Future directions of research

Laura

Structure: Higgs bundles, branes, Applications.

Σ will be a compact Riemann surface of genus $g \geq 2$ $K = T^*\Sigma$

(E, φ) Higgs bundles

Remark : One can consider principal $G_{\mathbb{C}}$ - Higgs bundles. For $G_{\mathbb{C}} < \mathrm{GL}_n\mathbb{C}$ we take classical (E, φ) and extend conditions reflecting the structure of the group.

For example $\mathrm{SL}_m\mathbb{C}$, then $E = \Lambda^m E \cong \mathcal{O}$ $\mathrm{Tr}(\varphi)$

If $G_{\mathbb{C}} = \mathrm{Sp}_{2m}\mathbb{C}$ then require symplectic and $\omega(\varphi???, w) = \omega(v, \varphi w)$ compatibility condition.

$\mathcal{M}_G$ be the moduli space of G-Higgs bundles.

Have that $\mathcal{M}_G$ is a hyperkahler structure. We inherit complex structure from Σ and denote it I, we inherit complex structure from $G_{\mathbb{C}}$ and denote it J. Together these generate the hyperkahler structure. Have the holomorphic symplectic structures Ω_i.

A Lagrangian subspace with respect to a holomorphic symplectic structure for an A-brane. A complex submanifold with respect to a complex structure is a B-brane.

Remark: We can then ask for, for example (A, B, B), that is an A-brane with respect to Ω_I, and B brane with respect to ??

Question: How to construct families of these branes.

We have the correspondence between $\mathcal{M}_{G_{\mathbb{C}}}$ and $\mathrm{Red}^+(\pi_1(\Sigma) \to G_{\mathbb{C}})/\sim$ What do branes correspond to in this space? Similarly, with $\mathcal{A}_{G_{\mathbb{C}}}$

Remark: Given $G_{\mathbb{C}}$ there is a Langlands dual group.

$$[[LISTOFDUALGROUPS]]GLGLSL \qquad PGLSPSO(2m+1)SO \qquad SO \tag{1}$$

Let $G^L_{\mathbb{C}}$ denote the Langlands dual. Mirror symmetry SYZ $\mathcal{M}_{G^L_{\mathbb{C}}} \to \mathcal{A}_{G^L_{\mathbb{C}}}$ and there is isomorphism $\mathcal{A}_{G_{\mathbb{C}}} \cong \mathcal{A}_{G^L_{\mathbb{C}}}$

Refrences : 02 06 07 Hitchin

Question: What are dual branes across a mirror symmetry correspondence.

Remark:

$$\mathcal{M}_{G_{\mathbb{C}}}\mathcal{M}_{G^L_{\mathbb{C}}} \tag{2}$$
$$(B, A, A)(B, B, B) \tag{3}$$
$$(A, B, A)(A, B, A) \tag{4}$$
$$[[OTHERES]] \tag{5}$$

Question: A new space such that all branes are of self dual type? In string theory appear branes appear as boundary conditions that come in pairs, that are often self dual.

[[CRUCIAL COMMUTATIVE DIAGRAM MISSING HERE]]

Suppose we endow Σ with real structure given locally by conjugation. Simultaneously consider antiholomorphic involution fixing G in $G_{\mathbb{C}}$. Ie involution fixing maximal compact subgroup.

$(E, \varphi) \to (fg(E), -fg(\varphi))$ this is an involution that fixes (A, B, A) -brane.

Hitchin 92, considering real Higgs bundles. Kupuscin and Witten later 96 saw fixes (B, A, A) branes.

[[COMMUTATIVE DIAGRAM WITH ALL MODULI SPACES]]

Conjecture: The dual (A, B, A) brane in the moduli space $\mathcal{M}_{G^L_{\mathbb{C}}}$ is the one fixed by involution induced.

Theorem 0.1. *...*

Given a 3 manifold for which $\partial M = \Sigma$. Which representations of $\pi_1(\Sigma)$ extend to $\pi_1(M)$. Relates to knots on S^3.

Let $M = \Sigma \times [-1, 1]/\sim$ where $(x, t) \sim (f(x), -t)$.

Theorem 0.2. *The fixed... (A, B, A)-branes for which ...*

[[TOO MUCH TO PIECE TOGETHER]]