Energy Functions for Galois ω-Automata

1 Context

In formal methods and theoretical computer science, weighted automata have long been a subject of intense study. These mathematical models extend traditional finite automata by associating input to weights, allowing for the quantitative analysis of system behaviors. See [4] for an introduction into the topic. Recent years have seen a growing interest in extensions of these automata, particularly in the context of energy constraints. Here the weight is interpreted as energy, which can be stored and accumulated. The energy is bounded from below in the sense that it must not drop below zero; it can also be bounded from above to model for instance a battery with a given capacity. These bounds can be interpreted as hard or weak bounds, depending on the semantics of the system and finding an energy feasible path in such automata is discussed in [2].

Building on these works, [5, 6] extend this approach to search for energy feasible paths in Büchi- and ω-automata. In this setting, finding and extracting such a path becomes significantly more involved as shown in Fig. 1. This complexity stems from the fact that, in order for a run to be valid, it needs to respect both the qualitative constraint given by the automaton (like the Büchi-condition shown here) and the quantitative constraint on the accumulated energy along the path. Due to the bounds on the energy, these constraints are tightly intertwined and can not be considered separately as in [3].

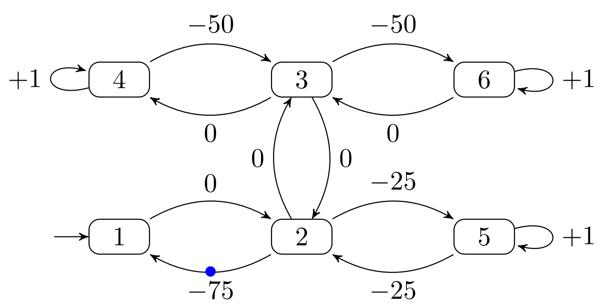

Figure 1: Non-trivial for ω-regular energy problems. Consider the energy to be bounded between 0 and 75. In order to satisfy the Büchi condition, every accepting path needs to take the accepting transition (The one carrying the blue dot) infinitely often. This transition however costs 75 energy, which gives and additional quantitative condition. To accumulate enough energy, we need to loop sufficiently often on the states 4, 5 and 6.

Independently, a generic decision procedure for energy games with energy-bounded attacker and reachability objective, moving beyond vector-valued energies and vector-addition updates was introduced in [7]. This approach allows fairly complex updates on two player game graphs, both of which is currently not possible with the existing approach for ω-regular energy problems, an example is shown in Fig. 2.

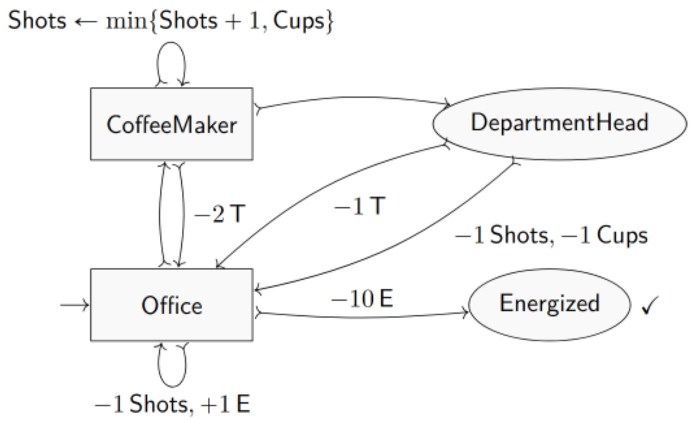

Figure 2: A Galois game with three "energies": Coffeeshots, Cups and Time . The node shape indicates the controlling player.

2 Objective

The objective of the internship is to unite the two approaches and develop a techniques to solve ω-regular Galois games. Ideally these techniques should be proven correct by formalizing them in Isabelle and applied in a prototype implementation, extending or replacing our current tool[1]. Also links to the recently introduced concept of energy functions (see here) should be explored. This work lifted the concept of energies with bounds to a proper semiring structure, circumventing the problem that energy propagation with bounds is not associative. With this extension it is possible to apply classical algorithms like Floyd-Warshall to energy problems.
By moving from a weighted graph to a game graph, we also open the door for reactive synthesis in this setting. That is we move from performing emptiness checks (Does a feasible run exist?) to solving synthesis problems (Can the system appropriately react to all possible environment behaviors?).
Indeed the winning strategy in such a game can be interpreted as a controller ensuring that the system operates in such a way that the energy constraints are always satisfied while fulfilling the ω-regular specification. Such correct-by-construction controllers are an important topic in formal methods and may find direct applications. In [1] it is shown that this synthesis problem can be solved in a symbolic fashion based on μ-calculus for ω-regular energy games and it would be interesting to compare the efficiency of both approaches.
The proposed internship covers a large spectrum of different activities, from theory to formal proofs of correctness to prototype implementation. Achieving all of the goals is highly unlikely in one internship, we therefore expect the candidate to choose a subset of the goals and detail

[1] `https://github.com/PhilippSchlehuberCaissier/wspot`

in your application letter why you have chosen them and how they fit your former studies and experiences.

3 Practical Information

The internship is supervised by Benjamin Bisping (PostDoc at Télécom SudParis RS2M), Sven Dziadek and Philipp Schlehuber-Caissier (both assistant professors at Télécom Sudparis and members of SAMOVAR). It will be located at the Télécom SudParis campus in Evry-Courcouronnes (9 rue Charles Fourier 91011 Evry-Courcouronnes) or in Palaiseau (19 place Marguerite Perey 91120 Palaiseau).
If you are interested in this subject, do not hesitate to get in touch with us for further details: benjamin.bisping@telecom-sudparis.eu sven.dziadek@telecom-sudparis.eu and philipp.schlehuber-caissier@telecom-sudparis.eu.

4 Qualifications

- Foundations of automata theory and regular languages
- C++ and / or Python and / or Isabelle

References

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[7] Caroline Lemke and Benjamin Bisping. Galois energy games: To solve all kinds of quantitative reachability problems, 2025.