A Comparison of Optimization Methods for Multi-Objective Constrained Bin Packing Problems

P. Olivier[1], A. Lodi[1], G. Pesant[1]

[1]École polytechnique de Montréal, Montreal, Canada
{philippe.olivier, andrea.lodi, gilles.pesant}@polymtl.ca

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Outline

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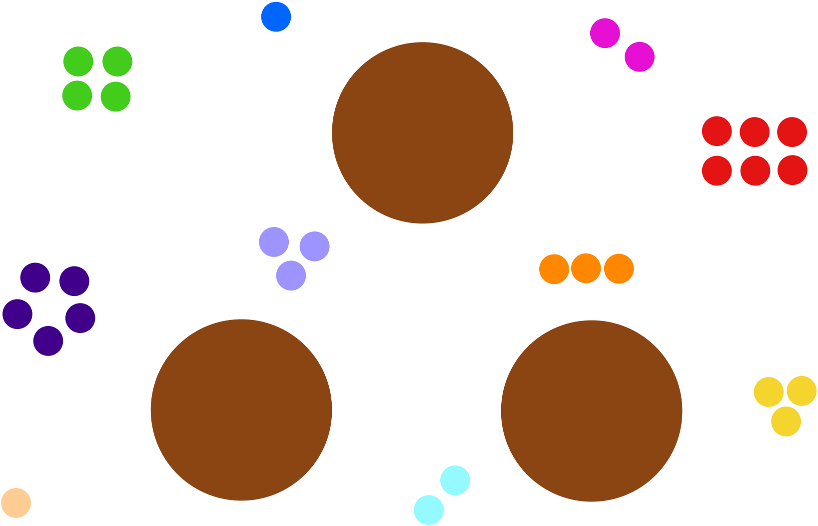
The Wedding Seating Problem



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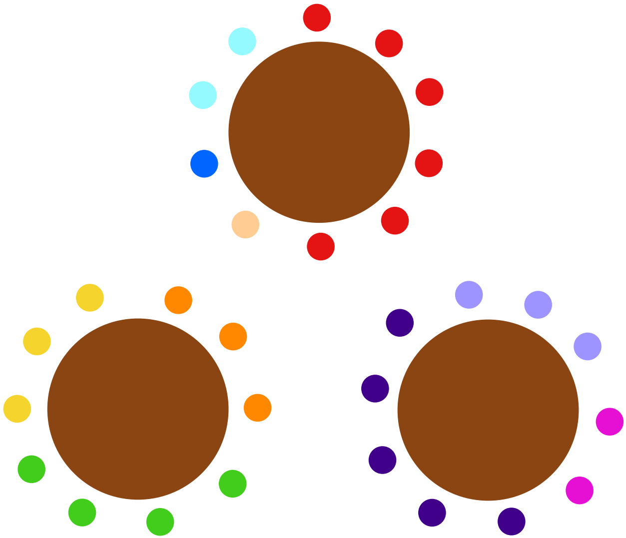

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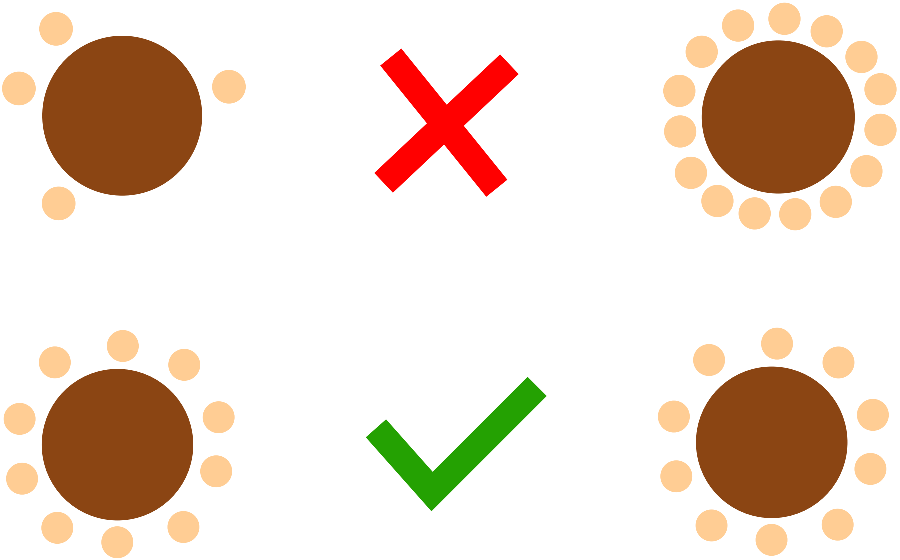


Outline

Existing Methods
Overview

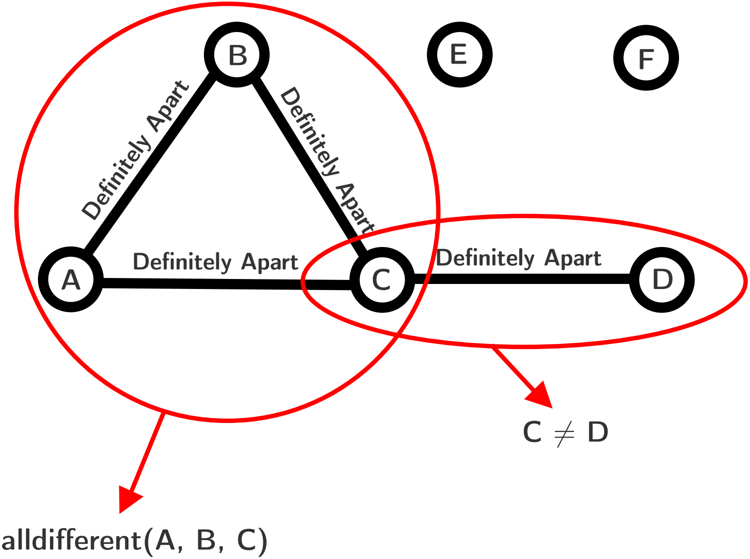
- Original IP model [Bellows and Petersen, *Annals of Improbable Research*, 2012].
- Two-stage algorithm using tabu search [Lewis, *WorldComp International Conference Proceedings*, 2013].
- Improved IP model [Lewis and Carroll, *Journal of the Operational Research Society*, 2016].

Build a graph where nodes represent groups, edges represent relations, and colors represent tables.

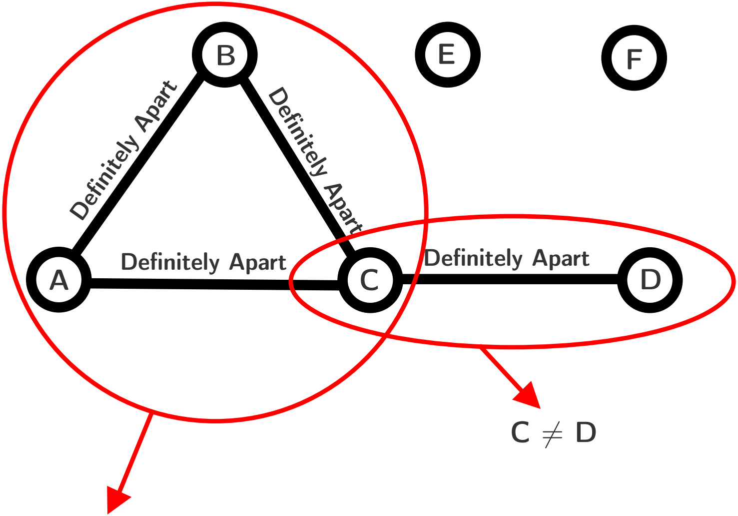
- Stage 1: Color nodes to find an initial feasible solution
- Stage 2: Improve this feasible solution with a tabu search

Outline

CP Model


A
B
C
D
E
F
Definitely Apart
Definitely Apart
Definitely Apart
Definitely Apart
$C \neq D$
alldifferent(A, B, C)

CP Model



$C \neq D$

alldifferent(A, B, C) ⟶ **Needs 3+ tables**

CP Model

 $\in \{ 1, 2, \ldots, m \}$

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binpacking( ,  **)**

CP Model

$\in \{1, 2, \ldots, m\}$

binpacking(,)

alldifferent( )

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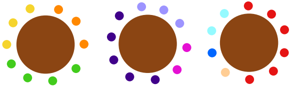
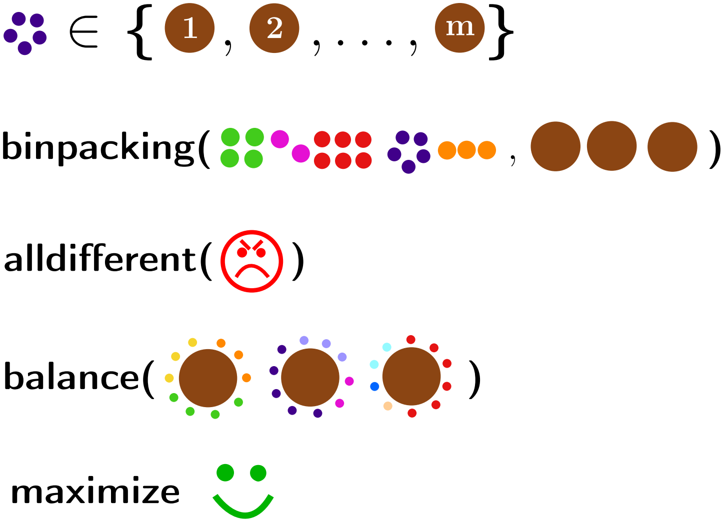
balance( **)**

CP Model



Branching heuristic (similar to *best fit decreasing*):

1. Pick the largest group yet unassigned
2. Assign that group at the best possible table
3. If no table has enough room, open a new table

CP Model
Branching and Search

Branching heuristic (similar to *best fit decreasing*):

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3. If no table has enough room, open a new table

Large Neighborhood Search (LNS):

- Freeze $\sim$1/3 of the tables for 10s
- The best tables are more likely to be frozen

Outline

IP Model A

$$x_{ik} := \begin{cases} 1, & \text{if group } i \text{ is assigned to table } k, \\ 0, & \text{otherwise.} \end{cases}$$

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$$x_{ik} = 0 \qquad \forall i \in \mathcal{G}, \quad \forall k \in \{i+1, \ldots, m\}$$

IP Model A

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \geq \ell \qquad \forall k \in \mathcal{T}$$

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \leq u \qquad \forall k \in \mathcal{T}$$

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$$\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \leq o_k \qquad \forall k \in \mathcal{T}$$

$$\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \geq -o_k \qquad \forall k \in \mathcal{T}$$

$$\sum_{k \in \mathcal{T}} o_k \geq d_{\min}$$

$$\sum_{k \in \mathcal{T}} o_k \leq d_{\max}$$

$$o_k \in \{\ell, \dots, u\} \qquad \forall k \in \mathcal{T}$$

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$$o_k \in \{\ell, \ldots, u\} \qquad \forall k \in \mathcal{T}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in \mathcal{G}, \quad \forall k \in \mathcal{T}$$

Outline

IP Model B

Master Problem

$\mathbb{S}$: All subsets of groups that can be assigned at the same table.

$$x_S := \begin{cases} 1, & \text{if table/pattern } S \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

IP Model B
Master Problem

$$\min \quad \sum_{S \in \mathbb{S}} \alpha_S x_S$$

IP Model B

Master Problem

$$\begin{aligned}
\text{min} \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S} : i \in S} x_S = 1 && \forall i \in \mathcal{G} \\
& \sum_{S \in \mathbb{S}} x_S = m
\end{aligned}$$

IP Model B

Master Problem

$$
\begin{aligned}
\min \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S} : i \in S} x_S = 1 \qquad \forall i \in \mathcal{G} \\
& \sum_{S \in \mathbb{S}} x_S = m \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\min} \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \leq d_{\max}
\end{aligned}
$$

IP Model B

Master Problem

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\begin{aligned}
\min \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S} : i \in S} x_S = 1 \qquad && \forall i \in \mathcal{G} \\
& \sum_{S \in \mathbb{S}} x_S = m \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\min} \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \leq d_{\max} \\
& x_S \in \{0, 1\} && \forall S \in \mathbb{S}
\end{aligned}
$$

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Master Problem

$$
\begin{aligned}
\min \quad & \sum_{S \in \mathbb{S}} \alpha_S x_S \\
\text{s.t.} \quad & \sum_{S \in \mathbb{S}: i \in S} x_S = 1 && \forall i \in \mathcal{G} && \color{red}{(y)} \\
& \sum_{S \in \mathbb{S}} x_S = m && && \color{red}{(\zeta)} \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \geq d_{\min} && && \color{red}{(\gamma)} \\
& \sum_{S \in \mathbb{S}} \beta_S x_S \leq d_{\max} && && \color{red}{(\delta)} \\
& x_S \in \{0, 1\} && \forall S \in \mathbb{S}
\end{aligned}
$$

IP Model B
Dual

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} y_i + m\zeta + d_{\text{min}}\gamma + d_{\text{max}}\delta \\
\text{s.t.} \quad & \\
& \sum_{i \in S} y_i + \zeta + \beta_S(\gamma + \delta) \le \alpha_S \quad && \forall S \in \mathbb{S} \\
& y_i \text{ free} && \forall i \in S \\
& \zeta \text{ free} \\
& \gamma \ge 0 \\
& \delta \le 0
\end{aligned}
$$

IP Model B
Pricing Problem

$$z_i := \begin{cases} 1, & \text{if group } i \text{ is packed into the new table/pattern,} \\ 0, & \text{otherwise.} \end{cases}$$

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$$\sum_{i=1}^{n} w_i z_i \geq \ell$$

$$\sum_{i=1}^{n} w_i z_i \leq u$$

$$\sum_{i=1}^{n} w_i z_i - w/m \leq \beta$$

$$\sum_{i=1}^{n} w_i z_i - w/m \geq -\beta$$

IP Model B
Pricing Problem

$$z_i + z_j \leq 1 \quad \forall i, j \in \mathcal{G} : c_{ij} = \infty$$

$$z_i \in \{0, 1\} \quad \forall i \in \mathcal{G}$$

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$$\max \quad \sum_{i=1}^{n} y_i^* z_i + \zeta^* + \beta(\gamma^* + \delta^*) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} z_i z_j \quad \text{if } \gamma^* + \delta^* < 0$$

$$\max \quad \sum_{i=1}^{n} y_i^* z_i + \zeta^* - \beta(\gamma^* + \delta^*) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} z_i z_j \quad \text{if } \gamma^* + \delta^* > 0$$

IP Model B
Column Generation

1. Solve the continuous relaxation of the RMP to get the dual values.

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2. Solve the PP to generate the most promising new table/column (S^*).

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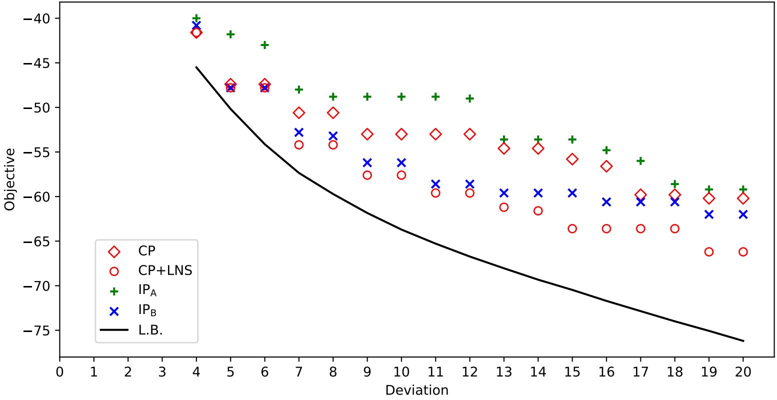
2. Solve the PP to generate the most promising new table/column (S^*).
3. Determine if this new column should be added to the RMP. If yes, compute α_{S^*} and β_{S^*}, and add column S^* to the RMP before going back to step 1. Otherwise, the current solution of the continuous relaxation of the RMP is the lower bound of the initial problem.

Outline

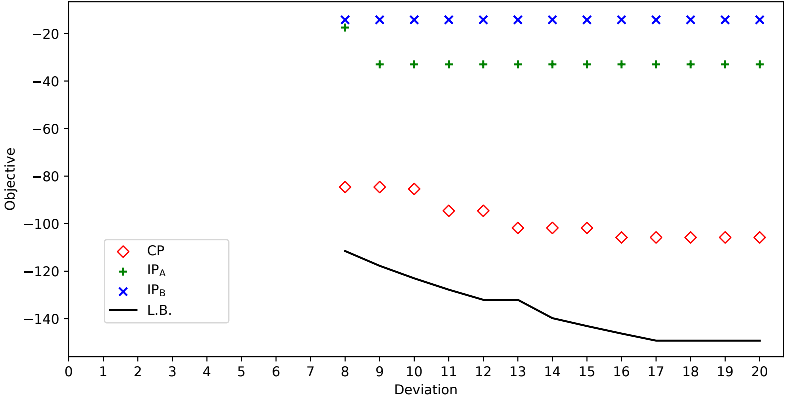
Results

25 groups, 600s time limit



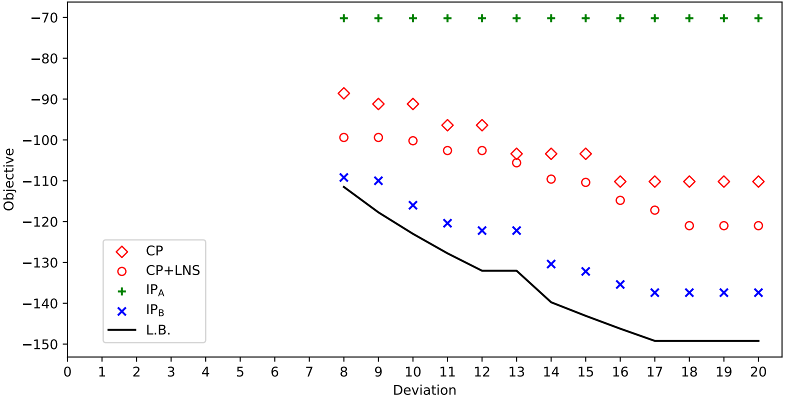
Results

50 groups, 6s time limit



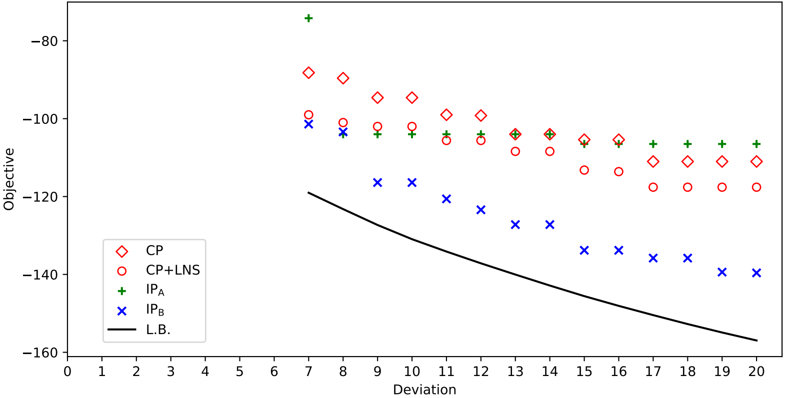
Results

50 groups, 600s time limit



Results

50 groups, 600s time limit (costs only)



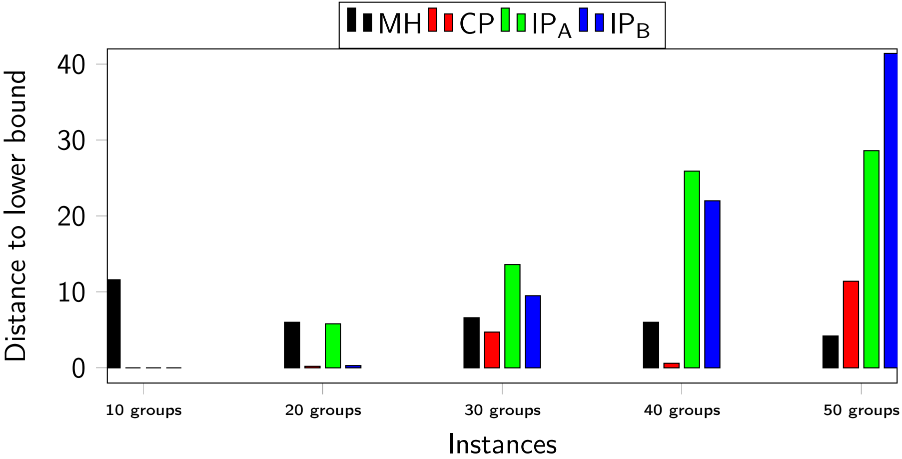
Results

50 groups, 600s time limit (conflicts only)

	50 groups
	Conflicts only
CP	0.03
CP+LNS	0.03
IP_A	601.11
IP_B	4.64

Results

Comparison With Metaheuristics



Outline

Conclusion

Metaheuristics Model

+ Good early solutions
+ Scales well

+ Good early solutions
+ Scales well

− No proof of optimality
− Poor balance

Conclusion
CP Model

- + Very good early solutions
- + Proves optimality for small instances
- + Quickly proves optimality or infeasibility for all instances when there are no costs

- \+ Very good early solutions
- \+ Proves optimality for small instances
- \+ Quickly proves optimality or infeasibility for all instances when there are no costs

- − Hard to optimize with costs
- − Limited symmetry breaking

Conclusion
IP Model A

+ Simple model
+ Proves optimality for small instances

Conclusion
IP Model A

+ Simple model
+ Proves optimality for small instances

− Inefficient
− Limited symmetry breaking

Conclusion
IP Model B

- Near-optimal solutions
- No symmetry
- Provides a good lower bound

Conclusion

IP Model B

+ Near-optimal solutions
+ No symmetry
+ Provides a good lower bound

– Complex
– No proof of optimality
– Poor early solutions