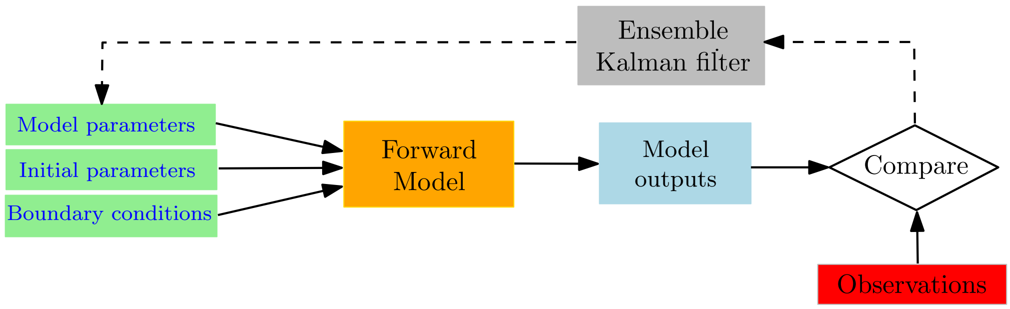

FIGURE 2.4: Parameter estimation flow chart using the ensemble Kalman filter.

2.7 An example of EnKF algorithm application: a test case

2.7.1 Parameter estimation of an advection-diffusion equation

The EnKF parameter estimation algorithm (Algorithm 1) is applied to a simple advection-diffusion problem. We consider the model equation:

$$\frac{\partial s}{\partial t} + v\frac{\partial s}{\partial z} = \mu\frac{\partial^2 s}{\partial z^2}, \tag{2.37}$$

where $s\,(z,t)$ is the model state variable of interest at position z and time t, v is the constant advection velocity, and μ is the viscosity (diffusion coefficient). Using finite difference method, Equation (2.37) is discretized over a spatial grid with N = 100 nodes, resulting in (at time $t = k+1$)

$$\frac{s^j_{k+1} - s^j_k}{\Delta t} = -v\frac{s^{j+1}_k - s^{j-1}_k}{2\Delta z} + \mu\frac{s^{j+1}_k - 2s^j_k + s^{j-1}_k}{(\Delta z)^2} + \mathcal{O}\left(\Delta t, (\Delta z)^2\right) \tag{2.38}$$

for $j = 2, 3, \ldots, N-1$. Model parameters are: the grid size $\triangle z$ = 0.01 m; the domain length L = 1 m; the advection velocity v = 1 ms^{-1} and the viscosity μ = 0.01 m^{2}s^{-1}. The computational domain is set to $[0,1]$. The initial condition is given by $s(z,0) = 1$, and the boundary conditions are defined by

$$s(0,t) = A + B\sin\omega t, \tag{2.39}$$
$$s(1,t) = 2\,s(0.99,t) - s(0.98,t), \tag{2.40}$$

where A = 1.0 m, B = 0.25 m and w = 10 rad·s^{-1}. The model time step, Δt, is chosen such that $\Delta t \leq \min\left\{\frac{\Delta z^2}{2\mu}, \frac{\Delta z}{\mu}\right\}$.