\frac{E[r_p] - r_f}{Std[E[r_p]]} \right) s.t. \sum_{i=1}^{K} w_i = 1$$

Or in vector notation:

$$Max \left(\frac{\mathbf{w^T r}}{\sqrt{\mathbf{w}^T \mathbf{\Sigma w}}} \right) s.t. \|\mathbf{w}\| = 1$$

Theoretical Solution (i.e. consider the actual mathematical result)

Discretized Dynamical System

Grid Search

Bisection Method

Gauss-Newton Method

Simulated Aneeling

Other Optimization Methods

There exist a plethora of possible methods to solve the optimization problem identified in this report. The two general contrasting methods discussed are Stochastic and Deterministic forms of optimization.

Discuss the Advantages and disadvantages of the two methods- Deterministic and stochastic optimization methodology and also of the methods in general.

The DDS Algorithm (R Package)

Pre-Processing Function

DDS Algorithm

- Discuss how the expected returns and then the portfolio standard deviation are ANNUALIZED- see Python for Finance course

Heat Maps and Plots

Testing

Unit Testing

Unit testing helps to create robust code. This ensures the code is robust to new versions of R, as well as enabling users to utilize the function correctly. This ensures the function generates the expected output and can deal with arguments of different structure, flagging problems if the structure of input arguments is not appropriate (boundary testing). This ensure code is robust and results are reproducible.

Boundary Testing

Simulation

Assessing Computational Efficiency

- discuss the computational efficiency of the different optimization methods- measure using the system.time function, for example.
- Also discuss how I improved the computational speed of the function using Parallelization and profiling.

Example Portfolio

We test and simulate using stock returns from 1 January 2010 to 1 December 2020 inclusive, for Delta Airlines, Walmart and Microsoft stock in a portfolio (could add many more)

Bibliography

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Appendix

Python Code

- Attach the equivalent Python code to the above problem, however, no 'package' has been created- BUT could look into this.

R code

- add any extra pieces of R code here