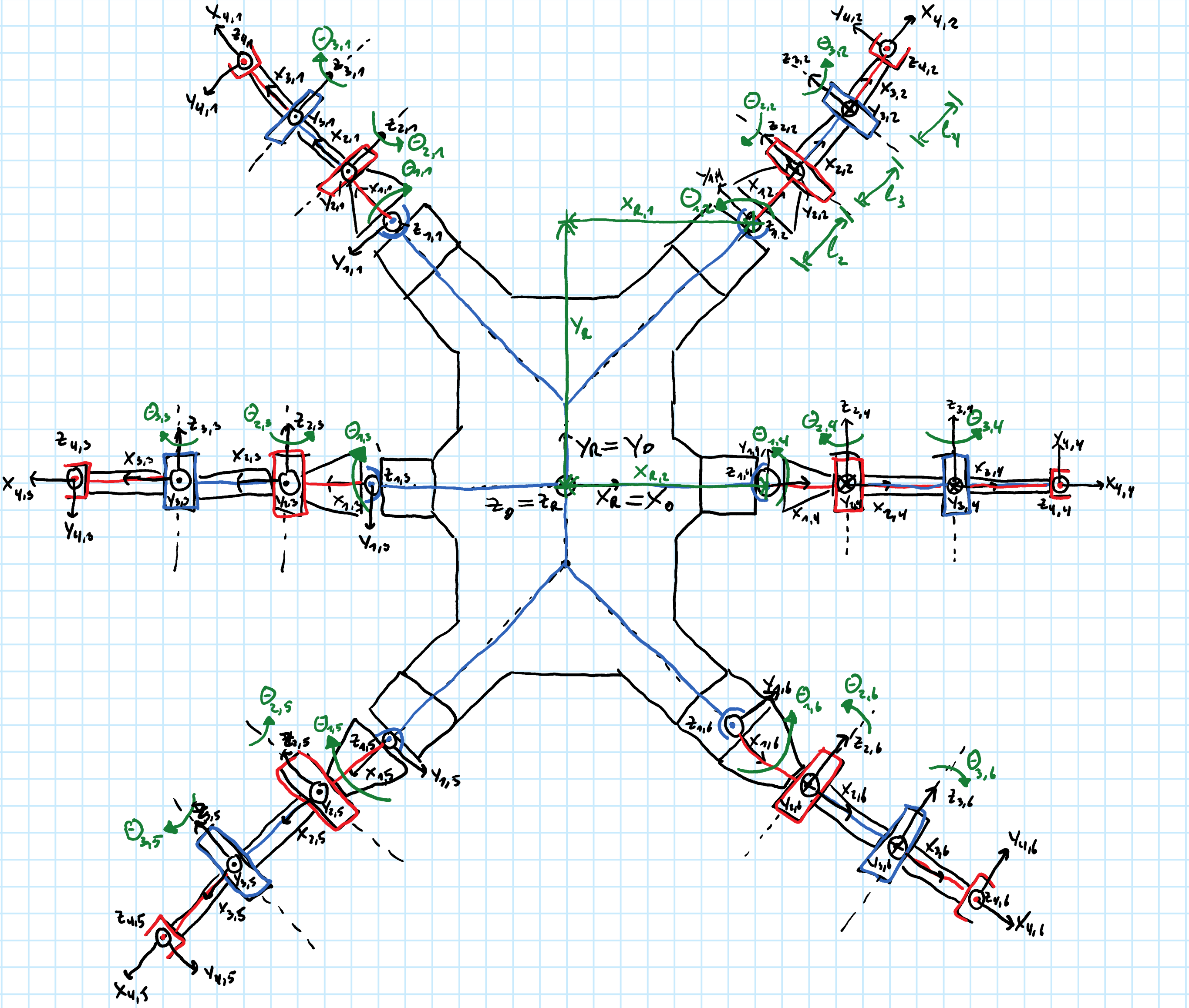
Transformation MK2.0 - Frames

Note: body is symmetric, dimensions of all legs are equal

Dienstag, 26. April 2022 16:01



<u>$T_{0,R}$:</u>

$$B_{0,R} = R_z(\gamma)\cdot R_y(\beta)\cdot R_x(\alpha) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

$$\boxed{T_{a,b} = \left[\begin{array}{c|c} B_{a,b} & t_{a,b} \\ \hline 0\;0\;0 & 1 \end{array}\right]}$$

$$t_{0,R} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\gamma)\cos(\beta) & -\sin(\gamma)\cos(\alpha)+\cos(\gamma)\sin(\beta)\sin(\alpha) & \sin(\gamma)\sin(\alpha)+\cos(\gamma)\sin(\beta)\cos(\alpha) \\ \sin(\gamma)\cos(\beta) & \cos(\gamma)\cos(\alpha)+\sin(\gamma)\sin(\beta)\sin(\alpha) & -\cos(\gamma)\sin(\alpha)+\sin(\gamma)\sin(\beta)\cos(\alpha) \\ -\sin(\beta) & \cos(\beta)\sin(\alpha) & \cos(\beta)\cos(\alpha) \end{pmatrix}$$

$$\Rightarrow T_{0,R} = \left(\begin{array}{ccc|c} \cos(\gamma)\cos(\beta) & -\sin(\gamma)\cos(\alpha)+\cos(\gamma)\sin(\beta)\sin(\alpha) & \sin(\gamma)\sin(\alpha)+\cos(\gamma)\sin(\beta)\cos(\alpha) & X_0 \\ \sin(\gamma)\cos(\beta) & \cos(\gamma)\cos(\alpha)+\sin(\gamma)\sin(\beta)\sin(\alpha) & -\cos(\gamma)\sin(\alpha)+\sin(\gamma)\sin(\beta)\cos(\alpha) & Y_0 \\ -\sin(\beta) & \cos(\beta)\sin(\alpha) & \cos(\beta)\cos(\alpha) & Z_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

<u>$T_{R,1,1}$:</u>

$$B_{R,1,1} = R_z(\gamma = 135^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},\quad t_{R,1,1} = \begin{pmatrix} -X_{R,1} \\ Y_R \\ Z_R \end{pmatrix}$$

$$\Rightarrow T_{R,1,1} = \left(\begin{array}{ccc|c} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -X_{R,1} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & Y_R \\ 0 & 0 & 1 & Z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$\underline{T_{R,1,2}:}$

$$B_{R,1,2} = R_z(\gamma = 45^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t_{R,1,2} = \begin{pmatrix} x_{R,1} \\ y_R \\ z_R \end{pmatrix} \Rightarrow T_{R,1,2} = \underline{\underline{\left(\begin{array}{ccc|c} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & x_{R,1} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & y_R \\ 0 & 0 & 1 & z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}}$$

$\underline{T_{R,1,3}:}$

$$B_{R,1,3} = R_z(\gamma = 180^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t_{R,1,3} = \begin{pmatrix} -x_{R,2} \\ 0 \\ z_R \end{pmatrix} \Rightarrow T_{R,1,3} = \underline{\underline{\left(\begin{array}{ccc|c} -1 & 0 & 0 & -x_{R,2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}}$$

$\underline{T_{R,1,4}:}$

$$B_{R,1,4} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t_{R,1,4} = \begin{pmatrix} x_{R,2} \\ 0 \\ z_R \end{pmatrix} \Rightarrow T_{R,1,4} = \underline{\underline{\left(\begin{array}{ccc|c} 1 & 0 & 0 & x_{R,2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}}$$

$\underline{T_{R,1,5}:}$

$$B_{R,1,5} = R_z(\gamma = 225^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t_{R,1,5} = \begin{pmatrix} -x_{R,1} \\ -y_R \\ z_R \end{pmatrix} \Rightarrow T_{R,1,5} = \underline{\underline{\left(\begin{array}{ccc|c} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -x_{R,1} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -y_R \\ 0 & 0 & 1 & z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}}$$

$\underline{T_{R,1,6}:}$

$$B_{R,1,6} = R_z(\gamma = -45^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t_{R,1,6} = \begin{pmatrix} x_{R,1} \\ -y_R \\ z_R \end{pmatrix} \Rightarrow T_{R,1,6} = \underline{\underline{\left(\begin{array}{ccc|c} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & x_{R,1} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -y_R \\ 0 & 0 & 1 & z_R \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}}$$

$\underline{T_{1,2,2,2} = T_{1,4,2,4} = T_{1,6,2,6}:}$

$$B = R_z(\gamma = \Theta_1) \cdot R_x(\alpha = -90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\Theta_1) & -\sin(\Theta_1) & 0 \\ \sin(\Theta_1) & \cos(\Theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} \cos(\Theta_1) & 0 & -\sin(\Theta_1) \\ \sin(\Theta_1) & 0 & \cos(\Theta_1) \\ 0 & -1 & 0 \end{pmatrix},\ t = \begin{pmatrix} l_2 \cdot \cos(\Theta_1) \\ -l_2 \cdot \sin(\Theta_1) \\ 0 \end{pmatrix},\ \underline{\underline{T_{1,2,2,2} = \left(\begin{array}{ccc|c} \cos(\Theta_1) & 0 & -\sin(\Theta_1) & l_2 \cdot \cos(\Theta_1) \\ \sin(\Theta_1) & 0 & \cos(\Theta_1) & -l_2 \cdot \sin(\Theta_1) \\ 0 & -1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{1,4,2,4} = T_{1,6,2,6}$$

$\underline{T_{1,1,2,1} = T_{1,3,2,3} = T_{1,5,2,5}:}$

$$B = R_z(\gamma = -\Theta_1) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\Theta_1) & \sin(\Theta_1) & 0 \\ -\sin(\Theta_1) & \cos(\Theta_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} \cos(\Theta_1) & 0 & -\sin(\Theta_1) \\ -\sin(\Theta_1) & 0 & -\cos(\Theta_1) \\ 0 & 1 & 0 \end{pmatrix},\ t = \begin{pmatrix} l_2 \cdot \cos(\Theta_1) \\ l_2 \sin(\Theta_1) \\ 0 \end{pmatrix},\ \underline{\underline{T_{1,1,2,1} = \left(\begin{array}{ccc|c} \cos(\Theta_1) & 0 & -\sin(\Theta_1) & l_2 \cos(\Theta_1) \\ -\sin(\Theta_1) & 0 & -\cos(\Theta_1) & l_2 \cdot \sin(\Theta_1) \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{1,3,2,3} = T_{1,5,2,5}$$

$\underline{T_{2,1,3,1} = T_{2,3,3,3} = T_{2,5,3,5}:}$

$$B = R_z(\gamma = \Theta_2) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(\Theta_2) & -\sin(\Theta_2) & 0 \\ \sin(\Theta_2) & \cos(\Theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t = \begin{pmatrix} l_3 \cdot \cos(\Theta_2) \\ l_3 \cdot \sin(\Theta_2) \\ 0 \end{pmatrix} \Rightarrow \underline{\underline{T_{2,1,3,1} = \left(\begin{array}{ccc|c} \cos(\Theta_2) & -\sin(\Theta_2) & 0 & l_3 \cdot \cos(\Theta_2) \\ \sin(\Theta_2) & \cos(\Theta_2) & 0 & l_3 \cdot \sin(\Theta_2) \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{2,3,3,3} = T_{2,5,3,5}$$

$\underline{T_{2,2,3,2} = T_{2,4,3,4} = T_{2,6,3,6}:}$

$$B = R_z(\gamma = -\Theta_2) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(\Theta_2) & \sin(\Theta_2) & 0 \\ -\sin(\Theta_2) & \cos(\Theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix},\ t = \begin{pmatrix} l_3 \cdot \cos(\Theta_2) \\ -l_3 \cdot \sin(\Theta_2) \\ 0 \end{pmatrix} \Rightarrow \underline{\underline{T_{2,2,3,2} = \left(\begin{array}{ccc|c} \cos(\Theta_2) & \sin(\Theta_2) & 0 & l_3 \cdot \cos(\Theta_2) \\ -\sin(\Theta_2) & \cos(\Theta_2) & 0 & -l_3 \cdot \sin(\Theta_2) \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{2,4,3,4} = T_{2,6,3,6}$$

$\underline{T_{3,1,4,1} = T_{3,3,4,3} = T_{3,5,4,5}:}$

$$R = R_z(\gamma = \theta_3) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) \\ 0 & -1 & 0 \end{pmatrix}, \quad t = \begin{pmatrix} l_4 \cdot \cos(\theta_3) \\ -l_4 \cdot \sin(\theta_3) \\ 0 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{T_{3,1,4,1} = \left(\begin{array}{ccc|c} \cos(\theta_3) & 0 & \sin(\theta_3) & l_4 \cdot \cos(\theta_3) \\ -\sin(\theta_3) & 0 & \cos(\theta_3) & -l_4 \cdot \sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{3,3,4,3} = T_{3,5,4,5}$$

$\underline{T_{3,2,4,2} = T_{3,4,4,4} = T_{3,6,4,6}:}$

$$R = R_z(\gamma = \theta_3) \cdot R_x(\alpha = 90^\circ) = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) \\ 0 & 1 & 0 \end{pmatrix}, \quad t = \begin{pmatrix} l_4 \cdot \cos(\theta_3) \\ l_4 \cdot \sin(\theta_3) \\ 0 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{T_{3,2,4,2} = \left(\begin{array}{ccc|c} \cos(\theta_3) & 0 & \sin(\theta_3) & l_4 \cdot \cos(\theta_3) \\ \sin(\theta_3) & 0 & -\cos(\theta_3) & l_4 \cdot \sin(\theta_3) \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)}} = T_{3,4,4,4} = T_{3,6,4,6}$$