Energy Functions for
Galois ω-Automata

1 Context

In formal methods and theoretical computer science, weighted automata have long been a subject of intense study. These mathematical models extend traditional finite automata with weights, allowing for the quantitative analysis of system behaviors [4]. Recent years have seen a growing interest in extensions of these automata, particularly in the context of energy constraints. Here, the weight is interpreted as energy that can be stored and accumulated. The energy is bounded from below in the sense that it must not drop below zero; it can also be bounded from above to model, for instance, a battery with a given capacity. These bounds can be interpreted as hard or weak bounds, depending on the semantics of the system and finding an energy-feasible path in such automata is discussed in [2].
Building on these works, [5, 6] extend this approach to search for energy-feasible paths in Büchi- and ω-automata. In this setting, finding and extracting such a path becomes significantly more involved as shown in Fig. 1. The complexity stems from the fact that, in order for a run to be valid, it needs to respect both the qualitative constraint given by the automaton (like the Büchi-condition shown here) and the quantitative constraint on the accumulated energy along the path. Due to the bounds on the energy, these constraints are tightly intertwined and can not be considered separately as in [3].

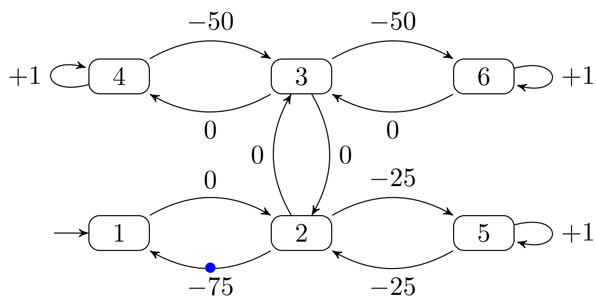

Figure 1: Non-trivial for ω-regular energy problems. Consider the energy to be bounded between 0 and 75. In order to satisfy the Büchi condition, every accepting path needs to take the accepting transition (The one carrying the blue dot) infinitely often. This transition however costs 75 energy, which gives and additional quantitative condition. To accumulate enough energy, we need to loop sufficiently often on the states 4, 5 and 6.

Independently, another line of research [7] extends the notion of energy to vectors making it multidimensional energies. A simple 4-dimensional example is shown in Fig. 2. The game in the example models the situation of a researcher, who hosts a coauthor at his office and wants to energize her with ten shots of espresso. For this, he leaves his Office for the CoffeeMaker and brews some Shots (bounded by the number of Cups he carries). Upon returning to the Office, the shots can be transformed into energization E. The return usually takes two units of time T. But there is a quicker way, passing by the office of the Department Head. Unfortunately, the department head will either cost our host some time by chatting or help herself to one of the filled coffee cups. (Which of the two happens is not under his control.) Depending on the number of cups, he has to take several rounds until energization E reaches level 10 and the game can be won by moving to Energized.

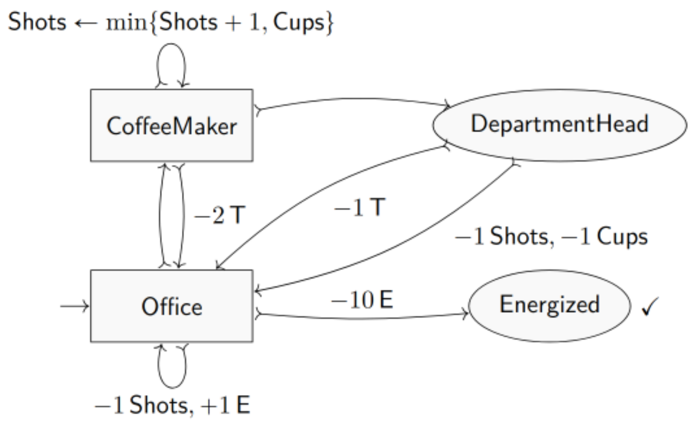

Figure 2: A Galois game with four "energies": Coffee shots, Cups, Time T and Energization E. The node shape indicates the controlling player with most of the game being controlled by the attacker (rectangular nodes).

This approach allows fairly complex updates on 2-player game graphs considering multi-dimensional energy vectors and other variants where energy levels are only partially ordered. For the moment, neither 2-player games nor multidimensional energies are currently supported by our existing approach for ω-regular energy problems.

2 Objective

The objective of the internship is to unite the two approaches and develop techniques to solve ω-regular Galois energy games. Ideally these techniques should be proven correct by formalizing them in Isabelle and applied in a prototype implementation, extending or replacing our current tool[1].

By moving from a weighted graph to a game graph, we also open the door for reactive synthesis in this setting. That is we move from performing emptiness checks (Does a feasible run exist?) to solving synthesis problems (Can the system appropriately react to all possible environment behaviors?).

Indeed the winning strategy in such a game can be interpreted as a controller ensuring that the system operates in such a way that the energy constraints are always satisfied while fulfilling the ω-regular specification. Such correct-by-construction controllers are an important topic in

[1] `https://github.com/PhilippSchlehuberCaissier/wspot`

formal methods and may find direct applications. In [1], it is shown that this synthesis problem can be solved in a symbolic fashion based on μ-calculus for ω-regular energy games and it would be interesting to compare the efficiency of both approaches.
The proposed internship covers a large spectrum of different activities, from theory to formal proofs of correctness to prototype implementation. Achieving all of the goals is beyond the scope of *one* internship, we therefore expect the candidate to choose a main goal and detail in their application letter why, especially with respect to fit to former studies and experiences.

3 Practical Information

The internship is supervised by

- Benjamin Bisping (PostDoc at Télécom SudParis RS2M),
- Sven Dziadek (assistant professor at Télécom SudParis, member of SAMOVAR), and
- Philipp Schlehuber-Caissier (assistant professor at Télécom SudParis, member of SAMOVAR)

It will be located at the Télécom SudParis campus in Evry-Courcouronnes (9 rue Charles Fourier, 91011 Evry-Courcouronnes) or in Palaiseau (19 place Marguerite Perey, 91120 Palaiseau).
If you are interested in this subject, do not hesitate to get in touch with us for further details:

- benjamin.bisping@telecom-sudparis.eu
- sven.dziadek@telecom-sudparis.eu
- philipp.schlehuber-caissier@telecom-sudparis.eu

4 Qualifications

- Foundations of automata theory and regular languages
- C++ and / or Python and / or Isabelle

References

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