Wavelets

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1. <u>Wavelets</u>

2.1 Introduction

The implementation of wavelets in signal processing allow us to analyse abrupt changes in transient responses more accurately compared to transforms such as Fourier and Gabor. This property is given by splitting the associated signal into subsequent stages, where each is down sampled by half the previous sampling frequency. Therefore, at each stage different characteristics can be analysed, and the frequency resolution is incremented.

The wavelet transform has a continuous and a discrete version, which differ on the way they discretize the scale and translation parameters. The discrete wavelet transform (DWT) on which we have focused has an aggressive down sampling that results in an efficient number of stage coefficients.
Fundamentally, the following components are applied in wavelet computation:

- *Mother and daughter wavelet.* These represent, respectively, the non-scaled and scaled functions of the wavelet transform. The mother wavelet has an oscillatory characteristic and its associated coefficients are referred to as detail coefficients, which constitute the high frequency part of the signal computed at every stage. These coefficients sustain the temporal resolution of the signal.

- *Father wavelet.* Also referred to as the scaling function. Represents the wavelet into which the projection is made to obtain the right approximation coefficients (low pass in nature), which maintain the frequency properties of the signal. [2]

2.2 Programming Exercises

2.2.1 ECG signal compression using wavelets

The goal of this exercise was to apply the discrete wavelet transform on a given ECG in order to reconstruct it using fewer coefficients. Compression of a signal is often found useful in file handling and could also serve for denoising purposes.

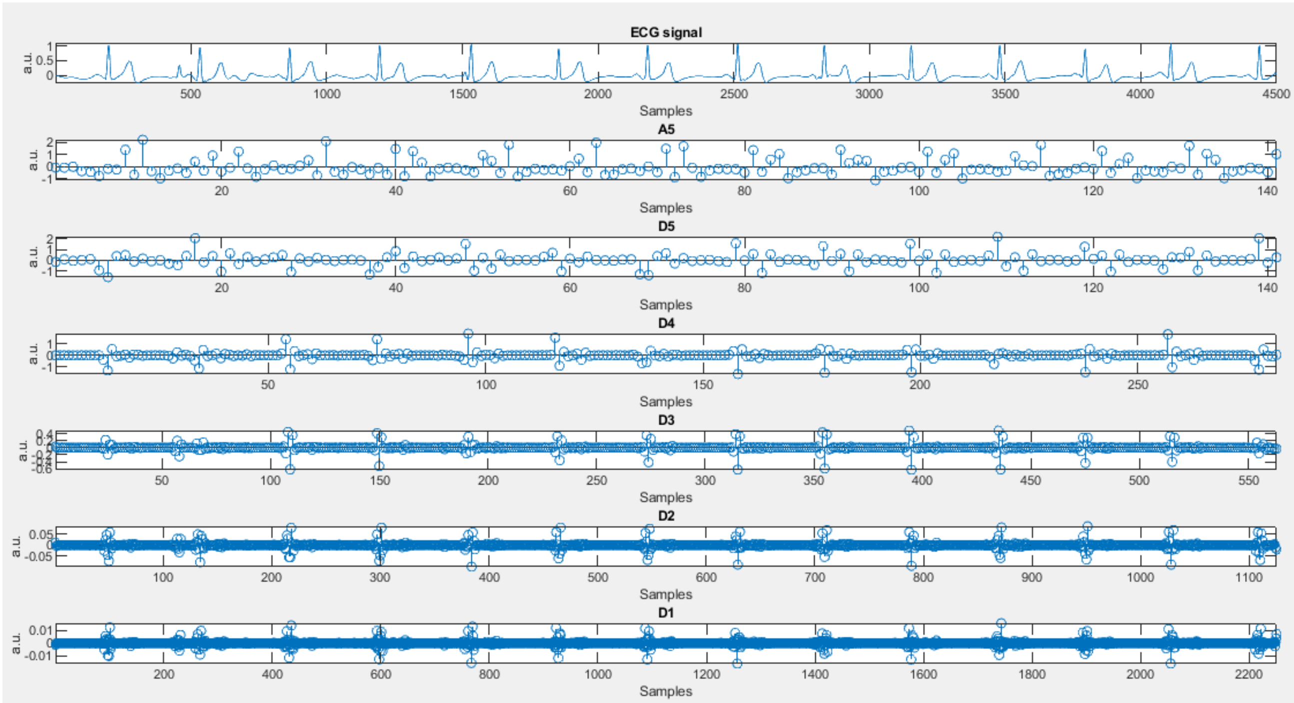

Figure 2.1 *Wavelet decomposition of the ECG Signal*

Figure 2.1 shows the initial wavelet decomposition performed on the ECG, obtained by using the Daubechies wavelet of fourth order in five levels.

__Question 1:__ *Given the wavelet decomposition, which coefficients (A5, D5, D4, D3, D2, D1) would you use to analyse the QRS complexes? Knowing that the sampling frequency is 300 Hz, what is the corresponding frequency range?*

The corresponding frequency ranges for this wavelet decomposition have been summarized in **Table 2.1.**

Level	Frequency Range	
	Min (Hz)	Max (Hz)
D1	75	150
D2	37.5	75
D3	18.75	37.5
D4	9.375	18.75
D5	4.6875	9.375
A5	0	4.6875

Table 2.1 *Frequency ranges of wavelet decomposition*

Knowing that, according to literature, the QRS complex frequency range lies between 0 and 20 Hz for frequency-based analysis [3] we can conclude that the coefficients A5, D5 and D4 would provide us the most discriminant information of QRS complexes. The use of these coefficients would serve to analyse their oscillatory and amplitude characteristics.

The first stages, such as D1 and D2, where the highest frequency ranges are present, would be useful to obtain an accurate moment in time of the QRS complexes occurrence, which could be of importance, for example, to compute the Heart Rate Variability (HRV) of a given subject.

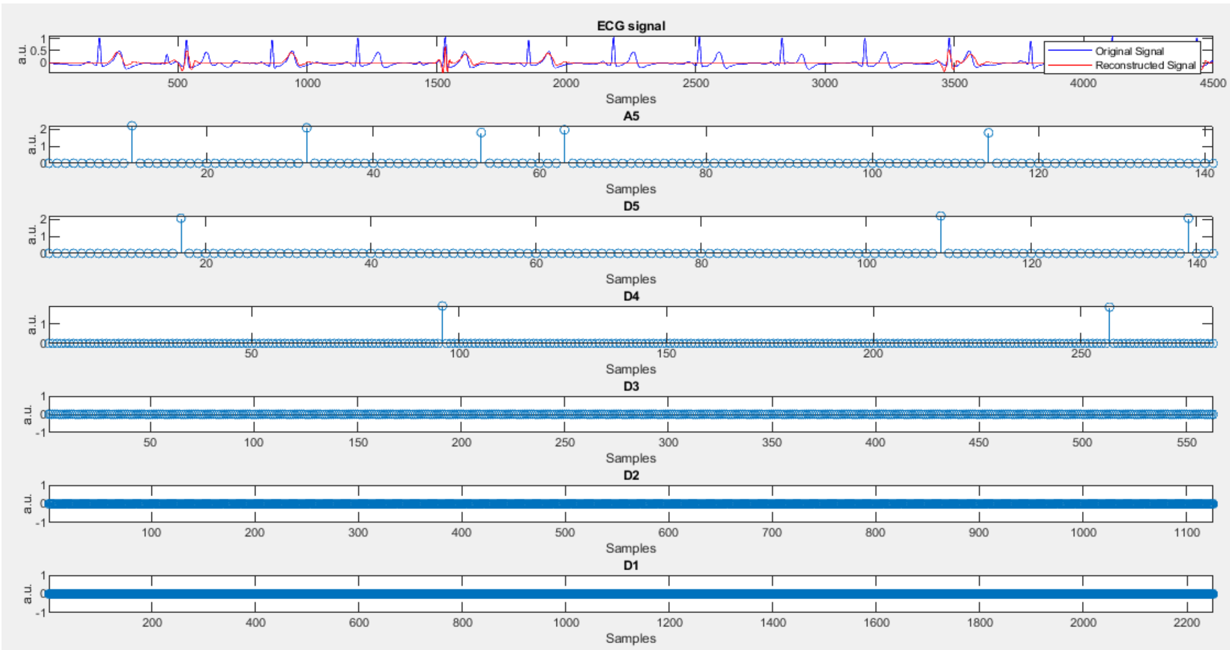

Figure 2.2. *Signal compression with 10 coefficients.*

A compressed reconstructed signal of the ECG has been obtained by retaining only the 10 highest coefficients across all levels. The resulting decomposition and subsequent reconstruction is shown in **Figure 2.2** and **Figure 2.3.**

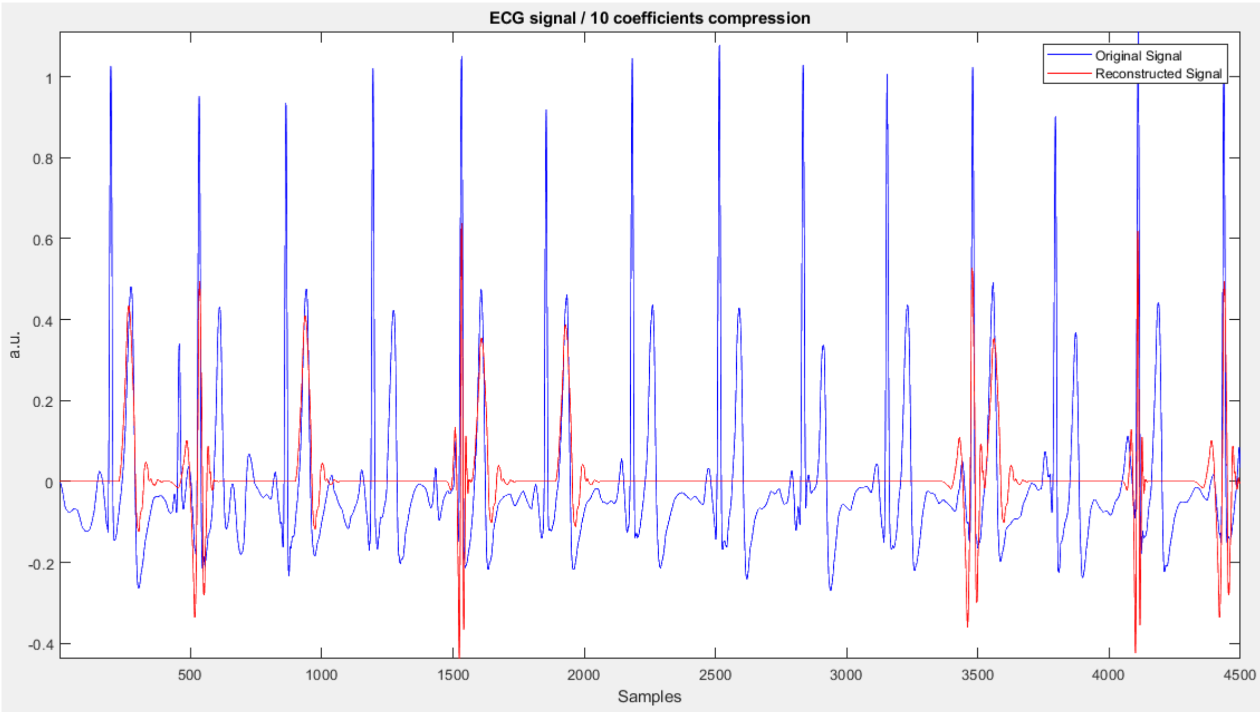

Figure 2.3. *Original and reconstructed signal with 10 coefficients compression.*

Question 2: *Compare the reconstructed ECG signal with 10 coefficients with the shapes of the scaling φ and the wavelet ψ functions of the Daubechies wavelet of order four. Why can you recognize φ at some places of the reconstructed ECG signal and ψ at other places?*

The scaling function represents the father wavelet, which corresponds to the approximation coefficients (A5 in this case) that exist to compensate for the gap left due to down sampling.

In the reconstructed signal in figure 2.3 the scaling function can be recognized 5 times while the wavelet function is present 5 other times. When compressing the signal with 10 coefficients, it is noticeable that 5 of the maximum absolute values are found in the A5 level (figure 2.2) and the rest between D5 and D4. Therefore, we can make a match between the coefficients and the purpose they serve in wavelet reconstruction.

The following figures (**Figure 2.4**, **Figure 2.5** and **Figure 2.6**) have been obtained from compressing the ECG signal with 25, 50 and 100 coefficients respectively. These graphs show that the wavelet transform allows us to obtain a sufficiently detailed reconstruction of a signal with a relatively small amount of computation expense and storage.

Additionally, it is noticeable that the most dominant coefficients (per their maximum absolute values) lay in the lowest frequency ranges, corresponding to A5, D5 and D4 (some in D3 for 100 coefficients). This is consistent with the known fact that the distinctive waves of an ECG signal (P, T and QRS) of a healthy subject have a frequency bandwidth below 50 Hz.

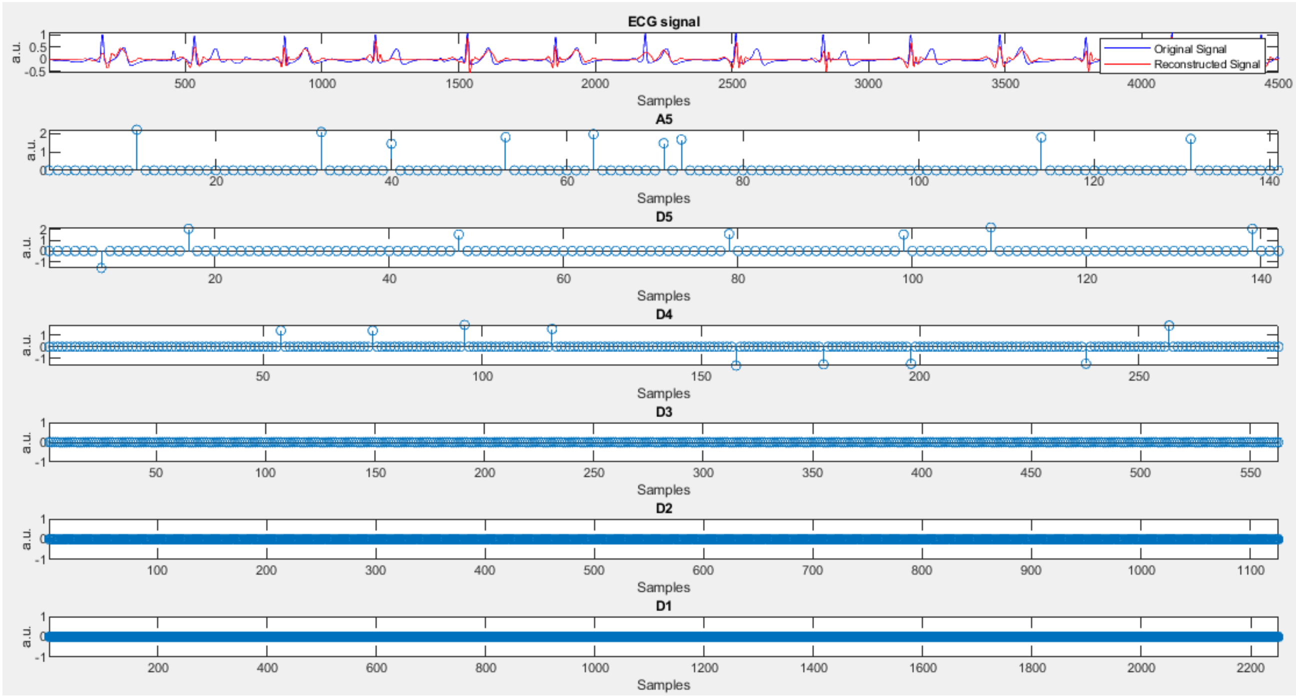

Figure 2.4. *Signal compression with 25 coefficients.*

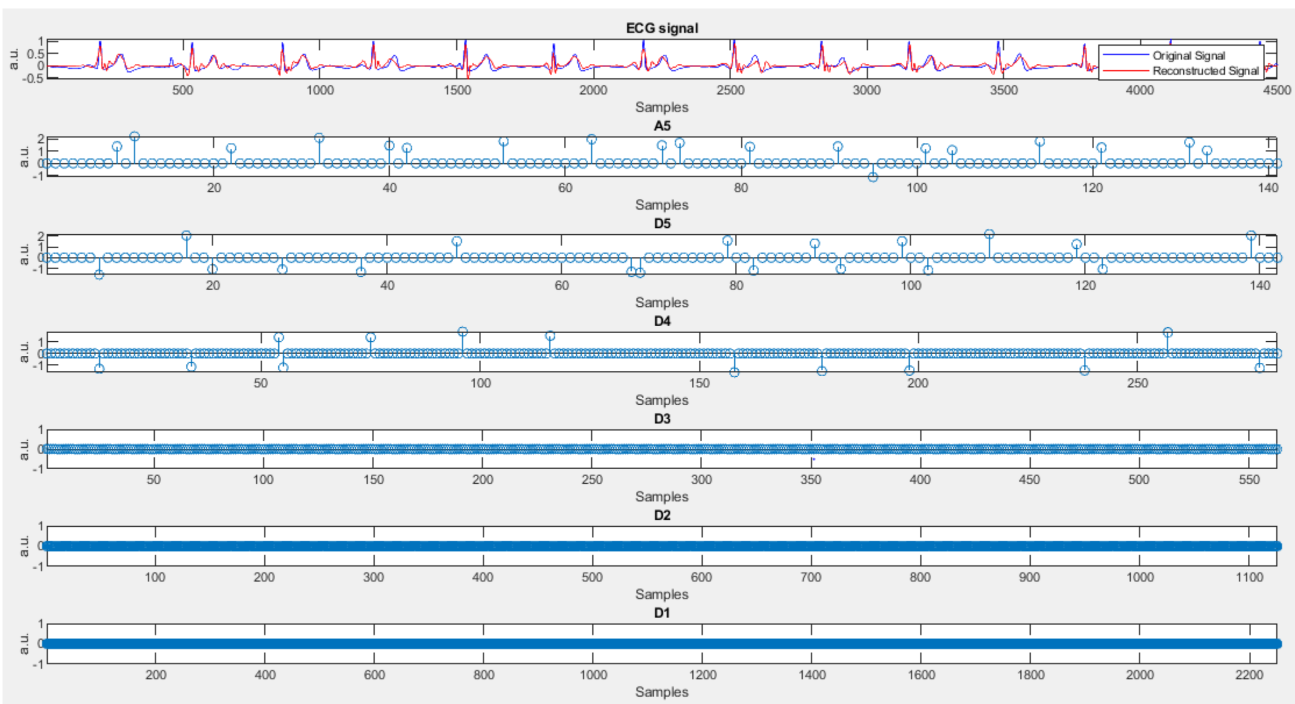

Figure 2.5. *Signal compression with 50 coefficients.*

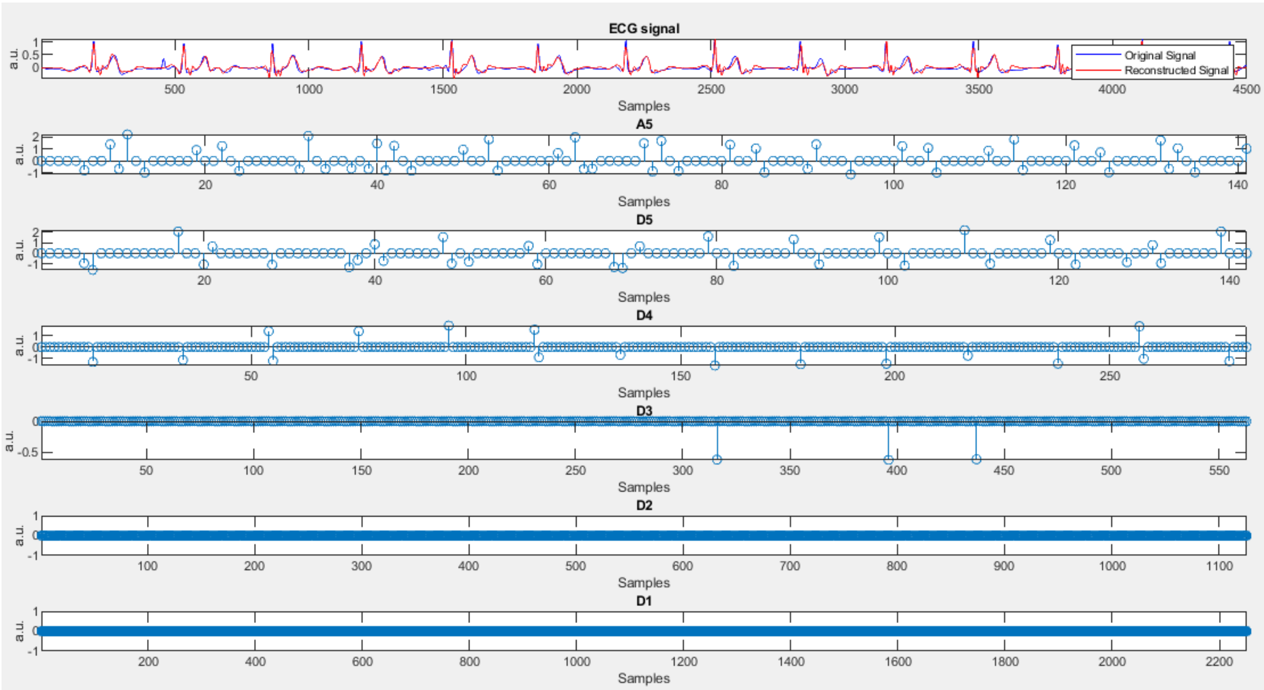

Figure 2.6. *Signal compression with 100 coefficients.*

In order to perform an objective analysis of the different compressions sizes, the Root Mean Squared Error (RMSE) and the Compression Rate (CR) have been computed as shown in **Table 2.2.**

The RMSE corresponds to the difference between the reconstructed and the original signal, and it can be noticed that it becomes lower as more coefficients are considered. The CR represents the ratio between the amount of the original coefficients and the retained coefficients after compression.

Reducing the compression rate 10 times (from 450 to 45) further reduces the RMSE by more than half. Also, we can appreciate that even with a compression rate of 45, where roughly only 2% of the original signal coefficients are kept, there's a RMSE value in the hundredths. These considerations show the strong wavelet properties for compression applications.

Compression Sizes	RMSE	CR
{'10' }	0.17366	450
{'25' }	0.14928	180
{'50' }	0.11784	90
{'100'}	0.080619	45

Table 2.2. *RMSE and CR for the different compression sizes.*

<u>Question 3:</u> *In this exercise, the Daubechies wavelet of order four has been used. Why would you prefer one wavelet over another?*

Following the wavelet selection general rule, we would prefer a wavelet that looks like the signal in question, which would result in high peaks in the cross-correlation. In this case, the Daubechies wavelet of fourth order has noticeable similarities with the QRS, P and T waves of an ECG.

In addition, the final objective should be considered. For instance, the Daubechies wavelet has important properties, such as its efficiency and energy preservation, that gives it an advantage for compression and denoising applications.

2.2.2 Epileptic seizure detection

This task's objective consisted in classifying EEG segments containing epileptic seizure activity. Training and test data with their related labels assignment (based on seizure or non-seizure presence) has been provided in order to perform supervised classification. Linear discriminant analysis has been implemented for the purpose of this exercise.

Initially, the given data has been plotted (**Figure 2.7**) to visualize the distinctive features between seizure and non-seizure segments.

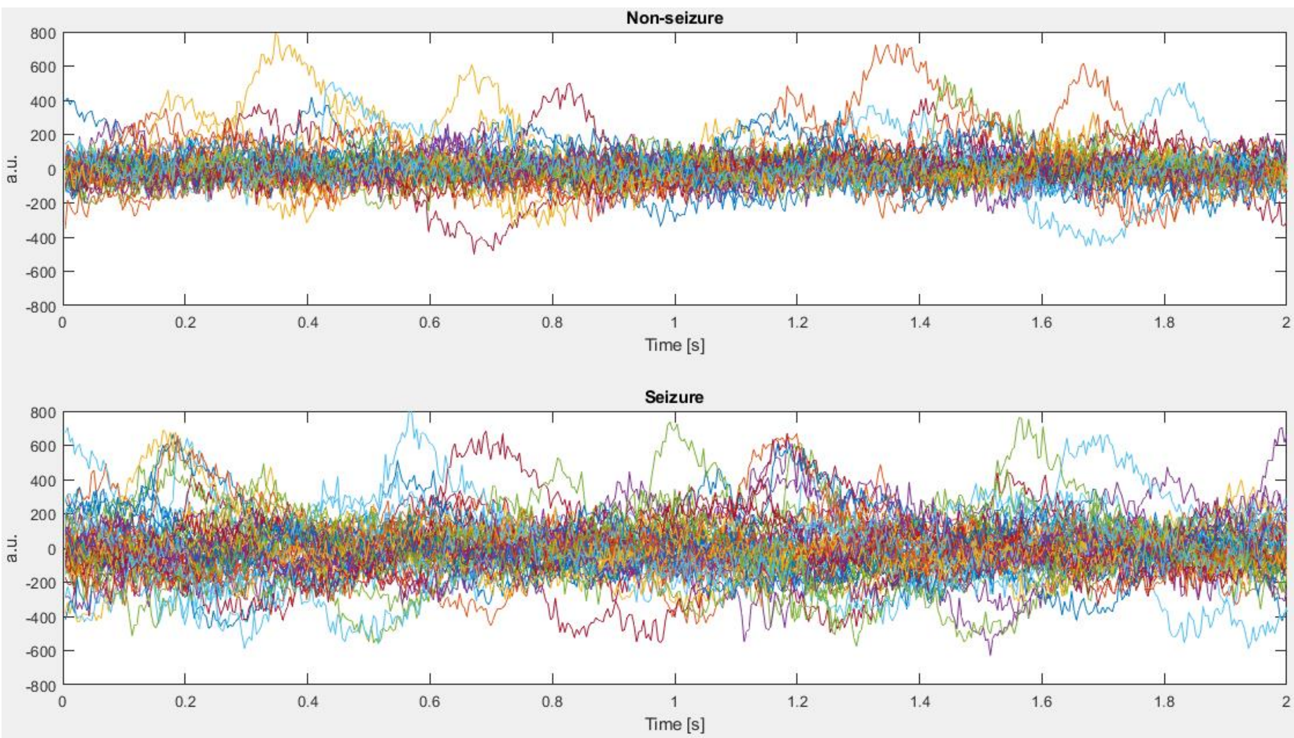

Figure 2.7. *Seizure and non-seizure segments.*

<u>Question 1:</u> *What discriminant property do you observe? How would you quantify this?*

By visualizing the X-axis, we can observe how in non-seizure activity higher frequencies (faster occurring oscillations) are present compared to seizure segments, which appear to be low frequency in nature. Additionally, the Y-axis of these graphs show a distinctive difference in amplitude between non-seizure and seizure segments, where the latter have higher amplitudes.

These properties could be quantified by computing the magnitude of the Y-axis ranges of both sets of variables. Moreover, the power spectral density (PSD) integral, which would give us the total signal power, could provide further insight.

Question 2: *Do you think we should normalize/standardize the raw data? Why (not)?*

After computing the mean and standard deviation of our given data segments, along with the data visualization, we have additionally centred the signal along the baseline by subtracting the mean of each segment, to account for any bias that may be present in the data. However, the raw data should not be standardized, since for this application the dynamic range that can be observed is in fact a discriminant feature. Therefore, by rescaling the data we could lose significant information that shows the variance between a seizure and non-seizure segment.

Two separate classifiers will be analysed in order to highlight how wavelets can be applied for classification purposes. In the first place, the full energy, which corresponds to the sum of the squared samples, has solely been taken as a feature. For the second classifier, we have computed the 6 sub band energies obtained by wavelet decomposition (Daubechies of order four) and used them as separate features.

The different features obtained can be visualized in the boxplots presented in **Figure 2.8.**

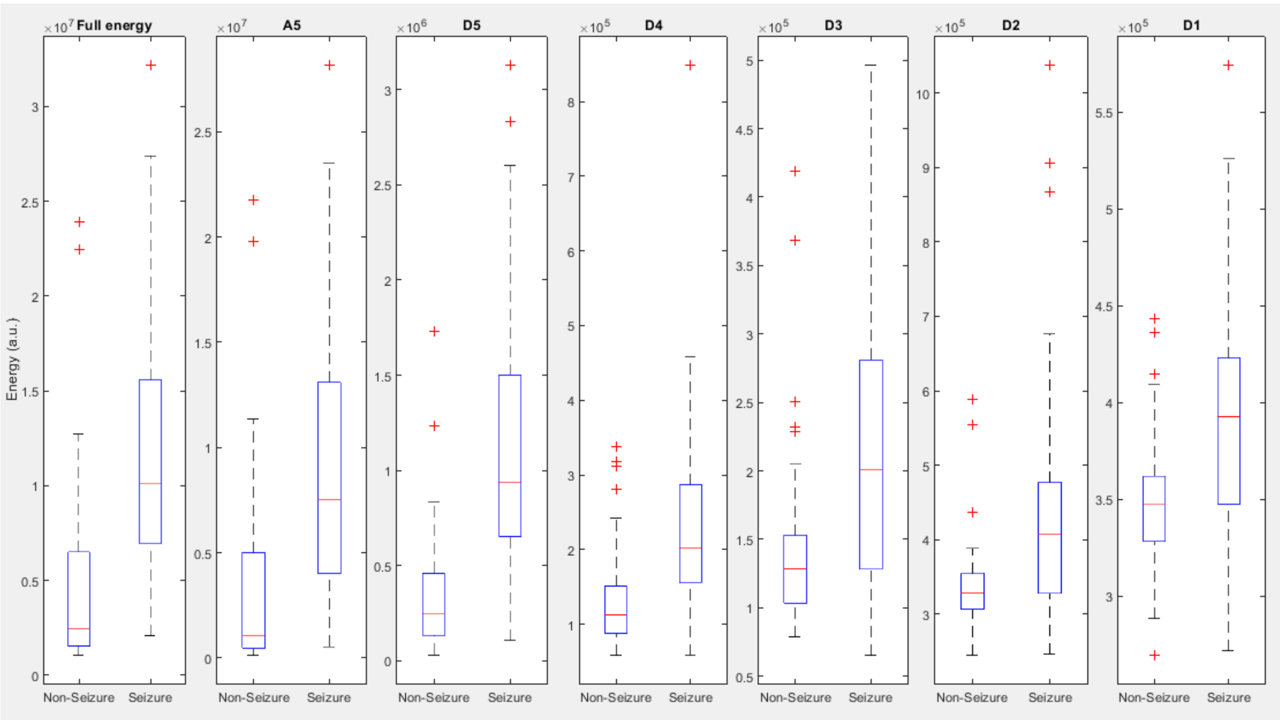

Figure 2.8. *Boxplots of different features.*

Question 3: *Given the wavelet decomposition, what are the frequency ranges corresponding to the approximation and detail coefficients? To what EEG rhythms do they correspond (approximately)?*

Knowing that the data is sampled at 256 Hz, we have summarized the corresponding down sampled ranges in **Table 2.3,** where it can also be seen the associated approximate EEG rhythms.

Level	Frequency Range		Approximate EEG Rhythms
	Min (Hz)	Max (Hz)	
D1	64	128	Gamma wave
D2	32	64	Gamma wave
D3	16	32	Beta wave
D4	8	16	Alpha wave
D5	4	8	Theta wave
A5	0	4	Delta wave

Table 2.3 *Frequency ranges of wavelet decomposition*

<u>Question 4:</u> *What frequency ranges/rhythms seem most interesting for classification?*

It can be observed from the boxplots shown in **Figure 2.8** how every sub band energy feature presents a bigger absolute energy when there is a seizure present compared to when there's no seizure activity, which is positive for classification purposes.

The first sub band, corresponding to A5 (lowest frequency range) has the largest median to median difference, which represents a strong differentiation between our two classes. This sub band is associated with the slowest oscillation, the delta wave, commonly related with seizure-like activity. Also interesting for classification would be the theta wave, which has a non-seizure third quartile below the seizure's 75th percentile. This characteristic shows there's a small overlap between both data. Additionally, these two frequency ranges present the least number of outliers.

In order to compare the two classifiers, performance measures for both have been obtained and documented under **Table 2.4.** The overall performance of the wavelet approach is better, with a higher accuracy, sensitivity and specificity. The sensitivity, which represents the classifiers capability to detect the presence of a seizure and is consequently an important measure, falls too low for the full energy classification method. However, the classifiers ability to identify a non-seizure event (specificity) has an effective performance for both methods.

Classification_method	Accuracy	Sensitivity	Specificity
{'Full Energy' }	0.79577	0.64789	0.94366
{'Subband Energies'}	0.89437	0.8169	0.97183

Table 2.4 *Performance measures of classifiers.*

Furthermore, we have applied a distinctive comparison method, the receiver operating characteristic (ROC) curves, which are plotted in **Figure 2.9.** The area under the curve (AUC) has been computed for both classifiers, obtaining 0.93 for the full energy method and 0.98 for the sub band energies method. The latter value, which is close to the ideal '1', demonstrate how applying wavelets to obtain our features has been an advantageous approach.

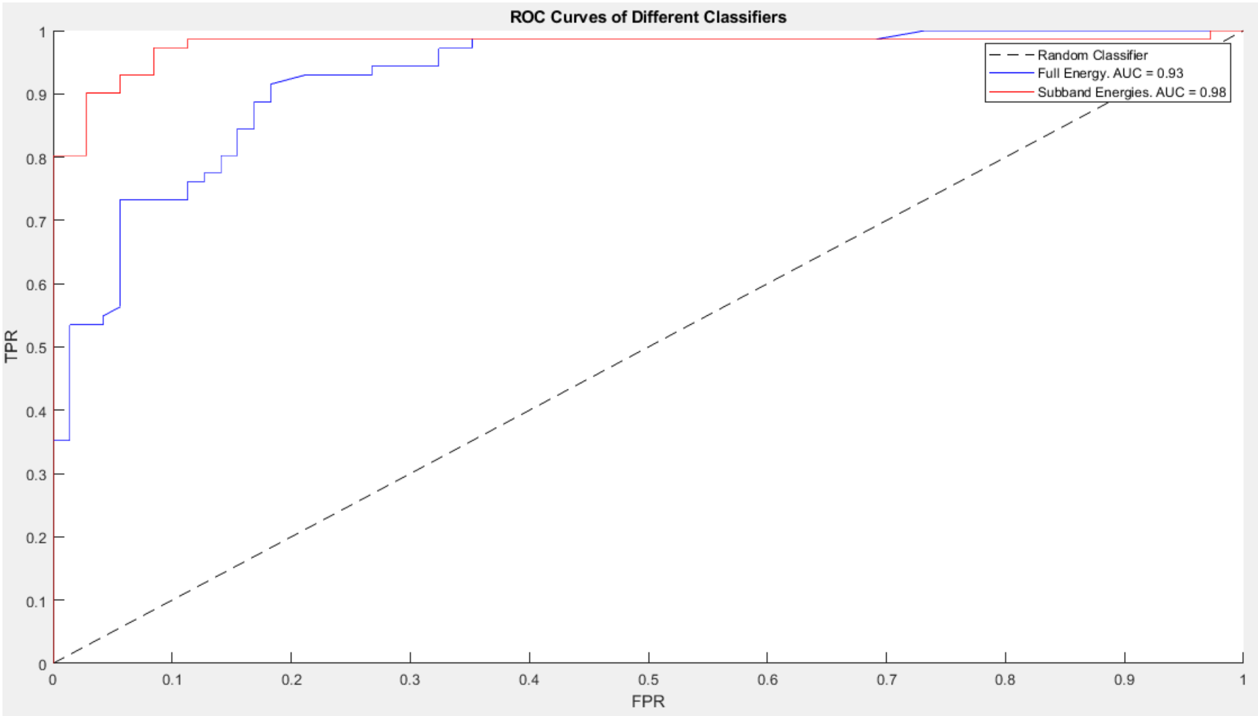

Figure 2.9. ROC curves of classifiers.

<u>**Question 5:**</u> *Do you expect an improvement of the performance when adding the total signal energy (per segment) as an additional feature to the sub band energies? Why (not)?*

Given the fact that the sub band energies are a subset of the total signal energy, adding the full energy may result in feature redundancy, which would not generate a performance improvement. This is because additional bias is included to the classifier. Also, in order to include the total signal energy we would need to rescale our features for structure consistency purposes, and this would cause loss of important variance information, which if anything, would worsen the performance instead of enhancing it.

2.3 Conclusions

Two different applications of wavelets have been implemented: signal reconstruction by compression and the use of wavelet properties to design a classifier. The wavelet transform is particularly useful when we are interested in analysing the characteristics of the different frequency ranges of a data set and for studying signals with sharp behaviours. The remarkable wavelet properties for denoising and compression that we have observed prove why it's a widely used tool in imaging applications.

2. References

[1] A. L. Goldberger, L. A. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley, **“PhysioBank, PhysioToolkit, and PhysioNet: components of a new research resource for complex physiologic signals,”** Circulation, vol. 101, no. 23, pp. e215–e220, 2000.

[2] Lin, Chia-Hung. **Frequency-domain features for ECG beat discrimination using grey relational analysis-based classifier (**2007). DOI: https://doi.org/10.1016/j.camwa.2007.04.035