A Comparison of Optimization Methods for Multi-Objective Constrained Bin Packing Problems

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JOPT 2018

8 May 2018

Outline

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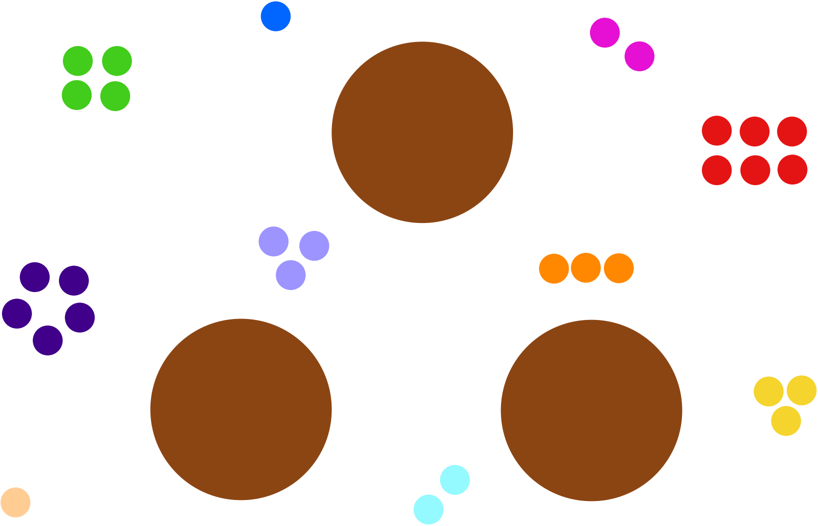
The Wedding Seating Problem



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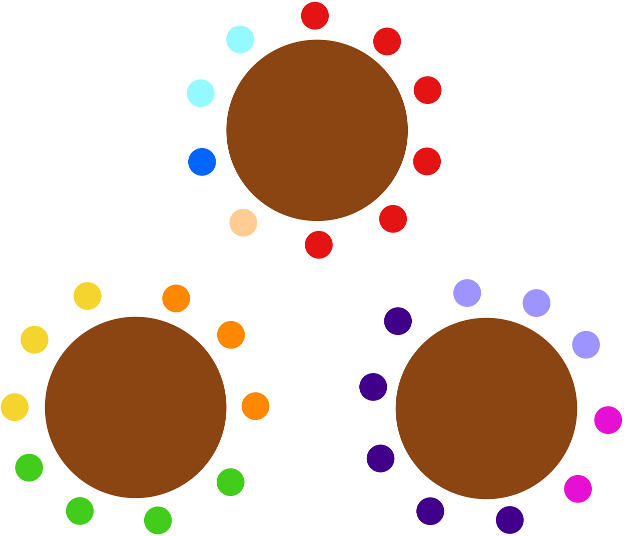

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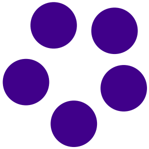

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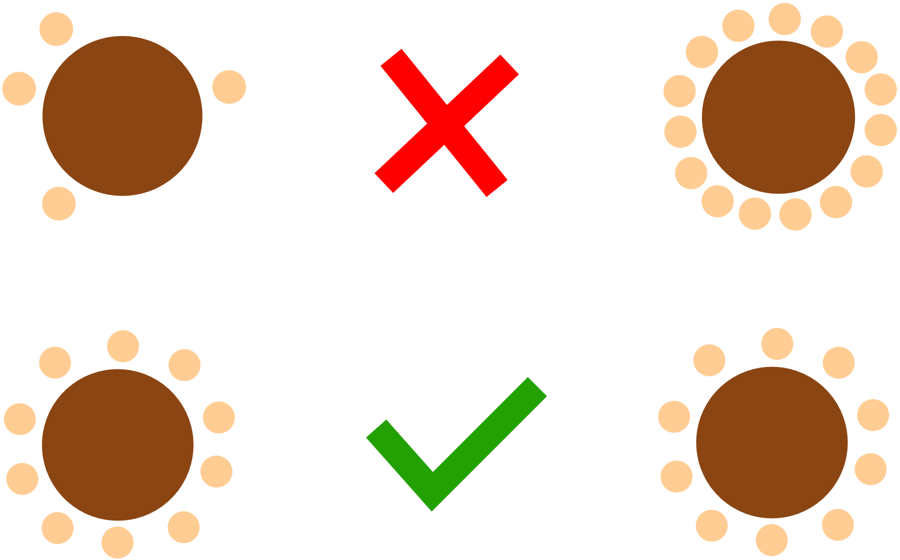

Outline

Existing Methods
Overview

- Original IP model [Bellows and Petersen, *Annals of Improbable Research*, 2012].

- Two-stage algorithm using tabu search [Lewis, *WorldComp International Conference Proceedings*, 2013].

- Improved IP model [Lewis and Carroll, *Journal of the Operational Research Society*, 2016].

Existing Methods

Two-stage algorithm using tabu search [Lewis, 2013]

Build a graph where nodes represent groups, edges represent relations, and colors represent tables.

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- Stage 1: Color nodes to find an initial feasible solution

Existing Methods

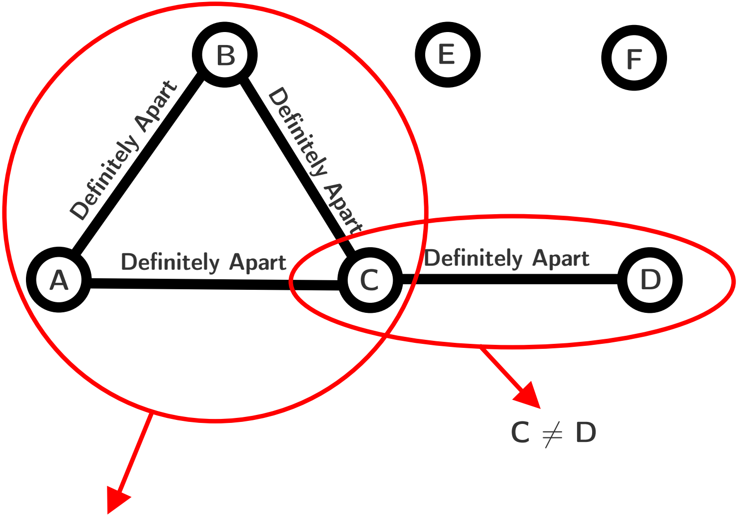
Two-stage algorithm using tabu search [Lewis, 2013]

Build a graph where nodes represent groups, edges represent relations, and colors represent tables.

- Stage 1: Color nodes to find an initial feasible solution
- Stage 2: Improve this feasible solution with a tabu search

Outline

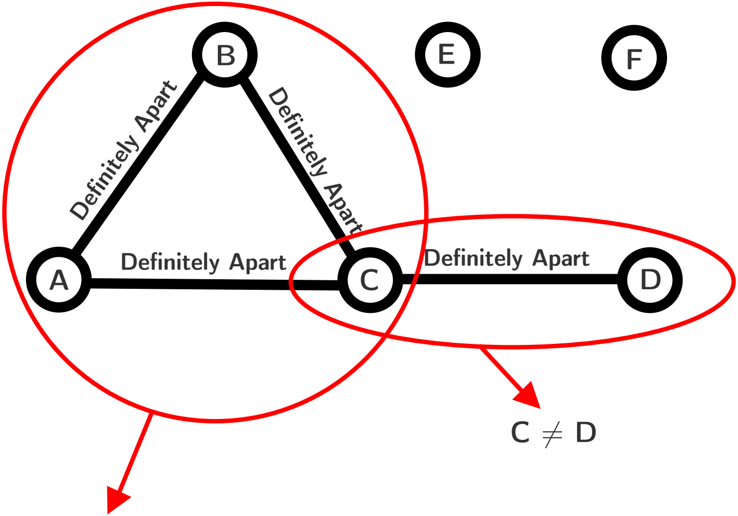
CP Model



$C \neq D$

alldifferent(A, B, C)

CP Model



C $\neq$ D

alldifferent(A, B, C) → Needs 3+ tables

CP Model

 $\in \{1, 2, \ldots, m\}$

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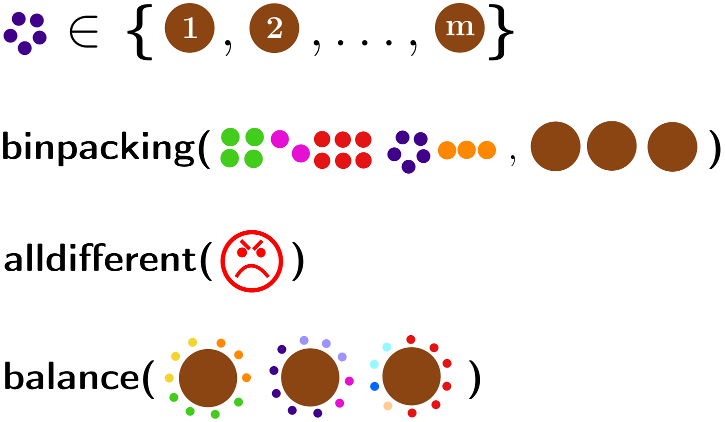
binpacking(,)

CP Model

 $\in \{ 1, 2, \ldots, m \}$

binpacking( ,  **)**

alldifferent( **)**


∈ { 1, 2, . . . , m }
binpacking(,)
alldifferent( )
balance( )

CP Model

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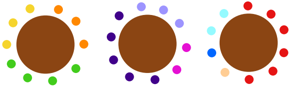
alldifferent( **)**

balance( **)**

maximize 

Similar to *best fit decreasing*:

CP Model
Branching Heuristic

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1. Pick the largest group yet unassigned

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Branching Heuristic

Similar to *best fit decreasing*:

1. Pick the largest group yet unassigned
2. Assign that group at the best possible table
3. If no table has enough room, open a new table

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$$x_{ik} := \begin{cases} 1, & \text{if group } i \text{ is assigned to table } k, \\ 0, & \text{otherwise.} \end{cases}$$

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$$x_{ik} = 0 \qquad \forall i \in \mathcal{G}, \quad \forall k \in \{i+1, \ldots, m\}$$

IP Model A

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \geq \ell \qquad \forall k \in \mathcal{T}$$

$$\sum_{i \in \mathcal{G}} x_{ik} w_i \leq u \qquad \forall k \in \mathcal{T}$$

IP Model A

$$
\begin{aligned}
&\sum_{i \in \mathcal{G}} x_{ik} w_i \geq \ell && \forall k \in \mathcal{T} \\
&\sum_{i \in \mathcal{G}} x_{ik} w_i \leq u && \forall k \in \mathcal{T} \\
&\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \leq o_k && \forall k \in \mathcal{T} \\
&\sum_{i \in \mathcal{G}} x_{ik} w_{ik} - w/m \geq -o_k && \forall k \in \mathcal{T} \\
&\sum_{k \in \mathcal{T}} o_k \geq d_{\min} \\
&\sum_{k \in \mathcal{T}} o_k \leq d_{\max} \\
&o_k \in \{\ell, \ldots, u\} && \forall k \in \mathcal{T}
\end{aligned}
$$

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$$\sum_{k \in \mathcal{T}} o_k \geq d_{\min}$$

$$\sum_{k \in \mathcal{T}} o_k \leq d_{\max}$$

$$o_k \in \{\ell, ..., u\} \qquad \forall k \in \mathcal{T}$$

$$x_{ik} \in \{0, 1\} \qquad \forall i \in \mathcal{G}, \quad \forall k \in \mathcal{T}$$

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IP Model B

$\mathbb{S}$: All subsets of groups that can be assigned at the same table.

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$$x_S := \begin{cases} 1, & \text{if pattern } S \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$