T_1 and T_2 time contributions in superconducting circuits

The longitudinal and transverse relaxation rates are given by their inverse time scales. Within the Bloch-Redfield description, the transverse relaxation is given by the longitudinal relaxation and dephasing. [4]

$$\Gamma_1 = \frac{1}{T_1} \tag{1}$$

$$\Gamma_2 = \frac{1}{T_2} = \frac{\Gamma_1}{2} + \Gamma_\varphi \tag{2}$$

Strictly speaking, due to $1/f$ noise, the decay functions of the off-diagonal terms are non-exponential and the system does not fall into the Bloch-Redfield regime. However for $T_\varphi \gtrsim T_1$, the decay can be approximated by an exponential function. If there is no pure dephasing, it holds that $T_2 = 2T_1$, otherwise we have $T_2 \leq 2T_1$. In the following we give an overview of noise contributions for superconducting qubits, which we could implement in CircuitQ.

1 T_1

Contribution	Formula	Remarks
Spontaneous emission	$T_1 = \frac{12\pi\epsilon_0\hbar c^3}{d}\frac{1}{\omega_q^3}$ [3]	$d = 2eL$: Dipole moment with $L \sim 15$ μm [3] ω_q: Qubit frequency For transmon, contribution to $T_1 = 0.3$ ms [3]
Quasiparticle tunneling	$T_1 = \left(\sum_j \lvert\langle g\rvert \sin(\hat{\varphi}_j/2)\lvert e\rangle\rvert^2 S_{qp,j}(\omega_q)\right)^{-1}$ [2]	$\hat{\varphi} = \hat{\Phi}/\Phi_0$: phase operator $S_{qp,j}(\omega_q) = x_{qp}\frac{E_{J,j}}{h}\sqrt{\frac{8\Delta}{\omega_q}}$ [6] $E_{J,j}$: Josephson energy of the i-th junction $x_{qp} = \sqrt{2\pi k_B T/\Delta} e^{-\Delta/k_B T}$: Quasiparticle density normalized by the density of Cooper pairs [1] $\Delta = 90$ GHz: superconducting gap [5]
Charge noise	$T_1 = \frac{\hbar^2}{S_Q(\omega_q)}\left\lvert\langle e\rvert\hat{V}\lvert g\rangle\right\rvert^{-2}$ [6]	$\hat{V}_i = \left(\hat{Q}C^{-1}\right)_i$: voltage operator $S_Q(\omega_q) = A_Q^2\left(\frac{2\pi\cdot 1Hz}{\omega_q}\right)^{\gamma_Q}$: noise spectral density with $A_Q^2 = (10^{-3}e)^2/\text{Hz}$ [4]
Flux noise	$T_1 = \frac{\hbar^2}{S_\Phi(\omega_q)}\left\lvert\langle e\rvert\hat{I}\lvert g\rangle\right\rvert^{-2}$ [6]	$\hat{I}_i$: current operator, which includes the sum of all $\frac{\Phi_i - \Phi_j}{L_{ij}}$ terms in the circuit that correspond to a distinct loop edge. $S_\Phi(\omega_q) = A_\Phi^2\left(\frac{2\pi\cdot 1Hz}{\omega_q}\right)^{\gamma_\Phi}$: noise spectral density with $A_\Phi^2 \approx (10^{-6}\Phi_0)^2/\text{Hz}$ [4]
Purcell effect	We don't include the purcell effect as we exclude external control and readout circuitry from our analysis.	For transmon, contribution to $T_1 \sim 16$ μs [3]

Table 1: Overview of the noise contributions to T_1.

2 T_2

The pure dephasing contributions listed in table 2 should vanish at the sweet spot. External parameters are used to perform gates, prepare states or tune the qubit. These parameters are used to manipulate the qubit and are no fundamental components. Therefore we do not consider these contributions for now.

Contribution	Formula	Remarks
Offset charge noise (pure dephasing)	$T_2 \sim \frac{\hbar}{A_q} \left\lvert \frac{\partial E_q}{\partial q_{off}} \right\rvert^{-1}$ [3]	This formula can be used for small fluctuations and small frequencies $A_q = 10^{-4}e - 10^{-3}e$: noise amplitude E_q: qubit energy
Offset flux noise (pure dephasing)	$T_2 \sim \frac{\hbar}{A_\phi} \left\lvert \frac{\partial E_q}{\partial \phi_{off}} \right\rvert^{-1}$ [3]	This formula can be used for small fluctuations and small frequencies $A_\phi = 10^{-6}\Phi_0 - 10^{-5}\Phi_0$: noise amplitude

Table 2: Overview of the noise contributions to T_2.

References

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