Transformation MK2.0

Note: body is symmetric, dimensions of all legs are equal

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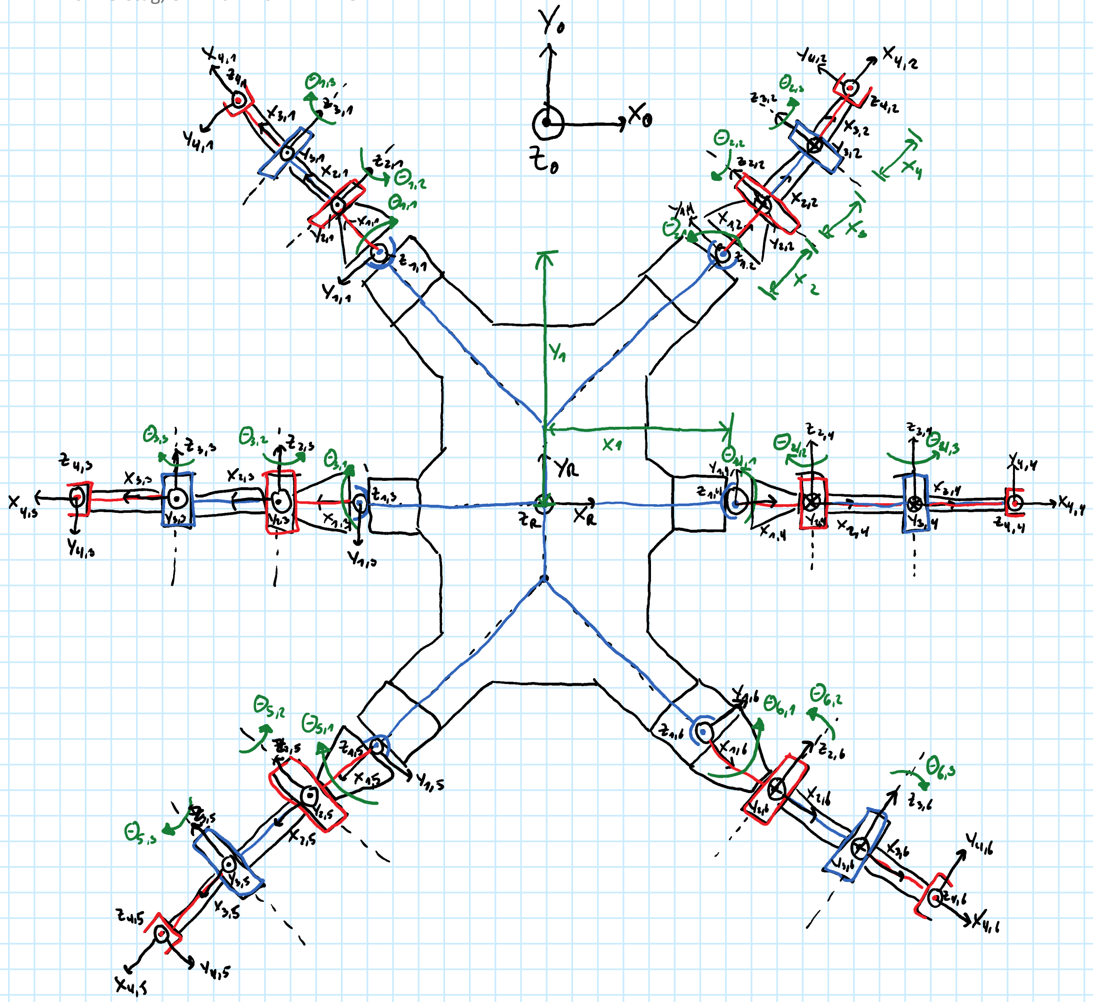

FRAMES

$T_{0,R}$:

$$B_{0,R} = R_z(\gamma)\cdot R_y(\beta)\cdot R_x(\alpha) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}, \quad T_{a,b} = \left[\begin{array}{c|c} B_{a,b} & t_{a,b} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$$t_{0,R} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\gamma)\cos(\beta) & -\sin(\gamma)\cos(\alpha)+\cos(\gamma)\sin(\beta)\sin(\alpha) & \sin(\gamma)\sin(\alpha)+\cos(\gamma)\sin(\beta)\cos(\alpha) \\ \sin(\gamma)\cos(\beta) & \cos(\gamma)\cos(\alpha)+\sin(\gamma)\sin(\beta)\sin(\alpha) & -\cos(\gamma)\sin(\alpha)+\sin(\gamma)\sin(\beta)\cos(\alpha) \\ -\sin(\beta) & \cos(\beta)\sin(\alpha) & \cos(\beta)\cos(\alpha) \end{pmatrix}$$

$$\Rightarrow T_{0,R} = \left(\begin{array}{ccc|c} \cos(\gamma)\cos(\beta) & -\sin(\gamma)\cos(\alpha)+\cos(\gamma)\sin(\beta)\sin(\alpha) & \sin(\gamma)\sin(\alpha)+\cos(\gamma)\sin(\beta)\cos(\alpha) & X_0 \\ \sin(\gamma)\cos(\beta) & \cos(\gamma)\cos(\alpha)+\sin(\gamma)\sin(\beta)\sin(\alpha) & -\cos(\gamma)\sin(\alpha)+\sin(\gamma)\sin(\beta)\cos(\alpha) & Y_0 \\ -\sin(\beta) & \cos(\beta)\sin(\alpha) & \cos(\beta)\cos(\alpha) & Z_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$T_{R,1,1}$:

$$B_{R,1,1} = R_z(\gamma=135^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_{R,1,1} = \begin{pmatrix} -X_1 \\ Y_1 \\ Z_1 \end{pmatrix} \Rightarrow T_{R,1,1} = \left(\begin{array}{ccc|c} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -X_1 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & Y_1 \\ 0 & 0 & 1 & Z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$T_{R,1,2}$:

$$B_{R,1,2} = R_z(\gamma=45^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_{R,1,2} = \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} \Rightarrow T_{R,1,2} = \left(\begin{array}{ccc|c} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & X_1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & Y_1 \\ 0 & 0 & 1 & Z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$T_{R,1,3}$:

$$B_{R,1,3} = R_z(\gamma=180^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_{R,1,3} = \begin{pmatrix} -X_1 \\ 0 \\ Z_1 \end{pmatrix} \Rightarrow T_{R,1,3} = \left(\begin{array}{ccc|c} -1 & 0 & 0 & -X_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & Z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$T_{R,1,4}$:

$$B_{R,1,4} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_{R,1,4} = \begin{pmatrix} X_1 \\ 0 \\ Z_1 \end{pmatrix} \Rightarrow T_{R,1,4} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & X_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$\underline{T_{R,1,5}:}$

$$B_{R,1,5} = R_z(\gamma = 225^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, t_{R,1,5} = \begin{pmatrix} -x_1 \\ -y_1 \\ z_1 \end{pmatrix} \Rightarrow T_{R,1,5} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -x_1 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\underline{T_{R,1,6}:}$

$$B_{R,1,6} = R_z(\gamma = -45^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, t_{R,1,6} = \begin{pmatrix} x_1 \\ -y_1 \\ z_1 \end{pmatrix} \Rightarrow T_{R,1,6} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & x_1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\underline{T_{1,2,2,2} = T_{1,4,2,4} = T_{1,6,2,6}:}$

$$B = R_z(\gamma = \theta_1) \cdot R_x(\alpha = -90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) \\ 0 & -1 & 0 \end{pmatrix}, t = \begin{pmatrix} x_2 \cdot \cos(\theta_1) \\ -x_2 \cdot \sin(\theta_1) \\ 0 \end{pmatrix}, T_{1,2,2,2} = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & x_2 \cdot \cos(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & -x_2 \cdot \sin(\theta_1) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{1,4,2,4} = T_{1,6,2,6}$$

$\underline{T_{1,1,2,1} = T_{1,3,2,3} = T_{1,5,2,5}:}$

$$B = R_z(\gamma = -\theta_1) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) \\ -\sin(\theta_1) & 0 & -\cos(\theta_1) \\ 0 & 1 & 0 \end{pmatrix}, t = \begin{pmatrix} x_2 \cos(\theta_1) \\ x_2 \sin(\theta_1) \\ 0 \end{pmatrix}, T_{1,1,2,1} = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & x_2 \cos(\theta_1) \\ -\sin(\theta_1) & 0 & -\cos(\theta_1) & x_2 \cdot \sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{1,3,2,3} = T_{1,5,2,5}$$

$\underline{T_{2,1,3,1} = T_{2,3,3,3} = T_{2,5,3,5}:}$

$$B = R_z(\gamma = \theta_2) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}, t = \begin{pmatrix} x_3 \cdot \cos(\theta_2) \\ x_3 \cdot \sin(\theta_2) \\ 0 \end{pmatrix} \Rightarrow T_{2,1,3,1} = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & x_3 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{2,3,3,3} = T_{2,5,3,5}$$

$\underline{T_{2,2,3,2} = T_{2,4,3,4} = T_{2,6,3,6}:}$

$$B = R_z(\gamma = -\theta_2) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}, t = \begin{pmatrix} x_3 \cdot \cos(\theta_2) \\ -x_3 \cdot \sin(\theta_2) \\ 0 \end{pmatrix} \Rightarrow T_{2,2,3,2} = \begin{pmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 & x_3 \cdot \cos(\theta_2) \\ -\sin(\theta_2) & \cos(\theta_2) & 0 & -x_3 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{2,4,3,4} = T_{2,6,3,6}$$

$\underline{T_{3,1,4,1} = T_{3,3,4,3} = T_{3,5,4,5}:}$

$$B = R_z(\gamma = -\theta_3) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) \\ 0 & -1 & 0 \end{pmatrix}, t = \begin{pmatrix} x_4 \cdot \cos(\theta_3) \\ -x_4 \cdot \sin(\theta_3) \\ 0 \end{pmatrix} \Rightarrow T_{3,1,4,1} = \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4 \cdot \cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4 \cdot \sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{3,3,4,3} = T_{3,5,4,5}$$

$\underline{T_{3,2,4,2} = T_{3,4,4,4} = T_{3,6,4,6}:}$

$$B = R_z(\gamma = \theta_3) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) \\ 0 & 1 & 0 \end{pmatrix}, t = \begin{pmatrix} x_4 \cdot \cos(\theta_3) \\ x_4 \cdot \sin(\theta_3) \\ 0 \end{pmatrix} \Rightarrow T_{3,2,4,2} = \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4 \cdot \cos(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) & x_4 \cdot \sin(\theta_3) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T_{3,4,4,4} = T_{3,6,4,6}$$

$\boxed{K_R \rightarrow K_4}$

$\underline{T_{R,1,4}:}$

$$T_{R,1,4} = T_{R,1,1} \cdot T_{1,1,2,1} \cdot T_{2,1,3,1} \cdot T_{3,1,4,1} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -x_1 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & x_2 \cos(\theta_1) \\ -\sin(\theta_1) & 0 & -\cos(\theta_1) & x_2 \cdot \sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & x_3 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4 \cdot \cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4 \cdot \sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2}(\sin(\theta_1) - \cos(\theta_1)) & 0 & \frac{\sqrt{2}}{2}(\sin(\theta_1) + \cos(\theta_1)) & -\left(\frac{\sqrt{2}}{2} x_2 (\cos(\theta_1) + \sin(\theta_1))\right) + x_1 \\ \frac{\sqrt{2}}{2}(\sin(\theta_1) + \cos(\theta_1)) & 0 & \frac{\sqrt{2}}{2}(-\sin(\theta_1) + \cos(\theta_1)) & \frac{\sqrt{2}}{2} x_2 (\cos(\theta_1) - \sin(\theta_1)) + y_1 \\ 0 & 1 & 0 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & x_3 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4 \cdot \cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4 \cdot \sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \left(\begin{array}{ccc|c} \frac{\sqrt2}{2}\cos(\theta_2)(\sin(\theta_1)-\cos(\theta_1)) & -\frac{\sqrt2}{2}\sin(\theta_2)(\sin(\theta_1)-\cos(\theta_1)) & \frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1)) & \frac{\sqrt2}{2}x_3(\sin(\theta_1)-\cos(\theta_1))-(\frac{\sqrt2}{2}x_2(\cos(\theta_1)+\sin(\theta_1)))+x_1 \\ \frac{\sqrt2}{2}\cos(\theta_2)(\sin(\theta_1)+\cos(\theta_1)) & -\frac{\sqrt2}{2}\sin(\theta_2)(\sin(\theta_1)+\cos(\theta_1)) & \frac{\sqrt2}{2}(-\sin(\theta_1)+\cos(\theta_1)) & \frac{\sqrt2}{2}x_3(\sin(\theta_1)+\cos(\theta_1))+\frac{\sqrt2}{2}x_2(\cos(\theta_1)-\sin(\theta_1))+y_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3\sin(\theta_2)+z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\cdot\left(\begin{array}{ccc|c} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4\cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{c|c} \frac{\sqrt2}{2}\cos(\theta_2)\cos(\theta_3)(\sin(\theta_1)-\cos(\theta_1))+\frac{\sqrt2}{2}\sin(\theta_2)\sin(\theta_3)(\sin(\theta_1)-\cos(\theta_1)) & -\frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1)) \\ \frac{\sqrt2}{2}\cos(\theta_2)\cos(\theta_3)(\sin(\theta_1)+\cos(\theta_1))-\frac{\sqrt2}{2}\sin(\theta_2)\sin(\theta_3)(\sin(\theta_1)+\cos(\theta_1)) & -\frac{\sqrt2}{2}(-\sin(\theta_1)+\cos(\theta_1)) \\ \sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3) & 0 \\ \hline 0 & 0 \end{array}\right. \ldots$$

$$\ldots \left.\begin{array}{c|c} \frac{\sqrt2}{2}\cos(\theta_2)\sin(\theta_3)(\sin(\theta_1)-\cos(\theta_1))-\frac{\sqrt2}{2}\sin(\theta_2)\cos(\theta_3)(\sin(\theta_1)-\cos(\theta_1)) & (x_4\cos(\theta_2)+x_3)\frac{\sqrt2}{2}(\sin(\theta_1)-\cos(\theta_1))-(\frac{\sqrt2}{2}x_2+x_1) \\ \frac{\sqrt2}{2}\cos(\theta_2)\sin(\theta_3)(\sin(\theta_1)+\cos(\theta_1))-\frac{\sqrt2}{2}\sin(\theta_2)\cos(\theta_3)(\sin(\theta_1)+\cos(\theta_1)) & (x_4\sin(\theta_2)+x_3)\frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1))+\frac{\sqrt2}{2}x_2+y_1 \\ \sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3) & x_4\sin(\theta_2)+z_1 \\ \hline 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} \frac{\sqrt2}{2}(\sin(\theta_1)-\cos(\theta_1))(\cos(\theta_2)\cos(\theta_3)+\sin(\theta_2)\sin(\theta_3)) & -\frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1)) & \frac{\sqrt2}{2}(\sin(\theta_1)-\cos(\theta_1))(\cos(\theta_2)\sin(\theta_3)-\sin(\theta_2)\cos(\theta_3)) & (x_4\cos(\theta_2)+x_3)\frac{\sqrt2}{2}(\sin(\theta_1)-\cos(\theta_1))-(\frac{\sqrt2}{2}x_2+x_1) \\ \frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1))(\cos(\theta_2)\cos(\theta_3)-\sin(\theta_2)\sin(\theta_3)) & \frac{\sqrt2}{2}(\sin(\theta_1)-\cos(\theta_1)) & \frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1))(\cos(\theta_2)\sin(\theta_3)-\sin(\theta_2)\cos(\theta_2)) & (x_4\sin(\theta_2)+x_3)\frac{\sqrt2}{2}(\sin(\theta_1)+\cos(\theta_1))+\frac{\sqrt2}{2}x_2+y_1 \\ \sin(\theta_2)\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3) & 0 & \sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3) & x_4\sin(\theta_2)+z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$= \left(\begin{array}{c|c|c} \frac{-\frac{\sqrt2}{2}\cos(\theta_1-\theta_2+\theta_3)-\frac{\sqrt2}{2}\cos(\theta_1+\theta_2-\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1+\theta_2-\theta_3)}{2} & -\frac{\sqrt2}{2}\cos(\theta_1)-\frac{\sqrt2}{2}\sin(\theta_1) & \frac{-\frac{\sqrt2}{2}\cos(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\cos(\theta_1+\theta_2-\theta_3)-\frac{\sqrt2}{2}\sin(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1+\theta_2-\theta_3)}{2} \\ \frac{\frac{\sqrt2}{2}\cos(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\cos(\theta_1+\theta_2-\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1+\theta_2-\theta_3)}{2} & -\frac{\sqrt2}{2}\cos(\theta_1)+\frac{\sqrt2}{2}\sin(\theta_1) & \frac{-\frac{\sqrt2}{2}\cos(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}\cos(\theta_1+\theta_2-\theta_3)+\frac{\sqrt2}{2}\sin(\theta_1-\theta_2+\theta_3)-\frac{\sqrt2}{2}\sin(\theta_1+\theta_2-\theta_3)}{2} \\ \sin(\theta_2-\theta_3) & 0 & \cos(\theta_2-\theta_3) \\ \hline 0 & 0 & 0 \end{array}\right.$$

$$\left.\begin{array}{l|l} x = (1) & \frac{-2x_1-2\frac{\sqrt2}{2}x_2\cos(\theta_1)-\frac{\sqrt2}{2}x_3\cos(\theta_1+\theta_2)-\frac{\sqrt2}{2}x_3\cos(\theta_1-\theta_2)-\frac{\sqrt2}{2}x_4\cos(\theta_1-\theta_2+\theta_3)-\frac{\sqrt2}{2}x_4\cos(\theta_1+\theta_2-\theta_3)-2\frac{\sqrt2}{2}x_2\sin(\theta_1)+\frac{\sqrt2}{2}x_3\sin(\theta_1+\theta_2)+\frac{\sqrt2}{2}x_3\sin(\theta_1-\theta_2)+\frac{\sqrt2}{2}x_4\sin(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}x_4\sin(\theta_1+\theta_2-\theta_3)}{2} \\ y = (2) & \frac{2y_1+2\frac{\sqrt2}{2}x_2\cos(\theta_1)+\frac{\sqrt2}{2}x_3\cos(\theta_1+\theta_2)+\frac{\sqrt2}{2}x_3\cos(\theta_1-\theta_2)+\frac{\sqrt2}{2}x_4\cos(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}x_4\cos(\theta_1+\theta_2-\theta_3)-2\frac{\sqrt2}{2}x_2\sin(\theta_1)+\frac{\sqrt2}{2}x_3\sin(\theta_1+\theta_2)+\frac{\sqrt2}{2}x_3\sin(\theta_1-\theta_2)+\frac{\sqrt2}{2}x_4\sin(\theta_1-\theta_2+\theta_3)+\frac{\sqrt2}{2}x_4\sin(\theta_1+\theta_2-\theta_3)}{2} \\ z = (3) & z_1+x_3\sin(\theta_2)+x_4\sin(\theta_2-\theta_3) \\ \hline & 1 \end{array}\right)$$

(1): $x=-x_1+\frac12\cdot\frac{\sqrt2}{2}\cdot[-2x_2\cos(\theta_1)-x_3\cos(\theta_1+\theta_2)-x_3\cos(\theta_1-\theta_2)-x_4\cos(\theta_1-\theta_2+\theta_3)-x_4\cos(\theta_1+\theta_2-\theta_3)-2x_2\sin(\theta_1)+x_3\sin(\theta_1+\theta_2)+x_3\sin(\theta_1-\theta_2)+x_4\sin(\theta_1-\theta_2+\theta_3)+x_4\sin(\theta_1+\theta_2-\theta_3)]$

$=-x_1+\frac{\sqrt2}{4}\cdot[x_2(-2\cos(\theta_1)-2\sin(\theta_1))+x_3\cdot(-\cos(\theta_1+\theta_2)-\cos(\theta_1-\theta_2)+\sin(\theta_1+\theta_2)+\sin(\theta_1-\theta_2))+x_4\cdot(-\cos(\theta_1-\theta_2+\theta_3)-\cos(\theta_1+\theta_2-\theta_3)+\sin(\theta_1-\theta_2+\theta_3)+\sin(\theta_1+\theta_2-\theta_3))]$

$=-x_1+\frac{\sqrt2}{4}\cdot[-2x_2(\cos(\theta_1)+\sin(\theta_1))+x_3\cdot(-(\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2))-(\cos(\theta_1)\cos(\theta_2)+\sin(\theta_1)\sin(\theta_2))+(\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2))+(\sin(\theta_1)\cos(\theta_2)-\cos(\theta_1)\sin(\theta_2)))$

$+x_4\cdot(-(\cos(\theta_1+\theta_3)\cos(\theta_2)+\sin(\theta_1+\theta_3)\sin(\theta_2))-(\cos(\theta_1+\theta_2)\cos(\theta_3)+\sin(\theta_1+\theta_2)\sin(\theta_3))+\sin(\theta_1+\theta_3)\cos(\theta_2)-\cos(\theta_1+\theta_3)\sin(\theta_2)+\sin(\theta_1+\theta_2)\cos(\theta_3)-\cos(\theta_1+\theta_2)\sin(\theta_3))]$

$=-x_1+\frac{\sqrt2}{4}\cdot[-2x_2(\cos(\theta_1)+\sin(\theta_1))+x_3\cdot(-(\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2))-(\cos(\theta_1)\cos(\theta_2)+\sin(\theta_1)\sin(\theta_2))+(\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2))+(\sin(\theta_1)\cos(\theta_2)-\cos(\theta_1)\sin(\theta_2)))$

$+x_4(-([\cos(\theta_1)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_3)]\cos(\theta_2)+[\sin(\theta_1)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_3)]\sin(\theta_2))-([\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2)]\cos(\theta_3)+[\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2)]\sin(\theta_3))$

$+[\sin(\theta_1)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_3)]\cos(\theta_2)-[\cos(\theta_1)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_3)]\sin(\theta_2)+[\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2)]\cos(\theta_3)-[\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2)]\sin(\theta_3))]$

$=-x_1+\frac{\sqrt2}{4}\cdot[-2x_2\cos(\theta_1)-2x_2\sin(\theta_1)-x_3\cos(\theta_1)\cos(\theta_2)+x_3\sin(\theta_1)\sin(\theta_2)-x_3\cos(\theta_1)\cos(\theta_2)-x_3\sin(\theta_1)\sin(\theta_2)+x_3\sin(\theta_1)\cos(\theta_2)+x_3\cos(\theta_1)\sin(\theta_2)+x_3\sin(\theta_1)\cos(\theta_2)-x_3\cos(\theta_1)\sin(\theta_2)$

$-x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)+x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)-x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)+x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$

$+x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+x_4\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-x_4\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)+x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+x_4\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)-x_4\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)]$

$=-x_1+\frac{\sqrt2}{4}\cdot[-2x_2\cos(\theta_1)-2x_2\sin(\theta_1)-2x_3\cos(\theta_1)\cos(\theta_2)+2x_3\sin(\theta_1)\cos(\theta_2)-2x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+2x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)+2x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)$

$+2x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)+2x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+2x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)]$

$=-x_1+\frac{\sqrt2}{2}\cdot[(-x_2-x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)+x_4\sin(\theta_2)\sin(\theta_3))\cdot\cos(\theta_1)+(-x_2+x_3\cos(\theta_2)+x_4\cos(\theta_2)\sin(\theta_3)+x_4\sin(\theta_2)\cos(\theta_3)+x_4\cos(\theta_2)\cos(\theta_3)+x_4\sin(\theta_2)\sin(\theta_3))\sin(\theta_1)]$

$\Rightarrow \sqrt2\cdot(x-x_1)=[-x_2-(x_3+x_4\cos(\theta_3))\cos(\theta_2)+x_4\sin(\theta_3)\sin(\theta_2)]\cos(\theta_1)+[-x_2+(x_3+x_4\sin(\theta_3)+x_4\cos(\theta_3))\cos(\theta_2)+(x_4\cos(\theta_3)+x_4\sin(\theta_3))\sin(\theta_2)]\cdot\sin(\theta_1)$

$\Leftrightarrow \sin(\theta_1)=\frac{\sqrt2(x-x_1)+[x_2+(x_3+x_4\cos(\theta_3))\cos(\theta_2)-x_4\sin(\theta_3)\sin(\theta_2)]\cos(\theta_1)}{-x_2+[x_3+x_4(\sin(\theta_3)+\cos(\theta_3))]\cos(\theta_2)+[x_4(\sin(\theta_3)+\cos(\theta_3))]\sin(\theta_2)}$ (4)

(2): $y=y_1+\frac12\cdot\frac{\sqrt2}{2}\cdot[2x_2\cos(\theta_1)+x_3\cos(\theta_1+\theta_2)+x_3\cos(\theta_1-\theta_2)+x_4\cos(\theta_1-\theta_2+\theta_3)+x_4\cos(\theta_1+\theta_2-\theta_3)-2x_2\sin(\theta_1)+x_3\sin(\theta_1+\theta_2)+x_3\sin(\theta_1-\theta_2)+x_4\sin(\theta_1-\theta_2+\theta_3)+x_4\sin(\theta_1+\theta_2-\theta_3)]$

$\Leftrightarrow\ = y_1+\frac{\sqrt2}{4}\cdot[2x_2\cos(\theta_1)+x_3(\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2))+x_3(\cos(\theta_1)\cos(\theta_2)+\sin(\theta_1)\sin(\theta_2))+x_4(\cos(\theta_1+\theta_3)\cos(\theta_2)-\sin(\theta_1+\theta_3)\sin(\theta_2))+x_4(\cos(\theta_1+\theta_2)\cos(\theta_3)-\sin(\theta_1+\theta_2)\sin(\theta_3))-2x_2\sin(\theta_1)$

$+x_3(\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2))+x_3(\sin(\theta_1)\cos(\theta_2)-\cos(\theta_1)\sin(\theta_2))+x_4(\sin(\theta_1+\theta_3)\cos(\theta_2)-\cos(\theta_1+\theta_3)\sin(\theta_2))+x_4(\sin(\theta_1+\theta_2)\cos(\theta_3)-\cos(\theta_1+\theta_2)\sin(\theta_3))]$

$= y_1+\frac{\sqrt2}{4}\cdot[2x_2\cos(\theta_1)+2x_3\cos(\theta_1)\cos(\theta_2)\cancel{-x_3\sin(\theta_1)\sin(\theta_2)}\cancel{+x_3\cos(\theta_1)\cos(\theta_2)+x_3\sin(\theta_1)\sin(\theta_2)}+x_4[(\cos(\theta_1)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_3))\cos(\theta_2)]-x_4[(\sin(\theta_1)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_3))\sin(\theta_2)]$

$$+\ x_4\left[(\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2))\cos(\theta_3)\right]-x_4\left[(\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2))\sin(\theta_3)\right]-2x_2\sin(\theta_1)+2x_3\sin(\theta_1)\cos(\theta_2)+\cancel{x_3\cos(\theta_1)\sin(\theta_2)}+\cancel{x_3\sin(\theta_1)\cos(\theta_2)}-\cancel{x_3\cos(\theta_1)\sin(\theta_2)}$$

$$+\ x_4\left[(\sin(\theta_1)\cos(\theta_3)+\cos(\theta_1)\sin(\theta_3))\cos(\theta_2)\right]-x_4\left[(\cos(\theta_1)\cos(\theta_3)-\sin(\theta_1)\sin(\theta_3))\sin(\theta_2)\right]+x_4\left[(\sin(\theta_1)\cos(\theta_2)+\cos(\theta_1)\sin(\theta_2))\cos(\theta_3)\right]-x_4\left[(\cos(\theta_1)\cos(\theta_2)-\sin(\theta_1)\sin(\theta_2))\sin(\theta_3)\right]\Big]$$

$$=y_1+\frac{\sqrt{2}}{4}\Big[2x_2\cos(\theta_1)+2x_3\cos(\theta_1)\cos(\theta_2)+2x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-2x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-2x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)-2x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)+\cancel{x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)}$$

$$-\cancel{x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)}-\cancel{x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)}-\cancel{x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)}-2x_2\sin(\theta_1)+2x_3\sin(\theta_1)\cos(\theta_2)+2x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+\cancel{x_4\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)}$$

$$-\cancel{x_4\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)}+2x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)+\cancel{x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)}+x_4\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)-\cancel{x_4\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)}+\cancel{x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)}\Big]$$

$$=y_1+\frac{\sqrt{2}}{4}\Big[2x_2\cos(\theta_1)+2x_3\cos(\theta_1)\cos(\theta_2)+2x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-2x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-2x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)-2x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)-2x_2\sin(\theta_1)+2x_3\sin(\theta_1)\cos(\theta_2)$$

$$+2x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)+2x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\Big]$$

$$=y_1+\frac{\sqrt{2}}{2}\left[x_2\cos(\theta_1)+x_3\cos(\theta_1)\cos(\theta_2)-x_2\sin(\theta_1)+x_3\sin(\theta_1)\cos(\theta_2)\right]+\frac{\sqrt{2}}{2}\Big[\big(x_4\cos(\theta_1)\cos(\theta_2)-x_4\sin(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\cos(\theta_2)\big)\cos(\theta_3)+\big(-x_4\sin(\theta_1)\cos(\theta_2)-x_4\cos(\theta_1)\sin(\theta_2)$$

$$+x_4\sin(\theta_1)\sin(\theta_2)\big)\sin(\theta_3)\Big]$$

$$\Rightarrow \sin(\theta_3)=\frac{\sqrt{2}(y-y_1)-x_2\cos(\theta_1)-x_3\cos(\theta_1)\cos(\theta_2)+x_2\sin(\theta_1)-x_3\sin(\theta_1)\cos(\theta_2)+\left[x_4\cos(\theta_1)\cos(\theta_2)-x_4\sin(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\cos(\theta_2)\right]\cos(\theta_3)}{-x_4\sin(\theta_1)\cos(\theta_2)-x_4\cos(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\sin(\theta_2)} \quad (5)$$

(3): $z=z_1+x_3\cdot\sin(\theta_2)+x_4\cdot\sin(\theta_2-\theta_3)=z_1+x_3\cdot\sin(\theta_2)+x_4\big(\sin(\theta_2)\cdot\cos(\theta_3)-\cos(\theta_2)\sin(\theta_3)\big)=z_1+x_3\cdot\sin(\theta_2)+x_4\cdot\sin(\theta_2)\cos(\theta_3)-x_4\cos(\theta_2)\sin(\theta_3)$

$\Leftrightarrow z=z_1+(x_3+x_4\cdot\cos(\theta_3))\cdot\sin(\theta_2)-(x_4\cdot\sin(\theta_3))\cos(\theta_2)$

$$\Leftrightarrow \sin(\theta_2)=\frac{z-z_1+x_4\cdot\sin(\theta_3)\cos(\theta_2)}{x_3+x_4\cdot\cos(\theta_3)} \quad (6)$$

Umstellung von (5) nach $\sin(\theta_1)$:

$$(5):\ \sin(\theta_3)=\frac{\sqrt{2}(y-y_1)-x_2\cos(\theta_1)-x_3\cos(\theta_1)\cos(\theta_2)+x_2\sin(\theta_1)-x_3\sin(\theta_1)\cos(\theta_2)+\left[x_4\cos(\theta_1)\cos(\theta_2)-x_4\sin(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\cos(\theta_2)\right]\cdot\cos(\theta_3)}{-x_4\sin(\theta_1)\cos(\theta_2)-x_4\cos(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\sin(\theta_2)}$$

$$\Leftrightarrow \big(-x_4\sin(\theta_1)\cos(\theta_2)-x_4\cos(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\sin(\theta_2)\big)\sin(\theta_3)=\sqrt{2}(y-y_1)-x_2\cos(\theta_1)-x_3\cos(\theta_1)\cos(\theta_2)+x_2\sin(\theta_1)-x_3\sin(\theta_1)\cos(\theta_2)$$

$$+\left[x_4\cos(\theta_1)\cos(\theta_2)-x_4\sin(\theta_1)\sin(\theta_2)+x_4\sin(\theta_1)\cos(\theta_2)\right]\cdot\cos(\theta_3)$$

$$\Leftrightarrow \sqrt{2}(y-y_1)=x_2\cos(\theta_1)+x_3\cos(\theta_1)\cos(\theta_2)-x_2\sin(\theta_1)+x_3\sin(\theta_1)\cos(\theta_2)-x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)-x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$-x_4\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)-x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)+x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$$

$$=\left[x_2+x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\sin(\theta_2)\sin(\theta_3)\right]\cos(\theta_1)+\left[-x_2+x_3\cdot\cos(\theta_2)+x_4\sin(\theta_2)\cos(\theta_3)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\cos(\theta_2)\sin(\theta_3)+x_4\sin(\theta_2)\sin(\theta_3)\right]\sin(\theta_1)$$

$$\Leftrightarrow \sin(\theta_1)=\frac{\sqrt{2}(y-y_1)-\left[x_2+x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\sin(\theta_2)\sin(\theta_3)\right]\cdot\cos(\theta_1)}{-x_2+x_3\cdot\cos(\theta_2)+x_4\sin(\theta_2)\cos(\theta_3)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\cos(\theta_2)\sin(\theta_3)+x_4\sin(\theta_2)\sin(\theta_3)}$$

$$\Leftrightarrow \sin(\theta_1)=\frac{\sqrt{2}(y-y_1)-\left[x_2+x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\sin(\theta_2)\sin(\theta_3)\right]\cdot\cos(\theta_1)}{-x_2+\left[x_3-x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)} \quad (7)$$

(4) = (7):

$$\frac{\overbrace{\sqrt{2}(x-x_1)}^{=a_1}+\overbrace{\left[x_2+(x_3+x_4\cos(\theta_3))\cos(\theta_2)-x_4\sin(\theta_3)\sin(\theta_2)\right]}^{b_1}\overbrace{\cos(\theta_1)}^{c_1}}{\underbrace{-x_2+\left[x_3+x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)}_{d_1}}=\frac{\overbrace{\sqrt{2}(y-y_1)}^{=a_2}-\overbrace{\left[x_2+x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\sin(\theta_2)\sin(\theta_3)\right]}^{b_2}\overbrace{\cos(\theta_1)}^{c_1}}{\underbrace{-x_2+\left[x_3-x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)}_{d_2}}$$

$$\hat{=}\ \frac{a_1+b_1\cdot c_1}{d_1}=\frac{a_2+b_2\cdot c_1}{d_2}\Leftrightarrow a_1d_2+b_1\cdot d_2\cdot c_1=a_2d_1+b_2d_1\cdot c_1\Leftrightarrow\left[b_1d_2-b_2d_1\right]c_1=a_2d_1-a_1d_2\Leftrightarrow c_1=\frac{a_2d_1-a_1d_2}{b_1d_2-b_2d_1}$$

$$\Rightarrow a_2d_1-a_1d_2=\left[\sqrt{2}(y-y_1)\right]\cdot\Big[-x_2+\left[x_3+x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)\Big]-\left[\sqrt{2}(x-x_1)\right]\cdot\Big[-x_2+\left[x_3-x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)\Big]$$

$$=-\sqrt{2}x_2y+\sqrt{2}x_2y_1+\sqrt{2}x_3y\cos(\theta_2)-\sqrt{2}x_3y_1\cos(\theta_2)+\sqrt{2}x_4y\sin(\theta_3)\cos(\theta_2)-\sqrt{2}x_4y_1\sin(\theta_3)\cos(\theta_2)+\sqrt{2}x_4y\cos(\theta_3)\cos(\theta_2)-\sqrt{2}x_4y_1\cos(\theta_3)\cos(\theta_2)+\sqrt{2}x_4y\sin(\theta_3)\sin(\theta_2)-\sqrt{2}x_4y_1\sin(\theta_3)\sin(\theta_2)$$

$$+\sqrt{2}x_4y\cos(\theta_3)\sin(\theta_2)-\sqrt{2}x_4y_1\cos(\theta_3)\sin(\theta_2)+\sqrt{2}xx_2-\sqrt{2}x_1x_2-\sqrt{2}xx_3+\sqrt{2}x_1x_3+\sqrt{2}xx_4\sin(\theta_3)\cos(\theta_2)-\sqrt{2}x_1x_4\sin(\theta_3)\cos(\theta_2)+\sqrt{2}xx_4\cos(\theta_3)\cos(\theta_2)-\sqrt{2}x_1x_4\cos(\theta_3)\cos(\theta_2)$$

$$-\sqrt{2}xx_4\sin(\theta_3)\sin(\theta_2)+\sqrt{2}x_1x_4\sin(\theta_3)\sin(\theta_2)-\sqrt{2}xx_4\cos(\theta_3)\sin(\theta_2)+\sqrt{2}x_1x_4\cos(\theta_3)\sin(\theta_2)$$

$$=x_2\sqrt{2}\cdot(y-y_1+x-x_1)+x_3\sqrt{2}(y-y_1-x+x_1)\cos(\theta_2)+x_4\sqrt{2}(y-y_1+x-x_1)\cos(\theta_2)\sin(\theta_3)+x_4\sqrt{2}(y-y_1+x-x_1)\cos(\theta_2)\cos(\theta_3)+x_4\sqrt{2}(y-y_1-x+x_1)\sin(\theta_2)\sin(\theta_3)$$

$$+x_4\sqrt{2}(y-y_1-x+x_1)\sin(\theta_2)\cos(\theta_3)$$

$$=\sqrt{2}\Big[\big(x_2+x_4\cos(\theta_2)\sin(\theta_3)+x_4\cos(\theta_2)\cos(\theta_3)\big)\cdot(y-y_1+x-x_1)+\big(x_3\cos(\theta_2)+x_4\sin(\theta_2)\sin(\theta_3)+x_4\sin(\theta_2)\cos(\theta_3)\big)\cdot(y-y_1-x+x_1)\Big]$$

$$\Rightarrow b_1d_2-b_2d_1=\left[x_2+(x_3+x_4\cos(\theta_3))\cos(\theta_2)-x_4\sin(\theta_3)\sin(\theta_2)\right]\cdot\Big[-x_2+\left[x_3-x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)\Big]$$

$$\cancel{-}+\Big(\cancel{-}\left[x_2+x_3\cos(\theta_2)-x_4\cos(\theta_2)\cos(\theta_3)-x_4\sin(\theta_2)\sin(\theta_3)\right]\Big)\cdot\Big[-x_2+\left[x_3+x_4(\sin(\theta_3)+\cos(\theta_3))\right]\cos(\theta_2)+\left[x_4(\sin(\theta_3)+\cos(\theta_3))\right]\sin(\theta_2)\Big]$$

$$
\begin{aligned}
&= \left[x_2 + x_3\cos(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)\right]\cdot\left[-x_2 + x_3\cos(\theta_2) - x_4\cos(\theta_2)\sin(\theta_3) - x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3) + x_4\sin(\theta_2)\cos(\theta_3)\right]\\
&\quad + \left[x_2 + x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)\right]\cdot\left[-x_2 + x_3\cos(\theta_2) + x_4\cos(\theta_2)\sin(\theta_3) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3) + x_4\sin(\theta_2)\cos(\theta_3)\right]\\
&= -2x_2^2 \cancel{+ x_2x_3\cos(\theta_2)} \cancel{- x_2x_4\sin(\theta_3)\cos(\theta_2)} \cancel{- x_2x_4\cos(\theta_3)\cos(\theta_2)} + x_2x_4\sin(\theta_3)\sin(\theta_2) + x_2x_4\cos(\theta_3)\sin(\theta_2) \cancel{- x_2x_3\cos(\theta_2)} + x_3^2\cos^2(\theta_2) \cancel{- x_3x_4\sin(\theta_3)\cos^2(\theta_2)} \cancel{- x_3x_4\cos(\theta_3)\cos^2(\theta_2)} \cancel{+ x_3x_4\cos(\theta_2)\sin(\theta_2)\sin(\theta_3)}\\
&\quad + x_3x_4\cos(\theta_2)\sin(\theta_2)\cos(\theta_3) \cancel{- x_2x_4\cos(\theta_3)} \cancel{+ x_3x_4\cos^2(\theta_2)\cos(\theta_3)} - x_4^2\cos^2(\theta_2)\cos(\theta_3)\sin(\theta_3) - x_4^2\cos^2(\theta_2)\cos^2(\theta_3) \cancel{+ x_4^2\cos(\theta_2)\sin(\theta_2)\cos(\theta_3)\sin(\theta_3)} \cancel{+ x_4^2\cos(\theta_2)\sin(\theta_2)\cos^2(\theta_3)} + x_2x_4\sin(\theta_2)\sin(\theta_3)\\
&\quad \cancel{- x_3x_4\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)} \cancel{+ x_4^2\sin(\theta_2)\cos(\theta_2)\sin^2(\theta_3)} \cancel{+ x_4^2\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)\cos(\theta_3)} - x_4^2\sin^2(\theta_2)\sin^2(\theta_3) - x_4^2\sin^2(\theta_2)\sin(\theta_3)\cos(\theta_3)\\
&\quad \cancel{-x_2^2} \cancel{+ x_2x_3\cos(\theta_2)} \cancel{+ x_2x_4\sin(\theta_3)\cos(\theta_2)} \cancel{+ x_2x_4\cos(\theta_3)\cos(\theta_2)} + x_2x_4\sin(\theta_3)\sin(\theta_2) + x_2x_4\cos(\theta_3)\sin(\theta_2) \cancel{- x_2x_3\cos(\theta_2)} + x_3^2\cos^2(\theta_2) \cancel{+ x_3x_4\sin(\theta_3)\cos^2(\theta_2)} \cancel{+ x_3x_4\cos(\theta_3)\cos^2(\theta_2)}\\
&\quad \cancel{+ x_3x_4\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)} \cancel{+ x_3x_4\sin(\theta_2)\cos(\theta_2)\cos(\theta_3)} + x_2x_4\cos(\theta_2)\cos(\theta_3) \cancel{- x_3x_4\cos^2(\theta_2)\cos(\theta_3)} \cancel{- x_4^2\cos^2(\theta_2)\sin(\theta_3)\cos(\theta_3)} - x_4^2\cos^2(\theta_2)\cos^2(\theta_3) \cancel{- x_4^2\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)\cos(\theta_3)}\\
&\quad \cancel{- x_4^2\sin(\theta_2)\cos(\theta_2)\cos^2(\theta_3)} + x_2x_4\sin(\theta_2)\sin(\theta_3) \cancel{- x_3x_4\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)} \cancel{- x_4^2\sin(\theta_2)\cos(\theta_2)\sin^2(\theta_3)} \cancel{- x_4^2\sin(\theta_2)\cos(\theta_2)\sin(\theta_3)\cos(\theta_3)} - x_4^2\sin^2(\theta_2)\sin^2(\theta_3) - x_4^2\sin^2(\theta_2)\sin(\theta_3)\cos(\theta_3)\\
&= -2x_2^2 + 4x_2x_4\sin(\theta_2)\sin(\theta_3) + 2x_2x_4\sin(\theta_2)\cos(\theta_3) + 2x_3^2\cos^2(\theta_2) + 2x_3x_4\sin(\theta_2)\cos(\theta_2)\cos(\theta_3) \underbrace{-2x_4^2\cos^2(\theta_2)\sin(\theta_3)\cos(\theta_3)}_{= -2x_4^2\sin(\theta_3)\cos(\theta_3)} - 2\cos^2(\theta_2)\cos^2(\theta_3) - 2\sin^2(\theta_2)\sin^2(\theta_3) - 2x_4^2\sin^2(\theta_2)\sin(\theta_3)\cos(\theta_3)
\end{aligned}
$$

$$
\Rightarrow \cos(\theta_1) = \frac{\sqrt{2}\left[\left(x_2 + x_4\cos(\theta_2)\sin(\theta_3) + x_4\cos(\theta_2)\cos(\theta_3)\right)\cdot(y - y_1 + x - x_1) + \left(x_3\cos(\theta_2) + x_4\sin(\theta_2)\sin(\theta_3) + x_4\sin(\theta_2)\cos(\theta_3)\right)\cdot(y - y_1 - x + x_1)\right]}{2\left(-x_2^2 + 2x_2x_4\sin(\theta_2)\sin(\theta_3) + x_2x_4\sin(\theta_2)\cos(\theta_3) + x_3^2\cos^2(\theta_2) + x_3x_4\sin(\theta_2)\cos(\theta_2)\cos(\theta_3) - x_4^2\sin(\theta_3)\cos(\theta_3) - \cos^2(\theta_2)\cos^2(\theta_3) - \sin^2(\theta_2)\sin^2(\theta_3)\right)} \qquad 8
$$

---

middle left leg (#3): $K_R \rightarrow K_{34}$

$\underline{T_{R,4,3}}$:

$$
T_{R,4,3} = T_{R,1,3}\cdot T_{1,3,2,3}\cdot T_{2,3,3,3}\cdot T_{3,3,4,3} = \begin{pmatrix} -1 & 0 & 0 & -x_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & x_2\cos(\theta_1) \\ -\sin(\theta_1) & 0 & -\cos(\theta_1) & x_2\sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & x_3\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4\cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
= \begin{pmatrix} -\cos(\theta_1) & 0 & \sin(\theta_1) & -x_1 - x_2\cos(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & -x_2\sin(\theta_1) \\ 0 & 1 & 0 & z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & x_3\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4\cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
= \begin{pmatrix} -\cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) & -x_1 - x_2\cos(\theta_1) - x_3\cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & -x_2\sin(\theta_1) + x_3\sin(\theta_1)\sin(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & x_3\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & x_4\cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -x_4\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
= \begin{pmatrix} -\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & \sin(\theta_1) & -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) & -x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - x_1 - x_2\cos(\theta_1) - x_3\cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & \cos(\theta_1) & \sin(\theta_1)\cos(\theta_2)\sin(\theta_3) - \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) & x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - x_2\sin(\theta_1) + x_3\sin(\theta_1)\sin(\theta_2) \\ \sin(\theta_2)\cos(\theta_3) - \cos(\theta_2)\sin(\theta_3) & 0 & \sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3) & x_4\sin(\theta_2)\cos(\theta_3) - x_4\cos(\theta_2)\sin(\theta_3) + x_3\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{matrix} = x \;\; 3.1 \\ = y \;\; 3.2 \\ = z \;\; 3.3 \\ \end{matrix}
$$

3.1: $x = -x_4\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - x_4\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - x_1 - x_2\cos(\theta_1) - x_3\cos(\theta_1)\cos(\theta_2)$

$\Leftrightarrow x + x_1 = \left[-x_2 - x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)\right]\cos(\theta_1) \Leftrightarrow \cos(\theta_1) = \dfrac{x + x_1}{-x_2 - x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)}$ 3.4

3.2: $y = x_4\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - x_2\sin(\theta_1) + x_3\sin(\theta_1)\sin(\theta_2)$

$\Leftrightarrow y = \left[-x_2 + x_3\sin(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3)\right]\sin(\theta_1) \Leftrightarrow \sin(\theta_1) = \dfrac{y}{-x_2 + x_3\sin(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3)}$ 3.5

3.3: $z = x_4\sin(\theta_2)\cos(\theta_3) - x_4\cos(\theta_2)\sin(\theta_3) + x_3\sin(\theta_2)$

$\Leftrightarrow z + x_4\cos(\theta_2)\sin(\theta_3) = \left[x_3 + x_4\cos(\theta_3)\right]\sin(\theta_2) \Leftrightarrow \sin(\theta_2) = \dfrac{z + x_4\sin(\theta_3)\cos(\theta_2)}{x_3 + x_4\cos(\theta_3)}$

$\Leftrightarrow z - x_4\sin(\theta_2)\cos(\theta_3) - x_3\sin(\theta_2) = \left[x_4\sin(\theta_3)\right]\cos(\theta_2) \Leftrightarrow \cos(\theta_2) = \dfrac{\left[x_3 + x_4\cos(\theta_3)\right]\sin(\theta_2) - z}{x_4\sin(\theta_3)}$

$$
\frac{3.5}{3.4}: \quad \frac{\sin(\theta_1)}{\cos(\theta_1)} = \tan(\theta_1) = \frac{\dfrac{y}{-x_2 + x_3\sin(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3)}}{\dfrac{x + x_1}{-x_2 - x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)}} = \frac{y\cdot\left[-x_2 - x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)\right]}{(x + x_1)\cdot\left[-x_2 + x_3\sin(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3)\right]}
$$

$$
\Rightarrow \underline{\underline{\theta_1 = \operatorname{arctan2}\left(\frac{y}{x + x_1}\cdot\frac{-x_2 - x_3\cos(\theta_2) - x_4\cos(\theta_2)\cos(\theta_3) - x_4\sin(\theta_2)\sin(\theta_3)}{-x_2 + x_3\sin(\theta_2) + x_4\cos(\theta_2)\cos(\theta_3) + x_4\sin(\theta_2)\sin(\theta_3)}\right)}} \qquad 3.6
$$