OLS

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***Abstract*—Implementation of linear regression using the closed form and the gradient descent solutions. Incorporate the ridge regularization from scratch and used the lasso implementation from scikit-learn [1].**

I. Introduction

Website popularity prediction is important because ...

Simple tools like OLS have a surprising power, particularly when couple with regularization techniques such as the lasso or ridge.

II. Implementation of OLS

$$Y = X\beta + \epsilon \tag{1}$$

A. Closed Form

With the traditional assumption of $X^T\epsilon = 0$ [**?**], i.e. that the error is uncorrelated with the matrix X, it is easy to solve for the weights, the resulting equation is given by

$$\begin{aligned} Y &= X\beta + \epsilon && (2)\\ X^TY &= X^TX\beta + X^T\epsilon && (3)\\ \hat{\beta} &= (X^TX)^{-1}X^TY && (4)\\ & && (5) \end{aligned}$$

B. Gradient Descent

It is computationally inefficient to invert large matrices such as the one provided for this exercise. It is more efficient to minimize the sum of squares $SSR(\beta) = \sum_{i=1}^{n}(Y - X\beta)^2$. We need to take the derivative to

$$\frac{\partial SSR(\beta)}{\partial \beta} = -2X^T(Y - X\beta) \tag{6}$$

cite Joelle's slides lecture 2

while $\epsilon > 0.1$ and $i <$ max_iterations **do**
  hypothesis $\leftarrow X^T\beta$
  loss $\leftarrow$ hypothesis - Y
  gradient $\leftarrow$ 2 X^T loss
  $\beta_{\text{new}} \leftarrow \beta - \frac{\alpha * \text{gradient}}{n}$
  $\epsilon \leftarrow \beta_{\text{new}} - \beta$
  $i \leftarrow i + 1$
  $\beta_{\text{new}} \leftarrow \beta$
end while

C. Lasso and Ridge Regularization

To be able to generalize well to new data, we want to avoid over fitting. To do so we will penalize extreme weights for our β

Talk about Occam's razor

if $i \geq maxval$ **then**
  $i \leftarrow 0$
else
  if $i + k \leq maxval$ **then**
    $i \leftarrow i + k$
  end if
end if

III. Cross-Validation

k-fold validation
complete randomization of the fold, by a random variable

A. Hyperparameters Optimization

Feature selection using the lasso function from [1]

Trying to avoid overfitting to be able to generalize to new examples.

IV. Results

Talk about the mean squared error (MSE) obtain

V. Complementary Datasets

VI. Conclusion

The conclusion goes here.

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References

[1] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit-learn: Machine learning in Python," *Journal of Machine Learning Research*, vol. 12, pp. 2825–2830, 2011.