Model Predictive Control

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Contents

1 Motion Control

Motion Control deals with the last stage of an autonomous driving pipeline: the control module. The input to the control module will be provided by the output of the path planning module via a set of waypoints to follow as close as possible. The control module will have to provide the actuators commands (in our case steering angle and throttling; acceleration or deceleration) so that the automated driving comply with a set of rules:

- follow the planned waypoints as close as possible
- drives smoothly
- try to adjust the speed: as fast as a configurable reference when possible and driving more slowly during curves

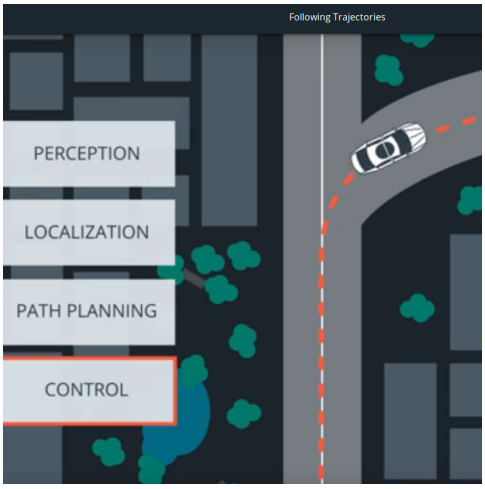


Figure 1: Autonomous Driving pipeline

2 Non linear optimization under constraints

2.1 Definition

In its most generic form we are dealing with the following problem:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f_0(x) \\ \text{subject to} \quad & lower_i \leq f_i(x) \leq upper_i,\ i = 1, \ldots, m. \end{aligned}$$

Note that by setting $lower_i = upper_i$ we can define constraints as equalities as well.

2.2 Example

$$\begin{array}{ll} \text{minimize} & x_1 * x_4 * (x_1 + x_2 + x_3) + x_3 \\ \text{subject to} & x_1 * x_2 * x_3 * x_4 \geq 25 \\ & x_1^2 + x_2^2 + x_3^2 + x_4^2 = 40 \\ & 1 \leq x_1, x_2, x_3, x_4 \leq 5 \end{array}$$

2.3 Solving with ipopt

ipopt and cppad are used to solve non-linear minimization problems. ipopt requires the computation of first order (Jacobians) and 2nd order derivatives (Hessians). These derivatives will be computed automatically thanks to cppad: providing automatic differentiation services.
The previous example is solved with ipopt and CppAD here: `https://www.coin-or.org/CppAD/Doc/ipopt_solve_get_started.cpp.htm`

Listing 1: Simple example with ipopt

```
1
2
3  # include <cppad/ipopt/solve.hpp>
4
5  namespace {
6        using CppAD::AD;
7
8        class FG_eval {
9        public:
10             typedef CPPAD_TESTVECTOR( AD<double> ) ADvector;
11             void operator()(ADvector& fg, const ADvector& x)
12             {      assert( fg.size() == 3 );
13                    assert( x.size()  == 4 );
14
15                    // Fortran style indexing
16                    AD<double> x1 = x[0];
17                    AD<double> x2 = x[1];
18                    AD<double> x3 = x[2];
19                    AD<double> x4 = x[3];
20                    // f(x)
21                    fg[0] = x1 * x4 * (x1 + x2 + x3) + x3;
```

```
22                  // g_1 (x)
23                  fg[1] = x1 * x2 * x3 * x4;
24                  // g_2 (x)
25                  fg[2] = x1 * x1 + x2 * x2 + x3 * x3 + x4 * x4;
26                  //
27                  return;
28              }
29      };
30  }
31
32  bool get_started(void)
33  {       bool ok = true;
34      size_t i;
35      typedef CPPAD_TESTVECTOR( double ) Dvector;
36
37      // number of independent variables (domain dimension for f and g)
38      size_t nx = 4;
39      // number of constraints (range dimension for g)
40      size_t ng = 2;
41      // initial value of the independent variables
42      Dvector xi(nx);
43      xi[0] = 1.0;
44      xi[1] = 5.0;
45      xi[2] = 5.0;
46      xi[3] = 1.0;
47      // lower and upper limits for x
48      Dvector xl(nx), xu(nx);
49      for(i = 0; i < nx; i++)
50      {       xl[i] = 1.0;
51              xu[i] = 5.0;
52      }
53      // lower and upper limits for g
54      Dvector gl(ng), gu(ng);
55      gl[0] = 25.0;       gu[0] = 1.0e19;
56      gl[1] = 40.0;       gu[1] = 40.0;
57
58      // object that computes objective and constraints
59      FG_eval fg_eval;
60
61      // options
```

```
62      std::string options;
63      // turn off any printing
64      options += "Integer print_level  0\n";
65      options += "String  sb           yes\n";
66      // maximum number of iterations
67      options += "Integer max_iter     10\n";
68      // approximate accuracy in first order necessary conditions;
69      // see Mathematical Programming, Volume 106, Number 1,
70      // Pages 25-57, Equation (6)
71      options += "Numeric tol          1e-6\n";
72      // derivative testing
73      options += "String  derivative_test            second-order\n";
74      // maximum amount of random pertubation; e.g.,
75      // when evaluation finite diff
76      options += "Numeric point_perturbation_radius  0.\n";
77
78      // place to return solution
79      CppAD::ipopt::solve_result<Dvector> solution;
80
81      // solve the problem
82      CppAD::ipopt::solve<Dvector, FG_eval>(
83           options, xi, xl, xu, gl, gu, fg_eval, solution
84      );
85      //
86      // Check some of the solution values
87      //
88      ok &= solution.status == CppAD::ipopt::solve_result<Dvector>::success
89      //
90      double check_x[]  = { 1.000000, 4.743000, 3.82115, 1.379408 };
91      double check_zl[] = { 1.087871, 0.,       0.,
    0.        };
92      double check_zu[] = { 0.,       0.,       0.,
    0.        };
93      double rel_tol    = 1e-6;  // relative tolerance
94      double abs_tol    = 1e-6;  // absolute tolerance
95      for(i = 0; i < nx; i++)
96      {     ok &= CppAD::NearEqual(
97                check_x[i],  solution.x[i],   rel_tol, abs_tol
98           );
99           ok &= CppAD::NearEqual(
```

```
100                 check_zl[i], solution.zl[i], rel_tol, abs_tol
101             );
102             ok &= CppAD::NearEqual(
103                 check_zu[i], solution.zu[i], rel_tol, abs_tol
104             );
105         }
106
107         return ok;
108 }
```

3 Basic Math utilities

3.1 Translate into vehicle's coordinate system

To make the computations more simple we will work in vehicule's coordinate system at time t: so we have to operate $Rot(-\psi_t) \circ Translation(\begin{bmatrix} x_t \\ y_t \end{bmatrix})$ on current world's coordinates to translate them into vehicule's coordinates

As a reminder, for a rotation matrix of angle θ, we have $Rot(\theta) = \begin{bmatrix} \cos(\theta) & -sin(\theta) \\ \sin(\theta) & cos(\theta) \end{bmatrix}$.
In our case we are dealing with $Rot(-\psi_t) = \begin{bmatrix} \cos(-\psi_t) & -sin(-\psi_t) \\ \sin(-\psi_t) & cos(-\psi_t) \end{bmatrix}$

So we have:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(-\psi_t) & -sin(-\psi_t) \\ \sin(-\psi_t) & cos(-\psi_t) \end{bmatrix} \begin{bmatrix} x - x_t \\ y - y_t \end{bmatrix}$$

3.2 Fit a polynomial to a set of waypoints

Let's assume the reference trajectory provided by the Path Planning module is a set of waypoints approximated or fitted by a (3rd order typically) polynomial $f(x_t) = a_0 + a_1 x_t + a_2 x_t^2 + a_3 x_t^3$ and our real position on the y-axis is y_t

How can we frame the problem ?
Our unknown vector corresponds to the unknown polynomial coefficients

$$X = [a_0, a_1, a_2, a_3]^T$$

Now let's assume we have 5 samples

$$A = \begin{bmatrix} 1 & x_t^{(1)} & x_t^{2(1)} & x_t^{3(1)} \\ 1 & x_t^{(2)} & x_t^{2(2)} & x_t^{3(2)} \\ 1 & x_t^{(3)} & x_t^{2(3)} & x_t^{3(3)} \\ 1 & x_t^{(4)} & x_t^{2(4)} & x_t^{3(4)} \\ 1 & x_t^{(5)} & x_t^{2(5)} & x_t^{3(5)} \end{bmatrix}$$

$$B = \begin{bmatrix} f(x_t^{(1)}) \\ f(x_t^{(2)}) \\ f(x_t^{(3)}) \\ f(x_t^{(4)}) \\ f(x_t^{(5)}) \end{bmatrix}$$

We have:

$$AX = B$$

A is not squarred, solve this problem with e.g. the pseudo-inverse.

Pseudo inverse quick recap:

$$AX = B \textit{ with } A \textit{ a non squarred matrix}$$

$$A^T AX = A^T B \textit{ with } A^T A \textit{ squarred}$$

So we may inverse it now if $A^T A$ is not degenerated (use samples that are different so that $A^T A$ is not degenerated)

$$X = (A^T A)^{-1} A^T B$$

The pseudo-inverse of A a non squarred matrix is

$$A^\dagger = (A^T A)^{-1} A^T$$

3.3 Compute the tangential angle of a polynomial curve

`https://en.wikipedia.org/wiki/Tangential_angle`
If the curve is given by $y = f(x)$, then we may take $(x, f(x))$ as the parametrization, and we may assume the tangential angle ψ is between $\pi/2$ and $\pi/2$. This produces the explicit expression:

$$\psi = arctan(f'(x))$$

4 Vehicle Models

4.1 Dynamic vs Kinematic Models

4.2 Kinematic Model

4.2.1 State

The state variables are the following:

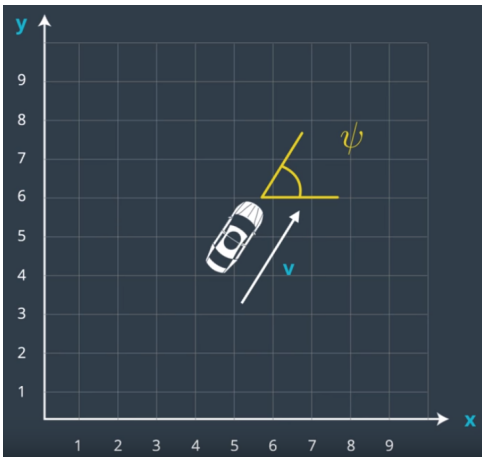


Figure 2: Model state

- x: x position
- y: y position
- ψ: angle between speed vector and x-axis
- v: speed vector

4.2.2 Deriving the kinematic model

Our state vector is

$$S_t = [x_t, y_t, \psi_t, v_t]$$

We derive an approximation model, kinematic, relating S_{t+1} and S_t. The smaller the dt the more accurate the model.

The model used id the kinematic bicycle model:

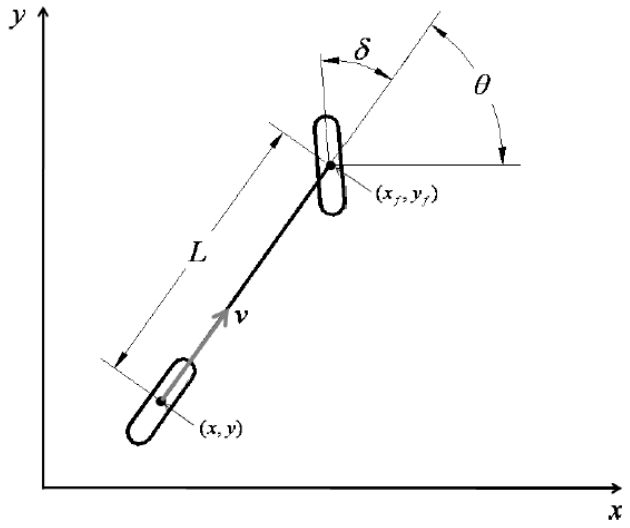


Figure 3: Kinematic bicycle model

Linear movement approximation: assuming during dt that v_t and ψ_t are constant:

$$x_{t+1} = x_t + v_t * \cos(\psi_t) * dt$$

$$y_{t+1} = y_t + v_t * \sin(\psi_t) * dt$$

Rotational movement approximation: assuming during dt that v_t and steering angle δ_t are constant we are creating a change from a starting ψ_t to end up with ψ_{t+1}:

Let first compute the radius of curvature R based on the steering angle δ and the distance L between rear and front wheels:

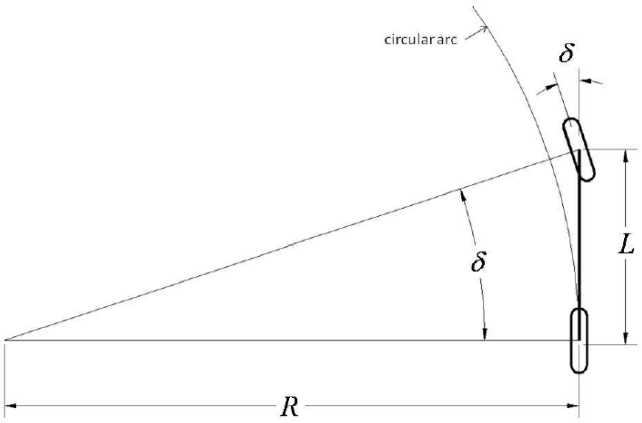


Figure 4: Radius of curvature R

We are moving on a circle from position M_t to position M_{t+1} at a constant speed v_t

$$M_{t+1}M_t = R * (\psi_{t+1} - \psi_t) \approx v_t * dt$$

We have

$$\tan(\delta) \approx L/R$$

Check that the 2 δ drawn are indeed the same. In the triangle we have $\pi/2 + \delta_{circle} + \alpha = \pi$ and along the line from the center of the circle of radius R going through the center of the front wheel we have $\pi/2 + \delta + \alpha = \pi$. So we have:

$$\delta_{circle} = \delta$$

And then:

$$\psi_{t+1} = \psi_t + (v_t/R) * dt$$
$$\psi_{t+1} = \psi_t + (v_t/L) * \tan(\delta_t) * dt$$

Note that for small δ_t *we have* $\tan(\delta_t) \approx \delta_t$

Speed update: assuming during dt that a_t is constant:

$$v_{t+1} = v_t + a_t * dt$$

So to summarize our kinematic bicycle model which is a **kinematic constraint model** is:

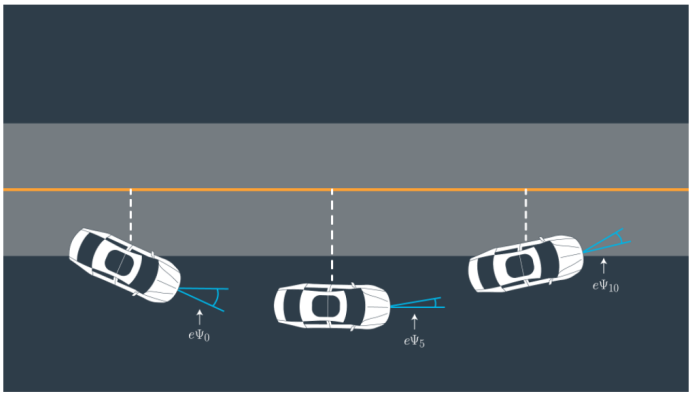
$$\boxed{\begin{aligned} x_{t+1} &= x_t + v_t * \cos(\psi_t) * dt \\ y_{t+1} &= y_t + v_t * \sin(\psi_t) * dt \\ \psi_{t+1} &= \psi_t + (v_t/L) * \tan(\delta_t) * dt \\ v_{t+1} &= v_t + a_t * dt \end{aligned}}$$

The state vector is $S_t = [x_t, y_t, \psi_t, v_t]$.
The actuator command $A_t = [a_t, \delta_t]$ defines a **constraint** between S_{t+1} and S_t.

Note also that for a bicycle we have $\delta_t \in [-\pi/2, \pi/2]$ whereas for a car we have $\delta_t \in (-\delta_{max}, \delta_{max})$ with $\delta_{max} < \pi/2$.

4.2.3 Errors

The errors variables are the following:



The dashed white line is the cross track error.

Figure 5: Model errors

- *cte*: cross track error. It corresponds to distance of vehicle from the planned trajectory (as planned by path planning module)
- $e\Psi$: psie error is the angle difference of the vehicle trajectory with the planned trajectory (as planned by path planning module)

The new state vector is $[x_t, y_t, \psi_t, v_t, cte_t, e\Psi_t]$**.**

Let's assume the reference trajectory provided by the Path Planning module is a set of waypoints approximated or fitted by a (3rd order typically) polynomial $f(x_t) = a_0 + a_1 x_t + a_2 x_t^2 + a_3 x_t^3$ and our real position on the y-axis is y_t

Compute current Cross Track Error:
The current Cross Track Error is:
$cte_t = y_t - f(x_t)$
Note: we are dealing with a trajectory which is a set of (x, y, t) or (x_t, y_t) coordinates. We are not dealing with a path which a set of (x, y) coordinates.

Compute current Orientation Error:
The current Orientation Error is:
$e\Psi_t = \psi_t -$ tangential angle of curve of f at $x_t = \psi_t - \arctan(f^{'}(x_t))$

Update Cross Track Error:
The Cross Track Error will increase during dt proportionnaly to the speed and Orientation error (along local y-axis):
$cte_{t+1} = cte_t + v_t * sin(e\Psi_t) * dt$

Update Orientation Error:
The update rule for $e\Psi_t$ is the same as for ψ_t:
$e\Psi_{t+1} = e\Psi_t + (v_t/L) * \tan(\delta_t) * dt$

As we have:

$$\psi_{t+1} = \psi_t + (v_t/L) * \tan(\delta_t) * dt$$
$$e\Psi_t = \psi_t - \arctan(f^{'}(x_t))$$
$$e\Psi_{t+1} = \psi_{t+1} - \arctan(f^{'}(x_{t+1}))$$

We get:

$$e\Psi_{t+1} = \psi_t + (v_t/L) * \tan(\delta_t) * dt - \arctan(f^{'}(x_{t+1}))$$

$$e\Psi_{t+1} = \psi_t - \arctan(f^{'}(x_{t+1})) + (v_t/L) * \tan(\delta_t) * dt$$

And assuming our tangential angle remains constant during dt our orientation error will evolve such that:

$$e\Psi_{t+1} = \psi_t - \arctan(f^{'}(x_t)) + (v_t/L) * \tan(\delta_t) * dt$$

$$e\Psi_{t+1} = e\Psi_t + (v_t/L) * \tan(\delta_t) * dt$$

So to summarize our kinematic bicycle error model which is a **kinematic error model constraint** is:

$$\boxed{\begin{aligned} cte_{t+1} &= cte_t + +v_t * sin(e\Psi_t) * dt \\ e\Psi_{t+1} &= e\Psi_t + (v_t/L) * \tan(\delta_t) * dt \end{aligned}}$$

The underlying model is an approximation. Better models exist and are used in practice. Also the smaller the dt considered, the better the approximation. But in any cases no models are perfect (they are just usefull) and there will anyways be a delta (the smaller the better) between where we predict the vehicle will go and where we will end up. As an extreme case consider we are driving over some ice or oil that was not modelled. So the model will be reactive and corrective. Commands for the actuators will be reevaluated at every timestep even if we do predictions and plan actuator commands over multiple timesteps.

4.2.4 Actuator Constraints

In a real vehicle, actuators are limited by the design of the vehicle and fundamental physics. For example, a vehicle can't have a steering angle of 90 degrees. It is impossible. Thus it does not make sense for us to even consider these kinds of inputs. There is actually a vocabulary for describing this constraint. We call this model **nonholomonic** because the vehicle can't move in any arbitrary directions. It's limited by steering angle constraints. We can solve this by setting lower and upper bounds for the actuators.
We have such **actuator constraints** as:

$$\boxed{\begin{aligned} -30^o \leq &\delta_t \leq +30^o \\ -1 \leq &a_t \leq +1 \end{aligned}}$$

where -1 is full brake and $+1$ is full acceleration.

4.2.5 Constraints summary

We have now specified a set of constraints acounting for vehicle constraints. To summarize our kinematic model constraints are:

$$
\begin{aligned}
x_{t+1} &= x_t + v_t * \cos(\psi_t) * dt \\
y_{t+1} &= y_t + v_t * \sin(\psi_t) * dt \\
\psi_{t+1} &= \psi_t + (v_t/L) * \tan(\delta_t) * dt \\
v_{t+1} &= v_t + a_t * dt \\
cte_{t+1} &= cte_t + + v_t * sin(e\Psi_t) * dt \\
e\Psi_{t+1} &= e\Psi_t + (v_t/L) * \tan(\delta_t) * dt
\end{aligned}
$$

And our actuator constraints are:

$$
\begin{aligned}
-30^o \leq \delta_t \leq +30^o \\
-1 \leq a_t \leq +1
\end{aligned}
$$

So far these contraints are defined for a 1-step duration of dt milliseconds. But we can rollout these constraints over N-steps. Typically we may use $N = 10$ and $dt = 20$ ms.

4.3 Dynamic Models

Forces, Slip Angle, Slip ratio and Tire Models

5 Model Predictive Control

MPC reframes the task of following a trajectory as an optimization problem. The solution to the optimization problem is the optimal trajectory.

MPC involves simulating different actuator inputs, predicting the resulting trajectory and minimizing a set of constraints .

Once we found the lowest cost trajectory, we implement the very first set of actuation commands. Then we throw away the rest of the trajectory we calculated. Instead of using the old trajectory we predicted, we take our new state and use that to calculate a new optimal trajectory. In that sense, we are constantly calculating inputs over a future horizon. That's why this approach is also called Receding Horizon Control. We constantly reevaluate the trajectory because our vehicle model is not perfect and the

next predicted (or planned) state may (slightly...) differ with our prediction (in the sense of a consequence of a command sent).

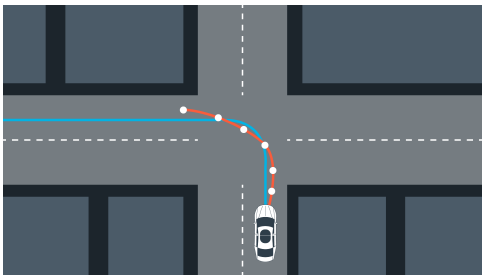


Figure 6: Minimization problem

5.1 Minimization of a cost function

We typically want to minimize things related to Tracking Error and comfort:

- Cross Track Error
- Orientation Error
- speed delta vs a target reference speed
- use of actuators (you prefer to use low values of a_t and δ_t as long as you can remain on track)
- limit the rate of change of actuator commands from one timestep to another

To account for these different objectives, we define different sub cost functions and weight them according to our priorities.

Listing 2: Cost function example

```
1       for (size_t t = 0; t < N; t++) {
2         fg[0] += 4 * 2000 * CppAD::pow(vars[cte_start + t], 2);
3         fg[0] += 4 * 2000 * CppAD::pow(vars[epsi_start + t], 2);
4         fg[0] += CppAD::pow(vars[v_start + t] - ref_v, 2);
5       }
6
7       // Minimize the use of actuators.
8       for (size_t t = 0; t < N - 1; t++) {
```

```
 9         fg[0] += 5 * CppAD::pow(vars[delta_start + t], 2);
10         fg[0] += 5 * CppAD::pow(vars[a_start + t], 2);
11       }
12
13       // smooth
14       for (size_t t = 0; t < N - 2; t++) {
15         fg[0] += 200 * CppAD::pow(vars[delta_start + t + 1] - vars[delta_sta
16         fg[0] += 10 * CppAD::pow(vars[a_start + t + 1] - vars[a_start + t],
17 }
```

5.2 Timsestep length and Elapsed duration

N=10 and dt=100 ms are used so that we are working on 1 second of data. This is a trade-off: we need enough data visibility to ensure a good prediction, but we also have to limit the amount of computation. In general, smaller dt gives better accuracy, but that will require higher N for given horizon (N*dt). However, increasing N will result in longer computational time which increases the latency. The most common choice of values is N=10 and dt=0.1 but anything between N=20, dt=0.05 should work.

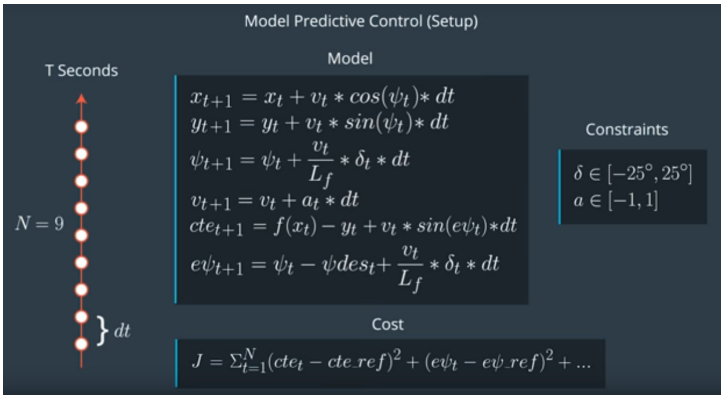


Figure 7: Solver setup with N*dt time horizon

5.3 Latency handling

A contributing factor to latency is actuator dynamics. For example the time elapsed between when you command a steering angle to when that angle is actually achieved. This could easily be modeled by a simple dynamic system

and incorporated into the vehicle model. One approach would be running a simulation using the vehicle model starting from the current state for the duration of the latency. The resulting state from the simulation is the new initial state for MPC.
Thus, MPC can deal with latency much more effectively, by explicitly taking it into account, than a PID controller.

5.4 MPC Solver algorithm

To summarize **we have defined a cost function we want to minimize under a set of constraints**. The cost function accounts for what we want to produce ideally: a comfortable trajectory that follows a reference path as close as possible. Whereas the constraints account for the constraints we are dealing with: typicaly at this stage of the Autonomous Driving pipeline, we are dealing with vehicle model constraints. So our non-linear optimization problem is defined and can be summarized visually as:

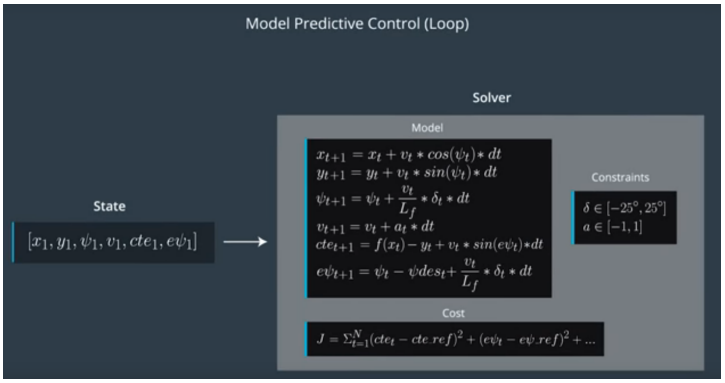


Figure 8: Solver input

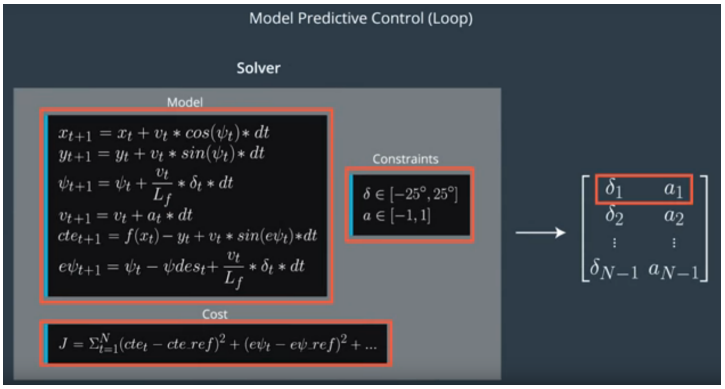


Figure 9: Solver output

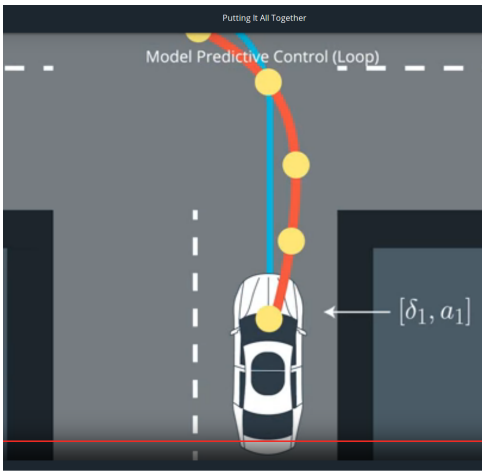


Figure 10: Solver actuator commands

The next step is to solve it in software. The model, a cost function to minimize and a set of constraints to comply with, will be described in software and a mathematical library will be used to solve this non-linear optimization problem.

5.5 MPC Solver code

Listing 3: MPC solver with ipopt

```
1  #include "MPC.h"
2  #include <cppad/cppad.hpp>
3  #include <cppad/ipopt/solve.hpp>
4  #include "Eigen-3.3/Eigen/Core"
5  #include "Eigen-3.3/Eigen/QR"
6
7  using CppAD::AD;
8
9  // TODO: Set the timestep length and duration
10 size_t N = 10;
11 double dt = 0.1;
12
13 // This value assumes the model presented in the classroom is used.
14 //
15 // It was obtained by measuring the radius formed by running the vehicle in
16 // simulator around in a circle with a constant steering angle and velocity
17 // flat terrain.
18 //
19 // Lf was tuned until the the radius formed by the simulating the model
20 // presented in the classroom matched the previous radius.
21 //
22 // This is the length from front to CoG that has a similar radius.
23 const double Lf = 2.67;
24
25 // NOTE: feel free to play around with this
26 // or do something completely different
27
28 double ref_v = 120;
29
30 // The solver takes all the state variables and actuator
31 // variables in a singular vector. Thus, we should to establish
```

```
32 // when one variable starts and another ends to make our lifes easier.
33 size_t x_start = 0;
34 size_t y_start = x_start + N;
35 size_t psi_start = y_start + N;
36 size_t v_start = psi_start + N;
37 size_t cte_start = v_start + N;
38 size_t epsi_start = cte_start + N;
39 size_t delta_start = epsi_start + N;
40 size_t a_start = delta_start + N - 1;
41
42 class FG_eval {
43  public:
44   // Fitted polynomial coefficients
45   Eigen::VectorXd coeffs;
46   FG_eval(Eigen::VectorXd coeffs) { this->coeffs = coeffs; }
47
48   typedef CPPAD_TESTVECTOR(AD<double>) ADvector;
49   void operator()(ADvector& fg, const ADvector& vars) {
50     // TODO: implement MPC
51     // `fg` a vector of the cost constraints, `vars` is a vector of variab
52     // NOTE: You'll probably go back and forth between this function and
53     // the Solver function below.
54
55     // The cost is stored is the first element of `fg`.
56     // Any additions to the cost should be added to `fg[0]`.
57     fg[0] = 0;
58
59     // Reference State Cost
60     // TODO: Define the cost related the reference state and
61     // any anything you think may be beneficial.
62     for (size_t t = 0; t < N; t++) {
63       fg[0] += 4 * 2000 * CppAD::pow(vars[cte_start + t], 2);
64       fg[0] += 4 * 2000 * CppAD::pow(vars[epsi_start + t], 2);
65       fg[0] += CppAD::pow(vars[v_start + t] - ref_v, 2);
66     }
67
68     // Minimize the use of actuators.
69     for (size_t t = 0; t < N - 1; t++) {
70       fg[0] += 5 * CppAD::pow(vars[delta_start + t], 2);
71       fg[0] += 5 * CppAD::pow(vars[a_start + t], 2);
```

```
72      }
73
74      // smooth
75      for (size_t t = 0; t < N - 2; t++) {
76        fg[0] += 200 * CppAD::pow(vars[delta_start + t + 1] - vars[delta_sta
77        fg[0] += 10 * CppAD::pow(vars[a_start + t + 1] - vars[a_start + t],
78      }
79
80      //
81      // Setup Constraints
82      //
83      // NOTE: In this section you'll setup the model constraints.
84
85      // Initial constraints
86      //
87      // We add 1 to each of the starting indices due to cost being located a
88      // index 0 of `fg`.
89      // This bumps up the position of all the other values.
90      fg[1 + x_start] = vars[x_start];
91      fg[1 + y_start] = vars[y_start];
92      fg[1 + psi_start] = vars[psi_start];
93      fg[1 + v_start] = vars[v_start];
94      fg[1 + cte_start] = vars[cte_start];
95      fg[1 + epsi_start] = vars[epsi_start];
96
97      // The rest of the constraints
98      for (size_t t = 1; t < N; t++) {
99        // at time t+1
100       AD<double> x1 = vars[x_start + t];
101       AD<double> y1 = vars[y_start + t];
102       AD<double> psi1 = vars[psi_start + t];
103       AD<double> v1 = vars[v_start + t];
104       AD<double> cte1 = vars[cte_start + t];
105       AD<double> epsi1 = vars[epsi_start + t];
106
107       // at time t
108       AD<double> x0 = vars[x_start + t - 1];
109       AD<double> y0 = vars[y_start + t - 1];
110       AD<double> psi0 = vars[psi_start + t - 1];
111       AD<double> v0 = vars[v_start + t - 1];
```

```
112         AD<double> cte0 = vars[cte_start + t - 1];
113         AD<double> epsi0 = vars[epsi_start + t - 1];
114 
115         AD<double> delta0 = vars[delta_start + t - 1];
116         AD<double> a0 = vars[a_start + t - 1];
117 
118         // XXX: to be updated
119         AD<double> f0 = coeffs[0] + coeffs[1] * x0 + coeffs[2] * x0 * x0 + c
120         AD<double> psides0 = CppAD::atan(coeffs[1] + 2 * coeffs[2] * x0 + 3 *
121 
122         // Here's `x` to get you started.
123         // The idea here is to constraint this value to be 0.
124         //
125         // NOTE: The use of `AD<double>` and use of `CppAD`!
126         // This is also CppAD can compute derivatives and pass
127         // these to the solver.
128 
129         // TODO: Setup the rest of the model constraints
130         fg[1 + x_start + t] = x1 - (x0 + v0 * CppAD::cos(psi0) * dt);
131         fg[1 + y_start + t] = y1 - (y0 + v0 * CppAD::sin(psi0) * dt);
132         fg[1 + psi_start + t] = psi1 - (psi0 - v0 * delta0 / Lf * dt); // XX
133         fg[1 + v_start + t] = v1 - (v0 + a0 * dt);
134 
135         // BUG fg[1 + cte_start + t] = cte1 - (cte0 + v0 * CppAD::sin(epsi0)
136         // BUG fg[1 + epsi_start + t] = epsi1 - (epsi0 + v0 * delta0 / Lf *
137         fg[1 + cte_start + t] = cte1 - ((f0 - y0) + (v0 * CppAD::sin(epsi0) *
138         fg[1 + epsi_start + t] = epsi1 - ((psi0 - psides0) - v0 * delta0 / L
139       }
140     }
141 };
142 
143 //
144 // MPC class definition implementation.
145 //
146 MPC::MPC() {}
147 MPC::~MPC() {}
148 
149 vector<double> MPC::Solve(Eigen::VectorXd state, Eigen::VectorXd coeffs) {
150   bool ok = true;
151   size_t i;
```

```
152    typedef CPPAD_TESTVECTOR(double) Dvector;
153
154    double x = state[0];
155    double y = state[1];
156    double psi = state[2];
157    double v = state[3];
158    double cte = state[4];
159    double epsi = state[5];
160
161    // TODO: Set the number of model variables (includes both states and inpu
162    // For example: If the state is a 4 element vector, the actuators is a 2
163    // element vector and there are 10 timesteps. The number of variables is
164    //
165    // 4 * 10 + 2 * 9
166    size_t n_vars = N * 6 + (N - 1) * 2;
167    // TODO: Set the number of constraints
168    size_t n_constraints = N * 6;
169
170    // Initial value of the independent variables.
171    // Should be 0 except for the initial values.
172    Dvector vars(n_vars);
173    for (i = 0; i < n_vars; i++) {
174      vars[i] = 0.0;
175    }
176    // Set the initial variable values
177    vars[x_start] = x;
178    vars[y_start] = y;
179    vars[psi_start] = psi;
180    vars[v_start] = v;
181    vars[cte_start] = cte;
182    vars[epsi_start] = epsi;
183
184    // Lower and upper limits for x
185    Dvector vars_lowerbound(n_vars);
186    Dvector vars_upperbound(n_vars);
187    // TODO: Set lower and upper limits for variables.
188    // Set all non-actuators upper and lowerlimits
189    // to the max negative and positive values.
190    for (i = 0; i < delta_start; i++) {
191      vars_lowerbound[i] = -1.0e19;
```

```
192       vars_upperbound[i] = 1.0e19;
193     }
194
195     // The upper and lower limits of delta are set to -25 and 25
196     // degrees (values in radians).
197     // NOTE: Feel free to change this to something else.
198     for (i = delta_start; i < a_start; i++) {
199       vars_lowerbound[i] = -0.436332 * Lf; // *Lf ? XXX
200       vars_upperbound[i] = 0.436332 * Lf;  // *Lf ?
201     }
202
203     // Acceleration/decceleration upper and lower limits.
204     // NOTE: Feel free to change this to something else.
205     for (i = a_start; i < n_vars; i++) {
206       vars_lowerbound[i] = -1.0;
207       vars_upperbound[i] = 1.0;
208     }
209
210
211     // Lower and upper limits for the constraints
212     // Should be 0 besides initial state.
213     Dvector constraints_lowerbound(n_constraints);
214     Dvector constraints_upperbound(n_constraints);
215     for (i = 0; i < n_constraints; i++) {
216       constraints_lowerbound[i] = 0;
217       constraints_upperbound[i] = 0;
218     }
219     constraints_lowerbound[x_start] = x;
220     constraints_lowerbound[y_start] = y;
221     constraints_lowerbound[psi_start] = psi;
222     constraints_lowerbound[v_start] = v;
223     constraints_lowerbound[cte_start] = cte;
224     constraints_lowerbound[epsi_start] = epsi;
225
226     constraints_upperbound[x_start] = x;
227     constraints_upperbound[y_start] = y;
228     constraints_upperbound[psi_start] = psi;
229     constraints_upperbound[v_start] = v;
230     constraints_upperbound[cte_start] = cte;
231     constraints_upperbound[epsi_start] = epsi;
```

```
232
233     // Object that computes objective and constraints
234     FG_eval fg_eval(coeffs);
235
236     //
237     // NOTE: You don't have to worry about these options
238     //
239     // options for IPOPT solver
240     std::string options;
241     // Uncomment this if you'd like more print information
242     options += "Integer print_level  0\n";
243     // NOTE: Setting sparse to true allows the solver to take advantage
244     // of sparse routines, this makes the computation MUCH FASTER. If you
245     // can uncomment 1 of these and see if it makes a difference or not but
246     // if you uncomment both the computation time should go up in orders of
247     // magnitude.
248     options += "Sparse  true        forward\n";
249     options += "Sparse  true        reverse\n";
250     // NOTE: Currently the solver has a maximum time limit of 0.5 seconds.
251     // Change this as you see fit.
252     options += "Numeric max_cpu_time          0.5\n";
253
254     // place to return solution
255     CppAD::ipopt::solve_result<Dvector> solution;
256
257     // solve the problem
258     CppAD::ipopt::solve<Dvector, FG_eval>(
259         options, vars, vars_lowerbound, vars_upperbound, constraints_lowerbou
260         constraints_upperbound, fg_eval, solution);
261
262     //
263     // Check some of the solution values
264     ok &= solution.status == CppAD::ipopt::solve_result<Dvector>::success;
265
266     // Cost
267     auto cost = solution.obj_value;
268     //std::cout << "Cost " << cost << std::endl;
269
270     // TODO: Return the first actuator values. The variables can be accessed
271     // `solution.x[i]`.
```

```
272     //
273     // {...} is shorthand for creating a vector, so auto x1 = {1.0,2.0}
274     // creates a 2 element double vector.
275     //return {solution.x[delta_start],   solution.x[a_start]};
276
277     vector<double> result;
278
279     result.push_back(solution.x[delta_start]);
280     result.push_back(solution.x[a_start]);
281
282     for (size_t i = 0; i < N - 1; i++) {
283       result.push_back(solution.x[x_start + i + 1]);
284       result.push_back(solution.x[y_start + i + 1]);
285     }
286
287     return result;
288 }
```