Tensor Study of Quantum Link Model

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1 Hamiltonian of QLM as a spin system

We define the Square Ice Hamiltonian

$$H = \sum_{\square}(-f_{\square} + \lambda f_{\square}^2)\,, \tag{1}$$

as the sum over all plaquettes:

$$f_{\square} = \sigma^+_{\mu_1}\sigma^+_{\mu_2}\sigma^-_{\mu_3}\sigma^-_{\mu_4} + \sigma^-_{\mu_1}\sigma^-_{\mu_2}\sigma^+_{\mu_3}\sigma^+_{\mu_4}. \tag{2}$$

Plaquette-operators To identify the different local interaction terms in the Hamilton operator (**??**) with (**??**) we rewrite the plaquette-operator into our computational basis $|i_n\rangle$. A plaquette operator defines our nearest neighbor interaction between state $|i_n\rangle$ and $|i_{n+1}\rangle$

$$f_{\square} = f_{\sqsubset,n,m} \otimes f_{\sqsupset,n,m} + h.c. \tag{3}$$

$$f_{\sqsubset,n,m} = \sigma^-_{r,n,m+1}\sigma^-_{v,n,m+1}\sigma^+_{r,n,m} \tag{4}$$

$$f_{\sqsupset,n+1,m} = \sigma^+_{l,n+1,m+1}\sigma^+_{v,n+1,m+1}\sigma^-_{l,n+1,m} \tag{5}$$

Comparing this to (**??**) yields:

$$h^{(1)}_{\sqsubset,n,m} = -f_{\sqsubset,n,m} \qquad h^{(1)}_{\sqsupset,n+1,m} = f_{\sqsupset,n+1,m} \tag{6}$$

$$h^{(2)}_{\sqsubset,n,m} = -f^\dagger_{\sqsubset,n,m} \qquad h^{(2)}_{\sqsupset,n+1,m} = f^\dagger_{\sqsupset,n+1,m} \tag{7}$$

$$h^{(3)}_{\sqsubset,n,m} = \lambda f^\dagger_{\sqsubset,n,m} f_{\sqsubset,n,m} \qquad h^{(3)}_{\sqsupset,n+1,m} = f^\dagger_{\sqsupset,n+1,m} f_{\sqsupset,n+1,m} \tag{8}$$

$$h^{(4)}_{\sqsubset,n,m} = \lambda f_{\sqsubset,n,m} f^\dagger_{\sqsubset,n,m} \qquad h^{(4)}_{\sqsupset,n+1,m} = f_{\sqsupset,n+1,m} f^\dagger_{\sqsupset,n+1,m} \tag{9}$$

$$\tag{10}$$

For example in our $L_y = 2$ system we get 64×64 size Operators :

$$h^{(1)}_{\sqsubset,n,m} = -\sigma^+ \otimes \sigma^- \otimes \sigma^+ \otimes I_2 \otimes I_2 \otimes I_2 \in \mathbb{R}^{2^6,2^6} \tag{11}$$
$$h^{(1)}_{\sqsupset,n,m} = I_2 \otimes I_2 \otimes \sigma^+ \otimes I_2 \otimes \sigma^- \otimes \sigma^+ \in \mathbb{R}^{2^6,2^6} \tag{12}$$
$$\tag{13}$$

Note that this allready inherits the periodicity in $\hat{y}$. For the choosen up/down ([1 0]/[0 1]) basis the link operators are given by:

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$

on a long cylindrical lattice:

$$\Omega = \{\nu = (n, m)| \qquad n \in \{1, \ldots, L_x\},\ m \in \{1, \ldots L_y\}\}. \tag{15}$$

In all our lattices $L_y \ll L_x$. For now we fix $L_y = 2$.
The Hamiltonian eq.(**??**) is invariant under the local symmetry:

$$G_\nu = \sum_{\hat{i} \in \{\hat{x}, \hat{y}\}} (\sigma_{\nu - \hat{i}/2} - \sigma_{\nu + \hat{i}/2}) \tag{16}$$

which counts the difference between in and outgoing arrows at vertex ν. In this work

$$G_\nu = 0 \qquad \text{for all} \qquad \nu \in \Omega \tag{17}$$

$$f^2_\square = \sigma^+_{\mu_1}\sigma^-_{\mu_1}\ \sigma^+_{\mu_2}\sigma^-_{\mu_2}\ \sigma^-_{\mu_3}\sigma^+_{\mu_3}\ \sigma^-_{\mu_4}\sigma^+_{\mu_4}\ + hc \tag{18}$$

If we define $p+$ and $p+$ as:

$$p+ = \frac{\mathbb{1} + \sigma^z}{2}\ ;\ p- = \frac{\mathbb{1} - \sigma^z}{2} \tag{19}$$

I have:

$$f^2_\square = p^+_{\mu_1}p^+_{\mu_2}p^-_{\mu_3}p^-_{\mu_4}\ + p^-_{\mu_1}p^-_{\mu_2}p^+_{\mu_3}p^+_{\mu_4} \tag{20}$$

1.1 Todos

- Hamiltonian in external magnetic field, $\phi_\square \in \mathbb{R}$. Therefore we define the generalized plaquette operator

$$f(\phi_\square) := u_\square e^{i\phi_\square} + u^\dagger_\square e^{-i\phi_\square} \tag{21}$$

  and plug it in (**??**)

- Winding number operators

$$W_y = \tag{22}$$