T_1 and T_2 time contributions in superconducting circuits

The longitudinal and transverse relaxation rates are given by their inverse time scales. Within the Bloch-Redfield description, the transverse relaxation is given by the longitudinal relaxation and dephasing. [2]

$$\Gamma_1 = \frac{1}{T_1} \tag{1}$$

$$\Gamma_2 = \frac{1}{T_2} = \frac{\Gamma_1}{2} + \Gamma_\varphi \tag{2}$$

Strictly speaking, due to $1/f$ noise, the decay functions of the off-diagonal terms are non-exponential and the system does not fall into the Bloch-Redfield regime. However for $T_\varphi \gtrsim T_1$, the decay can be approximated by an exponential function. If there is no pure dephasing, it holds that $T_2 = 2T_1$, otherwise we have $T_2 \leq 2T_1$. In the following we give an overview of noise contributions for superconducting qubits, which we could implement in CircuitQ.

1 T_1

Contribution	Formula	Remarks
Spontaneous emission	$T_1 = \frac{12\pi\epsilon_0\hbar c^3}{d}\frac{1}{\omega_q^3}$ [1]	$d = 2eL$: Dipole moment with $L \sim 15$ μm [1] ω_q: Qubit frequency For transmon, contribution to $T_1 = 0.3$ ms [1]
Quasiparticle tunneling (Transmon paper)	In transmon regime [1] $T_1 = \left(\Gamma_{qp}N_{qp}\sqrt{\frac{k_bT}{\hbar\omega_q}}\lvert\langle g, n_g \pm \frac{1}{2}\lvert e, n_g\rangle\rvert^2\right)^{-1}$ with number of quasiparticles: $N_{qp} = 1 + N_e e^{-\Delta/k_BT}\frac{3\sqrt{2\pi}\sqrt{\Delta k_bT}}{2E_F}$	$\Gamma_{qp} = \delta g_T/4\Pi\hbar$: quasiparticle tunneling rate $\delta = 1/\nu V$: mean level spacing of reservoir $\nu = 3n/2E_F$: density of states $n = 18.1 \cdot 10^{22}$ cm^{-3}: conduction electron density $E_F = 11.7$ eV: for aluminum (as n) $V = 150$ μm^3: metal volume $g_T = 1e^2/h$: junction conductance $\Delta = 90$ GHz: superconducting gap [3] $N_e = nV$: number of conduction elecctrons $T = 20$ mK: temperature For transmon, contribution to $T_1 \sim 1$ s [1]
Quasiparticle tunneling (more general expression)	$T_1 = \left(\sum_j \lvert\langle g\rvert\sin(\hat{\varphi}_j/2)\lvert e\rangle\rvert^2 E_{Jj}\tilde{S}_{qp}(\omega_q)\right)^{-1}$	[?]
Charge noise	$T_1 = \frac{\hbar^2}{S_Q(\omega_q)}\left\lvert\langle e\rvert\hat{V}\lvert g\rangle\right\rvert^{-2}$ [4]	$\hat{V}_i = \left(\hat{Q}C^{-1}\right)_i$: voltage operator $S_Q(\omega_q) = A_Q^2\left(\frac{2\pi\cdot 1Hz}{\omega_q}\right)^{\gamma_Q}$: noise spectral density with $A_Q^2 = (10^{-3}e)^2$/Hz [2]
Flux noise	$T_1 = \frac{\hbar^2}{S_\Phi(\omega_q)}\left\lvert\langle e\rvert\hat{I}\lvert g\rangle\right\rvert^{-2}$ [4]	$\hat{I}_i$: current operator, which includes the sum of all $\frac{\Phi_i - \Phi_j}{L_{ij}}$ terms in the circuit. $S_\Phi(\omega_q) = A_\Phi^2\left(\frac{2\pi\cdot 1Hz}{\omega_q}\right)^{\gamma_\Phi}$: noise spectral density with $A_\Phi^2 \approx (10^{-6}\Phi_0)^2$/Hz [2]
Purcell effect	We don't include the purcell effect as we exclude external control and readout circuitry from our analysis.	For transmon, contribution to $T_1 \sim 16$ μs [1]

Table 1: Overview of the noise contributions to T_1.

2 T_2

Contribution	Formula	Remarks
Offset charge noise (pure dephasing)	$T_2 \sim \frac{\hbar}{A_q} \left\| \frac{\partial E_q}{\partial q_{off}} \right\|^{-1}$ [1]	This formula can be used for small fluctuations and small frequencies $A_q = 10^{-4}e - 10^{-3}e$: noise amplitude E_q: qubit energy
Offset flux noise (pure dephasing)	$T_2 \sim \frac{\hbar}{A_\phi} \left\| \frac{\partial E_q}{\partial \phi_{off}} \right\|^{-1}$ [1]	This formula can be used for small fluctuations and small frequencies $A_\phi = 10^{-6}\Phi_0 - 10^{-5}\Phi_0$: noise amplitude

Table 2: Overview of the noise contributions to T_2.

References

[1] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Charge-insensitive qubit design derived from the cooper pair box. *Phys. Rev. A*, 76:042319, Oct 2007. URL: `https://link.aps.org/doi/10.1103/PhysRevA.76.042319`, `doi:10.1103/PhysRevA.76.042319`.

[2] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver. A quantum engineer's guide to superconducting qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. `arXiv:1904.06560`, `doi:10.1063/1.5089550`.

[3] Hans Mooij. *Superconducting quantum bits*, 2004. URL: `https://physicsworld.com/a/superconducting-quantum-bits/`.

[4] Fei Yan, Simon Gustavsson, Archana Kamal, Jeffrey Birenbaum, Adam P Sears, David Hover, Ted J. Gudmundsen, Danna Rosenberg, Gabriel Samach, S. Weber, Jonilyn L. Yoder, Terry P. Orlando, John Clarke, Andrew J. Kerman, and William D. Oliver. The flux qubit revisited to enhance coherence and reproducibility. *Nature Communications*, 7(1):12964, 2016. URL: `https://doi.org/10.1038/ncomms12964`, `doi:10.1038/ncomms12964`.