The Configuration Space

Algorithms and Data Structures 2 – Motion Planning and its applications

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Introduction

- Motion planning is not done in the workspace.
- Motion planning is done in the so-called configuration space.
- The configuration space is an additional space in the algorithm.
- All algorithms are defined in the configuration space.

Definition of the configuration space

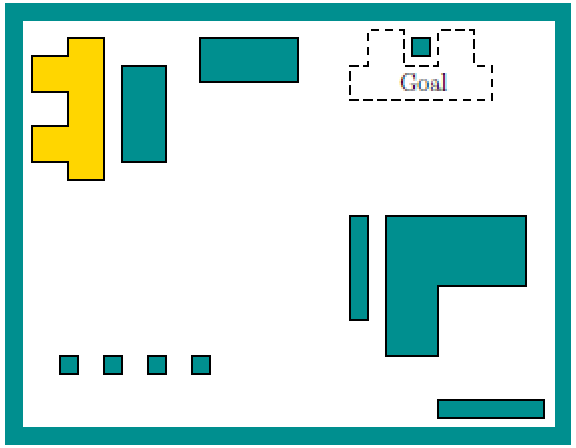
Definition: Configuration space

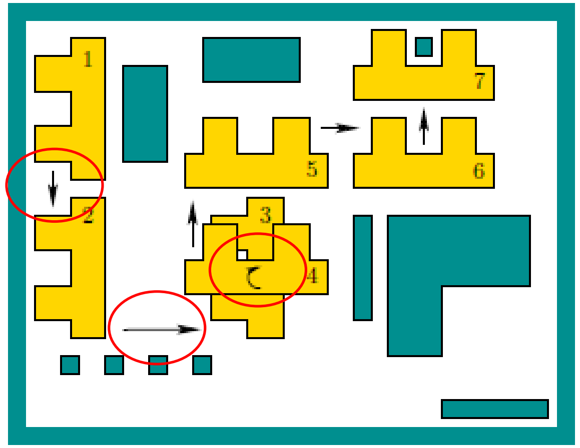
For a robot $\mathcal{A} \subset \mathcal{W}$ with n total motion DOFs, the configuration space $\mathcal{C}$ is a space with one dimension per DOF.

Remarks:

- This implies that the configuration space is not related to the workspace $\mathcal{W}$ itself, but to the motion definition of the robot $\mathcal{A}$.
- In most cases the configuration space is consists of:
  - Rotational DOFs
  - Translational DOFs

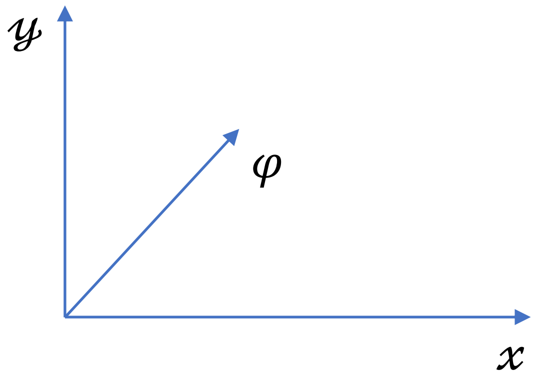
What is the configuration space of our examples?





Sources:
Motion Planning: The Essentials – LaValle - http://msl.cs.illinois.edu/~lavalle/papers/Lav11b.pdf

First DOF: Translation in x
Second DOF: Translation in y
Third DOF: Rotation in φ
Amount of DOF: 3
Notation: $SE(2)$



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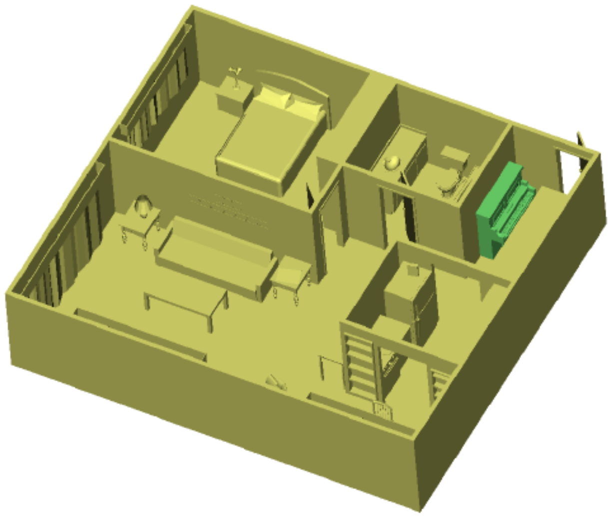

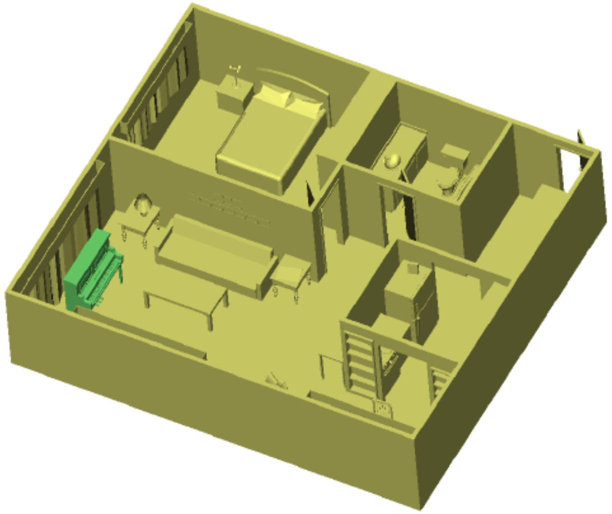

First DOF: Translation in x
Second DOF: Translation in y
Third DOF: Translation in z
Fourth DOF: Rotation in φ
Fifth DOF: Rotation in ϑ
Sixth DOF: Rotation in ψ
Amount of DOF: 6
Notation: $SE(3)$

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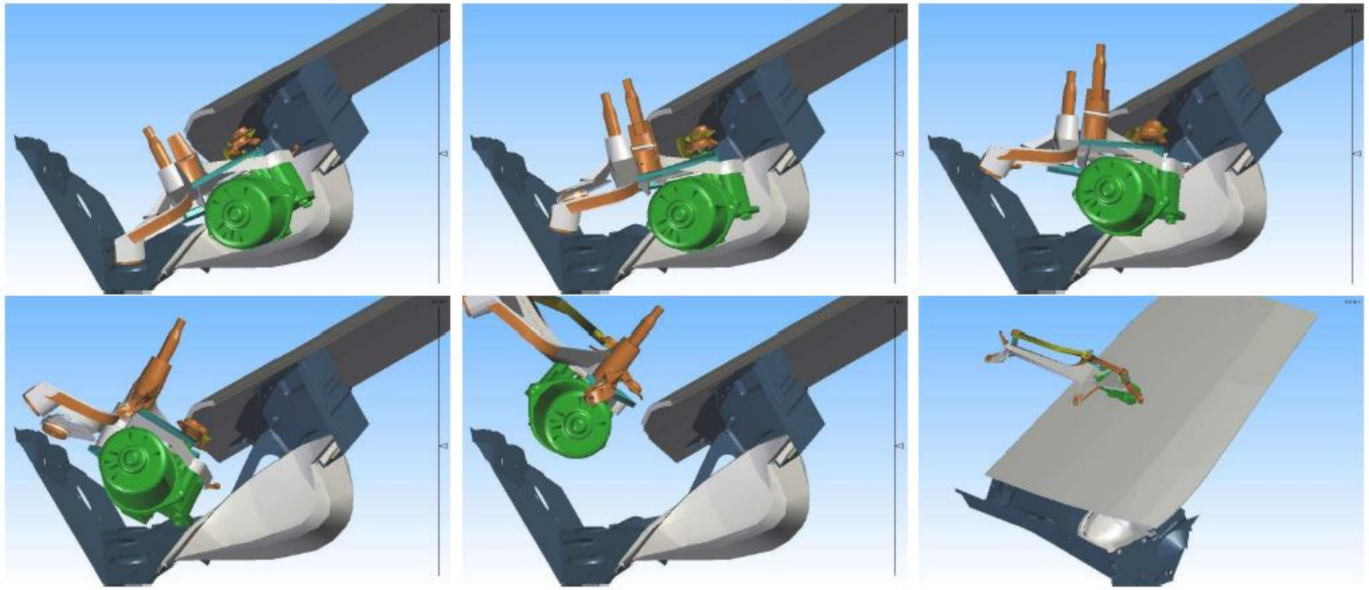

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Sources:
Planning Algorithms– LaValle - http://planning.cs.uiuc.edu/

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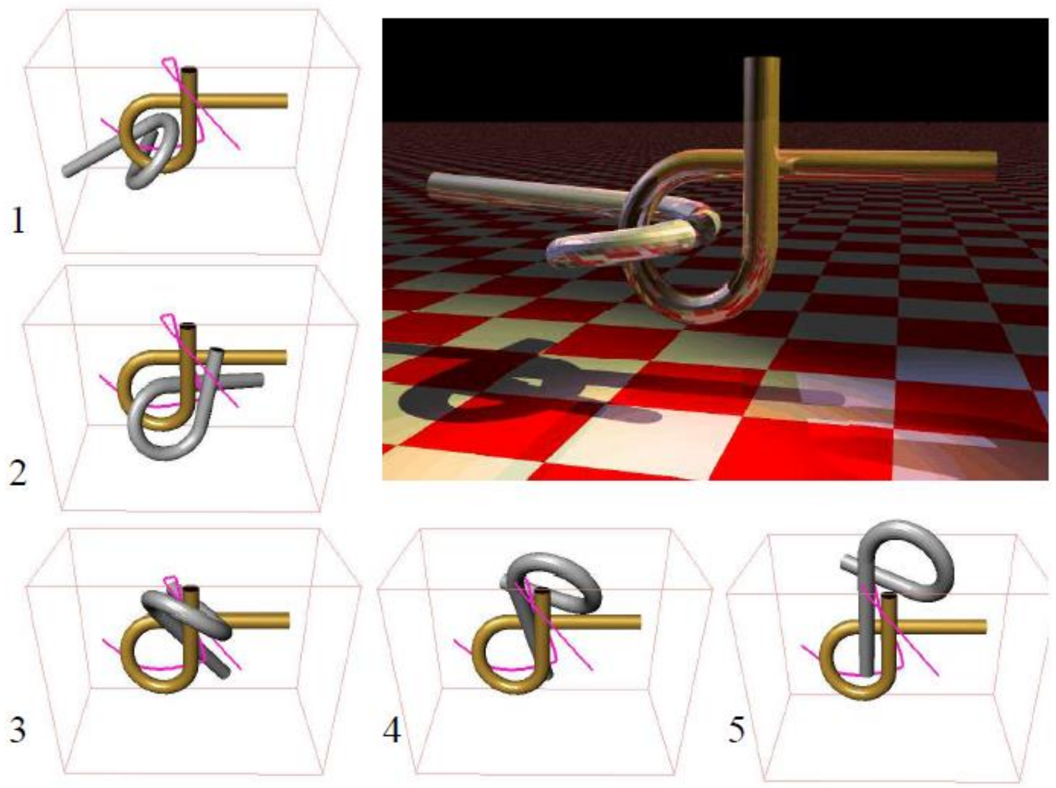

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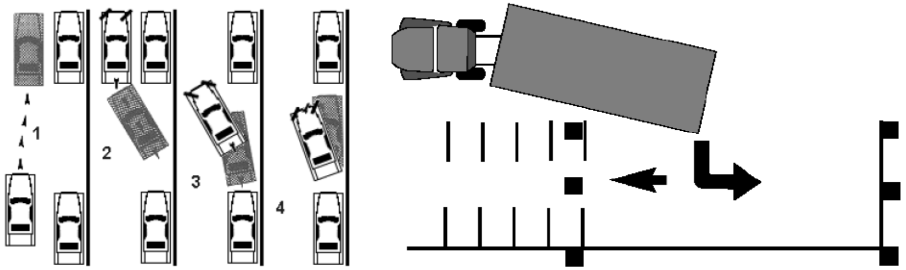


Sources:
Planning Algorithms– LaValle - http://planning.cs.uiuc.edu/

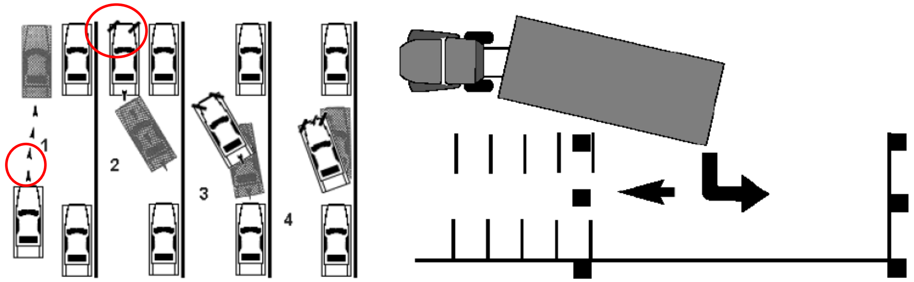
Exercise:

What are the configuration spaces of these two examples?

What is the configuration space of our examples?

For the car:
First DOF: Translation in x
Second DOF: Rotation in φ
Amount of DOF: 2

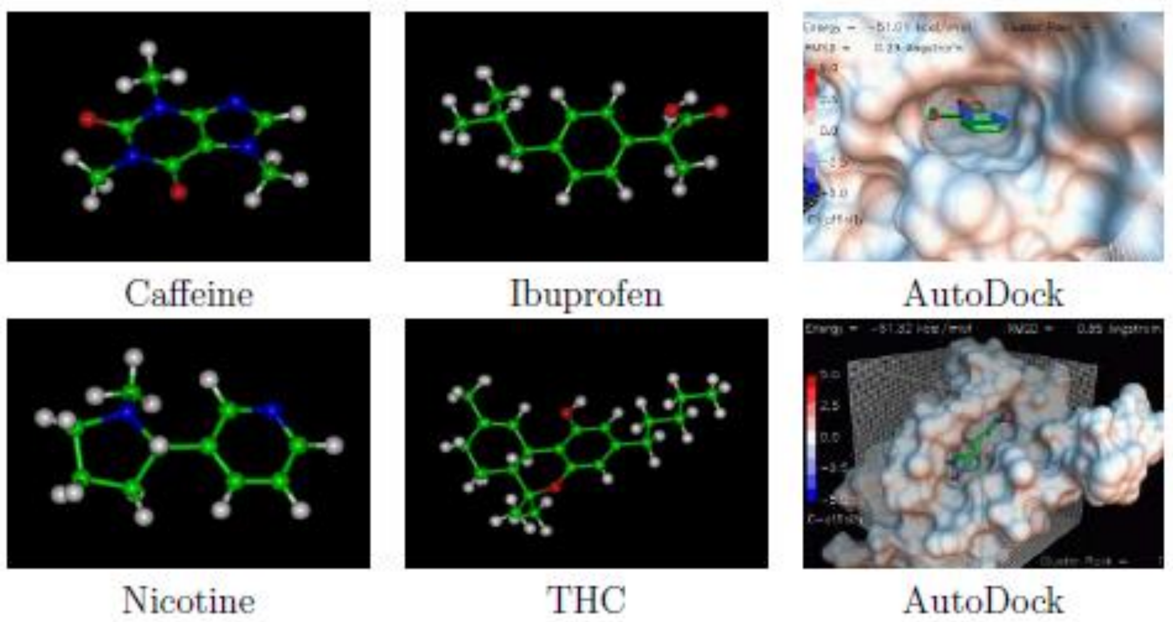
For the trailer:
First DOF: Translation in x
Second DOF: Rotation in φ
Amount of DOF: 2



Sources:
Planning Algorithms– LaValle - http://planning.cs.uiuc.edu/

Car:
The car can turn its front wheels and the engines turn one pair of wheels.
Trailer:
This is the same for the trailer. The angle between trailer and truck is not a DOF. It is resulting from the forces of the truck from start to end.

What is the configuration space of our examples?



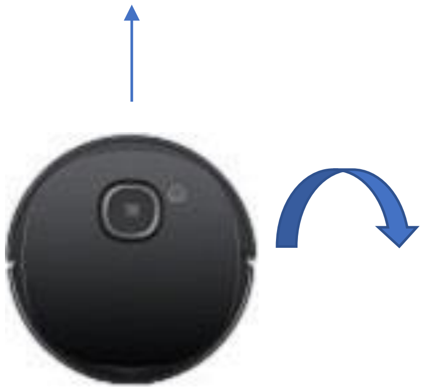
Sources:
Planning Algorithms– LaValle - http://planning.cs.uiuc.edu/

Docking process:
First DOF: Translation in x
Second DOF: Translation in y
Third DOF: Translation in z
Fourth DOF: Rotation in φ
Fifth DOF: Rotation in ϑ
Sixth DOF: Rotation in ψ
Amount of DOF: 6
Notation: $SE(3)$

BUT:
The motion is constrainted by energy functions → very challenging.
Note: Constraints → workspace.

What is the configuration space of our examples?

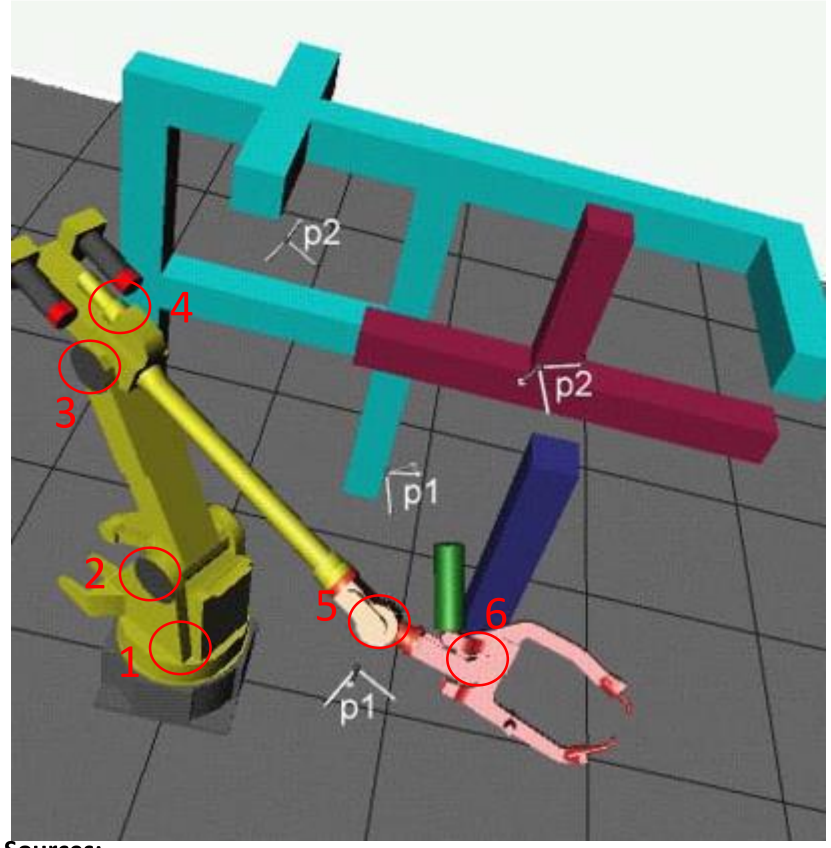
One turning wheel:
First DOF: Translation in x
Second DOF: Rotation in φ
Amount of DOF: 2



Two turning wheels:
First DOF: Translation in x
Second DOF: Translation in y
Amount of DOF: 2

Sources:
Robot Motion Planning– Wolfram Burgard et al. –
http://ais.informatik.uni-freiburg.de/teaching/ss11/robotics/slides/18-robot-motion-planning.pdf

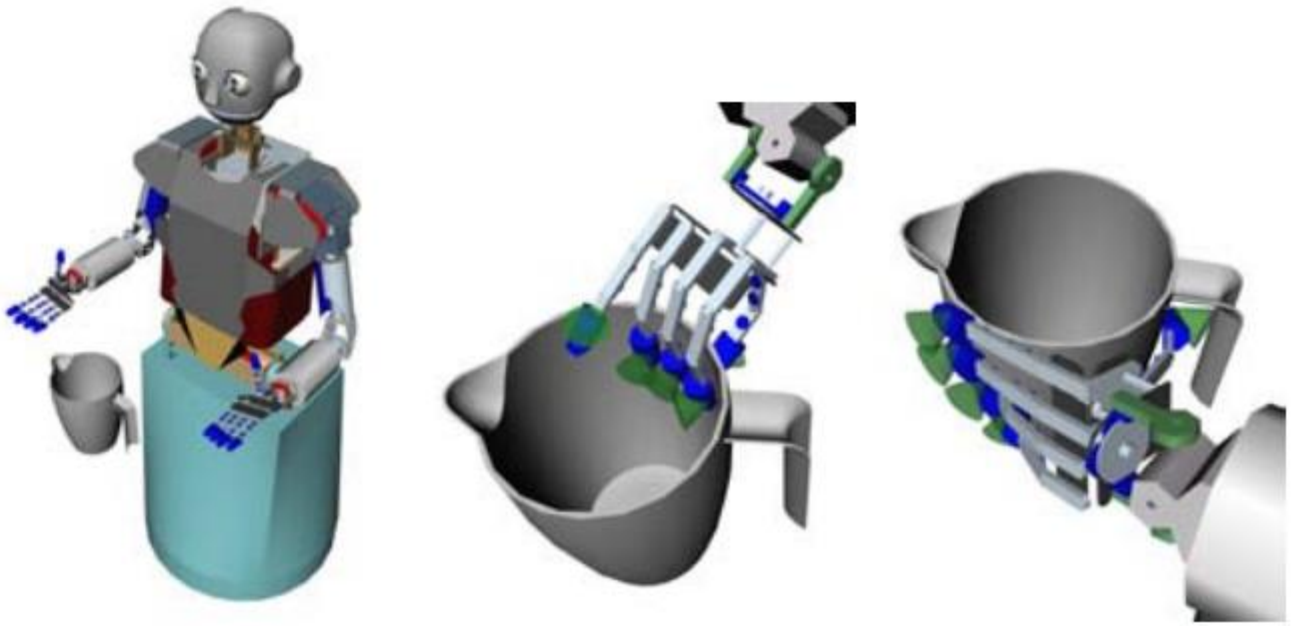
What is the configuration space of our examples?



Sources:
A Journey of Robots, Digital Actors, Molecules and Other Artifacts – Jean Claude Latombe – https://robotics.stanford.edu/~latombe/projects/motion-planning.ppt

First DOF: Rotation in φ
Second DOF: Rotation in ϑ
Third DOF: Rotation in ψ
Fourth DOF: Translation in x
Fifth DOF: Rotation in ω
Sixth DOF: Translation in y
Amount of DOF: 6

What is the configuration space of our examples?



1-5: Rotational DOFs for finger
6: Rotational DOF for hand
7: Rotational DOF for elbow
8: Rotational DOF for shoulder
Amount of DOF: 8

BUT:
The motion is constrained by a physical force simulation→ very challenging.
Note: Constraints → workspace.

Sources:
Simultaneous Grasp and Motion Planning – Nikolaus Vahrenkamp et al– https://h2t.anthropomatik.kit.edu/pdf/Vahrenkamp2012.pdf

Notes

- When programming not the whole configuration space is programmed.
- In most cases the workspace has some limits. These limits are then applied to the possible motions of the robot. → So the configuration space is also limited.

What is $\mathcal{C}_{free}$ and $\mathcal{C}_{obs}$

- **Recap:** The configuration space $\mathcal{C}$ is the space of all possible configurations (movements) of the robot $\mathcal{A}$.
- The configurations space is subdivided into two sets of configurations.
  1. There are configurations where the robot is in collision with its environment. $\mathcal{C}_{obs}$
  2. And in addition there are the configuration where the robot is free of collision. $\mathcal{C}_{free}$

Definition of $\mathcal{C}_{free}$ and $\mathcal{C}_{obs}$

Definition 2.2 (C_{obs}, C_{free}) *Let $A(q)$ be the robot in configuration $q \in C$ and O the forbidden region in W. The obstacle region $C_{obs} \subset C$ is defined by*

$$C_{obs} = \{q \in C : A(q) \cap O \neq \emptyset\}. \tag{2.1}$$

The leftover configurations of C are defined as $C_{free} = C \setminus C_{obs}$.

- **Note:** Each configuration is either part of $\mathcal{C}_{free}$ **or** $\mathcal{C}_{obs}$. The sets are disjunct.

Special configurations, c_{init}, c_{goal} and C_{goal}

- In the motion planning problem the robot starts it journey at a given configuration. This configuration is called c_{init}.
- Moreover, the goal is to reach one certain configuration c_{goal} or a set of configurations C_{goal}.

Definition of a path, the solution to the motion planning problem.

Definition 2.3 (Path) *A function* $\sigma[0,1] \rightarrow \mathbb{R}^d$ *is called,*

1. *a path, if it is continuous,*
2. *a collision–free path, if it is a path and* $\sigma(\tau) \in C_{free} \forall \tau \in [0,1]$*,*
3. *and a feasible path, if it is a collision–free path and* $\sigma(0) = x_{init}$ *and* $\sigma(1) \in X_{goal}$*.*

Note:

Computing a continuous path in real world examples is not possible (in reasonable computing times) caused by the amount of DOFs. Therefore an sufficient precise discretization is computed.

Finally the definition of the motion planning problem

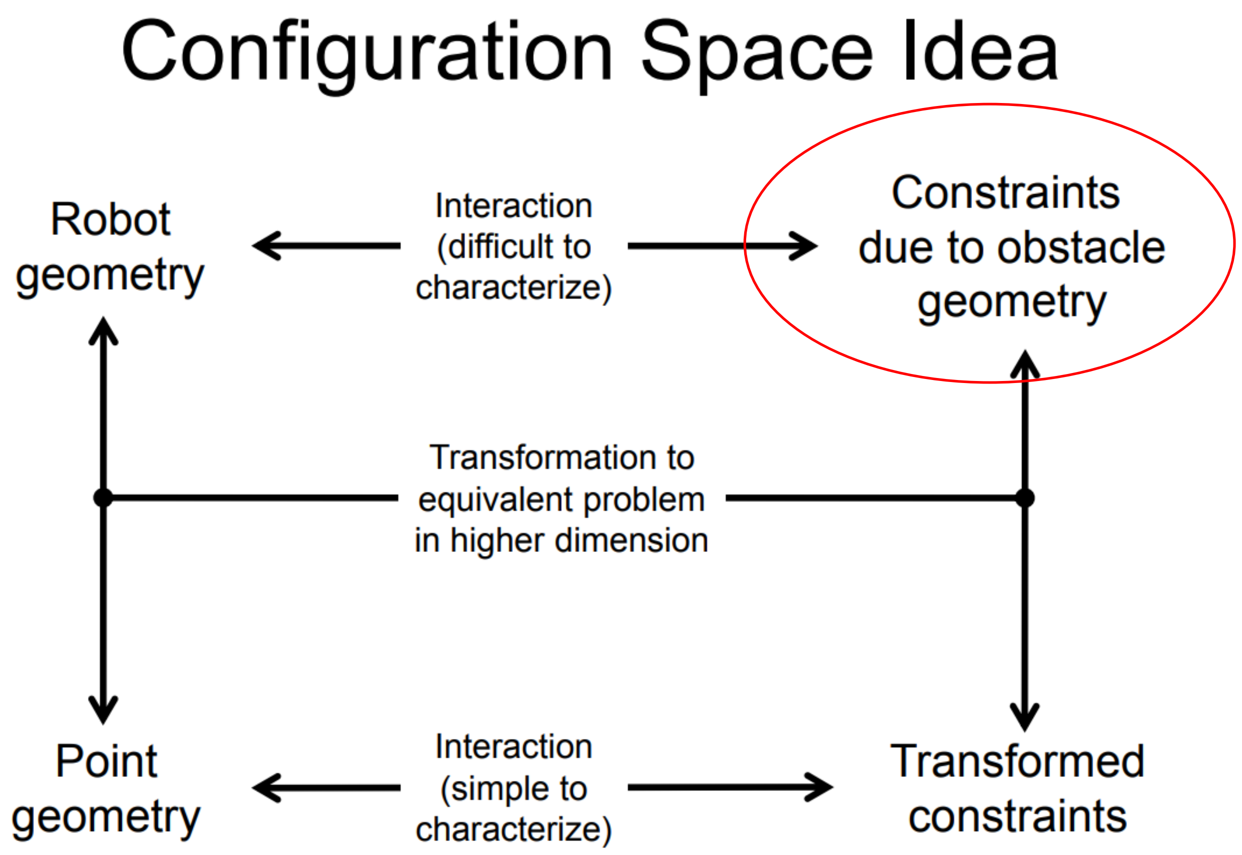
Definition 1 (*Motion Planning Problem (Single-Query)*)

Given $c_{init} \in C_{free}$, the valid initial configuration of D, and $c_{goal} \in C_{free}$, the valid goal configuration of D. The single-query motion planning problem is to find a continuous path $\sigma : [0,1] \rightarrow C_{free}$ with $\sigma(0) = c_{init}$ and $\sigma(1) = c_{goal}$, if one exists.

Note:

- We mainly consider the single-query motion planning problem. In this problem we are only interested in a single path from start to goal. In the multi-query variant we are interested in multiple paths.
- The difference is that some algorithms used in single-query motion planning are (due to performance) not used in multi-query motion planning.

Why do we introduce the configuration space at all?

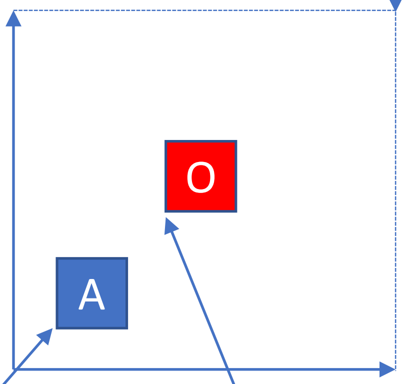


Sources:
Configuration Space Configuration Space for Motion Planning– Prof. Seth Teller – http://courses.csail.mit.edu/6.141/spring2010/pub/lectures/Lec10-ConfigurationSpace.pdf

How does the configuration space looks like?

Border of the workspace (5/5)

Workspace $\mathcal{W}$

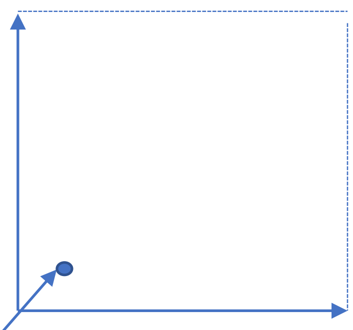


Origin of the robot (0,5/0,5)
Size of the robot 1x1

Origin of the obstacle (2/2)

Configuration Space $\mathcal{C}$

Robot can move 4 units at max
→ Therefore size of 4
(4/4)



Robot turns into a point at (0,5/0,5)
Size of the robot 1x1

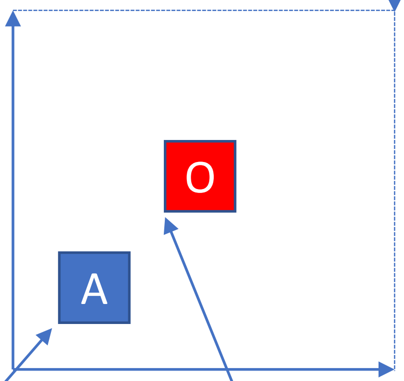
Exercise: Draw $\mathcal{C}_{obs}$

- Give coordinates of the lowest left point of $\mathcal{C}_{obs}$
- Give coordinates of the uppest right point of $\mathcal{C}_{obs}$

How does the configuration space looks like?

Border of the workspace (5/5)

Workspace $\mathcal{W}$



Origin of the robot (0,5/0,5)
Size of the robot 1x1

Origin of the obstacle (2/2)

Configuration Space $\mathcal{C}$

Robot can move 4 units at max
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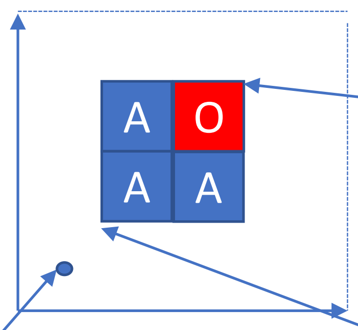
(4/4)



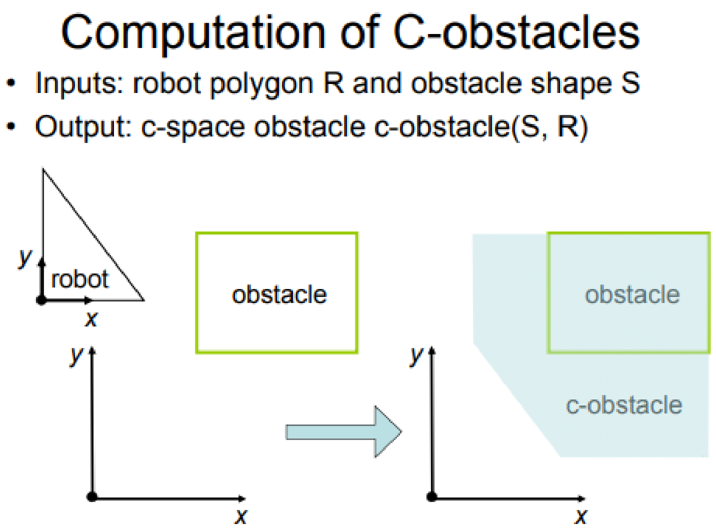
3/3, the last point
where the robot collides

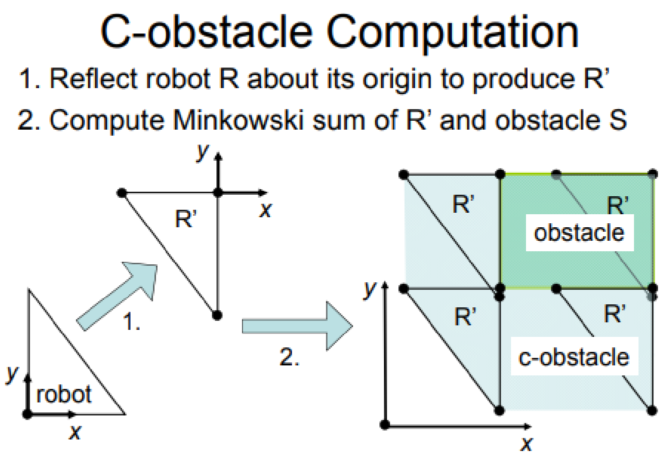
Robot turns into a point at (0,5/0,5)
Size of the robot 1x1

1/1, the closest point
where the robot collides

Some Theory

- What you have just done is computing the Minkowski Sum of the robot and its obstacle.
- Note that for solving motion planning problem with the Minkowski Sum for non symmetric robots → the robot has to be mirrored to the origin.





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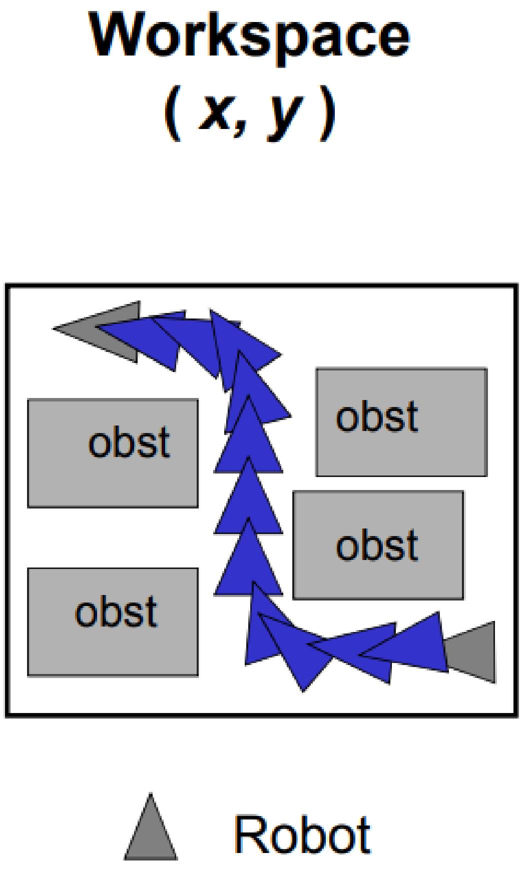
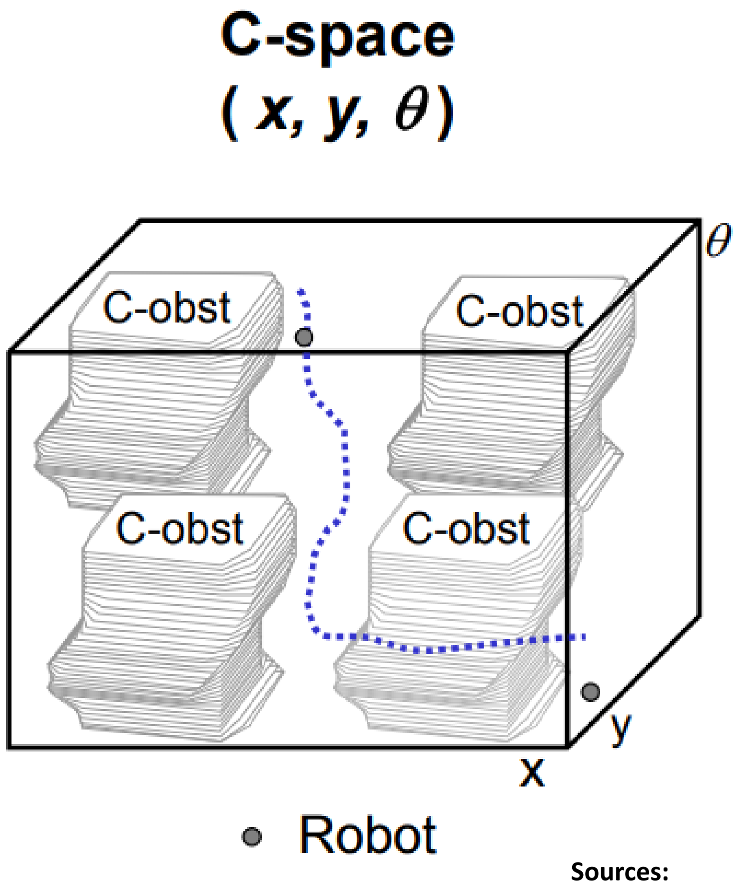
Configuration Space Configuration Space for Motion Planning– Prof. Seth Teller – http://courses.csail.mit.edu/6.141/spring2010/pub/lectures/Lec10-ConfigurationSpace.pdf

How does the configuration space looks like in higher Dimensions?





Sources:
Configuration Space Configuration Space for Motion Planning– Prof. Seth Teller –
http://courses.csail.mit.edu/6.141/spring2010/pub/lectures/Lec10-ConfigurationSpace.pdf

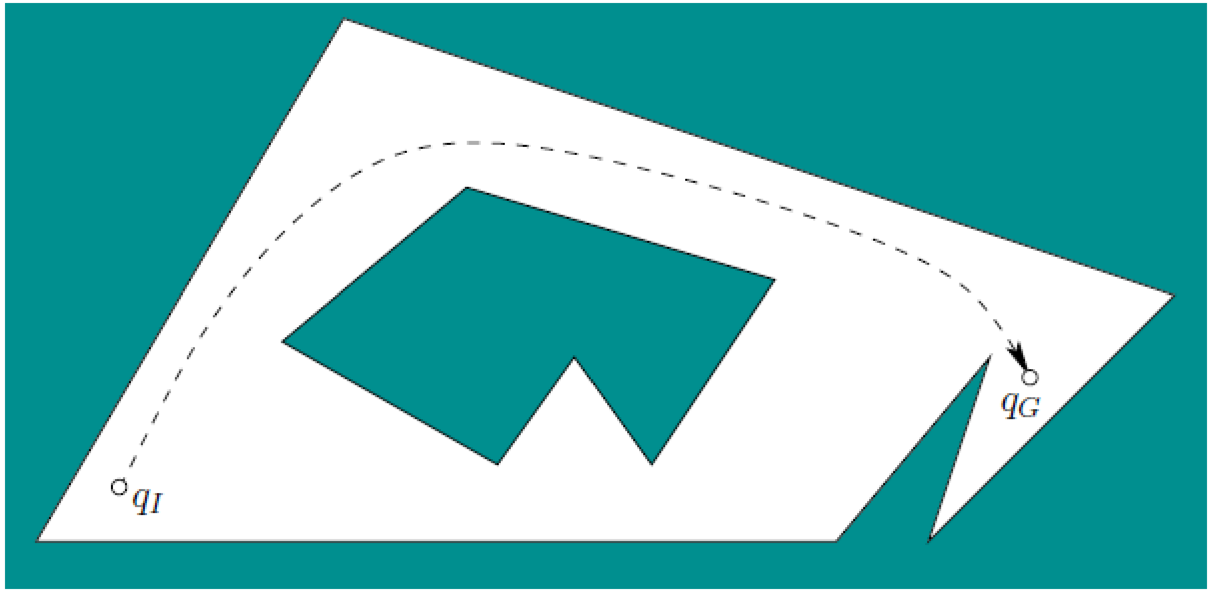
Video from Dror Atariah

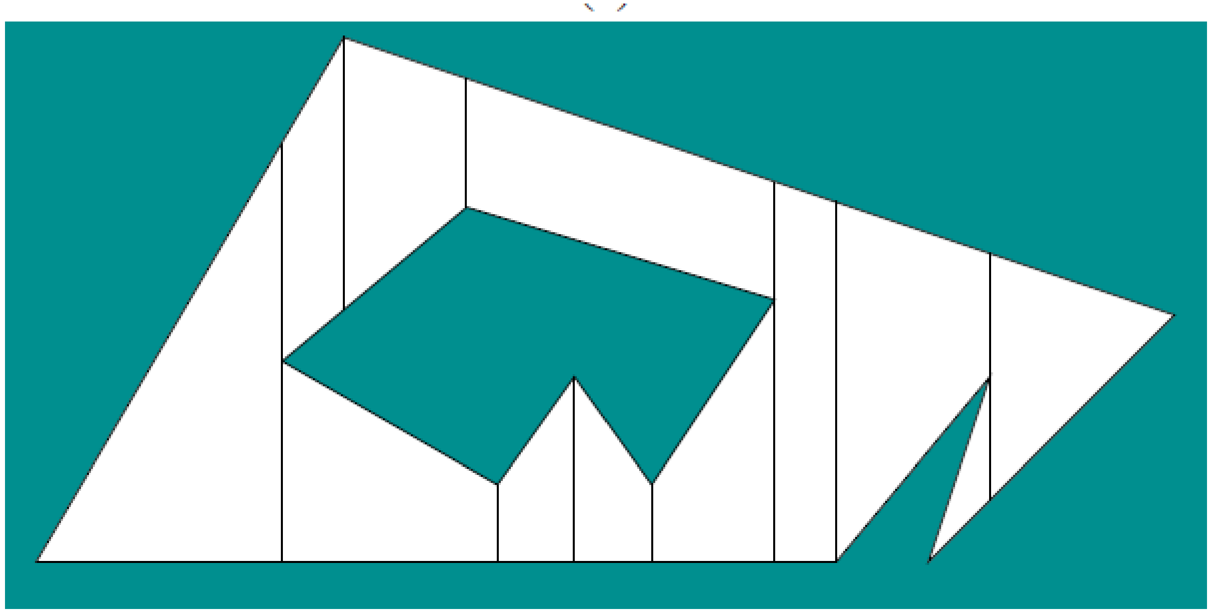
https://www.youtube.com/watch?v=SBFwgR4K1Gk

Summary

- In lower dimensions(<=3) $\mathcal{C}_{obs}$ can be computed <=3.
- This is very complex and starting from 3 DOFs it is getting hard to imagine.
- Until the late 90s, computing $\mathcal{C}_{obs}$ was the way to go.
- This is often called "continuous motion planning" or "combinatorial motion planning" or "classical motion planning".
- Newer algorithms are not longer computing $\mathcal{C}_{obs}$ → These algorithms we will consider.

How to find a path if $\mathcal{C}_{obs}$ computed





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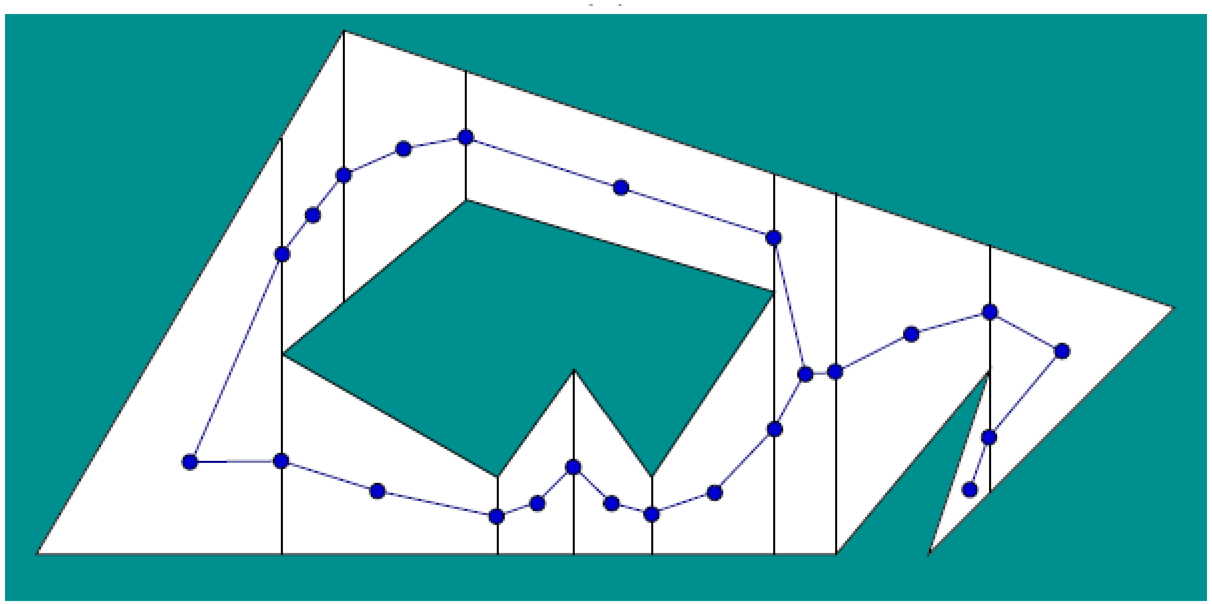

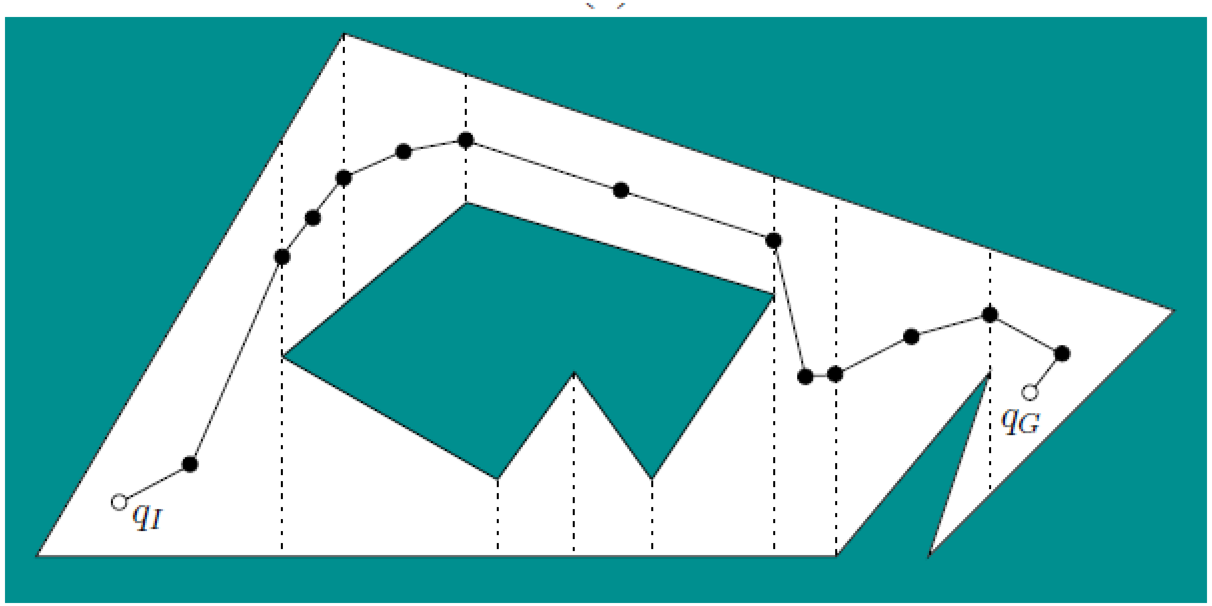

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