
Trajectory Optimization with Dynamic Obstacles Avoidance

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Abstract

We study the problem of Trajectory Optimization.

1 Introduction

This project investigates trajectory optimization in the presence of obstacles [11, 2, 5, 12]. One such application for this class of problems is that of autonomous driving, where we have an ego vehicle and dynamic obstacles (vehicles, pedestrians) which may intersect our desired trajectory and which we wish to avoid using motion planning and control. Trajectory optimization problems minimize a cost function which takes into account start and terminal states, as well as cost along the trajectory path. The design space is subject to constraints on the states and control input at sampled time points.

2 Related Work

Todo ...

3 Problem Formulation

We define a MPC problem over 20 time steps of 250 ms each with a Quadratic Cost function with $x \in \mathbb{R}^{60}$ and 160 constraints. We have 120 linear and nonlinear ($\|x_{\text{ego}} - x_{\text{obj}}\| \geq d_{\text{saf}}$) inequality constraints and 40 linear equality constraints (Dynamics Model).

$$\begin{aligned} \min_{u_0, \dots, u_{T-1}} & (x_T - x_{\text{ref}})^T Q_T (x_T - x_{\text{ref}}) + \sum_{k=0}^{T-1} (x_k - x_{\text{ref}})^T Q (x_k - x_{\text{ref}}) + u_k^T R u_k \\ \text{subject to} & \begin{cases} x_{k,\min} \leq x_k \leq x_{k,\max} \\ u_{k,\min} \leq u_k \leq u_{k,\max} \\ x_{k+1} = A_d x_k + B_d u_k \\ x_0 = x_{\text{init}} \\ \forall (t_{\text{col}}, s_{\text{col}})_{i \in [1,10]} \quad x_{t_{\text{col}}^{(i)}}[1] < s_{\text{col}}^{(i)} - \Delta_{\text{safety}} \text{ or } x_{t_{\text{col}}^{(i)}}[1] > s_{\text{col}}^{(i)} + \Delta_{\text{safety}} \end{cases} \end{aligned}$$

Linear Dynamics with Constant Acceleration model in between 2 time steps

$$\begin{bmatrix} s \\ \dot{s} \end{bmatrix}_{k+1} = A_d \begin{bmatrix} s \\ \dot{s} \end{bmatrix}_k + B_d [\ddot{s}]_k \text{ with } A_d = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, B_d = \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}$$

4 Methods

4.1 Collision Avoidance Model

- Elastic Model
- Disjunctive Constraints Handling

4.2 Optimization Algorithms

4.3 Penalty Methods

$$p_{\text{quadratic}}(x) = \sum_i \max(g_i(x), 0)^2 + \sum_j h_j(x)$$

$$p_{\text{Lagrange}}(x) = \frac{1}{2}\rho \sum_i h_i(x)^2 - \sum_i \lambda_i h_i(x)$$

4.3.1 Interior Point Method with Inequality and Equality Constraints

$$\min_{\text{subject to}} \begin{cases} \hat{f}(x+v) = f(x) + \nabla f(x)^T v + \frac{1}{2}v^T \nabla^2 f(x) v & \text{Taylor 2nd order approx} \\ A(x+v) = b \end{cases}$$

$$\text{Via optimality conditions on } \mathcal{L}(x, \lambda) : \begin{bmatrix} \Delta x_{\text{newton_step}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \nabla^2 f(x) & A^T \\ A & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\nabla f(x) \\ -(Ax - b) \end{bmatrix}$$

4.3.2 Simplex Algorithm

Simplex is fast. We investigate how to bootstrap the feasibility search phase of an Interior Point method with a simplex algorithm.

4.4 Optimization under Uncertainty

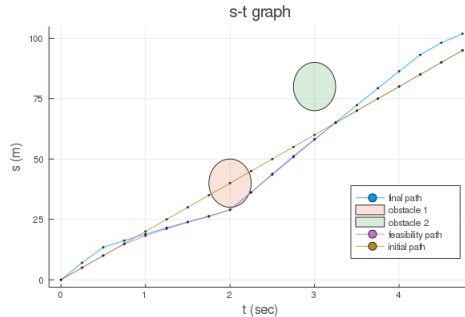
$$\min_{x \in \mathcal{X}} \max_{z \in \mathcal{Z}} f(x, z)$$

5 Experiments

The github repo is AA222-project.

5.1 ST Graphs Analysis

- Runtime: ≤ 250 ms for real time applicability
- Feasibility constraints compliance: check safety & dynamics constraints
- Cost value: efficiency and comfort (lower cost function)



5.2 Anti Collision Tests Benchmarks

We use five metrics to evaluate the performance of our different approaches. (1) The main success metric is the percentage of cases where we reach a target state without collision. (2) The second metric is the agent runtime. (3) The third metric is a comfort metric: the number of hard braking decisions. (4) The fourth metric relates to efficiency: how fast we reach a target while complying to some speed limitation. (5) The last metric is a safety metric: for some of our randomly generated test cases, a collision is unavoidable. In these cases, we aim for a lower speed at collision.

6 Conclusion

It works even better than expected ...

References

- [1] John T. Betts. *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*. Cambridge University Press, USA, 2nd edition, 2009.
- [2] Haoyang Fan, Fan Zhu, Changchun Liu, Liangliang Zhang, Li Zhuang, Dong Li, Weicheng Zhu, Jiangtao Hu, Hongye Li, and Qi Kong. Baidu apollo em motion planner. *ArXiv*, abs/1807.08048, 2018. 1
- [3] T. Gu, J. M. Dolan, and J. Lee. Runtime-bounded tunable motion planning for autonomous driving. In *2016 IEEE Intelligent Vehicles Symposium (IV)*, pages 1301–1306, 2016.
- [4] C. R. Hargraves and S. W. Paris. Direct trajectory optimization using nonlinear programming and collocation. In *Astrodynamics 1985*, pages 3–12, August 1986.
- [5] Christos Katrakazas, Mohammed A. Quddus, Wen hua Chen, and Lipika Deka. Real-time motion planning methods for autonomous on-road driving: State-of-the-art and future research directions. 2015. 1
- [6] Mykel J. Kochenderfer and Tim A. Wheeler. *Algorithms for Optimization*. The MIT Press, 2019.
- [7] X. Li, Z. Sun, Z. He, Q. Zhu, and D. Liu. A practical trajectory planning framework for autonomous ground vehicles driving in urban environments. In *2015 IEEE Intelligent Vehicles Symposium (IV)*, pages 1160–1166, 2015.
- [8] C. Liu, W. Zhan, and M. Tomizuka. Speed profile planning in dynamic environments via temporal optimization. In *2017 IEEE Intelligent Vehicles Symposium (IV)*, pages 154–159, 2017.
- [9] Changliu Liu, Chung-Yen Lin, and Masayoshi Tomizuka. The convex feasible set algorithm for real time optimization in motion planning. *SIAM J. Control and Optimization*, 56:2712–2733, 2017.
- [10] Tianyu Gu, J. Atwood, Chiyu Dong, J. M. Dolan, and Jin-Woo Lee. Tunable and stable real-time trajectory planning for urban autonomous driving. In *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 250–256, 2015.
- [11] Andreas Wachter and Lorenz Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical programming*, 106:25–57, 03 2006. 1
- [12] Xiaojing Zhang, Alexander Liniger, and Francesco Borrelli. Optimization-based collision avoidance. *ArXiv*, abs/1711.03449, 2020. 1