Karlsruher Institut für Technologie

Institute for Operations Research
Chair of Stochastic Optimization
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Computer exercise on optimization approaches under uncertainty

Organizational information:

- The computer exercise can be completed in groups of up to 3 students.
- The processing and submission deadline isSunday, December 15, 2024, at 23:59 Clock.It is not possible to extend the submission deadline.
- The tax

  by email to Mr. Christian Füllner (christian.fuellner@kit.edu),

  should contain the answers to the questions from the task in a PDF document as well as the documented GAMS source code (.gms files),

  should be well commented and the results should be clearly presented,

  should contain all names, registration numbers and email addresses of the group members.
- By passing the calculator exercise, all group members receive a bonus of one grade step (0.3 or 0.4) on their exam. Bonuses are only credited to passed exams.
- A bonus earned in this semester also retains its value for future exams. Validity.
- The correction of the computer exercise only begins after the deadline.
- If the same solution is submitted twice, the computer exercise will be failed. of the groups.

Problem:

Given is the linear problem of optimizing the personal nutrition plan of a student.

The aim is to minimize nutritional costs while adhering to the recommended minimum and maximum amounts of various ingredients and taking into account some taste preferences.

For simplicity, we assume that only 9 different foods are available. The decision variables x_i, $i=1,\dots,9$, the amount of the i-th food in 100 g The foods associated with the decision variables are listed in Table 1.

Table 1: De nition of the decision variables

variable	amount of food in 100 g
x_1	apples
x_2	corn akes
x_3	carrots
x_4	potatoes
x_5	Cheese
x_6	Milk
x_7	Chocolate
x_8	spinach
x_9	steak

The optimization problem can then be formulated as follows:

min $0.22x_1+0.18x_2+0.07x_3+0.14x_4+0.55x_5+0.1x_6+0.54x_7+0.28x_8+3.2x_9$

st $52x_1+355x_2+26x_3+71x_4+354x_5+64x_6+536x_7+17x_8+121x_9\geq 2400$ (Minimum requirement calories in kcal)

$0.35x_1+7x_2+x_3+2x_4+25x_5+3.5x_6+9x_7+2.5x_8+21x_9\geq 56$ (Minimum protein requirement in G)

$18x_1+307x_2+7x_3+24.5x_4-177x_5-12x_6-52x_7-6.5x_8-60.5x_9\geq 0$ (Carbohydrates cover 50% of the calorie requirement)

$0.4x_1+0.6x_2+0.2x_3+0.11x_4+28.3x_5+3.5x_6+31.5x_7+0.3x_8+4x_9\geq 50$ (Minimum requirement of fats in G)

$0.4x_1+0.6x_2+0.2x_3+0.11x_4+28.3x_5+3.5x_6+31.5x_7+0.3x_8+4x_9\leq 70$ (Maximum consumption of fats in G)

$7x_1+13x_2+41x_3+6x_4+800x_5+120x_6+214x_7+126x_8+3x_9\geq 500$ (Minimum calcium requirement in mg)

$30x_1+60x_2+53x_3+47x_4+300x_5+170x_6+370x_7+230x_8+130x_9\geq 1100$

(Minimum requirement of vitamin B2 inμG) $x_1+x_3+x_8 \geq 4$ (At least400 g healthy) $x_1, \ldots, x_9 \leq 5$ $x_1, \ldots, x_9 \geq 0$

$x_2 \leq 3$

$x_6 \leq 2$

$x_7 \leq 3$

$x_9 \geq 1$

(barriers)

Explanation of the objective function:

The objective function coefficients describe the prices ineeach100 gGroceries.

Explanation of the constraints:

Minimum requirement of calories, proteins, fats, calcium and vitamin B2:To meet daily nutritional needs with moderate physical activity and average height, the student must consume at least2400 kcalThe recommended daily amounts for proteins, fats, calcium and vitamin B2 should also be adhered to.

Minimum carbohydrate requirement:The recommendation for carbohydrates is that at least 50% of the daily calorie intake should be covered by them. This also results in negative coefficients on the left side.

Maximum consumption of fats:The consumption of fats should be an amount of70 g per day.

Healthy eating:To keep the diet balanced, at least400 g of healthy foods (apples, carrots, spinach) are consumed.

Balanced diet and barriers:To avoid a very one-sided diet, the amount of each food should be500 gExceptions are corn akes with a maximum300 gand chocolate with a maximum300 gThere is a slight intolerance to milk, which is why only a maximum of 200 gper day. Since the student does not want to completely forego a juicy steak, this should be at least100 gare on the daily diet plan.

Task R.1 (LP modeling):

(a) Model the given optimization problem in GAMS.

Use index sets as described in the introduction to GAMS. Save your model under the nametask1.gms.

(b) Solve the problem using a suitable solver. State explicitly the optimal solution (optimum point and optimal value) of the problem.

Control result: The optimal value should be rounded5.17ebe.

Uncertainties that arise:

It is assumed that the data provided in the problem statement are to be regarded as uncertain due to various factors.

- Due to fluctuations in market prices and variations in type and quality of the food, it is to be expected that the objective function coefficients may be subject to fluctuations. Assume that the followingrelativeorabsolute Deviations from the initially assumed value are possible.

Table 2: Possible deviation of the objective function coefficients

	apples	corn akes	carrots	potatoes	Cheese
absolute deviation ineeach100 g	0.06			0.04	0.1
relative deviation in %		15	20		

	Milk	Chocolate	Spinach	Steak
absolute deviation ineeach100 g			0.1	
relative deviation in %	25	40		40

- Assume that the concentrations of proteins, fat, calcium and vitamin B2 in foods may vary. For simplification, the followingrelativesubject to deviations from their nominal values (in both directions).

Table 3: Possible relative deviations

ingredient	apples	corn akes	carrots	potatoes	Cheese
proteins	20%	10%	20%	5%	1%
fats	20%	20%	20%	2%	1%
calcium	20%	50%	20%	2%	10%
vitamin B2	15%	20%	20%	1%	5%

	Milk	Chocolate	Spinach	Steak
proteins	10%	1%	10%	15%
fats	10%	1%	15%	30%
calcium	20%	1%	10%	20%
vitamin B2	5%	1%	20%	15%

- The recommended minimum intake varies in different guides and depending on the constitution. The minimum information for calories and nutrients can vary considerably. Use the followingabsolutedeviations.

Table 4: Possible absolute deviations of the minimum requirement

Size	Calories	Proteins	Fats	Calcium	Vitamin B2
Unit	kcal	G	G	mg	μG
	350	10	10	200	300

ˆIn all uncertainties, assume that fluctuations in both directions are possible.

Task R.2 (Modeling of uncertainties):

The nutritional plan will then be protected against the existing uncertainties by means of robust optimization.

The uncertainty set is generally given by

$$U = \left\{ \underbrace{\left[\begin{array}{c|c} (c^0)^T & d^0 \\ \hline A^0 & b^0 \end{array}\right]}_{=:D^0} + \sum_{\ell=1}^{L} \zeta_\ell \underbrace{\left[\begin{array}{c|c} (c^\ell)^T & d^\ell \\ \hline A^\ell & b^\ell \end{array}\right]}_{=:D^\ell} \;\middle|\; \zeta \in Z \right\}$$

described.

(a) In your PDF submission, provide an example of the protein restriction (2nd restriction tion) the data matrices D^0 2, ..., D^ℓ explicitly.

When choosing L As usual in lectures and exercises, we assume that separate shift matrices and perturbation vectors are introduced for all columns of the problem, so that within each row the uncertain parameters can change independently of each other.

Task R.3 (Modeling of equivalents):

(a) First, we shall assume an interval uncertainty set.

(i) Formulate an LP equivalent of the robust counterpart of the problem in GAMS. Save your model under the nametask3a.gms.

(ii) Solve the problem using a suitable solver. State explicitly the optimal solution to the problem (optimum point and optimum value).

(iii) How many variables and restrictions does your model from (i) have? State explicitly how these numbers are composed.

(b) Now assume an ellipse uncertainty set.

(i) Formulate an equivalent of the robust counterpart of the problem in GAMS. Save your model under the nametask3b.gms.

(ii) Solve the problem using a suitable solver. State explicitly the optimal solution to the problem (optimum point and optimum value).

(c) Compare your solutions from task R.1(b), task R.3(a) and task R.3(b). Interpret the differences.

(d) Suppose that due to physical activity the calorie requirement can even be increased by750 g increase compared to the nominal value. What is special about solving the problem under interval uncertainty? Does this also apply under ellipse uncertainty?

Important: Specifications for modeling in GAMS:

ˆUse appropriate index sets in your modeling.

ˆModel the equivalents in the form presented in the lecture.

ˆTo do this, model the entries of the data matrices in a suitable form.
It is up to you whether you want to model the data matrices holistically or the entries c_ℓ, d_ℓ, b_ℓ, $A_\ell$define separately from each other.

ˆSolutions for the interval uncertainty, in which the coefficients are simply adjusted (naive approach from exercise 4) are not allowed!