



Overview of my PhD projects

Philipp Krah

Retreat DAEDALUS SS22,
09. May 2022


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Overview – "successful" DAEDALUS projects

Projects not included in thesis:

- ▶ U(1) Quantum Link Model with Matrix Product States
- ▶ Multizone Sound Field Reproduction

PhD Projects included in thesis:

- ▶ Wavelet Adaptive POD
- ▶ Non-linear Reduced Order Models

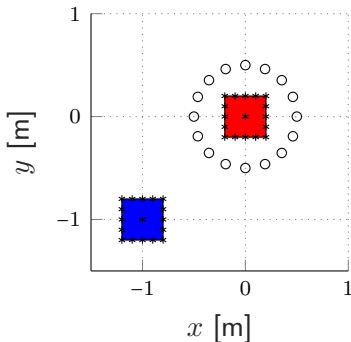
Supervised Learning for Multi Zone Sound Field Reproduction

Henry Sallandt, Mathias Lemke

[Sallandt, Krah, and Lemke 2021]



Sound Field Reproduction



Sound propagation

$$g_{nm} = a_{nm} \exp(ik\|\mathbf{x}_n - \mathbf{x}_m\|_2 + \phi_0)$$

$$p_n(t) = \sum_m g_{nm} w_m(t)$$

Bright Zone

$$\min_w \|p_{\text{goal},b} - \mathbf{G}_b \mathbf{w}\|_2^2$$

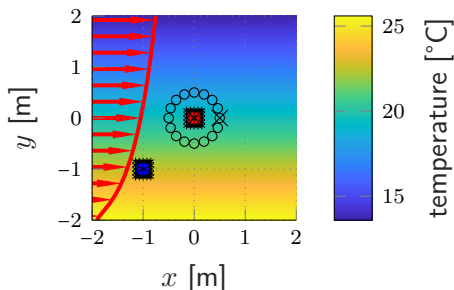
Dark Zone

$$\min_w \|p_{\text{goal},d} - \mathbf{G}_d \mathbf{w}\|_2^2$$

Optimization Problem

$$\min_w \|\hat{\mathbf{p}}_{\text{goal}} - \hat{\mathbf{G}} \mathbf{w}\|_2^2 + \lambda \|\mathbf{w}\|_2^2 \quad \hat{\mathbf{G}} = [\mathbf{G}_b, \mathbf{G}_d]$$

Sound Field Reproduction with Wind and Temperature

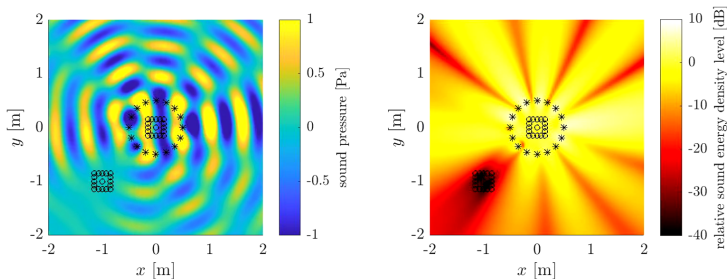


Sound propagation

$$g_{nm}^{\text{NN}}(f, Ma) = a_{nm}^{\text{NN}}(f, Ma) \cdot \exp \left(i \left(\phi_{nm}^{\text{NN}}(f, Ma) + k_{nm}^{\text{NN}}(Ma) k \Delta_{nm} \right) \right) .$$

$$\Delta_{nm} = \|\mathbf{x}_n - \mathbf{x}_m\|_2$$

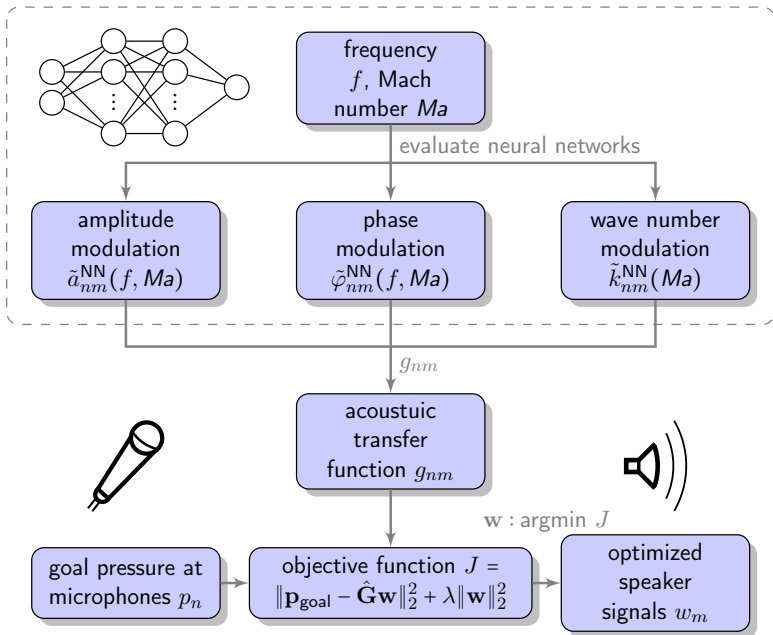
Sound Field Reproduction with Wind and Temperature



Sound propagation




$$g_{nm}^{\text{NN}}(f, Ma) = a_{nm}^{\text{NN}}(f, Ma) \cdot \exp \left(i \left(\phi_{nm}^{\text{NN}}(f, Ma) + k_{nm}^{\text{NN}}(Ma) k \Delta_{nm} \right) \right) .$$

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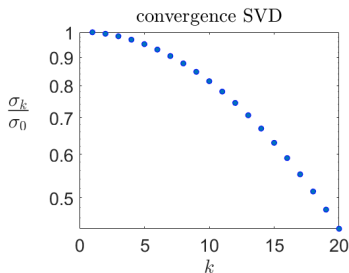
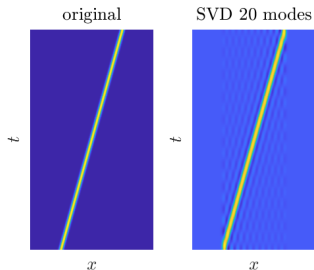


Model Order Reduction for Transport Dominated Fluid Systems

Shubhaditya Burela, Stefen Büchholz, Thomas Engels, Miriam Goldack, Matthias Häring, Martin Isoz, Anna Kovárnová, Julius Reiss, Kai Schneider, Mario Sroka

- ▶ shifted POD-ANN [Kovárnová et al. 2022] 
- ▶ shifted robust PCA [Krah et al. 2022a] 
- ▶ Front Transport Reduction 
[Krah, Sroka, and Reiss 2020]
[Krah et al. 2022b]

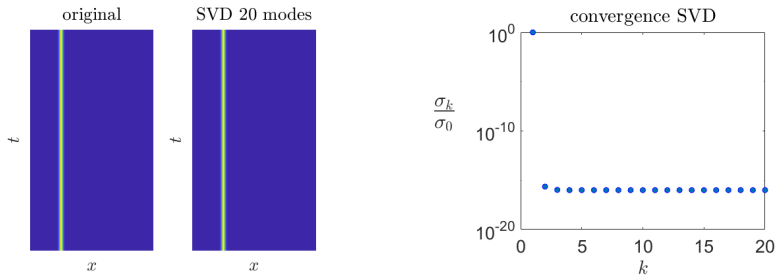
POD/SVD fails for transport dominated problems



$$q(x, t) \approx \tilde{q}(x, t) = \mathbf{U}_r \mathbf{a}(t) = \sum_{l=1}^r u_l(x) a_l(t)$$

Problem: slow singular value decay σ_k

Idea sPOD: Transport compensation [Reiss et al. 2018]



$$q(x + \Delta(t), t) \approx f(x) = \sum_{l=1}^r u_l(x) a_l(t)$$

$$\Rightarrow q(x, t) \approx T^{\Delta(t)} f(x) = f(x - \Delta(t))$$

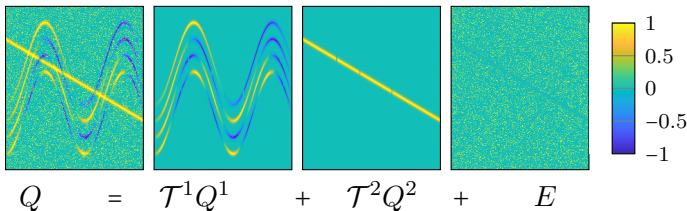
Idea: apply time-dependent shift

shifted robust PCA

For given shifts $\{\Delta_k\}$, $\lambda > 0$ and $Q \in \mathbb{R}^{M,N}$, $M > N$ with $Q_{ij} = q(x_i, t_j)$ find $\{Q^k \in \mathbb{R}^{M,N}\}$

$$\min_{Q^k} \sum_k \|Q^k\|_* + \lambda \|E\|_1 \quad \text{s.t.} \quad Q = \sum_{k=1}^F \mathcal{T}^k(Q^k)$$

where $\mathcal{T}^k(Q)_{ij} = q(x_i - \Delta(t)_i, t_j)$.



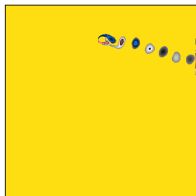
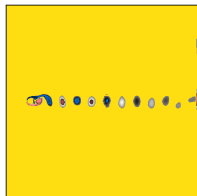
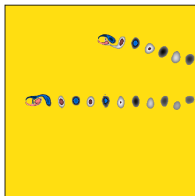
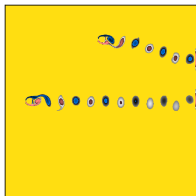
Incompressible Navier-Stokes

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \nu \nabla^2 \mathbf{u} = 0,$$

$$\nabla \cdot \mathbf{u} = 0,$$

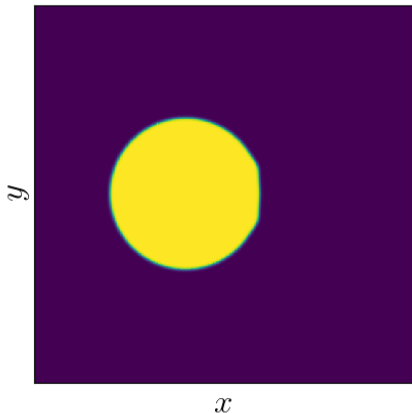
velocity $\mathbf{u} = \hat{\mathbf{u}}/\rho,$ pressure $p = \mathbf{p}/\rho,$

dynamic viscosity $\nu = \mu/\rho$



$$q(\mathbf{x}, t) \approx \tilde{q}(\mathbf{x}, t) = \mathcal{T}^1[q^1(\mathbf{x}, t)] + \mathcal{T}^2[q^2(\mathbf{x}, t)]$$

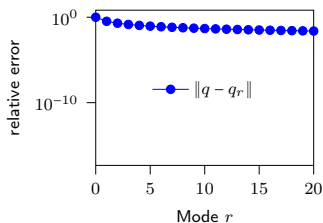
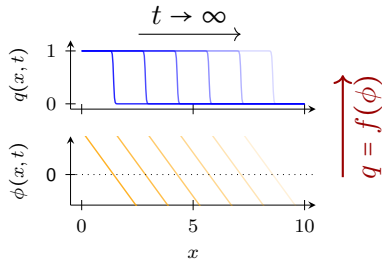
Often no 1 to 1 mapping possible:



Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x, t) \approx q_r(x, t) := \sum_{k=1}^r u_k(x) a_k(t)$$

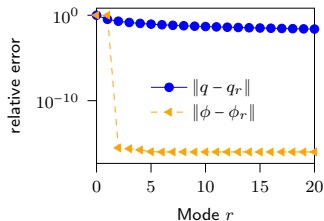
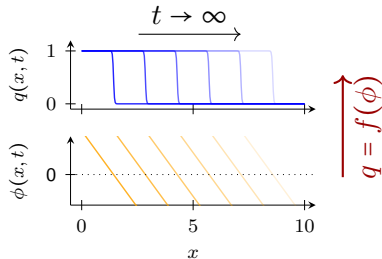


Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x, t) \approx q_r(x, t) := \sum_{k=1}^r u_k(x) a_k(t)$$

$$\phi(x, t) \approx \phi_r(x, t) := \sum_{k=1}^r \psi_k(x) \tilde{a}_k(t)$$



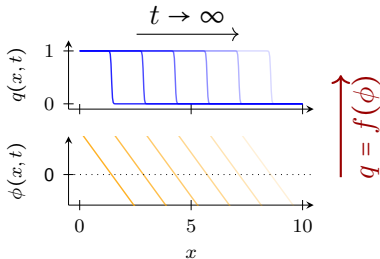
Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x, t) \approx q_r(x, t) := \sum_{k=1}^r u_k(x) a_k(t)$$

$$q(x, t) = f(\phi(x, t))$$

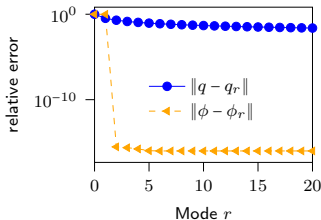
$$\phi(x, t) \approx \phi_r(x, t) := \sum_{k=1}^r \psi_k(x) \tilde{a}_k(t)$$



Transport compensation

$$\begin{aligned} q(x, t) &= T^{\Delta(t)}(f(x)) \\ &= f(x - \Delta(t)) \\ &= f(\phi(x, t)) \end{aligned}$$

$\Delta(t)$ front location

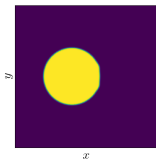


Advection-diffusion-reaction PDE (+ Periodic BC)

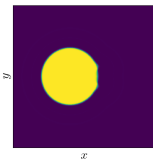
$$\begin{cases} \partial_t q(\mathbf{x}, t, \mu) &= -\mathbf{v} \cdot \nabla q + \kappa \Delta q - \mu q^2(q - 1) \\ q(\mathbf{x}, 0, \mu) &= q_0(\mathbf{x}) \end{cases}$$

with velocity field $\mathbf{v}(\mathbf{x}, t)$. Tuned to include topology change.

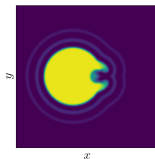
FOM
DoFs = 512^2 :



FTR-ROM
DoFs: = 6



POD-ROM
DoFs: = 6



References:



Kovárnová, A. et al. (2022). “Shifted Proper Orthogonal Decomposition and Artificial Neural Networks for Time-Continuous Reduced Order Models of Transport-Dominated Systems”. In: *Topical Problems of Fluid Mechanics 2022*. Prague. ISBN: 978-80-87012-77-2. DOI: 10.14311/TPFM.2022.016. URL: <https://doi.org/10.14311/TPFM.2022.016>.



Krah, P., M. Sroka, and J. Reiss (2020). “Model Order Reduction of Combustion Processes with Complex Front Dynamics”. In: *ENUMATH 2019*, 803–811.



Krah, P. et al. (Feb. 2022a). “Data-driven reduced order modeling for flows with moving geometries using shifted POD”. In: *10th Vienna International Conference on Mathematical Modelling, (MATHMOD 2022)*. Vienna, Austria. URL: <https://hal.archives-ouvertes.fr/hal-03396325>.



Krah, P. et al. (2022b). “Front Transport Reduction for Complex Moving Fronts”. In: *arXiv preprint arXiv:2202.08208*.



Krah, P. et al. (2022c). “Wavelet adaptive proper orthogonal decomposition for large-scale flow data”. In: *Advances in Computational Mathematics* 48.2, pp. 1–40.



Reiss, J. et al. (2018). “The shifted proper orthogonal decomposition: A mode decomposition for multiple transport phenomena”. In: *SIAM Journal on Scientific Computing* 40.3, A1322–A1344.



Sallandt, H., P. Krah, and M. Lemke (2021). “Supervised Learning for Multi Zone Sound Field Reproduction under Harsh Environmental Conditions”. In: *arXiv preprint arXiv:2112.07349*.



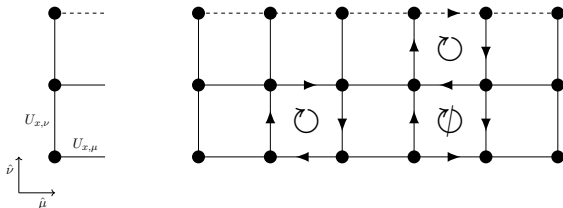
Stornati, P. et al. (Nov. 2021). “Phases at finite winding number of an Abelian lattice gauge theory”. In: *38th International Symposium on Lattice Field Theory*. arXiv: 2111.09364 [hep-lat].

U(1) Quantum Link Model with Matrix Product States

Debasish Banerjee, Karl Jansen, Paolo Stornati

[Stornati et al. 2021]





Hamiltonian

$$\mathcal{H} = -J \sum_{\square} (U_{\square} + U_{\square}^{\dagger}) + \sum_{\square} \lambda (U_{\square} + U_{\square}^{\dagger})^2,$$

Plaquette Operators

$$U_{\square} = U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^{\dagger} U_{x,\nu}^{\dagger}$$

Gauge Operator

$$G_x = \sum_{\mu} (E_{x-\hat{\mu},\mu} - E_{x,\mu}), \quad [\mathcal{H}, G_x] = 0$$

Quantum Links

$$U_{x,\mu} = S_{x,\mu}^{+}, \quad U_{x,\mu}^{\dagger} = S_{x,\mu}^{-}$$

Electric Field Operator

$$E_{x,\mu}^{\dagger} = S_{x,\mu}^z$$

Zero Charge Condition:

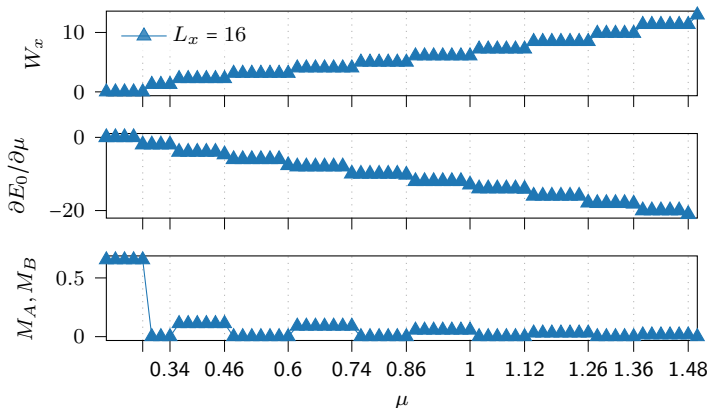
$$G_x |\Psi\rangle = 0$$

$$\text{MPS: } |\Psi\rangle = \sum_{j_1, \dots, j_N} A^{[1]j_1} A^{[2]j_2} \dots A^{[N]j_N} |j_1, \dots, j_N\rangle$$

Winding Number Sectors

$$W_y = \frac{1}{2L_x} \sum_x^{L_x} S_{(x,y),\hat{y}}^z \text{ with } [\mathcal{H}, W_y] = 0 \text{ therefore}$$

$$\mathcal{H}^{\text{chem}} = \mathcal{H} + \mu \sum_y W_y$$



Wavelet Adaptive POD for large-scale Flow Data

Thomas Engels, Kai Schneider, Julius Reiss

[Krah et al. 2022c]



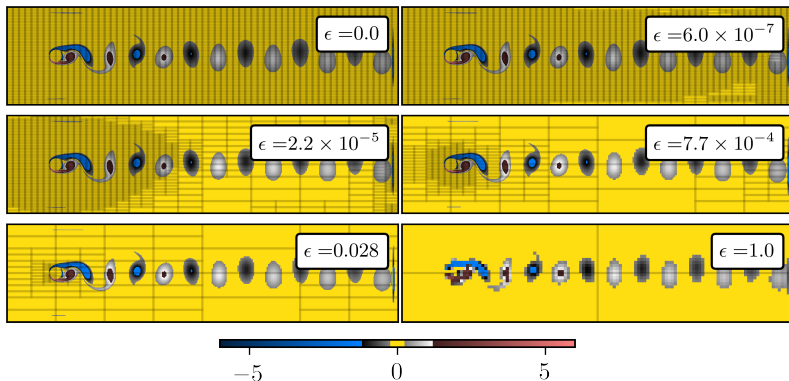
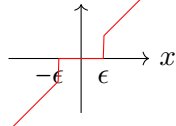
Biorthogonal, interpolating wavelets $\{\phi_\lambda^j, \psi_{\mu\lambda}^j\}_{j,\lambda,\mu}$ (CDF44)

$$q = \sum_{\lambda \in \overline{\Lambda}^{J_{\min}}} q_\lambda^{J_{\min}} \varphi_\lambda^{J_{\min}} + \sum_{j=J_{\min}}^{J_{\max}-1} \sum_{\lambda \in \overline{\Lambda}^j} \sum_{\mu=1}^3 d_{\mu\lambda}^j \psi_{\mu,\lambda}^j,$$

Biorthogonal, interpolating wavelets $\{\phi_\lambda^j, \psi_{\mu\lambda}^j\}_{j,\lambda,\mu}$ (CDF44)

$$q^\epsilon = \sum_{\lambda \in \bar{\Lambda}^{J_{\min}}} q_\lambda^{J_{\min}} \varphi_\lambda^{J_{\min}} + \sum_{j=J_{\min}}^{J_{\max}-1} \sum_{\lambda \in \bar{\Lambda}^j} \sum_{\mu=1}^3 \mathcal{T}_\epsilon(d_{\mu\lambda}^j) \psi_{\mu,\lambda}^j,$$

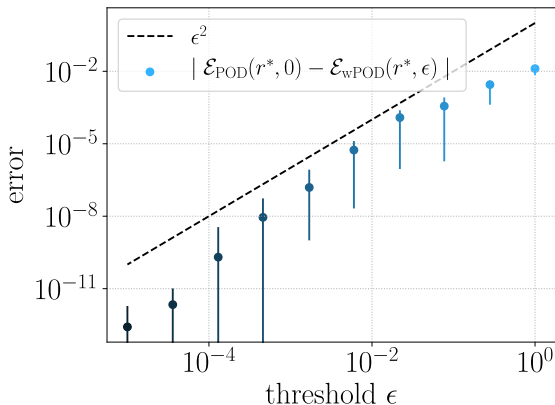
$$\mathcal{T}_\epsilon(x) = x \mathcal{O}(|x| - \epsilon)$$



Error Analysis – Total error

Under the assumption that the perturbation of eigenvalues is small in comparison to the the total energy $\mathcal{M}_r < \epsilon$

$$\mathcal{E}_{\text{wPOD}}(\epsilon, r) \lesssim \mathcal{E}_{\text{POD}}(0, r) + \epsilon^2$$



Bumblebee - Reconstruction

Reconstruction: $\tilde{q}^\epsilon(\mathbf{x}, t_i) = \sum_{k=1}^r a_k(t_i) \phi_k^\epsilon(\mathbf{x})$

