DAEDALUS
RTG 2433

Overview of my PhD projects

Philipp Krah

Retreat DAEDALUS SS22,
09. May 2022

Contact: `krah@math.tu-berlin.de`
Follow: `philipp137.github.io`

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Overview – "successfull" DAEDALUS projects

Projects not included in thesis:

- U(1) Quantum Link Model with Matrix Product States
- Multizone Sound Field Reproduction

PhD Projects included in thesis:

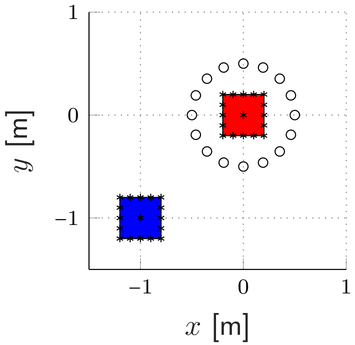
- Wavelet Adaptive POD
- Non-linear Reduced Order Models

Supervised Learning for Multi Zone Sound Field Reproduction

Henry Sallandt, Mathias Lemke

[Sallandt, Krah, and Lemke 2021]

Sound Field Reproduction



Sound propagation

$$g_{nm} = a_{nm} \exp(ik\|\boldsymbol{x}_n - \boldsymbol{x}_m\|_2 + \phi_0)$$

$$p_n(t) = \sum_m g_{nm} w_m(t)$$

Bright Zone

$$\min_{\boldsymbol{w}} \|\boldsymbol{p}_{\mathsf{goal,b}} - \mathbf{G}_{\mathsf{b}} \boldsymbol{w}\|_2^2$$

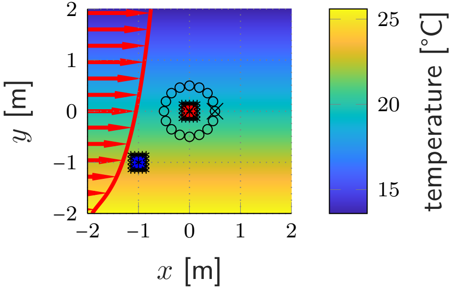
Dark Zone

$$\min_{\boldsymbol{w}} \|\boldsymbol{p}_{\mathsf{goal,d}} - \mathbf{G}_{\mathsf{d}} \boldsymbol{w}\|_2^2$$

Optimization Problem

$$\min_{\boldsymbol{w}} \|\hat{\boldsymbol{p}}_{\mathsf{goal}} - \hat{\mathbf{G}} \boldsymbol{w}\|_2^2 + \lambda \|\boldsymbol{w}\|_2^2 \qquad \hat{\mathbf{G}} = [\mathbf{G}_{\mathsf{b}}, \mathbf{G}_{\mathsf{d}}]$$

Sound Field Reproduction with Wind and Temperature

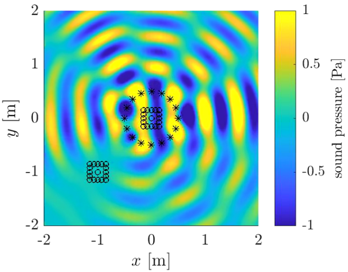


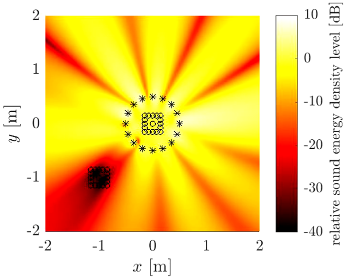
Sound propagation

$$g_{nm}^{\mathsf{NN}}(f,Ma) = a_{nm}^{\mathsf{NN}}(f,Ma) \cdot \exp\left(i\left(\phi_{nm}^{\mathsf{NN}}(f,Ma) + k_{nm}^{\mathsf{NN}}(Ma)k\Delta_{nm}\right)\right).$$

$$\Delta_{nm} = \|\boldsymbol{x}_n - \boldsymbol{x}_m\|_2$$

Sound Field Reproduction with Wind and Temperature





Sound propagation

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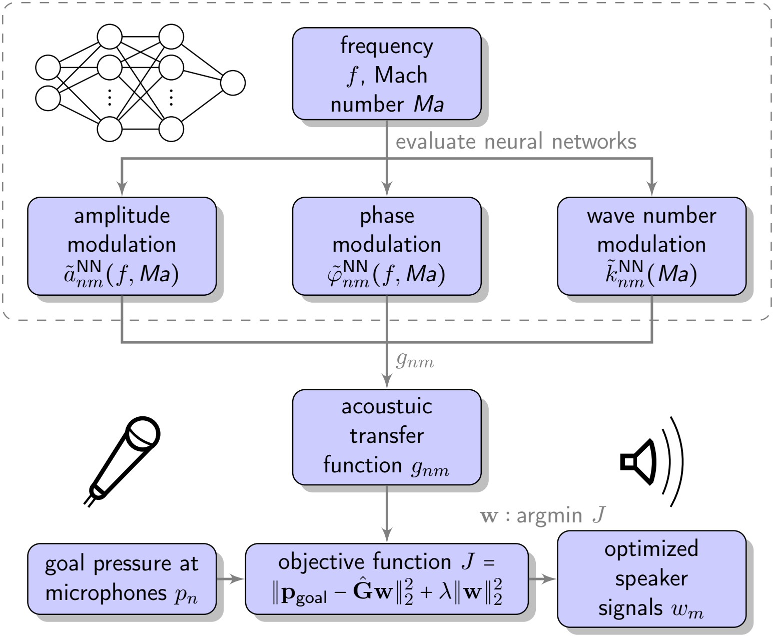


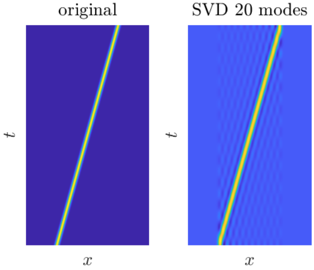
frequency f, Mach number *Ma*
evaluate neural networks
amplitude modulation $\tilde{a}_{nm}^{\mathrm{NN}}(f, Ma)$
phase modulation $\tilde{\varphi}_{nm}^{\mathrm{NN}}(f, Ma)$
wave number modulation $\tilde{k}_{nm}^{\mathrm{NN}}(Ma)$
g_{nm}
acoustuic transfer function g_{nm}
$\mathbf{w}$: argmin J
goal pressure at microphones p_n
objective function $J = \|\mathbf{p}_{\mathrm{goal}} - \hat{\mathbf{G}}\mathbf{w}\|_2^2 + \lambda\|\mathbf{w}\|_2^2$
optimized speaker signals w_m

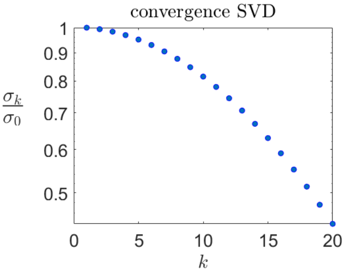
Model Order Reduction for Transport Dominated Fluid Systems

Shubhaditya Burela, Stefen Büchholz, Thomas Engels, Miriam Goldack, Matthias Häringer, Martin Isoz, Anna Kovárnová, Julius Reiss, Kai Schneider, Mario Sroka

- shifted POD-ANN [Kovárnová et al. 2022]
- shifted robust PCA [Krah et al. 2022a]
- Front Transport Reduction
  [Krah, Sroka, and Reiss 2020]
  [Krah et al. 2022b]

POD/SVD fails for transport dominated problems

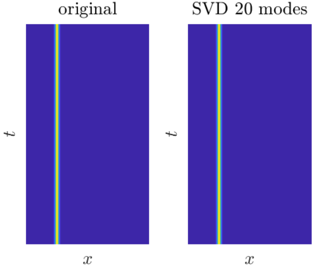


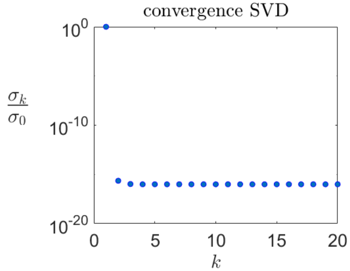


$$q(x,t) \approx \tilde{q}(x,t) = \mathbf{U}_r \boldsymbol{a}(t) = \sum_{l=1}^{r} u_l(x) a_l(t)$$

Problem: slow singular value decay σ_k

Idea sPOD: Transport compensation [Reiss et al. 2018]





$$q(x+\Delta(t),t) \approx f(x) = \sum_{l=1}^{r} u_l(x) a_l(t)$$

$$\Rightarrow q(x,t) \approx T^{\Delta(t)} f(x) = f(x-\Delta(t))$$

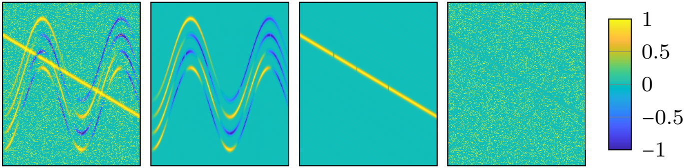
Idea: apply time-dependent shift

shifted robust PCA

For given shifts $\{\Delta_k\}$, $\lambda > 0$ and $Q \in \mathbb{R}^{M,N}, M > N$ with $Q_{ij} = q(x_i, t_j)$ find $\{Q^k \in \mathbb{R}^{M,N}\}$

$$\min_{Q^k} \sum_k \|Q^k\|_* + \lambda \|E\|_1 \quad \text{s.t.} \quad Q = \sum_{k=1}^{F} \mathcal{T}^k(Q^k)$$

where $\mathcal{T}^k(Q)_{ij} = q(x_i - \Delta(t)_i, t_j)$.



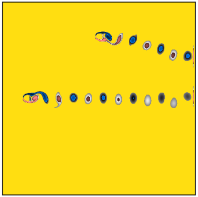
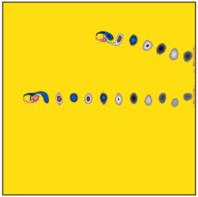
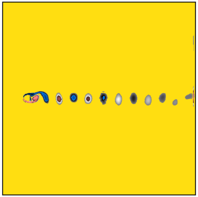
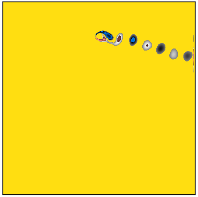
$Q = \mathcal{T}^1 Q^1 + \mathcal{T}^2 Q^2 + E$

Incompressible Navier-Stokes

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p - \nu \nabla^2 \boldsymbol{u} = 0\,,$$
$$\nabla \cdot \boldsymbol{u} = 0\,,$$

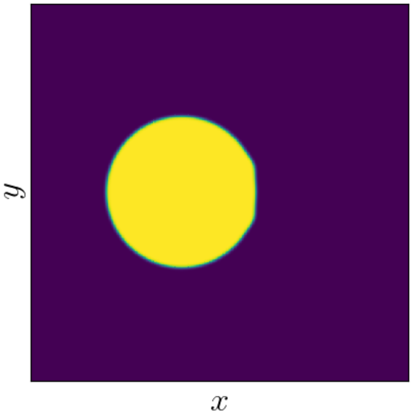
velocity $\boldsymbol{u} = \hat{\boldsymbol{u}}/\rho\,,$ pressure $p = \boldsymbol{p}/\rho,$

dynamic viscousity $\nu = \mu/\rho$

$$q(\boldsymbol{x},t) \approx \tilde{q}(\boldsymbol{x},t) = \mathcal{T}^1[q^1(\boldsymbol{x},t)] + \mathcal{T}^2[q^2(\boldsymbol{x},t)]$$

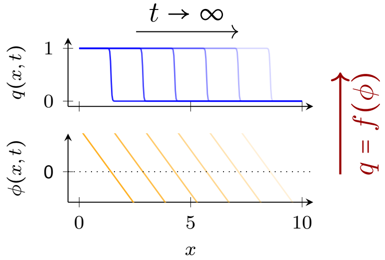
Often no 1 to 1 mapping possible:

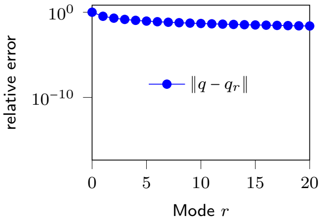


Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x,t) \approx q_r(x,t) \coloneqq \sum_{k=1}^{r} u_k(x) a_k(t)$$



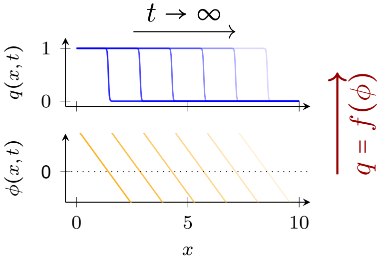


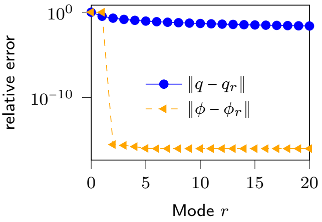
Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x,t) \approx q_r(x,t) \coloneqq \sum_{k=1}^{r} u_k(x) a_k(t)$$

$$\phi(x,t) \approx \phi_r(x,t) \coloneqq \sum_{k=1}^{r} \psi_k(x) \tilde{a}_k(t)$$





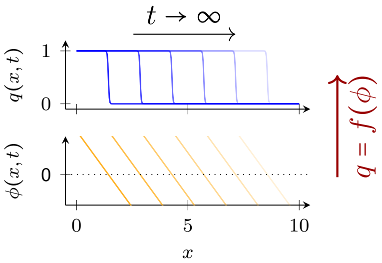
Idea: Front Transport Reduction (FTR)

[Krah, Sroka, and Reiss 2020, Krah et al. 2022b]

$$q(x,t) \approx q_r(x,t) \coloneqq \sum_{k=1}^{r} u_k(x) a_k(t)$$

$$q(x,t) = f(\phi(x,t))$$

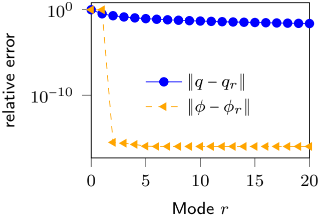
$$\phi(x,t) \approx \phi_r(x,t) \coloneqq \sum_{k=1}^{r} \psi_k(x) \tilde{a}_k(t)$$



Transport compensation

$$\begin{aligned} q(x,t) &= T^{\Delta(t)}(f(x)) \\ &= f(x - \Delta(t)) \\ &= f(\phi(x,t)) \end{aligned}$$

$\Delta(t)$ front location

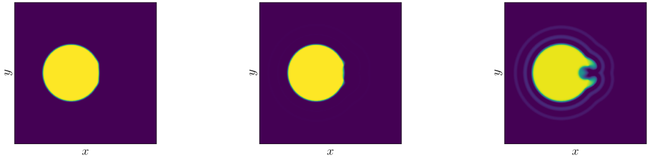


Advection-diffusion-reaction PDE (+ Periodic BC)

$$\begin{cases} \partial_t q(\boldsymbol{x}, t, \mu) & = -\boldsymbol{v} \cdot \nabla q + \kappa \Delta q - \mu q^2 (q-1) \\ q(\boldsymbol{x}, 0, \mu) & = q_0(\boldsymbol{x}) \end{cases}$$

with velocity field $\boldsymbol{v}(\boldsymbol{x}, t)$. Tuned to include topology change.

FOM	FTR-ROM	POD-ROM
DoFs $= 512^2$:	**DoFs:** $= 6$	**DoFs:** $= 6$



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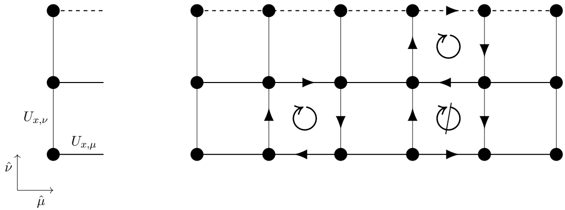
U(1) Quantum Link Model with Matrix Product States

Debasish Banerjee, Karl Jansen, Paolo Stornati

[Stornati et al. 2021]

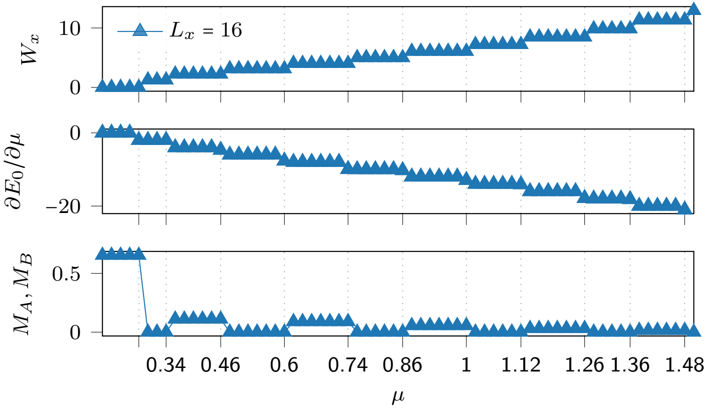


Hamiltonian	$\mathcal{H} = -J\sum_{\square}(U_{\square} + U_{\square}^{\dagger}) + \sum_{\square}\lambda(U_{\square} + U_{\square}^{\dagger})^2\,,$
Plaquette Operators	$U_{\square} = U_{x,\mu}U_{x+\hat{\mu},\nu}U^{\dagger}_{x+\hat{\nu},\mu}U^{\dagger}_{x,\nu}$
Gauge Operator	$G_x = \sum_{\mu}\left(E_{x-\hat{\mu},\mu} - E_{x,\mu}\right),\ [\mathcal{H}, G_x] = 0$
Quantum Links	$U_{x,\mu} = S^{+}_{x,\mu},\ U^{\dagger}_{x,\mu} = S^{-}_{x,\mu}$
Electric Field Operator	$E^{\dagger}_{x,\mu} = S^{z}_{x,\mu}$
Zero Charge Condition:	$G_x\lvert\Psi\rangle = 0$

MPS: $|\Psi\rangle = \sum_{j_1,\ldots,j_N} A^{[1]j_1}A^{[2],j_2}\ldots A^{[N]j_N}|j_1,\ldots,j_N\rangle$

Winding Number Sectors

$W_y = \frac{1}{2L_x} \sum_x^{L_x} S^z_{(x,y),\hat{y}}$ with $[\mathcal{H}, W_y] = 0$ therefore
$\mathcal{H}^{\text{chem}} = \mathcal{H} + \mu \sum_y W_y$



Wavelet Adaptive POD for large-scale Flow Data

Thomas Engels, Kai Schneider, Julius Reiss

[Krah et al. 2022c]

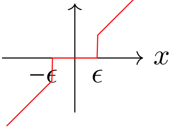
Biorthogonal, interpolating wavelets $\{\phi_\lambda^j, \psi_{\mu\lambda}^j\}_{j,\lambda,\mu}$ (CDF44)

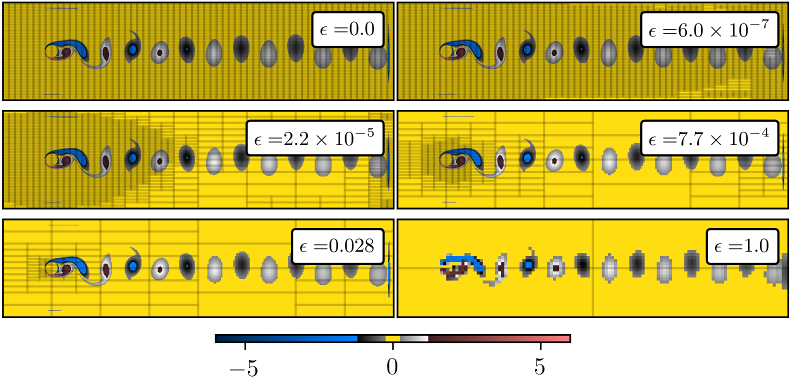
$$q = \sum_{\lambda \in \overline{\Lambda}^{J_{\min}}} \mathsf{q}_\lambda^{J_{\min}} \varphi_\lambda^{J_{\min}} + \sum_{j=J_{\min}}^{J_{\max}-1} \sum_{\lambda \in \overline{\Lambda}^j} \sum_{\mu=1}^{3} \mathsf{d}_{\mu\lambda}^j \ \psi_{\mu,\lambda}^j,$$

Biorthogonal, interpolating wavelets $\{\phi^j_\lambda, \psi^j_{\mu\lambda}\}_{j,\lambda,\mu}$ (CDF44)

$$\mathcal{T}_\epsilon(x) = x\mathcal{O}(|x| - \epsilon)$$

$$q^\epsilon = \sum_{\lambda\in\overline{\Lambda}^{J_{\min}}} \mathsf{q}^{J_{\min}}_\lambda \varphi^{J_{\min}}_\lambda + \sum_{j=J_{\min}}^{J_{\max}-1} \sum_{\lambda\in\overline{\Lambda}^j} \sum_{\mu=1}^{3} \mathcal{T}_\epsilon(\mathsf{d}^j_{\mu\lambda})\psi^j_{\mu,\lambda},$$

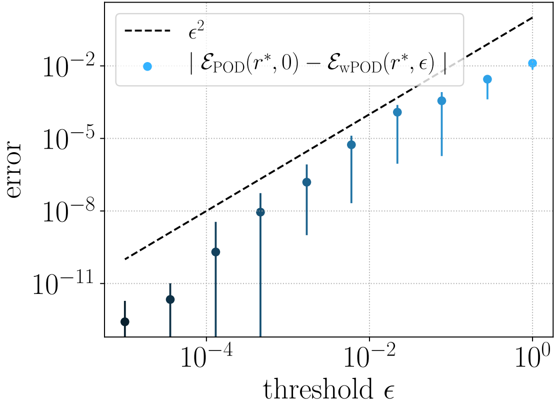




Error Analysis – Total error

Under the assumption that the perturbation of eigenvalues is small in comparison to the the total energy $\mathcal{M}_r < \epsilon$

$$\mathcal{E}_{\mathsf{wPOD}}(\epsilon, r) \lesssim \mathcal{E}_{\mathsf{POD}}(0, r) + \epsilon^2$$



Bumblebee - Reconstruction

Reconstruction: $\tilde{q}^{\epsilon}(\boldsymbol{x}, t_i) = \sum_{k=1}^{r} a_k(t_i)\phi_k^{\epsilon}(\boldsymbol{x})$