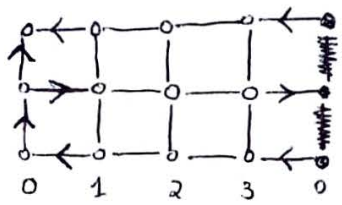
- Diagonalization of the 4x2 system with fixed boundary conditions – the aim is to have some exact results which we can compare with E.D. and the tensor network.

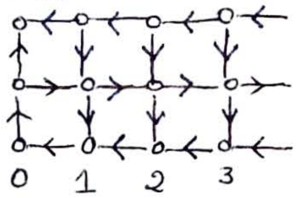
- The lattice is depicted as follows:



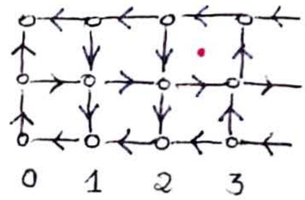
The arrows depicted on the links are already fixed. Then the ED code gives the following 14 states.

Basis states:

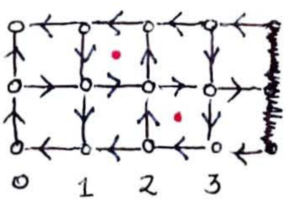
$|1\rangle =$



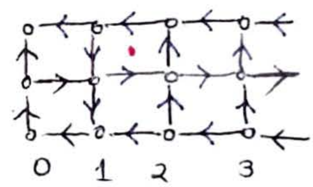
$|2\rangle =$



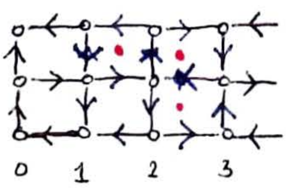
$|3\rangle =$



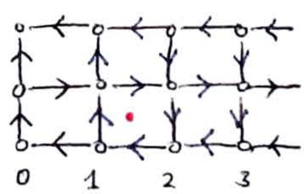
$|4\rangle =$



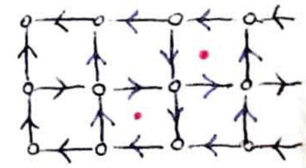
$|5\rangle =$



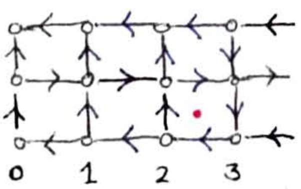
$|6\rangle =$



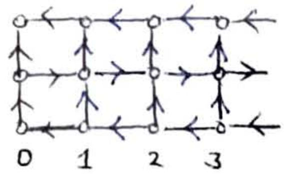
$|7\rangle =$



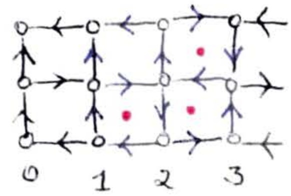
$|8\rangle =$



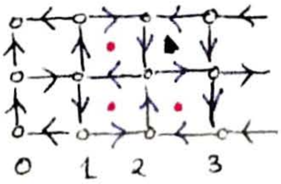
$|9\rangle =$



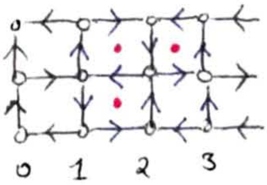
$|10\rangle =$



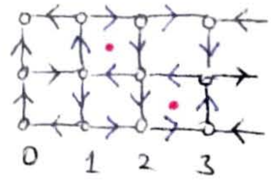
$|11\rangle =$



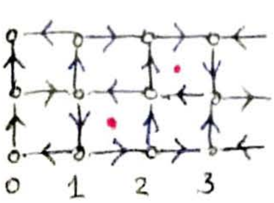
$|12\rangle =$



$|13\rangle =$



$|14\rangle =$



The plaquettes with dots in them are flippable ones. The Hamiltonian (with $\lambda = 0$) can only act on the flippable plaquettes.

Next, we ~~diagonalize the~~ need to compute the matrix elements for the system by acting the Hamiltonian.

$H|1\rangle = 0$; $H|9\rangle = 0$

State 1 $\Rightarrow$ winding no $=3$
State 2 $\Rightarrow$ winding no $=-3$

$H|2\rangle = |5\rangle$

$H|3\rangle = |11\rangle + |5\rangle$

$H|4\rangle = |12\rangle$

$H|5\rangle = |13\rangle + |2\rangle + |3\rangle$

$H|6\rangle = |11\rangle$

$H|7\rangle = |12\rangle + |10\rangle$

$H|8\rangle = |10\rangle$

$H|10\rangle = |8\rangle + |7\rangle + |14\rangle$

$H|11\rangle = |3\rangle + |6\rangle + |13\rangle$

$H|12\rangle = |4\rangle + |7\rangle + |14\rangle$

$H|13\rangle = |5\rangle + |11\rangle$

$H|14\rangle = |10\rangle + |12\rangle$

- The eigensystem would then be found by ~~solving~~ diagonalizing the Hamiltonian matrix. The elements are all zero; ~~going~~ except the ones connected by the non-zero matrix elements; which are $-J$.