Machine Learning of Many Body Localization

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(Dated: July 10, 2020)

Exact diagonalization was used to find the reduced density matrices of the lowest energy eigenstate of the Heisenberg Model with an additional field in z-direction at low and high disorder strength. The resulting dataset representing extended and localized phases, was used to train a neural network to deduct the critical disorder strength for a phase transition.

I. INTRODUCTION

Review Literature on task
Why is the topic interesting? => Finding Wc?

A. Hamiltonian of the Heisenberg model

What is it used for? With $J = 1$, $h_i \in [-W, W]$

$$H = \underbrace{J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}}_{\text{Exchange Energy}} - \underbrace{\sum_i h_i S_i^z}_{\text{Random Field}} \tag{1}$$

Outcome expectation:
In the ergodic phase (delocalized phase) $h < h_c$, the many-body eigenstates are thermal,20–23 so the isolated quantum system can relax to thermal equilibrium under the dynamics due to its Hamiltonian. In the thermodynamic limit $L \to \infty$, the system thus successfully serves as its own heat bath in the ergodic phase. In a thermal eigenstate, the reduced density operator of a finite subsystem converges to the equilibrium thermal distribution for $L \to \infty$. Thus the entanglement entropy between a finite subsystem and the remainder of the system is, for $L \to \infty$, the thermal equilibrium entropy of the subsystem. At nonzero temperature, this entanglement entropy is extensive, proportional to the number of degrees of freedom in the subsystem.

In the many-body localized phase $h > h_c$, on the other hand, the many-body eigenstates are not thermal:2 the "eigenstate-thermalization hypothesis"20–23 is false in the localized phase. Thus in the localized phase, the isolated quantum system does not relax to thermal equilibrium under the dynamics of its Hamiltonian. The infinite system fails to be a heat bath that can equilibrate itself. It is a "glass" whose local configurations at all times are set by the initial conditions. Here the eigenstates do not have extensive entanglement, making them accessible to density-matrixrenormalization-group–type numerical techniques.5 A limit of the localized phase that is simple is J= 0 with $h > 0$. Here the spins do not interact, all that happens dynamically is local Larmor precession of the spins about their localrandom fields. No transport of energy or spin happens and the many-body eigenstates are simply product states with each spin either "up" or "down." `https://doi.org/10.1103/PhysRevB.82.174411`

non-thermalising phase, in which the system violates the Eigenstate Thermalisation hypothesis `https://arxiv.org/pdf/1610.03042.pdf`

J<g: (large W) localized phase (adiabatically connected to trivial state), J>g extended phase (small W) (ordered??)

Is scaling important?

B. Exact Diagonalization

Introduce concepts: Exact Diagonalization,

C. Areal (reduced??) Density Matrix

areal Density Matrix: `http://www.thphys.nuim.ie/staff/jvala/Lecture_9.pdf`

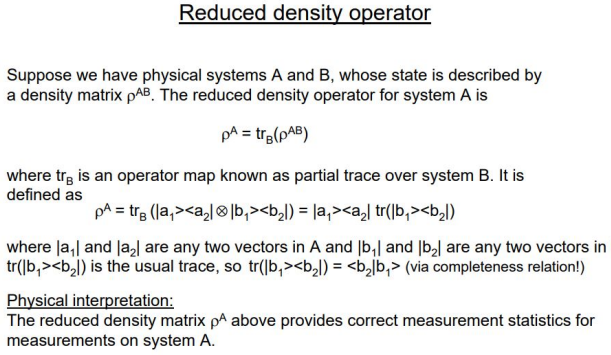

FIG. 1. Example of a figure [7].

vs

direct density matrix: (DOI: 10.1103/PhysRevB.99.054208 says Instead of dividing the system into two subsystems A and B to calculate the reduced density matrix of an eigenstate rho a and using the entanglement spectrum as the training data set 34,35, we directly feed the probability density of the eigenstate psi i computed in the spin basis to the machines as the training data set. The reason for doing so is that, although by preprocessing the training data can reduce the dimension and filter out redundant information, useful information contained in the wavefunction of the entire system can also be lost.)

Conclusion: => areal

D. Neural Networks and Convolutional Neural Networks

Neural Network, CNN

II. MATERIALS AND METHODS

A. Code Protocol

Explain Flow with figure

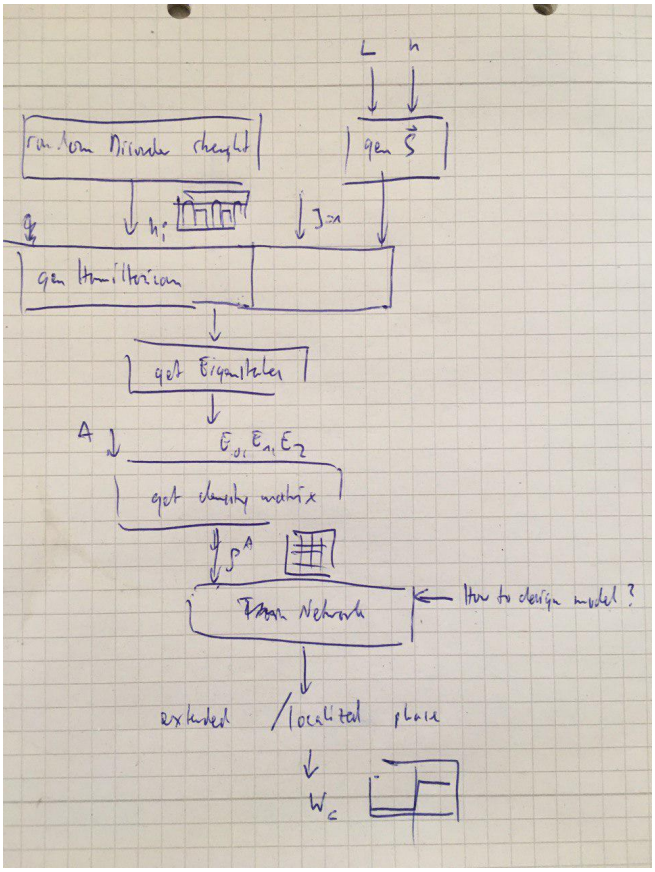

FIG. 2. Example of a figure [7].

Fig. 2

B. Machine Learning Models and Error Metrics

Loss: SparseCategoricalCrossentropy from `https://keras.io/api/losses/`
Optimizer: Adam
Explain metrics and errors and why they are used. Which ml models are used and why?
Hyperparameters? Amount of layers?

III. RESULTS

A. Generation of density matrix training set

Plots: What is computationally realizable in 1h concerning time? The training set was sufficiently large enough
We only need M Eigenstates
This is how corresponding density matrices look like
This will be our parameter space for n, L

B. Prediction of extended vs localized phase

Training and testing scores

C. W_c analysis

Those are our W_c depending on n, L.

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Appendix A: Code listing

Please copy your code in the appendix.

```
"""

Description

"""

import numpy as np

code
```