Tensor Study of Quantum Link Model

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Date: February 3, 2020

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1 Hamiltonian of QLM as a spin system

We want to study the Square Ice Hamiltonian

$$H = \sum_{\square} \left(-f_{\square} + \lambda f_{\square}^2\right), \tag{1}$$

as the sum over all plaquettes:

$$f_{\square} = \sigma_{\mu_1}^+ \sigma_{\mu_2}^+ \sigma_{\mu_3}^- \sigma_{\mu_4}^- + \sigma_{\mu_1}^- \sigma_{\mu_2}^- \sigma_{\mu_3}^+ \sigma_{\mu_4}^+. \tag{2}$$

on a long cylindrical lattice

$$\Omega = \{\mu = (n,m) \mid n \in \{1, \ldots, L_x\}, \ m \in \{1, \ldots L_y\}\} \tag{3}$$
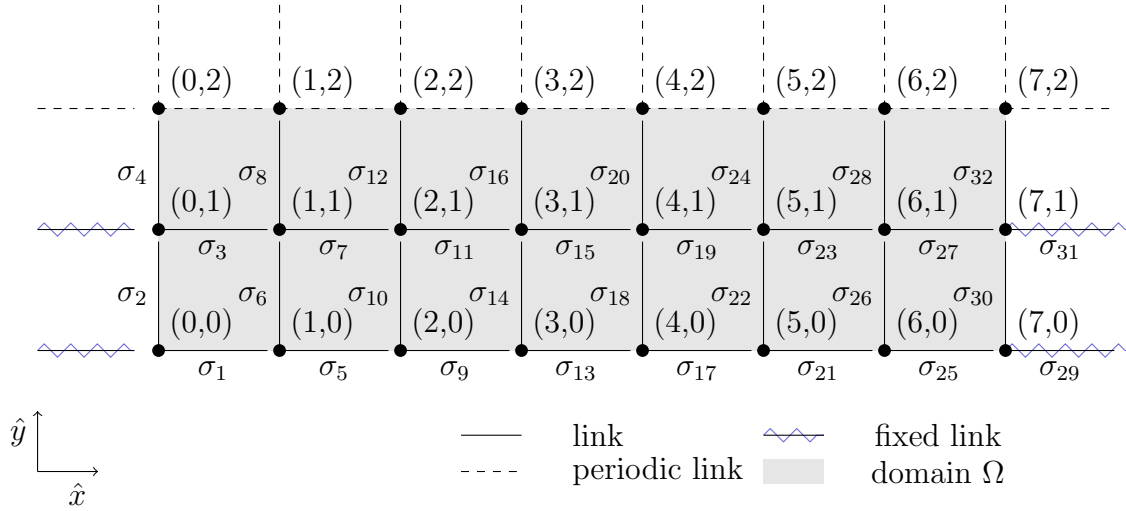
displayed in fig. 1.

Figure 1: Definition of the computational mesh

In all our lattices $L_y \ll L_x$. For now we fix $L_y = 2$. under the *Gauss law constraint*:

$$G_\mu = 0 \qquad \text{for all} \qquad \mu \in \Omega, \tag{4}$$

$$G_\mu := \sum_{\hat{i} \in \{\hat{x}, \hat{y}\}} \left(\sigma_{\nu - \hat{i}/2} - \sigma_{\nu + \hat{i}/2}\right). \tag{5}$$

Here G_μ counts the difference between in and outgoing arrows at vertex μ.

In the conventional up/down ([1 0]/[0 1]) basis the plaquette-operators $\sigma^\pm$ are given by:

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

To identify the different local interaction terms in the Hamilton operator (**??**) with (1) we rewrite the plaquette-operator into our computational basis $|i_n\rangle$. A plaquette operator defines our nearest neighbor interaction between state $|i_n\rangle$ and $|i_{n+1}\rangle$

$$f_\square = f_{\llcorner,n,m} \otimes f_{\lrcorner,n,m} + h.c. \tag{7}$$

$$f_{\llcorner,n,m} = \sigma^-_{r,n,m+1}\sigma^-_{v,n,m+1}\sigma^+_{r,n,m} \tag{8}$$

$$f_{\lrcorner,n+1,m} = \sigma^+_{l,n+1,m+1}\sigma^+_{v,n+1,m+1}\sigma^-_{l,n+1,m} \tag{9}$$

Comparing this to (**??**) yields:

$$h^{(1)}_{\llcorner,n,m} = -f_{\llcorner,n,m} \qquad\qquad h^{(1)}_{\lrcorner,n+1,m} = f_{\lrcorner,n+1,m} \tag{10}$$

$$h^{(2)}_{\llcorner,n,m} = -f^\dagger_{\llcorner,n,m} \qquad\qquad h^{(2)}_{\lrcorner,n+1,m} = f^\dagger_{\lrcorner,n+1,m} \tag{11}$$

$$h^{(3)}_{\llcorner,n,m} = \lambda f^\dagger_{\llcorner,n,m}f_{\llcorner,n,m} \qquad\qquad h^{(3)}_{\lrcorner,n+1,m} = f^\dagger_{\lrcorner,n+1,m}f_{\lrcorner,n+1,m} \tag{12}$$

$$h^{(4)}_{\llcorner,n,m} = \lambda f_{\llcorner,n,m}f^\dagger_{\llcorner,n,m} \qquad\qquad h^{(4)}_{\lrcorner,n+1,m} = f_{\lrcorner,n+1,m}f^\dagger_{\lrcorner,n+1,m} \tag{13}$$

$$\tag{14}$$

For example in our $L_y = 2$ system we get 64×64 size Operators :

$$h^{(1)}_{\llcorner,n,m} = -\sigma^+ \otimes \sigma^- \otimes \sigma^+ \otimes I_2 \otimes I_2 \otimes I_2 \in \mathbb{R}^{2^6,2^6} \tag{15}$$

$$h^{(1)}_{\lrcorner,n,m} = I_2 \otimes I_2 \otimes \sigma^+ \otimes I_2 \otimes \sigma^- \otimes \sigma^+ \in \mathbb{R}^{2^6,2^6} \tag{16}$$

$$\tag{17}$$

Note that this allready inherits the periodicity in $\hat{y}$. For the choosen up/down ([1 0]/[0 1]) basis the link operators are given by:

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{18}$$

$$f^2_\square = \sigma^+_{\mu_1}\sigma^-_{\mu_1}\,\sigma^+_{\mu_2}\sigma^-_{\mu_2}\,\sigma^-_{\mu_3}\sigma^+_{\mu_3}\,\sigma^-_{\mu_4}\sigma^+_{\mu_4} + hc \tag{19}$$

If we define $p+$ and $p+$ as:

$$p+ = \frac{\mathbb{1} + \sigma^z}{2} \;;\; p- = \frac{\mathbb{1} - \sigma^z}{2} \tag{20}$$

I have:

$$f^2_\square = p^+_{\mu_1}p^+_{\mu_2}p^-_{\mu_3}p^-_{\mu_4} + p^-_{\mu_1}p^-_{\mu_2}p^+_{\mu_3}p^+_{\mu_4} \tag{21}$$

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1.1 Todos

- Hamiltonian in external magnetic field, $\phi_\square \in \mathbb{R}$. Therefore we define the generalized plaquette operator

$$f(\phi_\square) := u_\square e^{i\phi_\square} + u_\square^\dagger e^{-i\phi_\square} \tag{22}$$

and plug it in (1)

- Winding number operators

$$W_y = \tag{23}$$