TEIL 1: Nichtlineare Optimierung

Aufgabe 1: optimale Dimensionierung eines Kühlkreislaufs in einem Kernkraftwerk

2 Kühlflüssigk. → untschl. Viskosität   2 getrennte Kreisläufen

umk proportional zu Querschnitt v. Viskosität

r_1 / r_2 → optimal

$$2r_1 + 2r_2 \geq 30 \text{ cm}$$
$$2r_1 + 2r_2 \leq 60 \text{ cm}$$

$$\max = \frac{1}{r_1^2 \pi} v_1 + \frac{1}{r_2^2 \pi} v_2$$

$$\Rightarrow \min \; r_1^2 \pi v_1 + r_2^2 \pi \cdot v_2$$

NB: $2r_1 + 2r_2 \geq 30$ $-2r_1 - 2r_2 + 30 \leq 0 = g_1(r_1, r_2)$

$2r_1 + 2r_2 \leq 60$ $2r_1 + 2r_2 - 60 \leq 0 = g_2(r_1, r_2)$

$r_1, r_2 \geq 0$

Lagrange Funktion

$$\mathcal{L}(r_1, r_2, \lambda_1, \lambda_2) = v_1 \cdot r_1^2 + v_2 \cdot r_2^2 + \lambda_1(-2r_1 - 2r_2 + 30) + \lambda_2 \cdot (2r_1 + 2r_2 - 60)$$

Regularitätsbedingung

$$\frac{\partial g_1(r_1, r_2)}{\partial r_1} = -2 \neq 0 \qquad \frac{\partial g_2(r_1, r_2)}{\partial r_1} = 2 \neq 0$$

$$\frac{\partial g_1(r_1, r_2)}{\partial r_2} = -2 \neq 0 \qquad \frac{\partial g_2(r_1, r_2)}{\partial r_2} = 2 \neq 0$$

KKT Bedingung

$$\frac{\partial \mathcal{L}}{\partial r_1} = 2r_1v_1 - 2\lambda_1 + 2\lambda_2 \overset{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial r_2} = 2r_2 \cdot v_2 - 2\lambda_1 + 2\lambda_2 \overset{!}{=} 0$$

Komplimentaritätsbedingung

$$\lambda_1(-2r_1 - 2r_2 + 30) \overset{!}{=} 0 \quad g_1(r_1, r_2)$$

$$\lambda_2(2r_1 + 2r_2 - 60) \overset{!}{=} 0 \quad g_2(r_1, r_2)$$

$$\lambda_1, \lambda_2 \geq 0$$

1. Fall

$\lambda_1 = \lambda_2 = 0$

$2r_1v_1 = 0 \qquad r_1 = 0$

$2r_2 \cdot v_2 = 0 \qquad r_2 = 0$

$g_1(r_1, r_2) \; -2 \cdot 0 - 2 \cdot 0 + 30 \leq \; ↯$

2. Fall

$\lambda_1 > 0, \; \lambda_2 > 0$

$-2r_1 - 2r_2 + 30 = 0 \Leftrightarrow 2r_1 + 2r_2 = 30$

$2r_1 + 2r_2 - 60 = 0 \Leftrightarrow 2r_1 + 2r_2 = 60 \; ↯$

Seite

3. Fall

$\lambda_1 > 0 \; , \; \lambda_2 = 0$

$-2r_1 - 2r_2 + 30 = 0$

$2r_1 + 2r_2 = 30 \iff r_2 = 15 - r_1 \; , \; r_1 = 15 - r_2$

$$\left.\begin{aligned} 2v_1 r_1 - 2\lambda_1 &= 0 \\ 2v_2 r_2 - 2\lambda_1 &= 0 \end{aligned}\right\} \text{Symmetrie}$$

$2v_1 r_1 = 2v_2 r_2$

$2v_1 r_1 = 2v_2 (15 - r_1)$

$2v_1 r_1 = 30v_2 - 2v_2 r_1$

$r_1 \cdot (2v_1 + 2v_2) = 30v_2$

$r_1 = \dfrac{30v_2}{2v_1 + 2v_2}$ $\quad r_1 = \dfrac{15v_2}{v_1 + v_2} \; , \quad r_2 = \dfrac{15v_1}{v_1 + v_2}$

$2v_1 \cdot r_1 - 2\lambda_1 = 0$

$2v_1 \left(\dfrac{15v_2}{v_1 + v_2}\right) - 2\lambda_1 = 0$

Passt

$\lambda_1 = v_1 \cdot \left(\dfrac{15v_1}{v_1 + v_2}\right) > 0$

4. Fall

$R_1 = 0 \; , \; R_2 > 0$

$2r_1 + 2r_2 - 60 = 0 \implies \underline{r_2 = 30 - r_1}$

$$\left.\begin{aligned} 2v_1 r_1 + 2R_2 &= 0 \\ 2v_2 r_2 + 2R_2 &= 0 \end{aligned}\right\} \text{Symmetrie}$$

$\implies 2v_1 r_1 = 2v_2 r_2$

$\implies v_1 r_1 = v_2 (30 - r_1)$

$v_1 r_1 = 30v_2 - v_2 r_1$

$r_1 (v_1 + v_2) = 30 v_2$

$\implies r_1 = \dfrac{30v_2}{v_1 + v_2} \; , \quad r_2 = \dfrac{30v_1}{v_1 + v_2}$

$2v_2 r_2 + 2R_2 = 0 \implies$

$2v_2 \dfrac{30v_1}{v_1 + v_2} + 2R_2 = 0 \implies$

$R_2 = \dfrac{-30 v_1 v_2}{v_1 + v_2} < 0 \; ↯$

Hinreichende Bedingung

$\dfrac{\partial^2 \mathcal{L}}{\partial r_1^2} = 2v_1 > 0 \qquad \dfrac{\partial^2 \mathcal{L}}{\partial r_2^2} = 2v_2 > 0$

$\rightarrow$ optimale Radien $\quad \dfrac{15v_2}{v_1 + v_2} = r_1 \qquad \dfrac{15v_1}{v_1 + v_2} = r_2$

Minimum $\quad \left(\dfrac{15v_1}{v_1 + v_2}\right)^2 \pi v_2 + \left(\dfrac{15v_2}{v_1 + v_2}\right)^2 \pi v_1$