A Comparison of Optimization Methods for Multi-Objective Constrained Bin Packing Problems

P. Olivier[1,2], A. Lodi[1,2], G. Pesant[1]

[1]École polytechnique de Montréal, Montreal, Canada
[2]CERC
{philippe.olivier, andrea.lodi, gilles.pesant}@polymtl.ca

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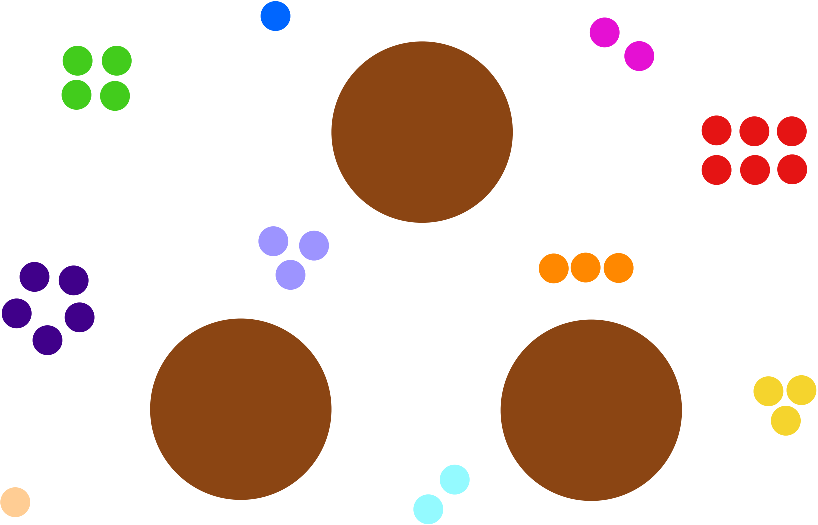
Outline

Outline

The Wedding Seating Problem



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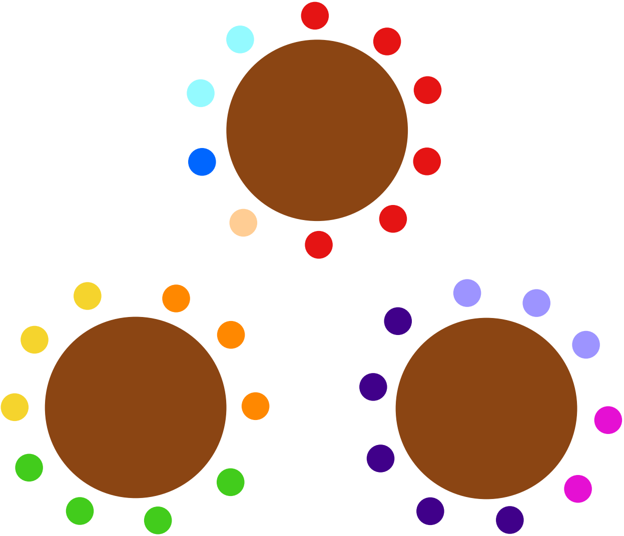

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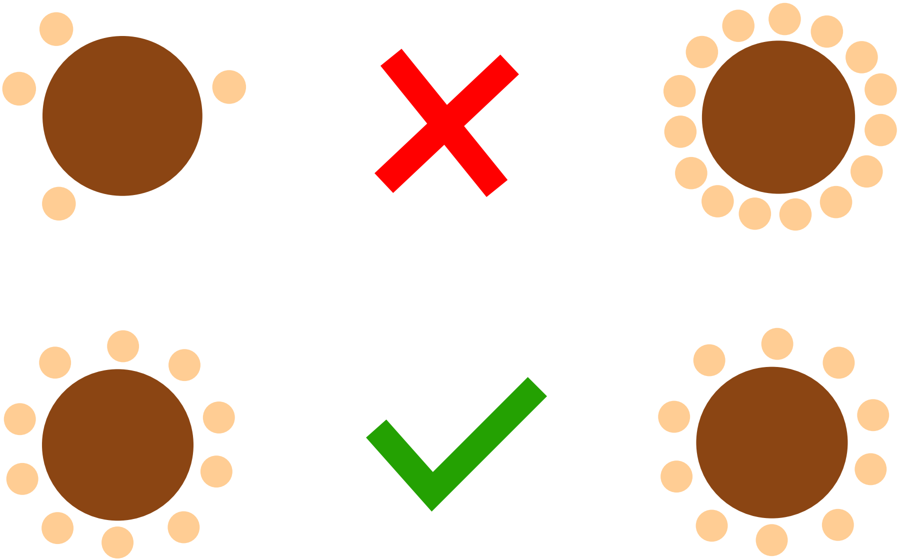


Outline

Existing Methods
Overview

- Original IP model [Bellows and Petersen, *Annals of Improbable Research*, 2012].
- Two-stage algorithm using tabu search [Lewis, *WorldComp International Conference Proceedings*, 2013].
- Improved IP model [Lewis and Carroll, *Journal of the Operational Research Society*, 2016].

Existing Methods

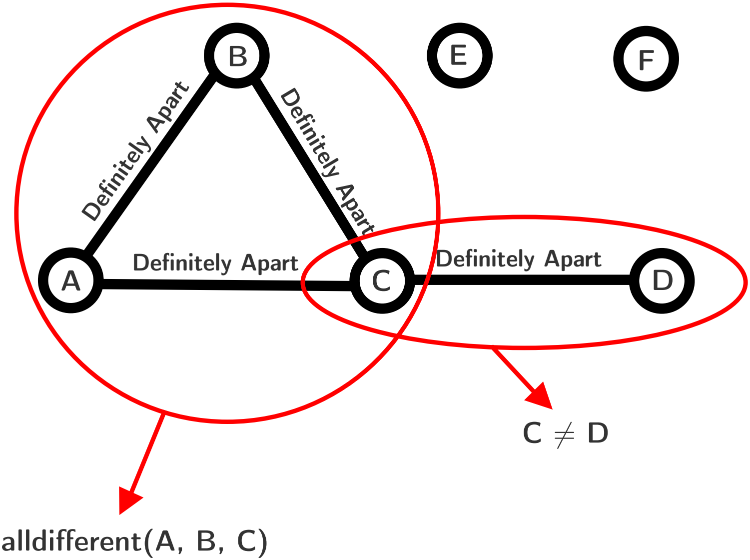
Two-stage algorithm using tabu search [Lewis, 2013]

Build a graph where nodes represent groups, edges represent relations, and colors represent tables.

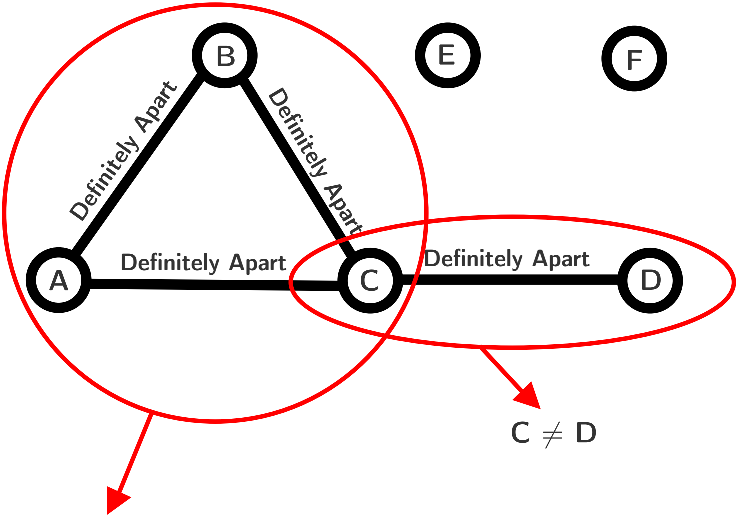
- Stage 1: Color nodes to find an initial feasible solution
- Stage 2: Improve this feasible solution with a tabu search

Outline

CP Model


B
E
F
Definitely Apart
Definitely Apart
A
Definitely Apart
C
Definitely Apart
D
C ≠ D
alldifferent(A, B, C)

CP Model



alldifferent(A, B, C) → **Needs 3+ tables**

CP Model

 $\in \{1, 2, \ldots, m\}$

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binpacking( ,  **)**

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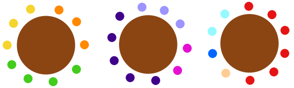
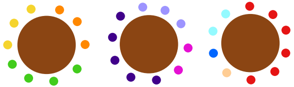
dispersion( **)**

CP Model

binpacking( ,  **)**

alldifferent( **)**

dispersion( **)**

maximize 

Branching heuristic (similar to *best fit decreasing*):

1. Pick the largest group yet unassigned
2. Assign that group at the best possible table
3. If no table has enough room, open a new table

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Large Neighborhood Search (LNS):

- Freeze $\sim$1/3 of the tables for 10s
- The best tables are more likely to be frozen

Outline

IP Model A

$$x_{ik} := \begin{cases} 1, & \text{if group } i \text{ is assigned to table } k, \\ 0, & \text{otherwise.} \end{cases}$$

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$$x_{ik} + x_{jk} \leq 1 \qquad \forall i, j \in \mathcal{G} : c_{ij} = \infty, \quad \forall k \in \mathcal{T}$$

$$x_{ik} = 0 \qquad \forall i \in \mathcal{G}, \quad \forall k \in \{i+1, \ldots, m\}$$

IP Model A

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \geq \ell \qquad \forall k \in \mathcal{T}$$

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \leq u \qquad \forall k \in \mathcal{T}$$

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$$\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \leq o_k \qquad \forall k \in \mathcal{T}$$

$$\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \geq -o_k \qquad \forall k \in \mathcal{T}$$

$$\sum_{k \in \mathcal{T}} o_k \geq d_{\min}$$

$$\sum_{k \in \mathcal{T}} o_k \leq d_{\max}$$

$$o_k \in \{\ell, \ldots, u\} \qquad \forall k \in \mathcal{T}$$

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$$o_k \in \{\ell, \dots, u\} \qquad \forall k \in \mathcal{T}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in \mathcal{G}, \quad \forall k \in \mathcal{T}$$

Outline

IP Model B

Master Problem

$\mathbb{S}$: All subsets of groups that can be assigned at the same table.

$$x_S := \begin{cases} 1, & \text{if table/pattern } S \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

IP Model B

Master Problem

$$\min \quad \sum_{S \in \mathbb{S}} \alpha_S x_S$$

IP Model B

Master Problem

$$
\begin{aligned}
\text{min} \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S} : i \in S} x_S = 1 && \forall i \in \mathcal{G} \\
& \sum_{S \in \mathbb{S}} x_S = m
\end{aligned}
$$

IP Model B

Master Problem

$$
\begin{aligned}
\text{min} \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S}: i \in S} x_S = 1 \qquad \forall i \in \mathcal{G} \\
& \sum_{S \in \mathbb{S}} x_S = m \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\text{min}} \\
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\end{aligned}
$$

IP Model B

Master Problem

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\begin{aligned}
\min \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
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& \sum_{S \in \mathbb{S}} x_S = m \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\min} \\
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& x_S \in \{0, 1\} && \forall S \in \mathbb{S}
\end{aligned}
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IP Model B

Master Problem

$$
\begin{aligned}
\min \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S}: i \in S} x_S = 1 && \forall i \in \mathcal{G} && (y) \\
& \sum_{S \in \mathbb{S}} x_S = m && && (\zeta) \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\min} && && (\gamma) \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \leq d_{\max} && && (\delta) \\
& x_S \in \{0, 1\} && \forall S \in \mathbb{S}
\end{aligned}
$$

IP Model B
Dual

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} y_i + m\zeta + d_{\min}\gamma + d_{\max}\delta \\
\text{s.t.} \quad & \\
& \sum_{i \in S} y_i + \zeta + \beta_S(\gamma + \delta) \leq \alpha_S \quad && \forall S \in \mathbb{S} \\
& y_i \text{ free} && \forall i \in S \\
& \zeta \text{ free} \\
& \gamma \geq 0 \\
& \delta \leq 0
\end{aligned}
$$

IP Model B

Pricing Problem

$$z_i := \begin{cases} 1, & \text{if group } i \text{ is packed into the new table/pattern,} \\ 0, & \text{otherwise.} \end{cases}$$

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$$\sum_{i=1}^{n} w_i z_i \geq \ell$$

$$\sum_{i=1}^{n} w_i z_i \leq u$$

$$\sum_{i=1}^{n} w_i z_i - w/m \leq \beta$$

$$\sum_{i=1}^{n} w_i z_i - w/m \geq -\beta$$

IP Model B

Pricing Problem

$$z_i + z_j \leq 1 \quad \forall i, j \in \mathcal{G} : c_{ij} = \infty$$

$$z_i \in \{0, 1\} \quad \forall i \in \mathcal{G}$$

IP Model B
Pricing Problem

$$z_i + z_j \leq 1 \quad \forall i, j \in \mathcal{G} : c_{ij} = \infty$$

$$z_i \in \{0, 1\} \quad \forall i \in \mathcal{G}$$

$$\max \quad \sum_{i=1}^{n} y_i^* z_i + \zeta^* + \beta(\gamma^* + \delta^*) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} z_i z_j \quad \text{if } \gamma^* + \delta^* < 0$$

$$\max \quad \sum_{i=1}^{n} y_i^* z_i + \zeta^* - \beta(\gamma^* + \delta^*) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} z_i z_j \quad \text{if } \gamma^* + \delta^* > 0$$

1. Solve the continuous relaxation of the RMP to get the dual values.

IP Model B

Column Generation

1. Solve the continuous relaxation of the RMP to get the dual values.
2. Solve the PP to generate the most promising new table/column (S^*).

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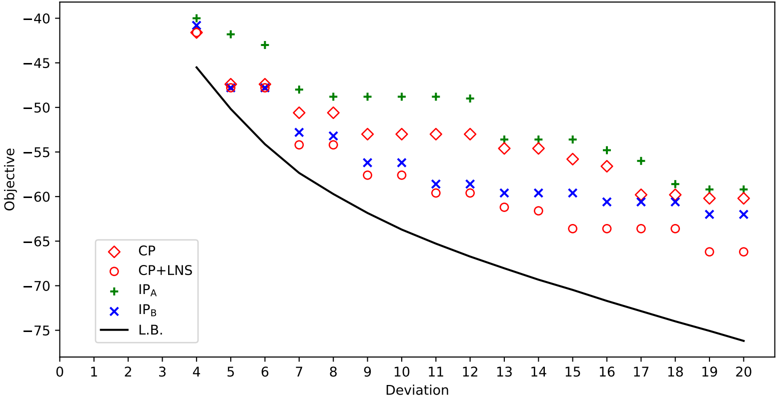
2. Solve the PP to generate the most promising new table/column (S^*).
3. Determine if this new column should be added to the RMP. If yes, compute α_{S^*} and β_{S^*}, and add column S^* to the RMP before going back to step 1. Otherwise, the current solution of the continuous relaxation of the RMP is the lower bound of the initial problem.

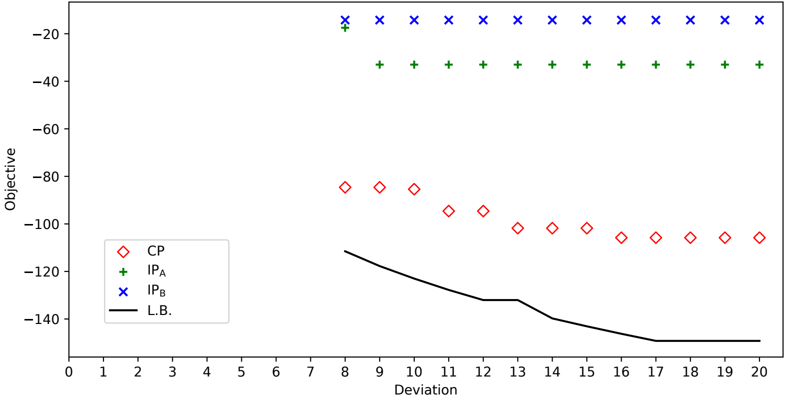
Outline

Results
25 groups, 600s time limit



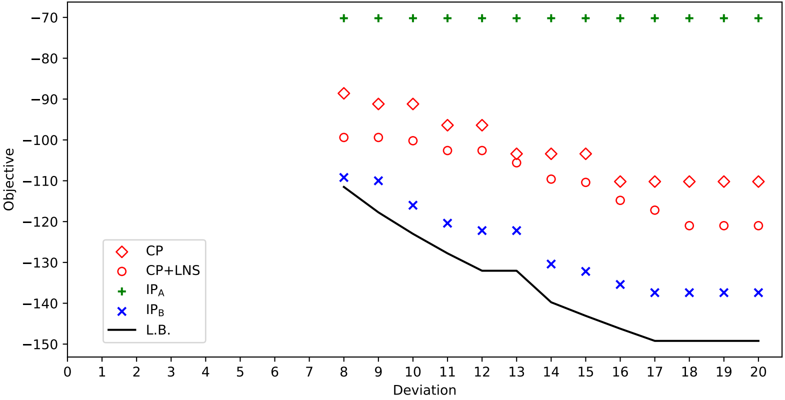
Results

50 groups, 6s time limit



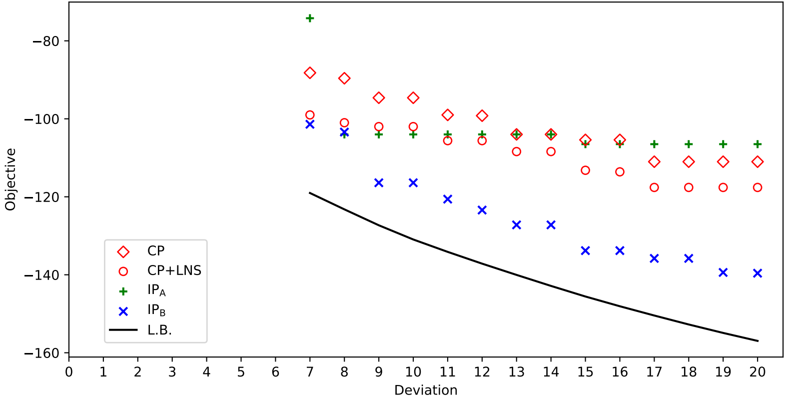
Results

50 groups, 600s time limit



Results

50 groups, 600s time limit (costs only)

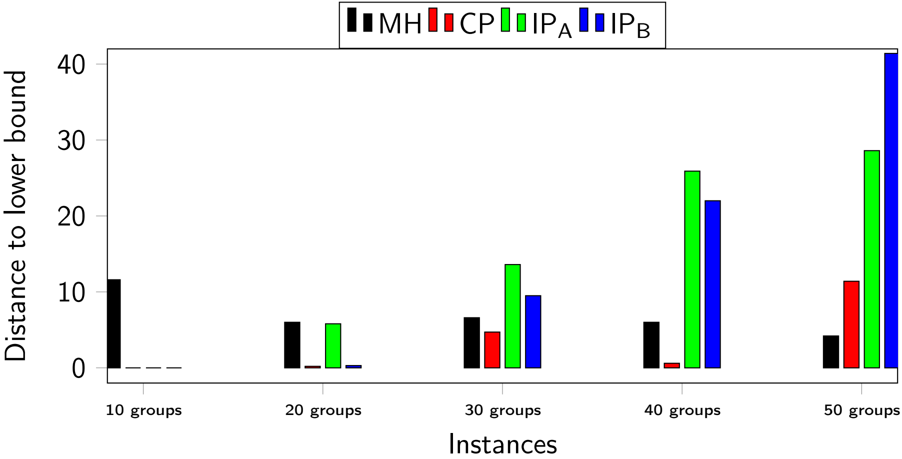


Results

50 groups, 600s time limit (conflicts only)

	50 groups
	Conflicts only
CP	0.03
CP+LNS	0.03
$\mathrm{IP_A}$	601.11
$\mathrm{IP_B}$	4.64

Results
Comparison With Metaheuristics



Outline

Conclusion

When to use each model?

Short time limit, or only conflicts: **CP Model**

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Small instance: **IP Model A**

Conclusion
When to use each model?

Short time limit, or only conflicts: **CP Model**

Small instance: **IP Model A**

Near-optimal solution: **IP Model B**

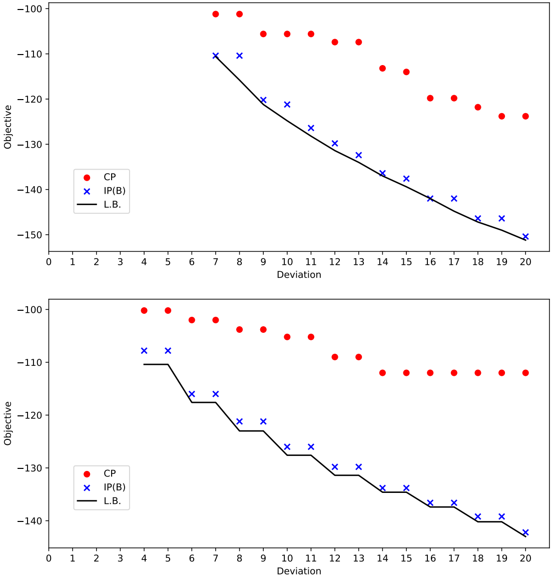
Short time limit, or only conflicts: **CP Model**

Small instance: **IP Model A**

Near-optimal solution: **IP Model B**

Large instance: **MH Model**

Ongoing Work



CP2018 Workshop on Constraints & AI Planning
Call for Abstracts

The CP2018 Workshop on Constraints and AI Planning will take place on August 27, 2018 in Lille, France as part of the CP2018.

Please submit an abstract on the intersection of constraint-based techniques (interpreted broadly to include mixed integer programming, SAT, SMT, etc. as well as CP) and AI planning.

Dates
Abstract submission: July 6, 2018
Notification: July 10, 2018
Workshop: August 27, 2018

More Info: pesantgilles.github.io/consAIplan18/